Non-perturbative QCD effective charges

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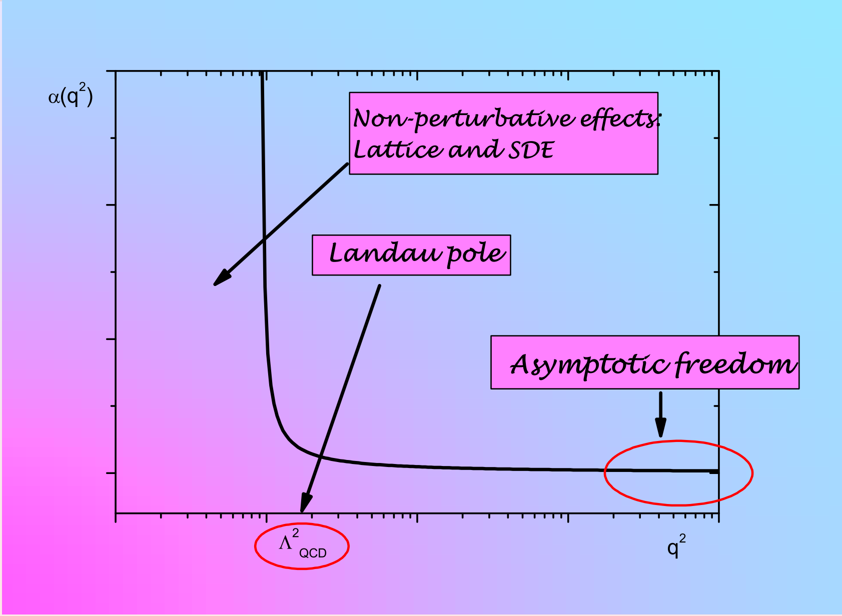
Based on:

A.C.A., D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008)
A.C.A., D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, arXiv:0906.2633 [hep-ph] .
A.C.A., D. Binosi and J. Papavassiliou, arXiv:0907.0153 [hep-ph]

Outline of the talk

- Motivation
- The pinch tecnhique effective charge
- The effective charge from the ghost-gluon vertex
- Comparing the two definitions
- Conclusions

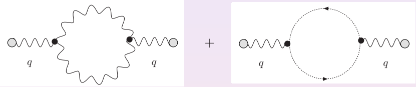
The QCD perturbative coupling



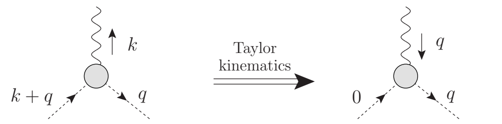
We will study two different definitions of the QCD effective charge

1. The first is based on the pinch-technique gluon self-energy, in close analogy to QED.

$$\widehat{\Pi}_{\mu\nu}(q) =$$



2. The second is based on the definition of ghost-gluon vertex, in a particular kinematic configuration.



The pinch technique effective charge

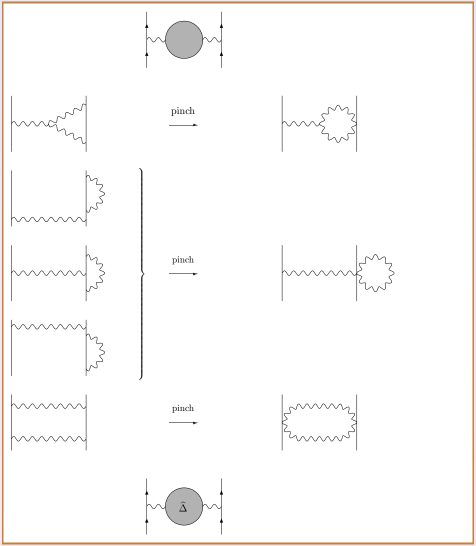
Diagrammatic rearrangement of perturbative expansion (to all orders) gives rise to effective Green's functions with special properties .

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
J. M. Cornwall and J.P. , Phys. Rev. D **40**, 3474 (1989)
D. Binosi and J.P. , Phys. Rev. D **66**, 111901 (2002)
M. Binger and S.J.Brodsky , Phys. Rev. D **74**:054016 (2006)

Longitudinal momenta trigger
Slavnov-Taylor identities inside diagrams:

$$\begin{aligned} k_\nu \gamma^\nu &= (\not k + \not p - m) - (\not p - m) \\ &= S_0^{-1}(k+p) - S_0^{-1}(p) \,, \end{aligned}$$

Pinch technique rearrangement:

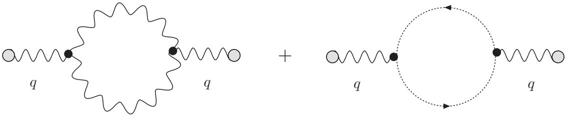


- Simple, QED-like Ward Identities , instead of Slavnov-Taylor Identities, to all orders

$$q_1^{\mu}\widetilde{\Gamma}^{abc}_{\mu\alpha\beta}(q_1,q_2,q_3) \quad = \quad gf^{abc}\left[\Delta^{-1}_{\alpha\beta}(q_2)-\Delta^{-1}_{\alpha\beta}(q_3)\right]$$

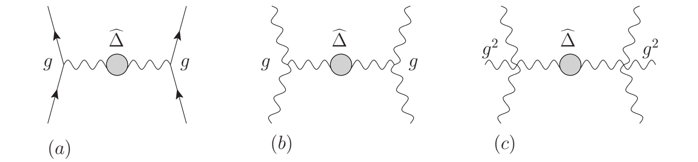
- Profound connection with Background Field Method

$\widehat{\Pi}_{\mu\nu}(q) =$



- Special transversality properties

The properties of the PT charge



- The PT construction involves a combination of two point functions (no explicit reference to any of the full vertices)
- As in QED, the effective charge so obtained is universal (process independent) ,
- It depends naturally on a single scale , namely the physical momentum exchange of a given process.

The pinch-technique effective charge

- Due to the fact that the Green's functions obey simple, QED-like Ward Identities

$$\widehat{Z}_1 = \widehat{Z}_2, \; Z_g = \widehat{Z}_A^{-1/2}$$

where, the renormalization constants

$$\begin{aligned} g(\mu^2) &= Z_g^{-1}(\mu^2) g_0, \\ \widehat{\Delta}(q^2, \mu^2) &= \widehat{Z}_A^{-1}(\mu^2) \widehat{\Delta}_0(q^2), \end{aligned}$$

- we have the renormalization-group invariant combination

$$\widehat{d}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{d}(q^2)$$

D. Binosi and J. P., Phys. Rev. D 66, 025024 (2002) .

At large momenta one may extract a dimensionless quantity,

$$\widehat{d}(q^2) = \frac{\overline{g}^2(q^2)}{q^2}$$

$\overline{g}^2(q^2)$ is the RG-invariant effective charge of QCD; at one-loop

$$\overline{g}^2(q^2) = \frac{g^2}{1 + \mathbf{b} g^2 \ln(q^2/\mu^2)} = \frac{1}{\mathbf{b} \ln\left(q^2/\Lambda^2_{\mathrm{QCD}}\right)}$$

$$\widehat{d}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{d}(q^2)$$

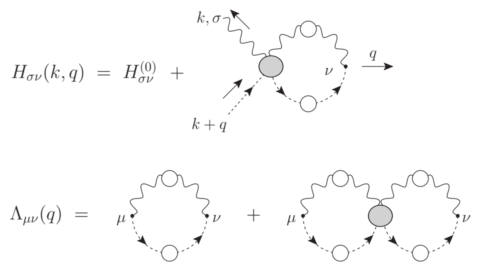
- is a non-perturbative relation , it can serve unaltered as the starting point for extracting a **non-perturbative effective charge** .
- we should have information on the IR behavior of $\widehat{\Delta}(q^2)$.

- The *conventional* gluon propagator $\Delta(q^2)$ and $\widehat{\Delta}(q^2)$ are connected by the formal relation:

$$\Delta(q^2) = \left[1 + G(q^2)\right]^2 \widehat{\Delta}(q^2)$$

P. A. Grassi, T. Hurth and M. Steinhauser, Annals Phys. **288**, 197 (2001)
D. Binosi and J. Papavassiliou, Phys. Rev. D **66**(R), 025024 (2002)

$G(q^2)$ is the $g_{\mu\nu}$ component of the two-point function $\Lambda_{\mu\nu}(q)$



$$\Lambda_{\mu\nu}(q) = -ig^2 C_A \int_k H^{(0)}_{\mu\rho} D(k+q) \Delta^{\rho\sigma}(k) H_{\sigma\nu}(k,q), = g_{\mu\nu} G(q^2) + \frac{q_\mu q_\nu}{q^2} L(q^2),$$

- The SDE for $G(q^2)$ is (Landau gauge)

$$G(q^2) = -\frac{C_{\rm A} g^2}{3} \int_k \left[2 + \frac{(k\cdot q)^2}{k^2 q^2}\right] \Delta(k) D(k+q)\,.$$

- In addition, the following relation is satisfied

$$1 + G(q^2) + L(q^2) = F^{-1}(q^2)$$

A.C.A., D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008) A.C.A., D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, arXiv:0906.2633 [hep-ph].

Checking the perturbative behavior of $\widehat{d}(q^2)$

$$g^2\widehat{\Delta}(q^2) = \frac{g^2\Delta(q^2)}{[1+G(q^2)]^2}$$

$$1+G(q^2) = 1 + \frac{9}{4}\frac{C_A g^2}{48\pi^2}\ln(q^2/\mu^2)$$

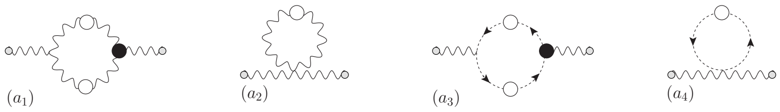
$$\Delta^{-1}(q^2) = q^2\left[1 + \frac{13}{2}\frac{C_A g^2}{48\pi^2}\ln(q^2/\mu^2)\right]$$

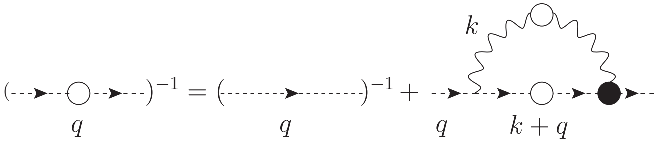
$$\Downarrow$$

$$\widehat{\Delta}^{-1}(q^2) = q^2\left[1 + b g^2\ln(q^2/\mu^2)\right]$$

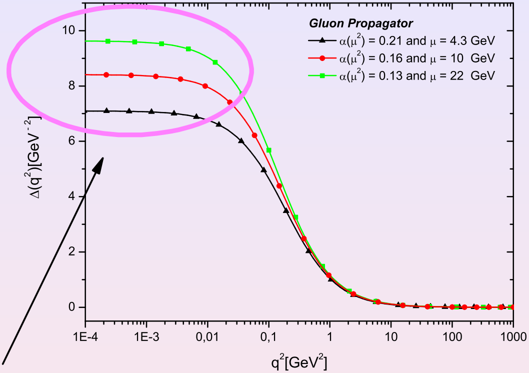
- Enforces β function coefficient in front of UV logarithm ($b = 11C_A/48\pi^2$).

Schwinger-Dyson system: Non-perturbative information for $\Delta(q^2)$ and $G(q^2)$





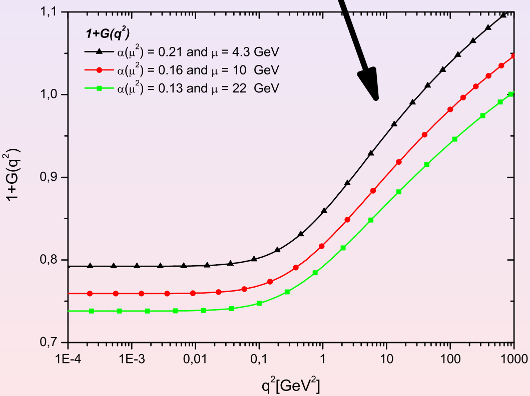
SD results for the gluon propagator



Different freezing values
Dependence on μ

SD result for the auxilary function $G(q^2)$

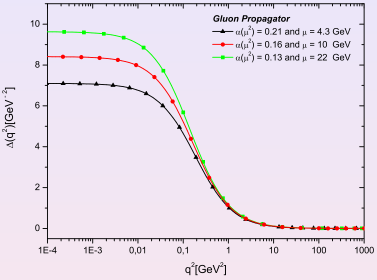
Dependence on μ



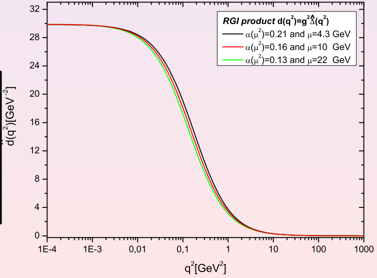
A.C.A., D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, arXiv:0906.2633 [hep-ph] .

Computing $\widehat{d}(q^2)$

$$\widehat{d}(q^2) = \frac{g^2(\mu)\Delta(q^2)}{[1+G(q^2)]^2}$$



$$\times \; \frac{g^2(\mu)}{[1+G(q^2)]^2}$$



All curves on top of each other

No μ - dependence !

Effective charge from massive propagator

- The dimensionful $\widehat{d}(q^2)$ can be written as

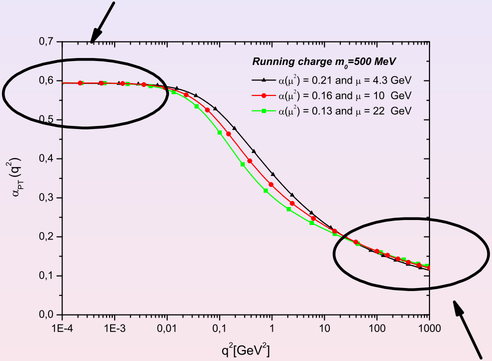
$$\widehat{d}(q^2) = \frac{4\pi\alpha_{\rm PT}(q^2)}{q^2 + m^2(q^2)}.$$

where the gluon mass has power-law running

$$m^2(q^2) = \frac{m_0^4}{q^2 + m_0^2}\left[\ln\left(\frac{q^2 + 2m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{2m_0^2}{\Lambda^2}\right)\right]^3$$

J. M. Cornwall , Phys. Rev. D **26**, 1453 (1982)
M. Lavelle , Phys. Rev. D **44**, 26 (1991)
A. C. Aguilar and JP , Eur. Phys. J. A **35**, 189 (2008)

Infrared fixed point

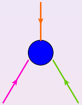


Asymptotic freedom

Effective charge from the ghost-gluon vertex

The effective charge from vertices

- A definition for the QCD effective charge can be obtained starting from the various QCD vertices .
- It involves more than one scales , and further assumptions are introduced, to express the charge as a function of a single variable .



$$Z_g = Z_V (Z_1 Z_2 Z_3)^{-1/2}$$

- The combination

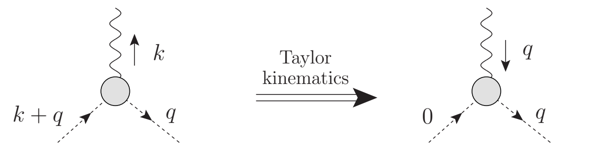
$$\widehat{r}(q_1, q_2, q_3) \equiv g^2 V^2(q_1, q_2, q_3)\Delta_1(q_1)\Delta_2(q_2)\Delta_3(q_3)$$

is a RG-invariant quantity.
Additional assumption: $q_1^2 = q_2^2 = q_3^2 = q^2$ (and therefore $q_1 \cdot q_2 = q_1 \cdot q_3 = q_2 \cdot q_3 = -q^2/2$)

Effective charge from the ghost-gluon vertex

$$\begin{aligned} \Delta(q^2,\mu^2) &= Z_A^{-1}(\mu^2)\Delta_0(q^2), \\ F(q^2,\mu^2) &= Z_c^{-1}(\mu^2)F_0(q^2), \\ \mathbb{\Gamma}^{\nu}(k,q,\mu^2) &= Z_1(\mu^2)\mathbb{\Gamma}_0^{\nu}(k,q), \\ g_0 &= Z_g(\mu^2)g \end{aligned}$$

- For the ghost-gluon vertex $Z_g = Z_1 Z_A^{-1/2} Z_c^{-1}$



- In the Taylor kinematics (incoming ghost momentum is zero), $Z_1 = 1$

$$Z_g = Z_A^{-1/2} Z_c^{-1}$$

$$\widehat{r}(q^2) \;=\; g^2\Delta(q^2;\mu^2)F^2(q^2;\mu^2) \;=\; g_0^2\Delta_0(q^2)F_0^2(q^2)$$

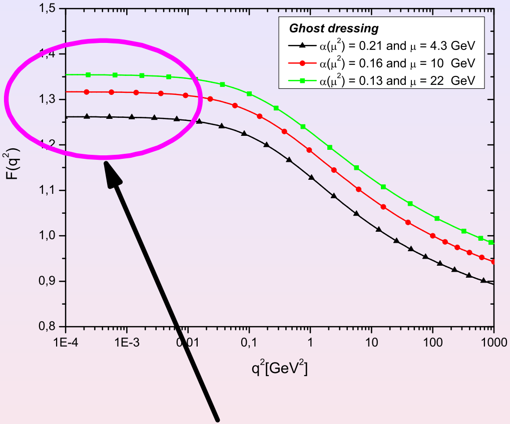
- The above combination is a dimensionful μ-independent
- For asymptotically large q^2 ,

$$\widehat{r}(q^2) = \frac{\overline{g}^2_{\rm gh}(q^2)}{q^2}.$$

Using the fact that

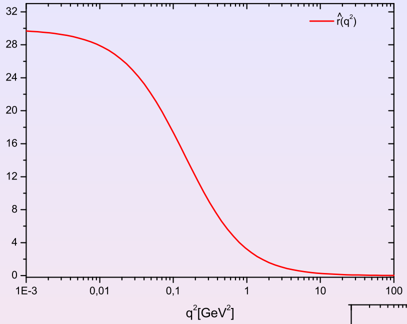
$$F^{-1}(q^2) = \left[1 + \frac{9}{4}\frac{C_{\rm A}g^2}{48\pi}\ln\left(\frac{q^2}{\mu^2}\right)\right]$$

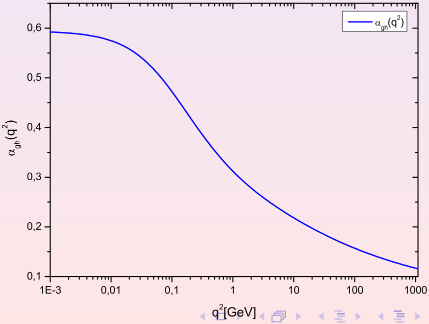
we verify that $\overline{g}_{\rm gh}(q^2)$ has the right one-loop behavior.



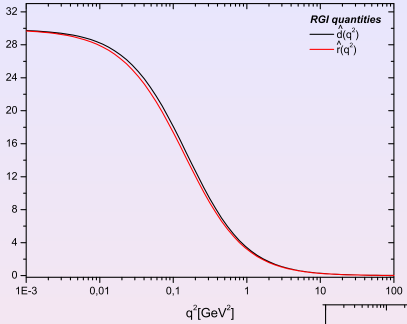
Different freezing value
Dependence on μ

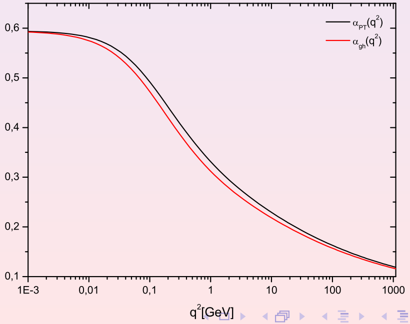
The effective charge from the ghost-gluon vertex





Comparison of the two definition

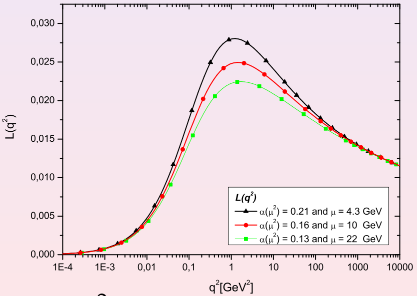




Comparison of the two definition

- Due to the relation $1 + G(q^2) + L(q^2) = F^{-1}(q^2)$
- Both effective charges are related by

$$\alpha_{\rm PT}(q^2) = \alpha_{\rm gh}(q^2)\left[1 + \frac{L(q^2)}{1 + G(q^2)}\right]^2$$



$L(q^2)$ is a small function.

Conclusions

- We have presented a nonperturbative comparison between the two QCD effective charges, $\alpha_{\mathrm{PT}}(q^2)$ and $\alpha_{\mathrm{gh}}(q^2)$.
- They were obtained within two completely different frameworks: the pinch-technique gluon self-energy and the ghost-gluon vertex .
- A common ingredient is the infrared finite gluon propagator $\Delta(q^2)$.
- By virtue of a special identity relating their non-common ingredients, in the deep infrared they coincide at the same finite value .

The W and Z bosons become massive at tree-level, through the standard Higgs mechanism

$$\widehat{d}_W(q^2) = \frac{\overline{g}_W^2(q^2)}{q^2 + M_W^2}$$

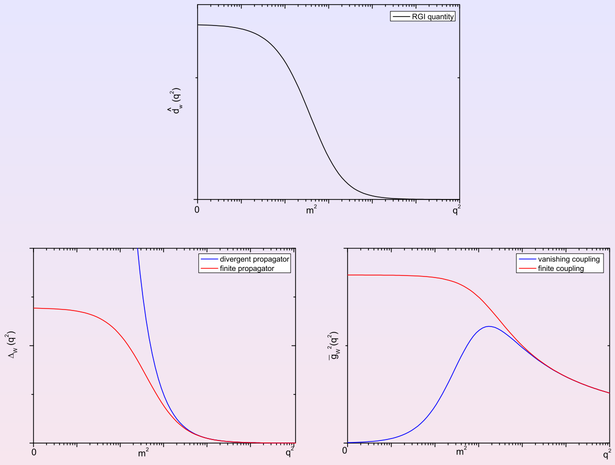
$$\overline{g}_W^2(q^2) = g_W^2(\mu)\left[1 + b_W g_W^2(\mu)\int_0^1 dx \ln\left(\frac{q^2 x(1-x) + M_W^2}{\mu^2}\right) - \ldots\right]^{-1}$$

where $b_W = 11/24\pi^2$, and the ellipses denote the contributions of the fermion families.

$\widehat{d}_W(0) = \overline{g}_W^2(0)/M_W^2$ with $\overline{g}_W^2(0) = g_W^2(\mu)[1 + b_W g_W^2(\mu)\ln(M_W^2/\mu^2)]^{-1}$

The coupling freezes at a constant value .

The Fermi's constant is determined as $4\sqrt{2}G_F = \overline{g}_W^2(0)/M_W^2$.



The same quantity decomposed in two different ways:

1. divergent propagator and a vanishing coupling
2. finite propagator and a finite coupling