Shear viscosity of the gluon plasma in the stochastic-vacuum approach

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Experimental indications

RHIC data exhibit strong collective phenomena in the asymmetric azimuthal distribution around the beam axis:

$$p_0 \frac{d^3N}{dp^3}\bigg|_{p_z=0} = v_0(p_\perp)\left[1 + 2v_2(p_\perp)\cos(2\phi) + 2v_4(p_\perp)\cos(4\phi) + \cdots\right],$$

where $(p_x, p_y, p_z) = (p_\perp \cos\phi, p_\perp \sin\phi, p_z)$.

The large elliptic flow $v_2 \simeq 0.06$ cannot be described just by two-body interactions between partons.

Particles of different mass are emitted from the fireball with a common fluid velocity.

Perfect-fluid dynamics (with zero shear and bulk viscosities) reproduces v_2 very well, up to $p_\perp \sim 1.5\,\mathrm{GeV}$ (P. Huovinen, U.W. Heinz, '01).

Experimental indications

$L_{\rm mfp}$ of a parton, which traverses an (ideal quantum) liquid is much smaller than the thermal wavelength $\sim \beta \equiv \frac{1}{T}$, i.e.

$$\frac{L_{\rm mfp}^{\rm liq.}}{\beta} \ll 1.$$

Instead, in the dilute-gas model of the QGP,

$$L_{\rm mfp}^{\rm gas} \sim (\rho\sigma_t)^{-1},$$

where $\rho \sim T^3$ is the particle-number density, $\sigma_t \sim g^4\beta^2 \ln g^{-1}$ is the Coulomb transport cross-section, and $g = g(T)$ is the perturbative finite-T QCD coupling

$$\Rightarrow \qquad \frac{L_{\rm mfp}^{\rm gas}}{\beta} \sim \frac{1}{g^4 \ln g^{-1}} \gg 1$$

$\Rightarrow$ the experimental results could have only been reproduced by the dilute-gas model if σ_t were larger by an order of magnitude (D. Molnar, M. Gyulassy, '02).

General properties of shear viscosity

Shear viscosity η represents the ability to transport momentum:

$$\frac{\eta}{s} \sim \frac{L_{\rm mfp}}{\beta},$$

where s is the entropy density $\Rightarrow \frac{\eta}{s}$ is large in the dilute-gas model of the QGP, and gets smaller for a strongly interacting QGP.

E.g., for $T \sim 200\,{\rm MeV}$ and $L_{\rm mfp} \sim 0.1\,{\rm fm}$, $\frac{\eta}{s} \sim 0.1$.

When a parton propagates through the QGP over the distance $L_{\rm mfp}$, its mean momentum change Δp is $\sim T$

$$\Rightarrow \qquad \frac{\eta}{s} \sim \frac{L_{\rm mfp}}{\beta} \sim L_{\rm mfp} \cdot \Delta p$$

is nonvanishing due to the Heisenberg uncertainty principle
$\Rightarrow \eta$ cannot vanish completely.

General properties of shear viscosity

- How small can $\frac{\eta}{s}$ be ?
- What is the temperature behavior of $\frac{\eta}{s}$?

The minimal possible value of $\frac{\eta}{s}$ is conjectured to be that in $\mathcal{N} = 4$ SYM (G. Policastro, D.T. Son, A.O. Starinets, '01):

$$\left.\frac{\eta}{s}\right|_{\mathcal{N}=4\,\mathrm{SYM}} = \frac{1}{4\pi} \simeq 0.08.$$

It is a temperature-independent constant.

Rather, in perturbative QCD (P. Arnold et al., '01, '03),

$$\left.\frac{\eta}{s}\right|_{\mathrm{pQCD}} \sim \frac{1}{g^4 \ln g^{-1}} \gg 1.$$

Note that plasma instabilities can generate an anomalous viscosity η_A (M. Asakawa, S.A. Bass, B. Müller, '06):

$$\frac{\eta_A}{s} \sim \frac{1}{g^{3/2}} < \left.\frac{\eta}{s}\right|_{\mathrm{pQCD}}, \quad \text{but} \quad \text{still} \quad \gg 1.$$

General properties of shear viscosity

For known liquids, $\frac{\eta}{s} \gg 1$ for small and large T, where η is dominated by potential- and kinetic-energy contributions, respectively.

Around T_c, these two contributions are nearly equal, and $\frac{\eta}{s}$ has a minimum, corresponding to the most difficult condition to transport momentum.

This behavior is exhibited by liquids of very different nature, such as helium, nitrogen, and water.

General properties of shear viscosity

The energy-momentum tensor of a perfect fluid is

$$T_{\mu\nu} = -p \cdot g_{\mu\nu} + Ts \cdot u_\mu u_\nu,$$

where u_μ is the local velocity of energy transport.

Shear viscosity η and bulk viscosity ζ are the coefficients of the 1st-order derivative terms:

$$\Delta T_{\mu\nu} = \eta \cdot (\Delta_\mu u_\nu + \Delta_\nu u_\mu) + \left(\frac{2}{3}\eta - \zeta\right) H_{\mu\nu}\partial_\rho u_\rho,$$

where $H_{\mu\nu} = u_\mu u_\nu - g_{\mu\nu}$, $\Delta_\mu = \partial_\mu - u_\mu u_\nu \partial_\nu$.

Note: this Landau-Lifshitz definition of u_μ (for which $u_\mu \cdot \Delta T_{\mu\nu} = 0$), is more suitable than the Eckart definition of u_μ as a velocity of the baryon-number flow (for which $u_\mu u_\nu \cdot \Delta T_{\mu\nu} = 0$), since the baryon number in HIC $\ll$ total number of particles.

Kubo formula and the SVM at $T > T_c$

Shear viscosity is defined via its spectral density $\rho(\omega, T)$ as

$$\eta = \pi \frac{d\rho}{d\omega}\bigg|_{\omega=0}.$$

The spectral density obeys the Kubo formula

$$\int_0^\infty d\omega\, \rho\, \frac{\cosh\left[\omega\left(x_4 - \frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty} \langle T_{12}(0)\, T_{12}(\mathbf{x}, x_4 + \beta n)\rangle\,,$$

where the Euclidean correlation function enters (F. Karsch & H.W. Wyld, '87; further lattice developments are due to G. Aarts & J.M. Martinez Resco, '02, and H.B. Meyer, '07).

Kubo formula and the SVM at $T > T_c$

$$\rho = \rho_{\rm nonpert} + \rho_{\rm pert},$$

where $\rho_{\rm pert} \propto g^4\omega^4$ can be isolated simultaneously with $\langle T_{12}(0)T_{12}(x)\rangle_{\rm pert} \propto g^4/|x|^8$.

The aim of this project is to calculate $\rho_{\rm nonpert}$ via the SVM at finite temperature (Yu.A. Simonov, '91 - present; N.O. Agasian, '03).

SVM. While QCD sum rules assume $\left\langle g^2(F^a_{\mu\nu})^2\right\rangle$, the SVM additionally assumes a finite correlation length of the vacuum, $\mu^{-1} < \infty$: $\left\langle F^a_{\mu\nu}(x)F^b_{\lambda\rho}(0)\right\rangle \sim {\rm e}^{-\mu|x|}$ (Pisa lattice group, '86 - '03) $\Rightarrow$ the SVM can quantitatively describe confinement: $\sigma \propto \mu^{-2}\left\langle g^2(F^a_{\mu\nu})^2\right\rangle$.

In the deconfinement phase ($T > T_c$), the spatial string tension

$$\sigma_s(T) \propto \mu^{-2}(T)\left\langle g^2(F^a_{ij})^2\right\rangle_T,$$

i.e. the chromomagnetic vacuum still confines.

Shear viscosity $\eta(T)$

$$\text{Kubo formula}: \quad T_{12} = g^2 F^a_{1\mu} F^a_{2\mu} \quad \Rightarrow \quad \eta \propto \left\langle g^2 (F^a_{ij})^2 \right\rangle_T^2 \; -$$

a general prediction of the SVM for all kinetic coefficients, as $\sigma^{\rm SVM}_{\rm total} \propto \left\langle g^2 (F^a_{\mu\nu})^2 \right\rangle^2$ (H.G. Dosch, O. Nachtmann et al., '91 - '03)
$\Rightarrow$ since $[\eta] = ({\rm mass})^3$, one expects

$$\eta \propto \mu^{-5}(T) \cdot \left\langle g^2 (F^a_{ij})^2 \right\rangle_T^2 .$$

At temperatures larger than the temperature of dimensional reduction, $T > T_*$,

$$\mu(T) \propto g^2 T, \quad \left\langle g^2 (F^a_{ij})^2 \right\rangle_T \propto (g^2 T)^4, \quad \text{but} \quad s(T) \propto T^3 \quad \Rightarrow$$

$$\frac{\eta}{s} \propto g^6(T) \quad \text{at} \quad T > T_*.$$

$\Rightarrow$ It is *a priori* clear that one cannot get a (slow) increase of $\frac{\eta}{s}$ at high temperatures. Nevertheless, a (rapid) decrease at temperatures close to T_c can be obtained, and the values of $\frac{\eta}{s}$ can be compared with $\frac{1}{4\pi}$.

Calculation of $\eta(T)$

Notations:

$$\langle g^2(F^a_{\mu\nu})^2\rangle \equiv \langle G^2\rangle, \quad \langle g^2(F^a_{ij})^2\rangle_T \equiv \langle G^2\rangle_T, \quad \rho_{\rm nonpert} \equiv \rho.$$

We assume at $T=0$ the correlation function of the form

$$\langle T_{12}(0)T_{12}(x)\rangle = N(\alpha)\langle G^2\rangle^2 \cdot \frac{K_{2-\alpha}(M|x|)}{(M|x|)^{2-\alpha}},$$

where α and $N(\alpha)$ will be determined.

An equation for the Fourier coefficients $(\sum_k {\rm e}^{i\omega_k x_4} f_k)$ of the Kubo formula:

$$\int_0^\infty d\omega\ \rho\ \frac{\omega}{\omega^2+\omega_k^2} = \pi^2 2^\alpha \Gamma(\alpha) N(\alpha)\langle G^2\rangle_T^2 \cdot \frac{M^{2\alpha-4}}{(\omega_k^2+M^2)^\alpha}, \qquad (*)$$

where $\alpha>0$; $\omega_k = 2\pi Tk$ is the k-th Matsubara frequency.

Calculation of $\eta(T)$

A Lorentzian-type *ansatz*

$$\rho = C(T) \cdot \frac{\omega}{[\omega^2 + M^2(T)]^{\alpha + \frac{1}{2}}} \qquad \left(\Rightarrow \quad \eta = \frac{\pi C}{M^{2\alpha+1}} \right)$$

guarantees that both sides of Eq. (*) have the same large-$|k|$ behavior.
$M(T) \sim \mu(T)$ is the momentum scale, below which PT breaks down.
$C(T)$ and $M(T)$ will be determined.

- For $|k| \gg 1$,

$$\text{LHS of Eq. } (*) = \frac{C}{\omega_k^{2\alpha}} \left[\frac{\pi}{2 \sin(\pi\alpha)} + \mathcal{O}\left(\frac{M^2}{\omega_k^2} \right) + \sum_{i=2}^{\infty} c_i \left(\frac{M}{\omega_k} \right)^{i-2\alpha} \right] \quad \Rightarrow$$

the leading term in the brackets is k-independent only for $\alpha < 1$.

$$\text{RHS of Eq. } (*) = \pi^2 2^\alpha \Gamma(\alpha) N(\alpha) \frac{\langle G^2 \rangle_T^2}{\omega_k^{2\alpha}} M^{2\alpha-4} \cdot \left[1 + \mathcal{O}\left(\frac{M^2}{\omega_k^2} \right) \right].$$

$$\Rightarrow \quad \eta(T)\Big|_{|k|\gg 1} = \pi^2 2^{\alpha+1}\Gamma(\alpha)N(\alpha)\sin(\pi\alpha)\frac{\langle G^2\rangle_T^2}{M^5(T)}.$$

Note: $|k| \gg 1$ means $|k| \geq 3$, since $\frac{M}{\omega_3} < 0.35$ for any $T > T_c$.

- For $|k| \sim 1$ (e.g. $k = 0$ for $T > T_*$), $\mathcal{O}\left(\frac{\omega_k^2}{M^2}\right)$-terms and higher can be disregarded $\Rightarrow$

$$\eta(T)\Big|_{|k|\sim 1} = \pi^{5/2} 2^{\alpha+1}\Gamma\left(\alpha+\frac{1}{2}\right)N(\alpha)\frac{\langle G^2\rangle_T^2}{M^5(T)}.$$

The ratio

$$\frac{\eta(T)\Big|_{|k|\gg 1}}{\eta(T)\Big|_{|k|\sim 1}} = \frac{\Gamma(\alpha)\sin(\pi\alpha)}{\sqrt{\pi}\Gamma\left(\alpha+\frac{1}{2}\right)} \quad \text{for} \quad 0 < \alpha < 1$$

is equal to 1 at $\alpha = \frac{1}{2}$, i.e. $\Rightarrow \eta\Big|_{\alpha=\frac{1}{2}}$ is k-independent.

Calculation of $\eta(T)$

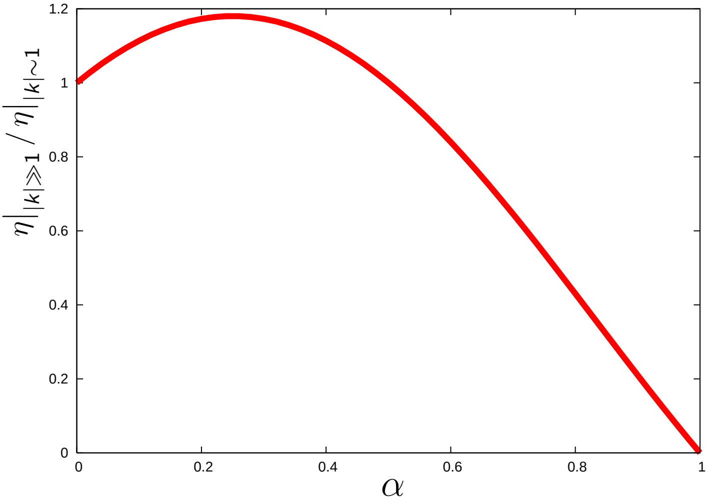

Figure: The ratio $\frac{\eta\big|_{|k|\gg 1}}{\eta\big|_{|k|\sim 1}}$.

Calculation of $\eta(T)$

For $\alpha = \frac{1}{2}$, the purely Lorentzian $\rho(\omega)$ is recovered.

To determine $N(\alpha)$ and $M(T)$, we impose the Gaussian-dominance hypothesis, which disregards the connected part of

$$\left\langle T_{12}(0) T_{12}(x) \right\rangle = \left\langle g^4 F^a_{1\mu}(0) F^a_{2\mu}(0) F^b_{1\nu}(x) F^b_{2\nu}(x) \right\rangle.$$

The SVM parametrizes confining interactions in the remaining two-point functions as

$$\left\langle g^2 F^a_{\mu\nu}(x) F^b_{\lambda\rho}(0) \right\rangle = (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}) \cdot \frac{\left\langle G^2 \right\rangle}{12(N_c^2-1)} \delta^{ab} D(x).$$

$\Rightarrow$ The only function $D(x)$ compatible with the Lorentzian $\rho(\omega)$ is

$$D(x) \propto \sqrt{\frac{K_{3/2}(2\mu|x|)}{(2\mu|x|)^{3/2}}} \propto \frac{\mathrm{e}^{-\mu|x|}}{\mu|x|} \sqrt{1 + \frac{1}{2\mu|x|}},$$

where the normalization constant can be fixed from $\sigma_{\mathrm{f}} = \frac{\left\langle G^2 \right\rangle}{144} \int d^2x D(x)$.

Calculation of $\eta(T)$

We obtain

$$M = 2\mu, \qquad N(1/2) = \frac{1}{36\left[\int_0^\infty dz \cdot z^{1/4} \cdot \sqrt{K_{3/2}(z)}\right]^2}.$$

$\Rightarrow$ The final result for the shear viscosity is

$$\eta(T) = \frac{\pi^{5/2} N(1/2)}{8\sqrt{2}} \cdot \frac{\left\langle G^2 \right\rangle_T^2}{\mu^5(T)}.$$

Calculation of $\eta(T)$

Ingredients for the numerical evaluation:

- $T_c = 270\,\mathrm{MeV}$.
- The two-loop running coupling in SU(3) YM:

$$g^{-2}(T) = 2b_0 \ln \frac{T}{\Lambda} + \frac{b_1}{b_0} \ln \left(2 \ln \frac{T}{\Lambda} \right),$$

$$b_0 = \frac{11 N_c}{48\pi^2},\ b_1 = \frac{34}{3} \left(\frac{N_c}{16\pi^2} \right)^2,\ N_c = 3,\ \Lambda = 0.104\, T_c.$$

- Parametrizations:

$$f(T) \equiv \begin{cases} 1 \ \ \mathrm{at} \ \ T_c < T < T_*, \\ \frac{g^2(T)T}{g^2(T_*)T_*} \ \ \mathrm{at} \ \ T > T_*, \end{cases}$$

$$\Rightarrow \mu(T) = \mu \cdot f(T),\ \sigma_{\mathrm{f}}(T) = \sigma_{\mathrm{f}} \cdot f^2(T),\ \left\langle G^2 \right\rangle_T = \left\langle G^2 \right\rangle \cdot f^4(T).$$

Calculation of $\eta(T)$

where $\mu = 894\,\mathrm{MeV}$ (Pisa group, '97), $\sigma_\mathrm{f} = (440\,\mathrm{MeV})^2$, $\langle G^2 \rangle = \frac{72}{\pi}\sigma_\mathrm{f}\mu^2$.

- Determining T_* from the equation $\sigma_\mathrm{f}(T_*) = \sigma_\mathrm{f}$, where $\sigma_\mathrm{f}(T) = [0.566 g^2(T) T]^2$ (Bielefeld group, '93, '96) $\Rightarrow T_* = 1.28\,T_c$.

- Entropy density $s(T) = \frac{dp_\mathrm{lat}}{dT} \Rightarrow s(T)/T^3$ is indeed nearly constant at $T \gtrsim 2T_c$.

Calculation of $\eta(T)$

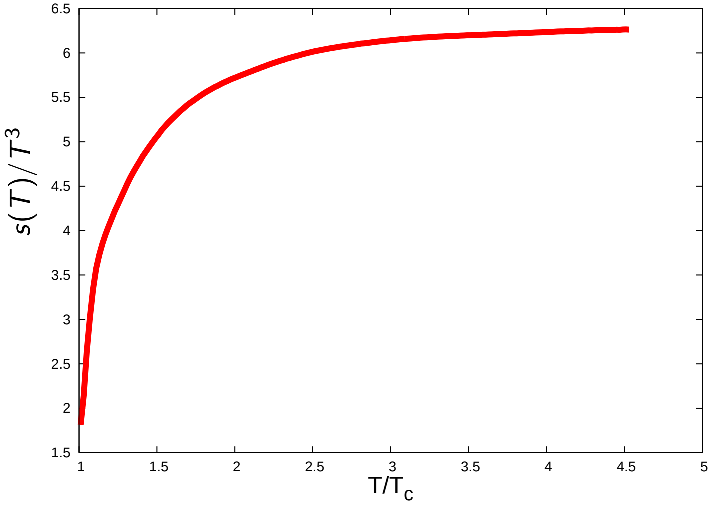

Figure: Entropy density $s(T)$ in the units of T^3 obtained from the lattice values for the pressure (G. Boyd et al., 1996; courtesy of F. Karsch).

Calculation of $\eta(T)$

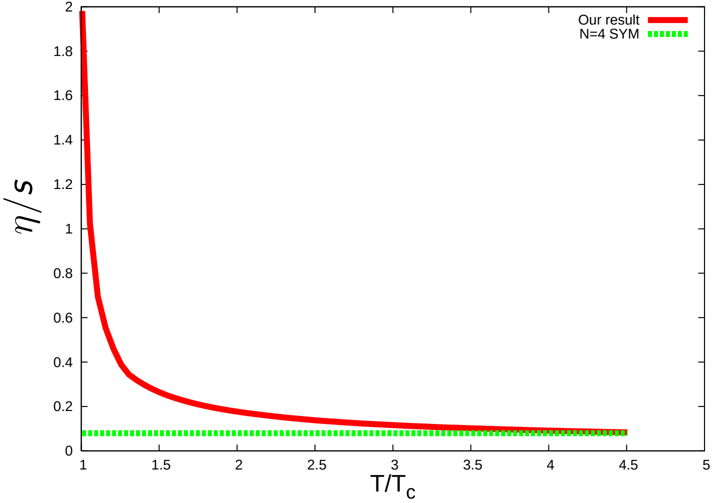

Figure: Calculated ratio η/s as a function of temperature. Also shown is the conjectured lower bound of $1/(4\pi)$ for this quantity, realized in $\mathcal{N} = 4$ SYM.

Calculation of $\eta(T)$

$\frac{\eta}{s}$ falls off rapidly at $T_c < T \lesssim 2T_c$, and then as $\mathcal{O}\left(g^6(T)\right)$, approaching $1/(4\pi)$.

Asymptotically, $\frac{\eta}{s}$ becomes smaller than $1/(4\pi)$, but never vanishes (cf. Heisenberg uncertainty principle).

At $T \gtrsim (5 \div 10)T_c$, the perturbative result (P. Arnold, G.D. Moore, L.G. Yaffe, '03) takes it over:

$$\eta_{\rm NLL} = \frac{T^3}{g^4} \cdot \frac{27.126}{\ln \frac{2.765}{g}}.$$

As was expected, the SVM does not yield a minimum of $\frac{\eta}{s}$.

Calculation of $\zeta(T)$

Bulk viscosity $\zeta(T)$ describes the degree of non-conformality of the QGP ($\zeta \equiv 0$ in any CFT, including $\mathcal{N} = 4$ SYM).

$\rho(\omega, T)$ for $\zeta_{\rm nonpert}$ can be derived from the corresponding Kubo formula

$$\int\limits_0^\infty d\omega \; \rho \; \frac{\cosh\left[\omega\left(x_4 - \frac{\beta}{2}\right)\right]}{\sinh(\omega\beta/2)} = \int d^3x \sum_{n=-\infty}^{+\infty} \langle \Theta(0)\Theta(\mathbf{x}, x_4 + \beta n)\rangle \,,$$

where $\Theta(x) = \frac{\beta(g)}{2g}[F^a_{\mu\nu}(x)]^2$ is the nonperturbative contribution to the trace of the YM energy-momentum tensor.

Calculation of $\zeta(T)$

The resulting

$$\zeta_{\rm nonpert} = \frac{N(1/2)}{3\sqrt{2\pi^3}} \left(\frac{11}{32}\right)^2 \frac{\left\langle G^2 \right\rangle_T^2}{\mu^5(T)}$$

is parametrically the same as $\eta(T)$.

$$\frac{\zeta_{\rm nonpert}}{T^3} \propto g^6(T) \quad \text{at} \quad T > T_*,$$

whereas (P. Arnold et al., '06)

$$\frac{\zeta_{\rm pert}}{T^3} = \frac{0.443\alpha_s^2}{\ln(7.14/g)} \sim g^4(T).$$

Calculation of $\zeta(T)$

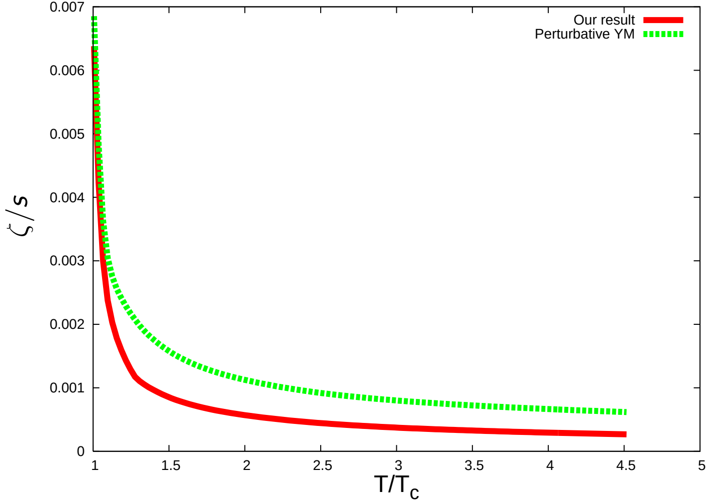

Figure: Calculated ratio ζ/s as a function of temperature. Also shown for comparison are perturbative values extrapolated down to T_c.

Outlook

• Accounting in $\left\langle g^2 F^a_{\mu\nu}(x) F^b_{\lambda\rho}(0) \right\rangle$ for non-confining non-perturbative interactions, which amount to 26%. (Work in progress.)

• Relaxing the Gaussian-dominance hypothesis via the parametrization of $\left\langle g^4 F^{a_1}_{\mu_1\nu_1}(x_1) F^{a_2}_{\mu_2\nu_2}(x_2) F^{a_3}_{\mu_3\nu_3}(x_3) F^{a_4}_{\mu_4\nu_4}(x_4) \right\rangle_{\rm conn}$ suggested by W. Kornelis and H.G. Dosch, '01.

Ref.: D.A., arXiv:0905.3329 [hep-ph].