Open issues for confinement, on the lattice and for center vortices

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The issues

1. In the center-vortex picture, where does the Lüscher term come from? Does it come from source-free center vortex solitons or from gluons with a Wilson loop as source? If the latter, how is it related to fishnet graphs and gluon chains? What lattice studies specifically address this, without having to simulate full QCD?

2. Approximate duality between fishnet graphs and vortex/nexus surfaces

3. The Lüscher term from fishnet graphs

4. Can we study the **pinch technique** on the lattice? Or how do we implement $R(\xi)$ gauges other than Landau gauge, and how do we implement the background-field Feynman gauge on the lattice?

First issue: Lüscher term and topology with center vortices

Center vortices, topological confinement, and the Lüscher potential

- Center vortices are very successful in explaining confinement as a topological property
- They explain the N-ality rule; the baryonic area law in QCD; thermal deconfinement; why G_2, F_4, and E_8 are screening, not confining; etc.
- But it is not so clear why there should be a Lüscher term and associated phenomena

The pinch-technique picture of center vortices

- The pinch technique Schwinger-Dyson equations show that infrared slavery demands a gluon mass, as abundantly confirmed by lattice simulations

- Given the dynamical mass, an infrared-effective and gauge-invariant massive action yields center vortices (and nexuses, monopole-like objects) as *quantum solitons*

- On the assumption of entropy dominance, center vortices condense—the condensate VEV $\langle TrG_{\mu\nu}^2 \rangle$ is self-consistently related to the gluon mass

- Mass generation plus local gauge invariance *requires* long-range pure-gauge excitations *that carry topology and confinement*

Lattice simulations all show a gluon mass $m \approx$ 600 MeV

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Basics of topological confinement

- Take a simple $d = 3$ Abelian massive model, with action:

$$I = \int d^3x \{ \frac{1}{4} F_{ij}^2 + \frac{m^2}{2} (A_i - \partial_i \phi)^2 - i \oint d\tau \dot{z}_i \delta(x - z(\tau)) A_i \}$$

- The classical solution is a sum (with given collective coordinates) of vortex terms of the form

$$A_i(x) = \pm \sum_V \pi \oint_V dz_k \epsilon_{ijk} \partial_j [\Delta_m(x - z) - \Delta_0(x - z)] \equiv V_i$$

plus the term $\Delta_m J_i \equiv \int (\nabla^2 + m^2)^{-1} J_i$ driven by the Wilson loop current $J_i = i \oint d\tau \dot{z}_i \delta(x - z(\tau))$

As m$\longrightarrow\infty$, two types of gluons

- **Type I:** The pure-gauge (Δ_0) part of V_i, called U_i, that survives $m \to \infty$:

$$U_i(x) = \pm \sum_V \pi \oint_V dz_k \epsilon_{ijk} \partial_j \Delta_0(x-z)$$

- **Type II:** Massive gluons; decouple in the limit

- Terms in the action involving the Wilson-loop current are:

$$I_J = \int \{ \frac{1}{2} J_i \frac{1}{\nabla^2 + m^2} J_i - U_i J_i \}$$

The difference between the types: Type I

- Type I gluons know nothing of the Wilson loop, but contribute to it *topologically*:

$$\langle W_I \rangle = \langle \exp[i \oint_\Gamma dz_i U_i(z)] \rangle = \langle \exp[i\pi \sum_V Lk_V] \rangle$$

- Lk_V is the Gauss link number:

$$Lk_V \equiv \oint_\Gamma dx_i \oint_V dz_k \epsilon_{ijk} \partial_j \Delta_0(x-z)$$

The area law

- By Stokes' theorem this is:

$$\langle \prod_{V \in S} (-1) \rangle$$

where $V \in S$ means vortex V pierces *any* surface S spanning the loop

- It is easy to see that there is an **area law**, and what the spanning surface must be, for a **flat** Wilson loop (next slide), but it is not obvious what happens for a **non-flat** loop[1]

1. Minimality may follow from a steepest-descent argument, but what about fluctuations?

Topological confinement—flat Wilson loop

- Latticize the loop into N squares of side λ; at most one vortex can pierce a square, with probability p. Vortex areal density $= p/\lambda^2 \equiv \xi^2$

- With $\bar{p} \equiv 1 - p$, and independent[1] piercings, the well-known result is:

$$\langle W_I \rangle = (\bar{p} - p)^N = \exp[\ln(1 - 2p)A/\lambda^2] \equiv e^{-\sigma A}$$

  where $A = N\lambda^2$ is the area of the flat spanning surface and $\sigma = -(1/\lambda^2)\ln(1 - 2p) \approx 2p/\lambda^2 = 2\xi^2$ is the string tension

- **Why do we assume a minimal (flat) surface, when any surface will do?** [Partial answer: Any surface will do, but the probability p will depend on the surface]

 1. Not really true; affects string tension, not the existence of an area law

Type II gluons

- In this simple Abelian model, Type II gluons **only** contribute to perimeter terms, not to the area law

- In a non-Abelian theory, they **also** give rise to fishnet graphs (and gluon chains) so they contribute to possible surface tension

Conjectures: Type I

- For a non-flat Wilson loop, the Type I contribution to the area law is:

$$\langle W_I \rangle = \exp[-\sigma A_{min}]$$

  where $A_{\rm min}$ is the area of an extremal spanning surface

- Fluctuations in the Type I vortex ensemble of configurations give rise to specifically a Lüscher term

- Because Type I gluons are purely topological, neither conjecture is at all obvious; they can be tested with fairly simple algorithms, such as used for polymers, on **large** lattices

Duality between gluon chains and vortex/nexus surfaces

Fishnet graphs and gluon chains

- An old subject—our contribution, if any, is to consider the gluons in fishnet graphs and chains to be massive, not relativistic
- People usually describe gluon chains at fixed time, but of course each gluon extends to world lines in the time direction
- The gluons interact, giving rise to fishnet graphs
- A gluon mass automatically localizes the gluons (as required for a gluon chain)

A picture of gluon chains

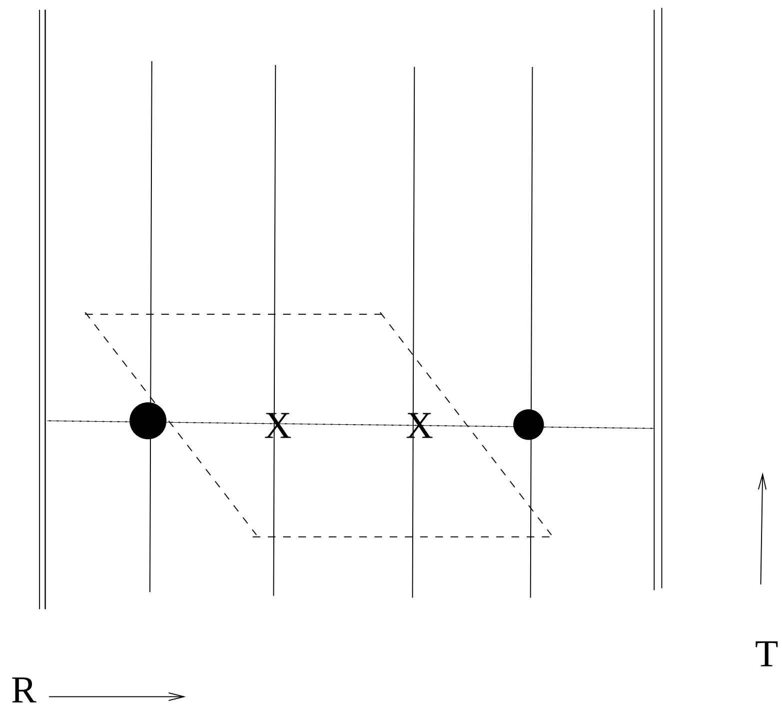

Figure 1: Wilson loop: Double lines. Gluon chain (with world lines): Black circles and Xs (that mark piercing of dual ('t Hooft) loop).

Dual gluon chain: Center vortices and nexuses

- A center vortex is a closed 2-surface of chromomagnetic flux, but it can have an 't Hooft loop as a boundary

- A center vortex is the home to **nexuses**, localized monopole-like configurations with chromomagnetic flux stretched into portions of a center vortex sheet

- Nexus mass $\sim 4\pi m/g^2$; $g^2/(4\pi) \simeq 1$ is the infrared coupling

- Energy stored between nexus and anti-nexus grows linearly with separation, as postulated for standard gluon chains

A nexus

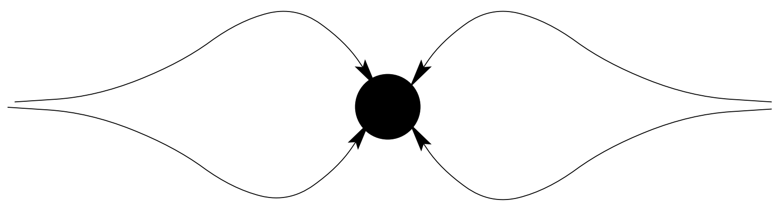

Figure 2: A nexus (monopole) with field lines compressed by the gluon mass (Meissner effect). The orientation of field lines on the left is different from that on the right, but each half contributes to topological confinement by linking exactly the same as a center vortex. The Polyakov picture of Georgi-Glashow confinement is actually in the same universality class as center vortices plus nexuses

't Hooft: Either electric or magnetic confinement, not both

- A dual gluon chain corresponds to a standard nexus-center-vortex configuration, which is condensed in the vacuum through entropy domination

- Center vortices tend to expand until stopped by excluded-volume effects

- The string between dual (magnetic) quarks is very long and loose, like having a 1000-m string between two people 10 m apart. Pull on one end, the other end doesn't even notice

- Electric strings are taut, because they don't condense in the vacuum; they need Wilson-loop sources

Back to (electric) massive gluon chains

- We assume that the properties of the gluon chain $d = 2$ “condensate” on the spanning surface are very like the properties of the $d = 3, 4$ space-filling vortex condensate (approximate duality in the regime $g^2/(4\pi) \simeq 1$)

- So the areal density $\rho_E = \xi_E^2 \approx \xi^2$; ξ_E is the intergluon spacing and ξ the vortex spacing

- Then the string energy is $\epsilon = \sigma R \approx R \cdot \xi \cdot m$

- But also $\sigma \approx 2\xi^2$, which yields $\xi \approx \frac{1}{2}m; \;\; \sigma \approx \frac{1}{2}m^2$

- This happens to work well for $m \approx$ 600 MeV

Fishnet graphs and surfaces under tension

Fishnet graphs

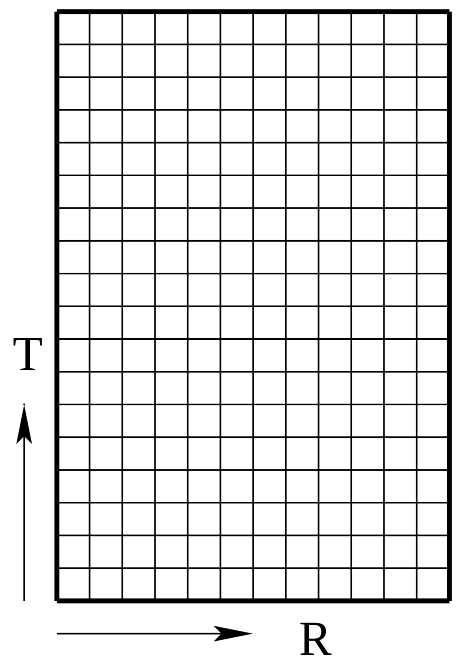

Figure 3: A fishnet graph in a Wilson loop (heavy lines). Vertices labeled by coordinates $Z_\beta(\sigma_1, \sigma_2)$; lines point along unit vectors $\hat{e}(R, T)$

Propagators

- We approximate gluons by scalars with quartic interactions
- The logarithm of the free propagator from a vertex to a neighbor is:

$$\ln \Delta[(x-y)^2] = \int_0^\infty \frac{ds}{s} \int_x^y (dz) \exp\{-\frac{m}{2} \int_0^s d\tau [\dot{z}^2 + 1]\}$$

$$x_\beta = Z_\beta(\sigma_a + \lambda \hat{e}_a^{R,T}), y_\beta = Z_\beta(\sigma_a)$$
$$x_\beta - y_\beta \approx \lambda \hat{e}(R,T) \cdot \partial Z_\beta(\sigma)$$

Semiclassical approximation to the fishnet graph

- Integrate over all vertices and sum over $\hat{e}(R,T)$:

$$const. \times \prod_{\sigma} \{ d^4 Z_\beta(\sigma) \Delta[(\partial_a Z_\beta)^2] \}$$

$$\approx \int (dZ_\beta) \exp\{ - \int \frac{d^2\sigma}{\lambda^2} \frac{m}{2\Delta s} [(\lambda \partial_a Z_\beta)^2] \}$$

where $\Delta s \simeq \lambda \simeq 1/m$

- Semiclassical: $(\partial_a)^2 Z_\beta = 0$

Partial vertex equilibrium

- Semiclassically, there is a force-balance condition at every vertex. We assume the gluon line spacing λ is small, so

$$Z_\beta(\sigma + \lambda\hat{e}) - Z_\beta(\sigma) \approx \lambda \hat{e}_a \partial_a Z_\beta(\sigma)$$

- The force lies along the classical gluon trajectory from one vertex to another, so:

$$Z_\beta(\sigma_1 + \lambda, \sigma_2) - Z_\beta(\sigma_1, \sigma_2) + Z_\beta(\sigma_1 - \lambda, \sigma_2) - Z_\beta(\sigma_1, \sigma_2)$$

$$+\text{two other terms} = 0 \rightarrow \lambda^2 (\partial_a)^2 Z_\beta = 0$$

Classical analog of vertex force equilibrium

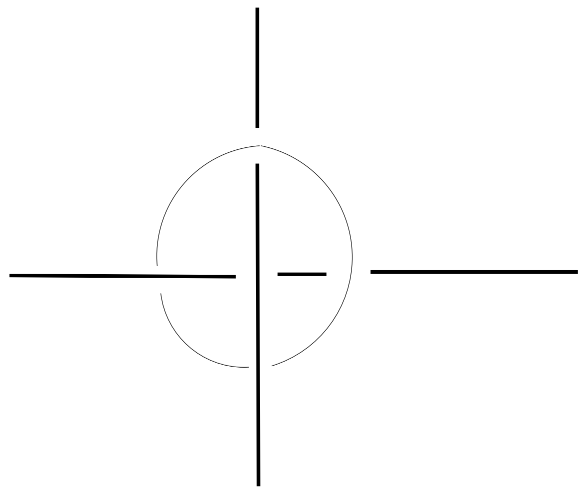

Figure 4: Heuristics of vertex equilibria: Like two strings held together by a frictionless ring. They can slide freely, but they can't separate

Equilibrium and conformal gauge

- Because the strings can slide, equilibrium requires:

$$\partial_a Z_\beta \partial_b Z_\beta = 0 \;\; (a \neq b)$$

and by isotropy

$$\partial_1 Z_\beta \partial_1 Z_\beta = \partial_2 Z_\beta \partial_2 Z_\beta \;\; [= f^2(\sigma)]$$

- So the induced metric η_{ab} on the surface is in **conformal gauge**:

$$\eta_{ab} = \partial_a Z_\beta \partial_b Z_\beta = \delta_{ab} f^2(\sigma)$$

Minimal surface

- The Dirichlet statement of the minimal surface problem: The minimum area of a surface spanning a contour Γ is the minimum over Z_β of the integral:
$$A_{min}\{\Gamma\} = \frac{1}{2}\int d^2\sigma[\partial_a Z_\beta]^2$$
provided that the (consequently harmonic) functions $Z_\beta(\sigma)$ are in conformal gauge

- **So fishnet graphs for massive gluons give a minimal surface, and there must be a Lüscher term as well**

Last issue: A return to gauge invariance on the lattice

To understand QCD we have to go off-shell

- The beauty of Wilson' formulation of gauge theory is that it is gauge-invariant
- But with present technology, to calculate an off-shell propagator **requires** gauge-fixing, even on the lattice
- There are many studies in Landau gauge on the lattice but much physics is hidden by gauge artifacts
- The first step toward true gauge invariance is to go to general $R(\xi)$ gauges, as Cucchieri *et al.* propose, and to the background field Feynman gauge ($\xi = 1$)

We have already returned to gauge invariance in the continuum

- The **pinch technique** re-assembles conventional Feynman-graph contributions to an off-shell Green's function that is part of a gauge-invariant quantity, such as the S-matrix
- The new combinations of graphs:
  - Are completely gauge-invariant, process-independent, and scheme-independent
  - Have the right analyticity with only physical threshholds (no ghosts)
  - Satisfy ghost-free Ward identities of QED type
  - Are, as proved by Papavassiliou and Binosi to all orders, precisely the Green's functions of the background-field Feynman gauge

How do we implement the background-field Feynman gauge on the lattice?

- Exploit the ideas of Cucchieri *et al.* [this Workshop] for latttice simulations in general $R(\xi)$ gauges

- Go to background-field Feynman gauge ($\xi = 1$)

- Dashen and Gross first studied this long ago, but only used it to lowest significant order in g^2; their work can be combined with that of Cucchieri *et al.*

- If we can do all this, we can simulate **gauge-invariant** off-shell Green's functions

Summary

- Try to understand the connection between a surface under tension and topological confinement (polymer-based simulations)

- Approximate duality between gluon chains or fishnet graphs and vortex/nexus surfaces

- Fishnet graphs imply (not necessarily Nambu-Goto) minimal surfaces under tension

- Off-shell gauge invariance on the lattice: **fix to background-field Feynman gauge**, yielding pinch-technique results