Magnetic monopoles in the deconfined phase of Yang-Mills theories

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1 – INTRODUCTION

Various models relate color confinement to the condensation of topological defects (monopoles, vortice) in the QCD vacuum. The condensation of abelian magnetic monopoles is thought to induce dual Meissner effect in the dual supercoductor model ('t Hooft, 1975, Mandelstam, 1976)

Recently the idea that such topological defects could play a relevant role also in the deconfined phase has been seriously considered: many strongly interacting properties of the QGP could possibly be explained in terms of such defects.

Among other topological defects, thermal monopoles "evaporating" from the zero T condensate have attracted a lot of attention

Chernodub-Zakharov, 2006-2008, Liao-Shuryak, 2006-2008, Ratti-Shuryak, 2008).

Monopoles are particle-like defects and therefore it is tempting to describe their properties as those of a particle ensemble.

- **Can we extract information about the properties of these thermal monopoles from lattice simulations?**
- **Can they be given an interpretation in terms of physical objects?**
- **Can we follow their way back to condensation as $T \to T_c$ from above?**

2 – Magnetic monopoles and confinement

Magnetic monopoles at work in the dual superconducting model of color confinement are of abelian nature.

An abelian subgroup of the original gauge group must be fixed so that a magnetic charge can be defined with respect to that group. This is usually called Abelian Projection. Let us consider the simple case of $SU(2)$ (which is the subject of our study):

- **fix gauge up to a $U(1)$ residual gauge freedom**

- **take the diagonal (abelian) part of gauge links in that gauge**

More on abelian projection ($SU(2)$ case)

Abelian projection is assigned in terms of an adjoint field $\vec{\phi}(x)$

$$\vec{\sigma} \cdot \vec{\phi}(x) \to G(x)(\vec{\sigma} \cdot \vec{\phi}(x))G^{\dagger}(x)$$

which defines 't Hooft tensor ($\hat{\phi} \equiv \vec{\phi}/|\vec{\phi}|$)

$$\begin{aligned} F_{\mu\nu} &= \hat{\phi} \cdot \vec{G}_{\mu\nu} - \frac{1}{g}\hat{\phi} \cdot (D_{\mu}\hat{\phi} \wedge D_{\nu}\hat{\phi}) \\ &= \partial_{\mu}(\hat{\phi} \cdot \vec{A}_{\nu}) - \partial_{\nu}(\hat{\phi} \cdot \vec{A}_{\mu}) - \frac{1}{g}\hat{\phi} \cdot (\partial_{\mu}\hat{\phi} \wedge \partial_{\nu}\hat{\phi}) \\ &= \partial_{\mu}A^{3}_{\nu} - \partial_{\nu}A^{3}_{\mu} \end{aligned}$$

't Hooft tensor defines an abelian, gauge independent field strength tensor which is diagonal in the gauge where $\hat{\phi} = (0, 0, 1)$ everywhere, i.e. the case last line refers to.

This gauge is fixed up to a $U(1)$ residual gauge freedom ($\hat{\phi} \in SO(3)/U(1)$). In this gauge abelian projection corresponds to taking the diagonal part of gauge links.

In usual QCD there is no natural adjoint field, but several adjoint fields can be constructed in terms of gauge fields, typically a closed path order product of gauge links like:

- open Polyakov loop (Polyakov gauge)
- open plaquette (spatial or temporal) (plaquette gauge)

Alternatively an implicit definition can be assigned by fixing $\hat{\phi} = (0, 0, 1)$ and constant in one particular gauge: that defines $\hat{\phi}(x)$ in every other gauge. An example is the so-called Maximal Abelian Gauge (MAG) where the gauge is fixed by maximizing:

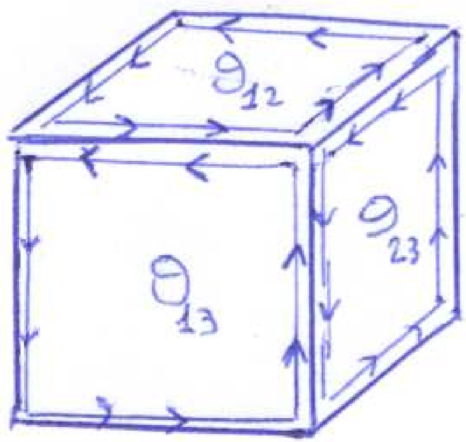
$$F_{\mathrm{MAG}} = \sum_{\mu,x} \mathrm{Re}\,\mathbf{tr}\left[U_\mu(x)\sigma_3 U^\dagger_\mu(x)\,\sigma_3\right]$$

which corresponds to maximizing the diagonal part of gauge links.

In the gauge where 't Hooft tensor is diagonal, abelian link phases are extracted as follows:

$$U_\mu = u_\mu^0 \mathrm{Id} + i\vec{\sigma}\cdot\vec{u}_\mu \to \mathrm{diag}(e^{i\theta_\mu}, e^{-i\theta_\mu}) \equiv \frac{(u_\mu^0 \mathrm{Id} + i\sigma_3 u_\mu^3)}{\sqrt{(u_\mu^0)^2 + (u_\mu^3)^2}}$$

from which abelian plaquettes can be constructed $\theta_{\mu\nu} \equiv \hat{\partial}_\mu \theta_\nu - \hat{\partial}_\nu \theta_\mu$



Monopole currents are then constructed by the usual De Grand - Toussaint construction.

$$m_\mu = \frac{1}{2\pi}\varepsilon_{\mu\nu\rho\sigma}\hat{\partial}_\nu \overline{\theta}_{\rho\sigma}$$

where $\overline{\theta}_{\rho\sigma}$ is the compactified part of the abelian plaquette phase

$$\theta_{\mu\nu} = \overline{\theta}_{\mu\nu} + 2\pi n_{\mu\nu}$$

and $n_{\mu\nu} \in \mathbf{N}$.

Monopole currents form closed loops, since $\hat{\partial}_\mu m_\mu = 0$. These loops may be either topologically trivial or wrapped around the lattice.

Below T_c typically one big cluster of monopole currents appears, percolating in all directions. Above T_c such cluster disappears, but monopole currents with a non-trivial wrapping in the temporal direction survive

V.G. Bornyakov, V.K. Mitrjushkin and M. Muller-Preussker, Phys. Lett. B284, 99 (1992);

S. Ejiri, Phys. Lett. B376, 163 (1996).

According to a recent proposal, such wrapping currents should be identified with thermal monopoles evaporating from the condensate

M. N. Chernodub and V. I. Zakharov, Phys. Rev. Lett. 98, 082002 (2007)

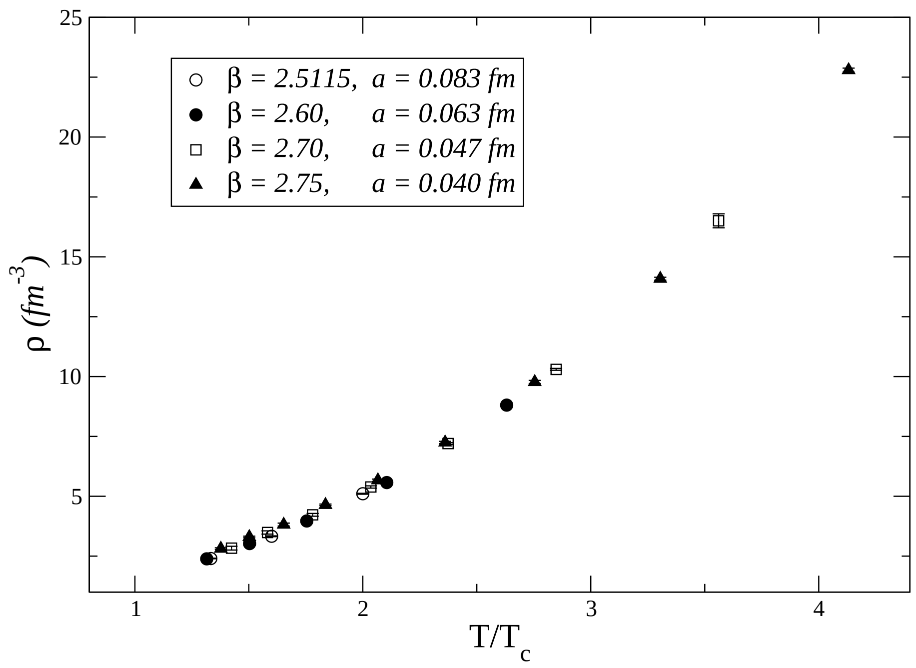
A systematic numerical study of the properties of such wrapped monopole trajectories has been performed recently in $SU(2)$ pure gauge theory in the MAG abelian projection. A. D'Alessandro, M. D'E., Nucl. Phys. B 799 241 (2008)

Monopole density

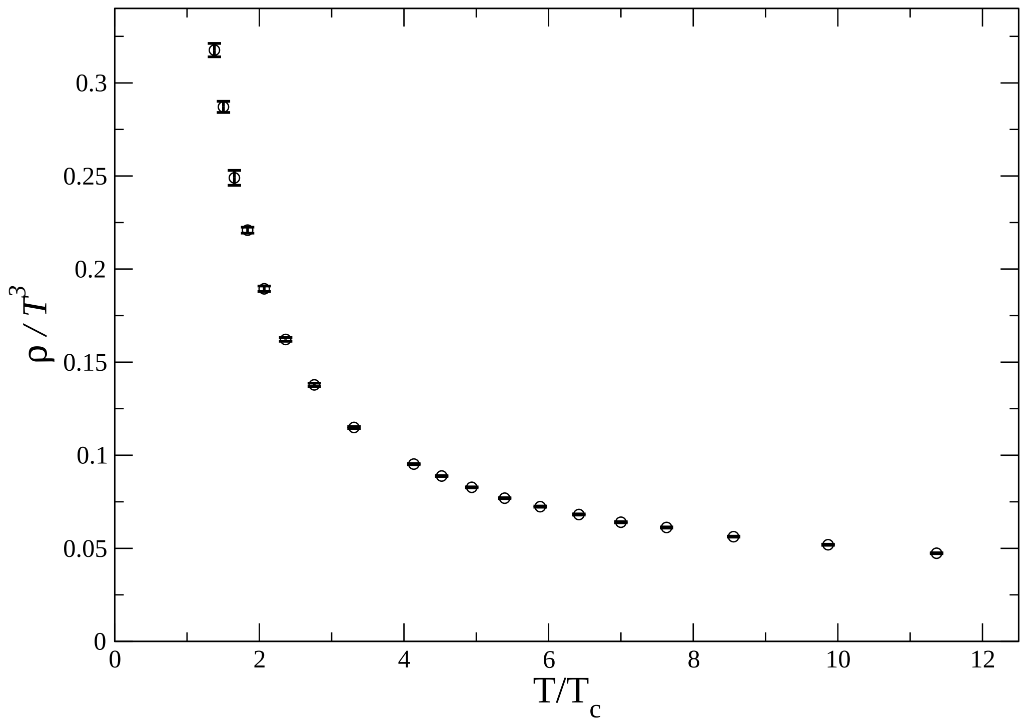
The thermal monopole density is then defined as

$$\rho = \frac{\langle \sum_{\vec{x}} |N_{wrap}(m_0(\vec{x},t))|\rangle}{V_s}$$

where $N_{wrap}(m_0(\vec{x},t))$ **is the temporal winding number of the monopole current** m_0 **at site** $(\vec{x},t)$**, while** $V_s = (L_s a)^3$ **is the spatial volume.**



density of wrapped trajectories scales well to the continuum limit



Density behaviour: not like that of free massless particles ($\rho \sim T^3$).

$$\frac{\rho}{T^3} = \frac{A}{(\log(T/\Lambda_{eff}))^\alpha}$$

Best fit for $T > 2\,T_c$: $A = 0.48(4)$, $T_c/\Lambda_{eff} = 2.48(3)$ and $\alpha = 1.89(6)$

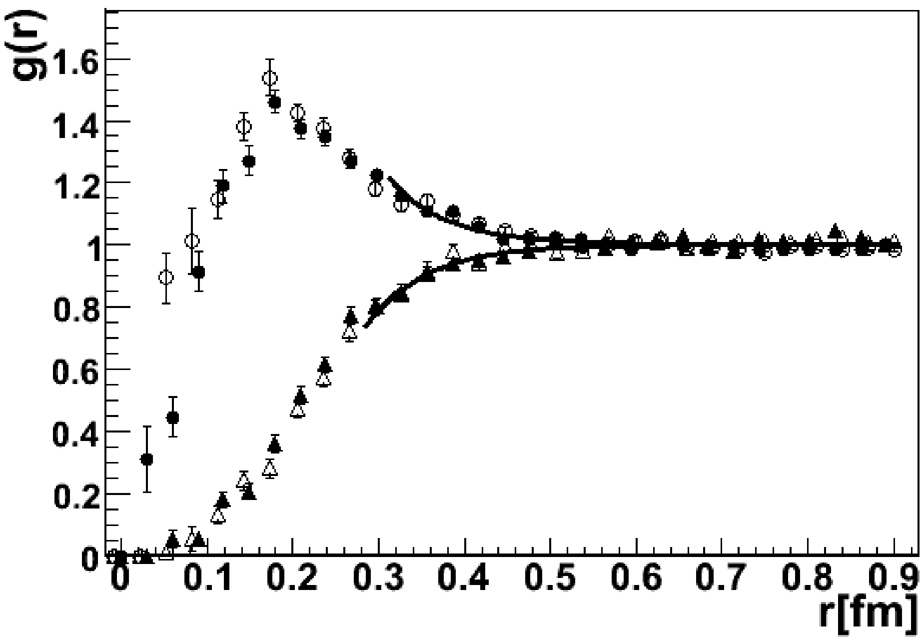
$\alpha = 3$ **(expected exponent in dimensional reduction) works fine for $T > 5\,T_c$.**

Monopole Interactions

Are wrapped trajectories (thermal monopole) randomly distributed in space? Or do they interact?

$\Longrightarrow$ **fix a reference monopole, count monopoles (antimonopoles) at distance** $\in [r, r + dr]$ **and normalize by the same number expected from random distribution: that gives the correlation function** $g(r)$

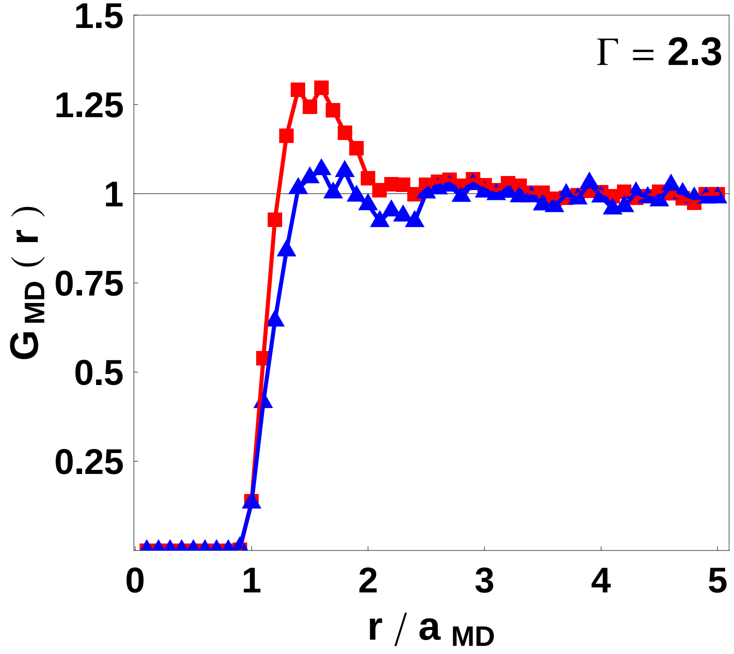
- $g(r) = 1 \;\Longrightarrow$ **no interaction**
- $g(r) \neq 1 \;\Longrightarrow$ **non-trivial interaction**

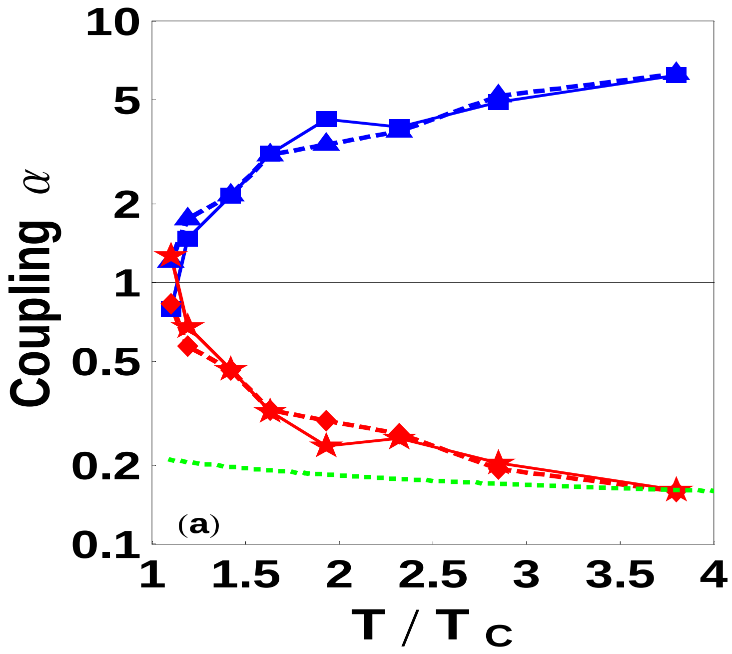


data for $T \simeq 2.85\, T_c$, $\beta = 2.7$ and 2.86.

- **nice scaling**
- **monopole-monopole repulsion**
- **monopole-antimonopole attraction + hard core repulsion**
- **single peak in $g(r)$: typical of liquid/gas behaviour**
- **At large distances $g(r) \simeq e^{-V(r)/T}$, where $V(r) = \alpha_M e^{-r/\lambda}/r$ is a screened Coulomb potential, $\lambda \sim$ 0.1 fm.**

Further analysis by E. Shuryak and J. Liao from Phys. Rev. Lett. 101, 162302 (2008)





- **molecular dynamics results for $g(r)$ look similar to ours**
- **magnetic coupling α_M growing at high temperatures: duality of electric-magnetic couplings at work.**
- $\Gamma = \alpha_M(4\pi\rho/3)^{1/3}/T$ **(which gives an estimate of interaction/kinetic energy ratio) stays above 1 down to T_c $\Longrightarrow$ liquid-like behaviour till close to T_c.**

Are thermal monopoles physical objects?

Unfortunately the density of thermal monopoles strongly depends on the chosen Abelian projection.

A. D'Alessandro, M. D'E., Nucl. Phys. B 799 241 (2008)

Studies are in progress regarding the correlation of MAG thermal monopoles with gauge-invariant quantities like the non-abelian action density

M. Chernodub, A. D'Alessandro, M.D'E and V. Zakharov, in progress

The answer is positive: the non-abelian action density is tipically peaked around the location of MAG thermal monopole trajectories.

Moreover, the magnitude of the electric and of the magnetic action density peaks are comparable

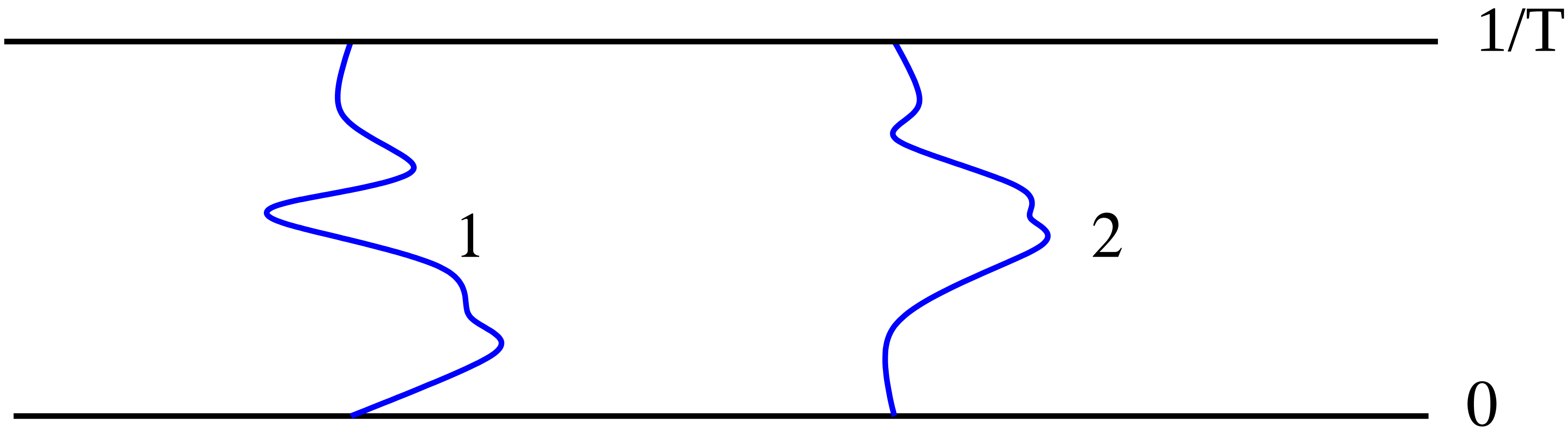
Are thermal monopoles similar to dyons?

Monopole condensation

If thermal monopoles above T_c are really the objects condensing below T_c, can we follow their way back to condensation as we approach T_c from above?

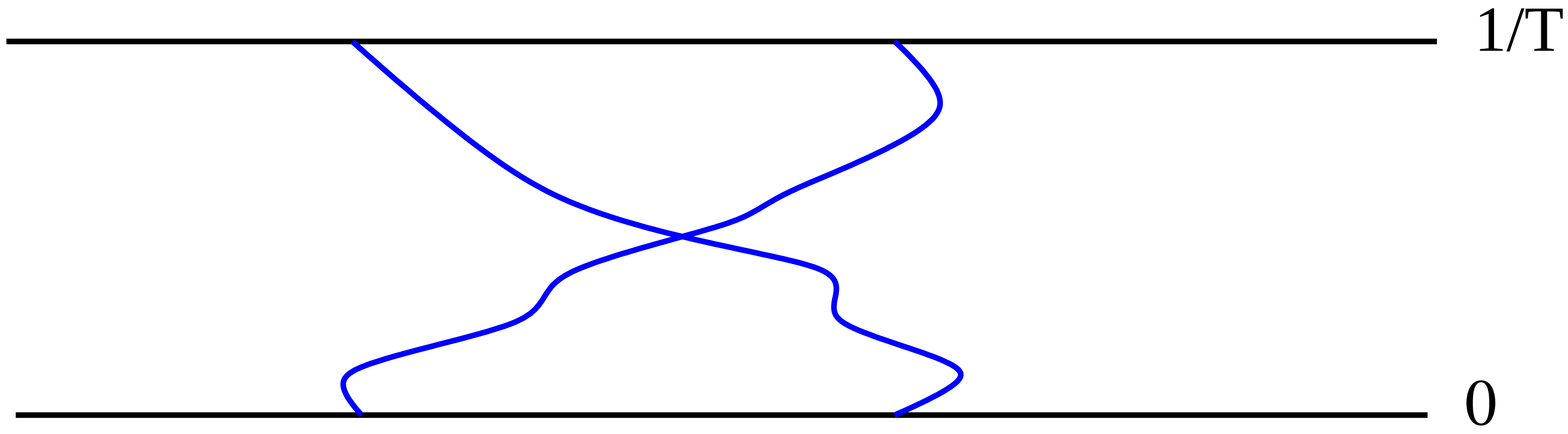
Such an approach complements standard studies about the validity of the dual superconductor model for color confinement which look at the spontaneous breaking of a magnetic symmetry below T_c (Pisa, Bari, Moscow groups).

Condensation is a phenomenon related to the identity of quantum particles. Quantum statistics properties are encoded in monopole trajectories wrapping two or more times in the Euclidean time direction.



What distinguishes the path integral representation of the thermal partition function for two distinguishable particles from that for two identical particles?

For non-identical particles we have to sum over all pairs of periodic (in euclidean time) disconnected paths ...



... instead for identical particles we have to include doubly wrapping paths corresponding to an exchange of the two particles

Such paths give an important contribution in the regime in which quantum statistic effects are important

3 – Bose-Condensation of free non-relativistic bosons

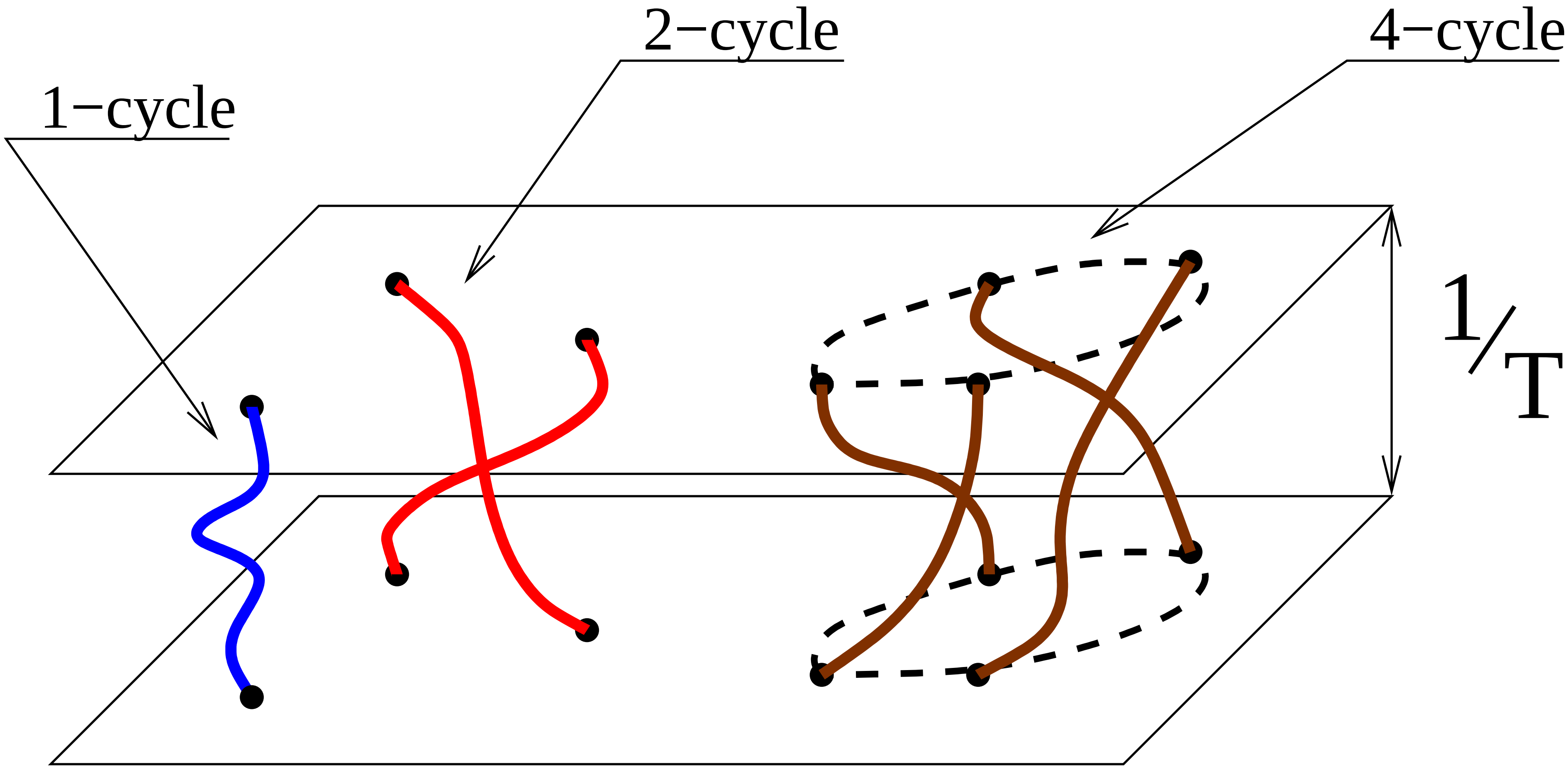
Consider the partition function for N identical particles, $Z = \mathrm{Tr}(e^{-H/T})$

$$Z = \frac{1}{N!} \sum_P \int d^3x_1 \ldots \int d^3x_N \langle x_{P_1} \ldots x_{P_N} | e^{-\beta H} | x_1 \ldots x_N \rangle$$

The sum is over permutations. For non-relativistic free bosons $H = \sum_i p_i^2/2m$

Each permutation can be conveniently decomposed in cycles, i.e. grouping together subsets of particles which undergo cyclic permutations.

Such subsets define loops in configuration space



A contribution to the path integral for 7 identical particles, corresponding to a permutation composed of a 1-cycle + a 2-cycle + a 4-cycle

A k-cycle corresponds to paths wrapping k times in time direction

The contribution from each permutation can be easily factorized into k-cycle contributions:

$$Z = \frac{1}{N!} \sum_P \prod_k z_k^{n_k}$$

and each k-cycle contribution is easily rewritten in terms of the 1-particle partition function:

$$\begin{aligned} z_k(T) &= \int \prod_j d^3 y_j \ldots \langle y_1 | e^{-p_k^2/(2mT)} | y_k \rangle \ldots \langle y_3 | e^{-p_2^2/(2mT)} | y_2 \rangle \langle y_2 | e^{-p_1^2/(2mT)} | y_1 \rangle \\ &= \int d^3 y_1 \langle y_1 | e^{-kp^2/(2mT)} | y_1 \rangle = z_1(T/k) \end{aligned}$$

$$z_1(T) = \frac{V}{\lambda^3} \qquad \lambda \equiv \sqrt{2\pi/(mT)} \qquad z_k(T) = z_1(T/k) = \frac{V}{\lambda^3 k^{3/2}}$$

After further elaborations:

- **Combinatorial factors;**
- **Relaxation on the constraint on N: Canonical $\longrightarrow$ Grand Canonical**

we get to the final expression for the density of k-cycles (i.e. density of paths wrapping k times) and for the total particle density

$$\rho_k \equiv \frac{\langle n_k \rangle}{V} = \frac{e^{-\hat{\mu}k}}{\lambda^3\, k^{5/2}} \qquad \rho = \sum_{k=1}^{\infty} k\rho_k = \frac{1}{\lambda^3} \sum_{k=1}^{\infty} \frac{e^{-\hat{\mu}k}}{k^{3/2}}$$

$\hat{\mu} \equiv -\mu/T$ **where μ is the usual chemical potential for bosons.**

This description is completely equivalent to the usual one made in momentum space **(** $\sum_k \frac{e^{-\hat{\mu}k}}{k^{3/2}} = \frac{2}{\sqrt{\pi}} \int_0^\infty dx \frac{\sqrt{x}}{e^{\hat{\mu}} e^x - 1}$ **)**

The total particle density reaches a maximum for $\hat{\mu} \longrightarrow 0$. Larger densities are accounted for by particles belonging to a macroscopic cycle (i.e. condensing in the fundamental state). Therefore condensation is signalled by $\hat{\mu} \longrightarrow 0$.

As $\hat{\mu} \longrightarrow 0$ the relative weight of paths wrapping k times is no more exponentially suppressed with k.

Such path integral approach can be applied to other systems

(see original papers by Feynman to treat ^{4}He in 1953, see also Elser's studies in 1984 and a recent paper by E. Shuryak and M. Cristoforetti, arXiv:0906.2019)

Suppose we make a path integral Monte-Carlo simulation of a free boson system for various temperatures. We can measure the densities ρ_k of trajectories wrapping k times and fit them to

$$\rho_k = \frac{e^{-\hat{\mu} k}}{\lambda^3 \, k^{5/2}}$$

to obtain $\hat{\mu}$ as a function of T, and then find the critical temperature T_{BEC} at which $\hat{\mu} \to 0$.

Can we do a similar thing for the ensemble of thermal monopoles extracted from Monte-Carlo gauge configurations?

The density of monopole trajectories wrapping k times is an observable which can be easily determined.

Our numerical simulations

- SU(2) pure gauge theory, standard Wilson action
- two different sets of lattices:

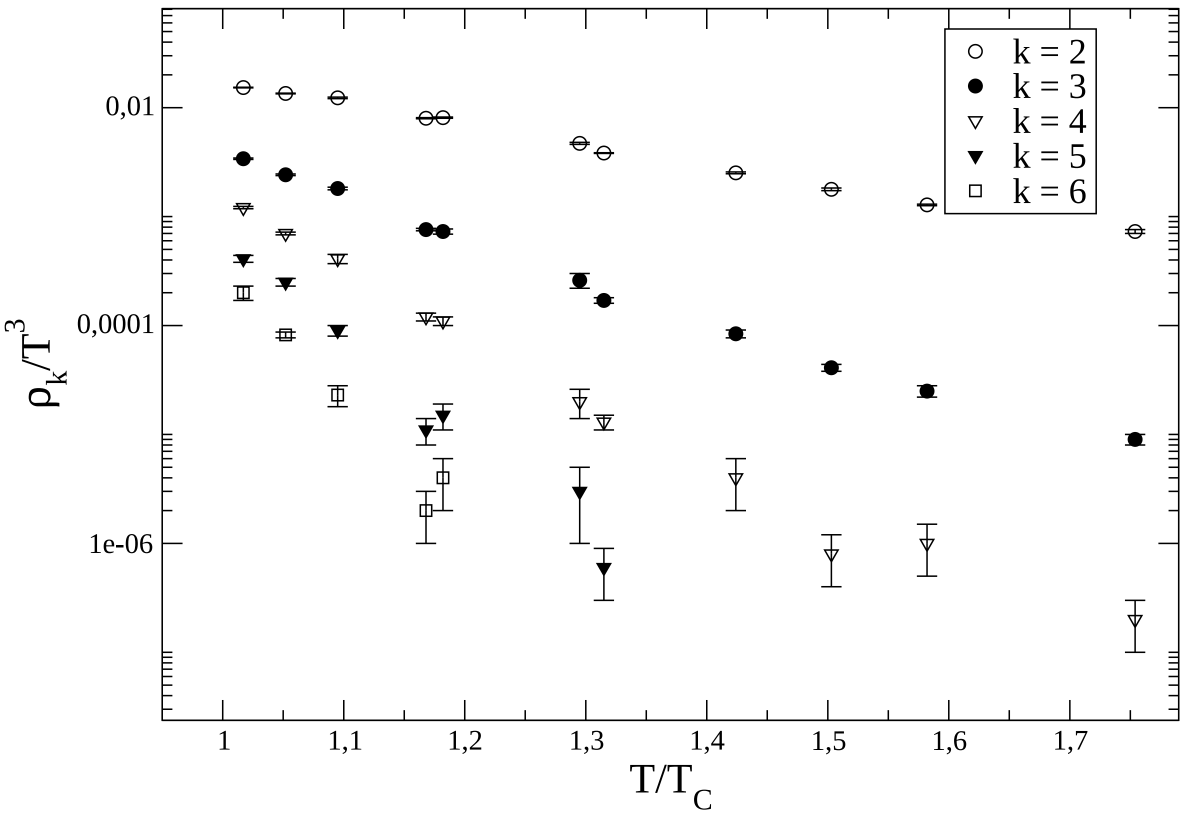
  $64^3 \times N_t$ with $N_t = 4, 14$ at $\beta = 2.7$, corresponding to a lattice spacing $a \simeq 0.47$ fm,

  $48^3 \times N_t$ with $N_t = 4, 10$ at $\beta = 2.6$, corresponding to a lattice spacing $a \simeq 0.63$ fm
- monopole trajectories extracted from samples of $O(10^3)$ independent gauge configurations after Maximal Abelian Projection

Table of measured densities

[a] $\longrightarrow a \simeq 0.047$ fm [b] $\longrightarrow a \simeq 0.063$ fm

T/T_c	ρ_1/T^3	ρ_2/T^3	ρ_3/T^3	ρ_4/T^3	ρ_5/T^3	ρ_6/T^3
1.017^a	**0.308(2)**	$1.53(1)\ 10^{-2}$	$3.40(5)\ 10^{-3}$	$1.21(3)\ 10^{-3}$	$4.1(3)\ 10^{-4}$	$2.0(3)\ 10^{-4}$
1.052^b	**0.315(5)**	$1.35(1)\ 10^{-2}$	$2.42(4)\ 10^{-3}$	$7.0(2)\ 10^{-4}$	$2.5(2)\ 10^{-4}$	$8.2(5)\ 10^{-5}$
1.095^a	**0.3395(15)**	$1.23(2)\ 10^{-2}$	$1.81(5)\ 10^{-3}$	$4.1(4)\ 10^{-4}$	$0.9(1)\ 10^{-4}$	$2.3(5)\ 10^{-5}$
1.168^b	**0.325(3)**	$8.0(1)\ 10^{-3}$	$7.6(2)\ 10^{-4}$	$1.2(1)\ 10^{-4}$	$1.1(3)\ 10^{-5}$	$0.2(1)\ 10^{-5}$
1.187^a	**0.337(2)**	$8.1(1)\ 10^{-3}$	$7.3(4)\ 10^{-4}$	$1.1(1)\ 10^{-4}$	$1.5(4)\ 10^{-5}$	$0.4(2)\ 10^{-5}$
1.295^a	**0.316(1)**	$4.72(10)\ 10^{-3}$	$2.6(3)\ 10^{-4}$	$2.0(6)\ 10^{-5}$	$0.3(2)\ 10^{-5}$	
1.315^b	**0.297(2)**	$3.83(3)\ 10^{-3}$	$1.7(1)\ 10^{-4}$	$1.3(2)\ 10^{-5}$	$0.6(3)\ 10^{-6}$	
1.424^a	**0.286(1)**	$2.52(5)\ 10^{-3}$	$8.4(7)\ 10^{-5}$	$0.4(2)\ 10^{-5}$		
1.503^b	**0.271(1)**	$1.78(5)\ 10^{-3}$	$4.1(3)\ 10^{-5}$	$0.8(4)\ 10^{-6}$		
1.582^a	**0.252(1)**	$1.28(2)\ 10^{-3}$	$2.5(3)\ 10^{-5}$	$1.0(5)\ 10^{-6}$		
1.754^b	**0.2134(10)**	$7.3(3)\ 10^{-4}$	$9(1)\ 10^{-6}$	$0.2(1)\ 10^{-6}$		
1.780^a	**0.2190(2)**	$6.26(7)\ 10^{-4}$	$8.3(8)\ 10^{-6}$	$0.2(1)\ 10^{-6}$		
2.034^a	**0.1870(4)**	$3.04(10)\ 10^{-4}$	$1.0(4)\ 10^{-6}$			



The relative weights of trajectories wrapping different number of times clearly approach each other as $T \to T_c$ from above. This is qualitatively consistent with monopole condensation at T_c

Can we be more quantitative?

The thermal monopole ensemble is surely quite far from a free particle ensemble.

Monopole-monopole repulsion is expected to disfavour multiple wrapping trajectories with respect to the free case

Monopole-antimonopole attractive interactions are also expected to play a role.

Taking interactions properly into account is a very difficult task. On general grounds one may expect some finite free energy cost needed to add one particle to a k-cycle plus some interaction dependent contribution:

$$\rho_k = e^{-\hat{\mu} k} f(k)$$

where $f(k)$ is some unknown function decreasing less than exponentially with k.

Our attitude in the following is to give some possible ansatz for $f(k)$ and then try it to fit our data.

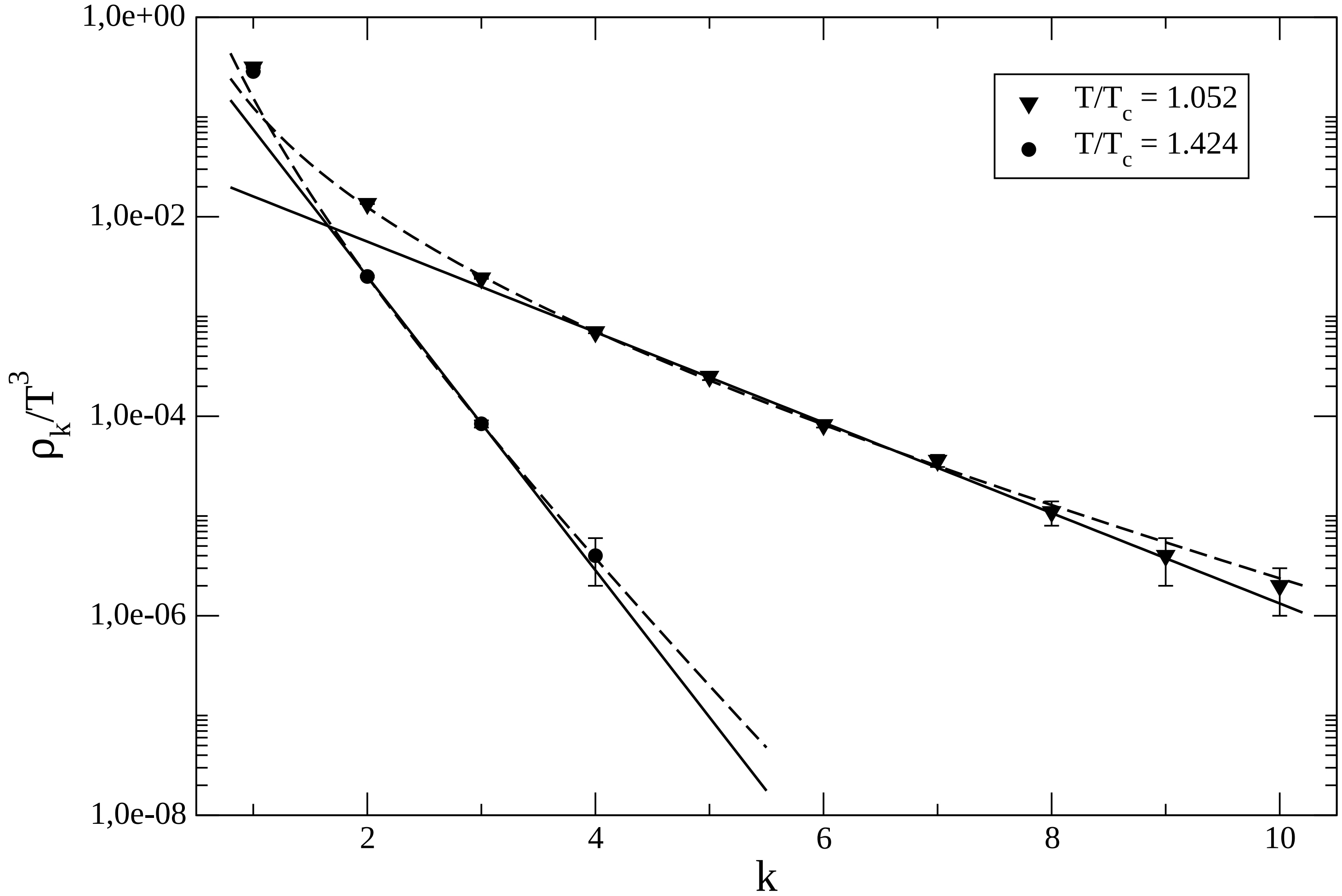
The simplest possibility is that asymptotically (i.e. for large k)

$$f(k) = (\lambda^3 k^{5/2})^{-1}$$

as in the free boson case, with some effective dynamical mass accounting for interactions.

As different possibilities we shall consider more general power law behaviours

$$f(k) \propto 1/k^\alpha$$



Fit of the densities ρ_k according to $e^{-\hat{\mu}k}/k^{5/2}$ (dashed line) and according to $e^{-\hat{\mu}k}$ (solid line) for two values of the temperature.

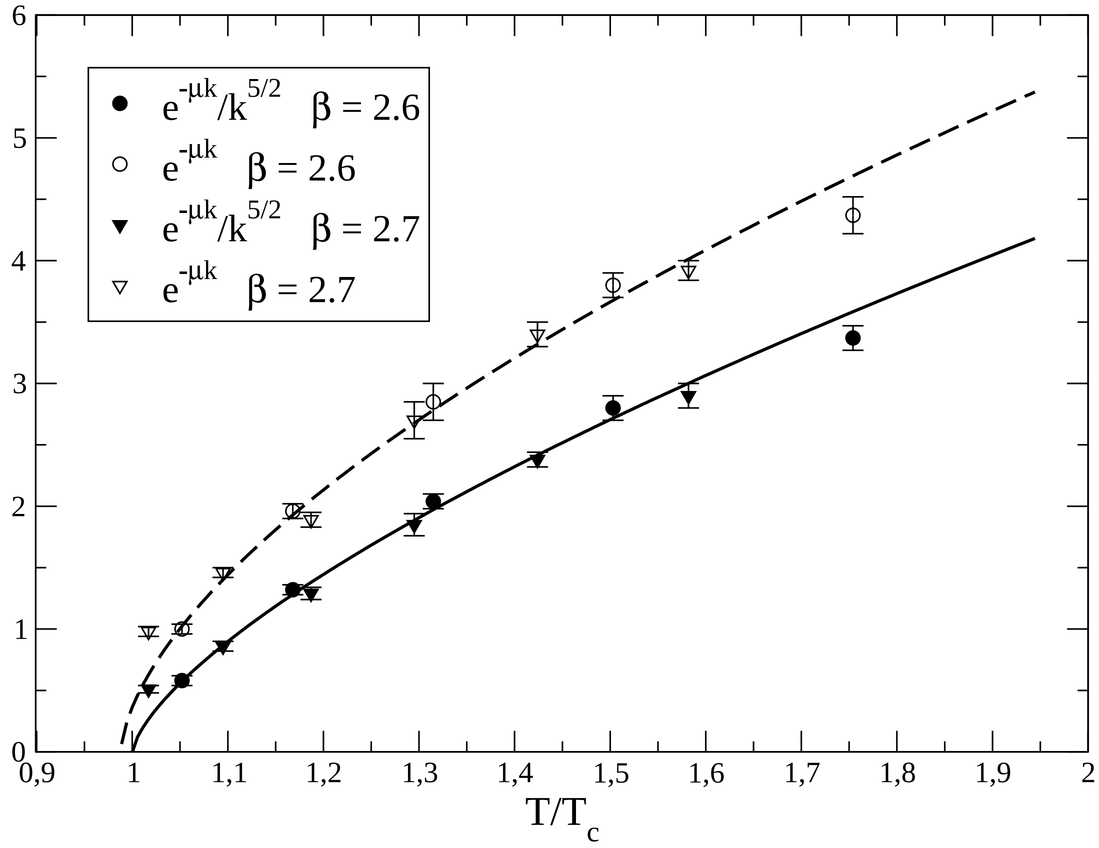
T/T_c	$\mu(\alpha=3)$	$\mu(\alpha=2.5)$	$\mu(\alpha=2)$	$\mu(\alpha=0)$
1.017^a	**0.43(4)**	**0.51(3)**	**0.61(4)**	**0.98(4)**
1.052^b	**0.49(5)**	**0.58(4)**	**0.68(3)**	**1.00(4)**
1.095^a	**0.75(5)**	**0.86(4)**	**1.02(4)**	**1.46(4)**
1.168^b	**1.16(4)**	**1.32(4)**	**1.44(5)**	**1.96(6)**
1.187^a	**1.11(6)**	**1.29(5)**	**1.36(6)**	**1.89(6)**
1.295^a	**1.67(6)**	**1.85(9)**	**2.03(9)**	**2.70(15)**
1.315^b	**1.89(4)**	**2.04(6)**	**2.20(8)**	**2.85(15)**
1.424^a	**2.18(6)**	**2.38(6)**	**2.58(6)**	**3.4(1)**
1.503^b	**2.64(10)**	**2.8(1)**	**3.0(1)**	**3.8(1)**
1.582^a	**2.69(10)**	**2.9(1)**	**3.1(1)**	**3.92(8)**
1.754^b	**3.16(8)**	**3.37(10)**	**3.57(10)**	**4.37(15)**

Depending on the ansatz, different chemical potentials are found

however if one tries to fit data for the chemical potentials according to

$$\hat{\mu} = A\,(T - T_{\rm BEC})^{\nu'}$$

results are quite independent of the ansatz



α	T_{BEC}/T_c	ν'	$\chi^2/\mathrm{d.o.f.}$
3	**1.005(13)**	**0.71(5)**	**2.24**
2.5	**1.000(12)**	**0.68(5)**	**1.23**
2	**0.989(13)**	**0.68(5)**	**1.72**
0	**0.988(15)**	**0.61(5)**	**2.34**

Our result: thermal monopoles condense at a temperature which coincides with T_c within errors.

Can we also understand the value of the critical index ν' ?

Assume the correlation length ξ grows proportionally to the typical spatial extension of k-cycles.

For a distribution of k-cycles $\rho_k \sim e^{-\hat{\mu}k}/k^\alpha$, we have $\langle k \rangle \propto 1/\hat{\mu}$

The typical spatial extension of a k-cycle may grow like k^ω where $\omega \sim 1/2$ for a typical random walk behaviour (no interactions) and $\omega \sim 1$ if permutating particles are typically ordered along linear structures by repulsive interactions.

Therefore $\xi \sim \hat{\mu}^{-\omega}$. On the other hand $\xi \sim (T - T_c)^{-\nu}$ ($\nu \sim 0.63$ for $SU(2)$, 3d Ising universality class), hence we expect

$$\hat{\mu} \sim (T - T_c)^{\nu/\omega} \qquad \nu' = \nu/\omega$$

Since $\nu' \sim \nu$, we conclude that $\omega \sim 1$, i.e.:

permutating particles are typically ordered along linear structures by repulsive interactions. A typical cluster of monopole currents wrapping several times in the time direction lies on a time oriented surface.

4 – Conclusions

Topological defects identified as thermal abelian monopoles are an important component of Yang-Mills theories above T_c and it is possible to study their condensation as $T \to T_c$ from above, thus directly relating them to the low temperature magnetic condensate.

More studies are in progress to study the mass of these particle-like objects and to extend results to SU(3)

Dependence on the choice of abelian projection is a problem

But scaling properties and the correlation of the unprojected non-abelian action with monopoles in the Maximal Abelian Projection suggests that they may indeed be related to physical objects (dyons?), whose nature should be better understood from a theoretical point of view.