Aspects of the Gribov-Zwanziger framework

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Overview

Overview

Gribov-Zwanziger

Choosing a gauge

- Classical $SU(N)$ Yang-Mills action has an “enormous” invariance

$$S_{YM} = \frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu}F^a_{\mu\nu}$$

w.r.t.

$$A^a_\mu \to A^a_\mu + D^{ab}_\mu \omega^b$$

- To quantize, one must choose a particular (unique) representant A^a_μ by fixing the gauge
- Popular choice: $\boxed{\text{Landau gauge, } \partial_\mu A^a_\mu = 0}$.

Gribov-Zwanziger

Choosing a gauge

- After some nice machinery, one ends up with the Faddeev-Popov gauge fixed action

$$S_{YM+FP} = \frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu}F^a_{\mu\nu} + \int \mathrm{d}^4x \,\left(b^a\partial_\mu A^a_\mu + \overline{c}^a\partial_\mu D^{ab}_\mu c^b\right)$$

- Gauge symmetry is lost, but it is "replaced" by the (rather important) nilpotent BRST invariance

$$sA^a_\mu = -(D_\mu c)^a \;, \quad sc^a = \frac{1}{2}gf^{abc}c^bc^c \;, \quad s\overline{c}^a = b^a \;, \quad sb^a = 0 \;, \quad s^2 = 0$$

Gribov-Zwanziger

Landau gauge and copies: the (partial) problem

- Take $\partial A = 0$ and $A' = A + D\omega$ an infinitesimally equivalent gauge field Is is possible to have

$$\partial A' = 0 \text{ or } \partial D\omega = 0\,? \qquad (*)$$

- To have (infinitesimally related) gauge copies, we need zero modes of the Faddeev-Popov operator

$$M^{ab} = -\partial_\mu D_\mu^{ab} = -\partial_\mu \left(\partial_\mu \delta^{ab} + g f^{acb} A_\mu^c\right)$$

- **Result Gribov: there are such (normalizable) zero modes!**
- **Problem for the Faddeev-Popov quantization** ($\det M = 0$, sign switching)

Gribov-Zwanziger

Landau gauge and copies: the (partial) solution

- **Only integrate** in A-space over **Gribov region** Ω

$$\Omega \equiv \{A_\mu^a,\ \partial_\mu A_\mu^a = 0,\ M^{ab} > 0\}\,.$$

- No more gauge copies of the type $(*)$.
- Sounds simple in theory, but how to do in practice?

Gribov-Zwanziger

The Gribov restriction: semiclassical level

- Partition function

$$Z = \int [D\Phi] e^{-S_{YM+FP}} \rightarrow Z = \int [D\Phi]\theta(M) e^{-S_{YM+FP}}$$

- how to characterize $\theta(M)$? We look at its inverse, written as

$$M^{-1} = (-\partial D)^{-1} = k^2(1 - \sigma(k^2, A))$$

- Gribov no pole condition: $\boxed{\sigma(0, A) < 1}$
- Partition function

$$Z = \int [D\Phi] e^{-S_{YM+FP}} \rightarrow Z = \int [D\Phi]\theta(1 - \sigma(0, A)) e^{-S_{YM+FP}}$$

Gribov-Zwanziger

The Gribov restriction: semiclassical level

- Semiclassical analysis in thermodynamic limit: $\theta \to \delta$
- Partition function (at the quadratic level)

$$Z = \int [D\Phi]\delta(1-\sigma(0,A))e^{-S_{YM+FP}} \to \int [D\Phi]e^{-S_{YM+FP}+\gamma^4 \int d^4x A \frac{1}{\partial^2} A}$$

- γ = **mass parameter**, fixed by gap equation,

$$\frac{3}{4}g^2 N \int \frac{d^4q}{(2\pi)^4}\frac{1}{q^4+\gamma^4} = 1 \qquad (\diamond)$$

- **New partition function** (or action) $\to$ **perfect tool to study theory with**

Gribov-Zwanziger

The Gribov restriction: propagators

- Gluon

$$D(q^2) = \frac{q^2}{q^4 + \gamma^4} \qquad \boxed{D(0) = 0}$$

- Ghost

$$G(q^2) = \frac{1}{q^2} \frac{1}{1 - \sigma(q^2)} \qquad \boxed{q^2 G(q^2) \to \infty \text{ if } q^2 \to 0}$$

  since $\sigma(0) = 1$ due to gap equation $(\diamond)$ $\Rightarrow$**IR ghost enhancement**.

- No surprise since the **gap equation** exactly **corresponds** to the **no-pole condition**

Gribov-Zwanziger

Gribov→Zwanziger: formal version

- Restricts the integration to the Gribov region to all orders (work of Zwanziger)
- Gribov region has several nice properties
- Implements the no-pole condition to all orders
- The Gribov-Zwanziger action is given by

$$S_h = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4x\, h(x)$$

with the horizon function

$$h(x) = g^2 f^{abc} A^b_\mu \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A^e_\mu$$

- horizon condition (= gap equation)

$$\langle h(x) \rangle = d(N^2-1)$$

Gribov-Zwanziger

Gribov→Zwanziger: formal version

- Is this a mathematically 100% rigorous derivation?
- As so many things in quantum field theory, **NO**
- But best thing on the analytical market concerning Gribov copies!

Gribov-Zwanziger

Gribov→Zwanziger: useful version

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM+GF} + S_h$$

with

$$\begin{aligned} S_h = & \int \mathrm{d}^4x \Big(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) - g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left(D_\nu c \right)^b \varphi_\mu^{mc} \\ & - \gamma^2 g \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \Big) \end{aligned}$$

- horizon condition (= gap equation)

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle g f^{abc} A_\mu^a (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2d(N^2-1)\gamma^2}_{d=2 \text{ condensate!!}}$$

Gribov-Zwanziger

Gribov→Zwanziger: useful version

- At lowest order, everything reduces to Gribov
- At $\gamma = 0$, everything reduces to Faddeev-Popov

Gribov-Zwanziger

Local GZ action

- Locality allows for the usual tools to study a quantum field theory
- E.g.: GZ action is **renormalizable** to all orders (Zwanziger, others)
- Local Ward identities
- We can also study/use the symmetries of the GZ theory

From now on, we shall only use the local formulation!

Overview

BRST

Back to the Gribov (quadratic) approximation

- Modified partition function
- Act with (free) BRST symmetry, $sA_\mu^a = -\partial_\mu c^a$

$$s(\text{action}) = s\left(\gamma^4 \int d^4x A_\mu^a \frac{1}{\partial^2} A_\mu^a\right) \propto \gamma^4 \int d^4x A_\mu \frac{1}{\partial^2}\partial_\mu c^a \neq 0$$

- $\boxed{\text{BRST is broken!?}}$

BRST

(soft) BRST breaking in the GZ theory

- Consider the GZ action S_{GZ}
- naturally extended BRST transformation

$$
\begin{aligned}
&sA^a_\mu = -(D_\mu c)^a \,, && sc^a = \frac{1}{2} g f^{abc} c^b c^c \,, && s\bar{c}^a = b^a \,, && sb^a = 0 \,, \\
&s\varphi^{ab}_\mu = \omega^{ab}_\mu \,, && s\omega^{ab}_\mu = 0 \,, && s\overline{\omega}^{ab}_\mu = \overline{\varphi}^{ab}_\mu \,, && s\overline{\varphi}^{ab}_\mu = 0 \,,
\end{aligned}
$$

- Then

$$
sS_{GZ} \;=\; g\gamma^2 \int \mathrm{d}^4x f^{abc} \left(A^a_\mu \omega^{bc}_\mu - \left(D^{am}_\mu c^m\right)\left(\overline{\varphi}^{bc}_\mu + \varphi^{bc}_\mu\right)\right) \;=\; \Delta_\gamma
$$

- BRST symmetry is softly broken
- softly, because breaking $\propto$ mass^2.

BRST

(soft) BRST breaking in the GZ theory

- the breaking is soft $\Rightarrow$ under control at quantum level
- $\Rightarrow$ softly broken Slavnov-Taylor identity

$$\int \mathrm{d}^4x \left(\frac{\delta\Gamma}{\delta K_\mu^a}\frac{\delta\Gamma}{\delta A_\mu^a} + \frac{\delta\Gamma}{\delta L^a}\frac{\delta\Gamma}{\delta c^a} + b^a\frac{\delta\Gamma}{\delta\overline{c}^a} + \overline{\varphi}_i^a\frac{\delta\Gamma}{\delta\overline{\omega}_i^a} + \omega_i^a\frac{\delta\Gamma}{\delta\varphi_i^a} \right) \quad = \quad -\left[\Delta_\gamma \cdot \Gamma\right]$$

  whereby $[\Delta_\gamma \cdot \Gamma]$ corresponds to insertion of the breaking operator Δ_γ.
- Nice application: **gluon self energy is not transverse** (explicitly checked by Gracey)

BRST

(soft) BRST breaking in the GZ theory: a problem?

- **Potential problem 1: renormalizability**
  this is no problem! softly broken BRST is sufficient to ensure this!!
- **Potential problem 2: physical states/unitarity/positiveness**
  not so easy to answer. Soft BRST breaking can play a role in definition of (would-be) physical states in the GZ formalism, see talk of Vandersickel
  $\longrightarrow$ this topic is under investigation.

BRST

Talking of physical states...

how does it work in usual (perturbative) gauge theories?

Overview

Kugo-Ojima in a nutshell

Starting points

- $4D$ Faddeev-Popov action in a covariant gauge,

$$S_{YM+FP} = \frac{1}{4}\int \mathrm{d}^4 x F^a_{\mu\nu}F^a_{\mu\nu} + \int \mathrm{d}^4 x \left(b^a\partial_\mu A^a_\mu + \overline{c}^a\partial_\mu D^{ab}_\mu c^b + \alpha b^a b^a\right)$$

- S enjoys nilpotent BRST symmetry s

$$sA^a_\mu = -(D_\mu c)^a \ , \quad sc^a = \frac{1}{2}gf^{abc}c^b c^c \ , \quad s\overline{c}^a = b^a \ , \quad sb^a = 0 \ , \quad s^2 = 0$$

- Based on operator formalism.

Kugo-Ojima in a nutshell

Result 1

- Define $|\psi_p\rangle$ **physical** state $\Leftrightarrow |\psi_p\rangle = \underbrace{|\ldots\rangle}_{Q_{BRST}|\ldots\rangle=0} + Q_{BRST}|\ldots\rangle$
- $|\psi_p\rangle \in cohom(Q_{BRST}) = \frac{\mathrm{Ker}Q_{BRST}}{\mathrm{Im}Q_{BRST}}$
- **unphysical states** $|\psi_u\rangle$ always come in **quartets**, and decouple from the physical spectrum.
- Example: longitudinal and temporal gauge polarization, ghost, antighost.
- 2 transverse gluons remain, with positive norm.
- KO analysis also guarantees **unitarity** (Q_{BRST} commutes with H as symmetry)

Kugo-Ojima in a nutshell

KO analysis

very clear what it means at perturbative level. Quartet mechanism quite general (basic properties of symmetry algebra).

Kugo-Ojima in a nutshell

Result 2

- global color charge reads

$$Q^a = \int d^3x \partial^i F^a_{i0} + \int d^3x \{Q_{BRST}, D_0\overline{c}^a\}$$

- assume that gluons are "massive" ($\rightarrow \int d^3x \partial^i F^a_{i0} = 0$) and that $\int d^3x \{Q_{BRST}, D_0\overline{c}^a\}$ is well defined
- $\Rightarrow$!!confinement!! (no colored physical objects)
- Physical meaning of such confinement is rather unclear.

Kugo-Ojima in a nutshell

Result 2

- KO showed $\int d^3x\{Q_{BRST}, D_0\overline{c}^a\}$ well defined $\Leftrightarrow u(0) = -1$ with

$$\int d^4x e^{iqx} \left\langle D_\mu c^a(x) D_\nu \overline{c}^b(0) \right\rangle = \delta^{ab} \left(g_{\mu\nu} - \frac{q_\nu q_\mu}{q^2} \right) u(q^2) - \frac{q_\mu q_\nu}{q^2}$$

KO confinement criterion $\boxed{u(0) = -1}$

Kugo-Ojima in a nutshell

Result 2

- Interesting relation (in Landau gauge $\alpha = 0$) (Kugo, Zwanziger, Kondo, Binosi et al)

$$G^{ab}(q) = \langle c^a \bar{c}^p \rangle_q = \frac{\delta^{ab}}{q^2} \frac{1}{1 + u(q^2) + q^2 v(q^2)}$$

- $u(0) = -1 \Leftrightarrow$ **ghost propagator** $G(q)$ **infrared enhanced**!

Kugo-Ojima in a nutshell

a few questions

- KO are silent about renormalization?
- $u = -1$ certainly **not realized in perturbation theory**
  $\rightarrow$ need for nonperturbative calculational scheme
- **trivial that BRST extends all the way down to $p^2 = 0$, a rather nontrivial region?**
- KO is based on Faddeev-Popov quantization $\Rightarrow$ what about the **gauge (Gribov) copy problem**?

Overview

GZKO

Gribov-Zwanziger = Kugo-Ojima?

- Gluon vanishes at zero momentum.
- Ghost is IR enhanced (proven, checked by Zwanziger, Gracey)
- **Equivalence KO criterion and no pole condition**

$$\sigma(0) = -u(0)(= 1)$$

$\Rightarrow$ Everything for the KO confinement scenario seems to be there!

GZKO

Gribov-Zwanziger = Kugo-Ojima?

- Can be formalized (using Ward identities)

$$\left\langle gf^{abc}A^a_\mu(\varphi+\overline{\varphi})^{bc}_\mu\right\rangle = 2\gamma^2\int d^4x\,\langle D_\mu c^a(x)D_\mu\overline{c}^a(0)\rangle = -2\gamma^2(N^2-1)((d-1)u(0)-1)$$

- $\rightarrow$ thus comes from (renormalizable) connection with correlator

$$\langle D_\mu c^a(x)D_\mu\overline{c}^a(y)\rangle$$

- but there is **no renormalizable** connection with

$$\left\langle(gf^{abc}A^b_\mu c^c(x))(gf^{apq}A^p_\mu\overline{c}^q)(y)\right\rangle$$

(cfr Kondo results)

GZKO

Gribov-Zwanziger = Kugo-Ojima?

- Equivalence horizon condition $\leftrightarrow$ KO criterion

$$\left\langle g f^{abc} A_\mu^a (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2\gamma^2 d(N^2 - 1)$$

$$\Updownarrow$$

$$u(0) = -1$$

GZKO

Gribov-Zwanziger = Kugo-Ojima?

- GZ + KO = GZKO framework
- enhanced ghost, vanishing gluon in infrared
- confinement

GZKO

Gribov-Zwanziger $\neq$ Kugo-Ojima

- **GZ $\leftrightarrow$ BRST breaking**
  vs
  KO $\leftrightarrow$ BRST indispensable
- GZ action $\neq$ FP action
  $\rightarrow$ also different version of global color charge, ...
- **Impossible** to just copy & paste KO into GZ

GZKO

Gribov-Zwanziger $\neq$ Kugo-Ojima

Rather unclear what KO criterion means in GZ formalism, in absence of BRST, a keystone of the whole KO construction.
Certainly no “proof” of confinement, nonperturbative unitarity, or whatsoever.

GZKO

Reconciling GZ with KO?

- find other GZ action, consistent with usual BRST $\rightarrow$ highly doubtful.
- find another BRST (cfr Kondo, Sorella) $\rightarrow$ BRST is necessarily nonlocal, and since it differs from original BRST, KO needs to be reworked anyhow (if even possible???)

Overview

KO as boundary

Imposing $u(0) = -1$

- Some Schwinger-Dyson approaches impose KO as boundary condition to solve equations
- Question: **does imposing $u(0) = -1$ harms symmetries of the starting FP action (or not)?**

KO as boundary

Imposing $u(0) = -1$

- We start from the FP action, written in the following form

$$S'_{YM+FP} = S_{YM+FP} + \int d^4x \Big(\overline{\varphi}^{ac}_\mu \partial_\nu \left(\partial_\nu \varphi^{ac}_\mu + g f^{abm} A^b_\nu \varphi^{mc}_\mu \right) - \overline{\omega}^{ac}_\mu \partial_\nu \left(\partial_\nu \omega^{ac}_\mu + g f^{abm} A^b_\nu \omega^{mc}_\mu \right) \Big)$$

- We recall that

$$u(0) = -1 \Leftrightarrow \left\langle g f^{abc} A^a_\mu (\varphi + \overline{\varphi}^{bc}_\mu) \right\rangle = 2d(N^2-1)\gamma^2$$

- We borrow something from statistical mechanics: we **enforce the constraint with a multiplier**. More precisely

$$S'_{YM+FP} \rightarrow S_{KO} \equiv S'_{YM+FP} - \int d^4x \Big(\gamma^2 g \Big(f^{abc} A^a_\mu (\varphi^{bc}_\mu + \overline{\varphi}^{bc}_\mu) + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \Big) \Big)$$

with self-consistency condition $\partial\Gamma / \partial\gamma^2 = 0$.

KO as boundary

Do boundary conditions come as a free lunch?

- Clearly,

$$S_{KO} \equiv S_{GZ}$$

$\Rightarrow S_{KO}$ shares all the properties of the GZ action (renormalizability!, ...).

- **We now have an action at our disposal that automatically implements KO boundary**, in way stable under radiative corrections!

KO boundary condition does not come as a free lunch

- Imposing (in a at quantum level consistent way) KO breaks usual BRST
- In fact, the eventual action is equivalent to the GZ action (although rather different starting philosophy).
- KO boundary $\overset{\text{mystery}}{\leftrightarrow}$ KO confinement (criterion)

Overview

GZ dynamics

Comparison GZ with lattice Landau gauge

	GZ	Lattice
$D(0)$	0	$\rightarrow c_1 > 0$
$G(p^2)$	enhanced	$\rightarrow \frac{c_2}{p^2}$
Pos.violation	yes	yes

  To be complete:
- $D(0) = 0$ is not settled according to e.g. Oliveira/Silva (see talk of Oliveira)
- There are other "views" on lattice data on the physics market (see e.g. Maas, Von Smekal)

Some plots for the ghost propagator

Ghost dressing function

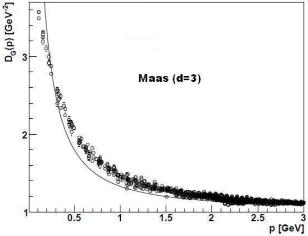


FIG. 16: The lattice results in absolute Landau gauge from a 64^3 lattice at $\beta = 4.24$ compared to results from DSE calculations [20].

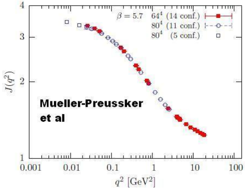


Figure 4: Bare ghost dressing function $J(q^2)$ versus q^2 for $L = 64, 80$ at $\beta = 5.70$. Errors are not shown at the two lowest q^2 (squares).

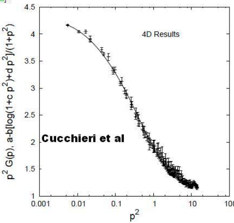


FIG. 2: The ghost dressing function $p^2 G(p^2)$ as a function of p^2 (in GeV) for the 2d case (top, at $\beta = 10$ with volume 320^2), 3d case (center, at $\beta = 3$, with volume 240^3) and the 4d

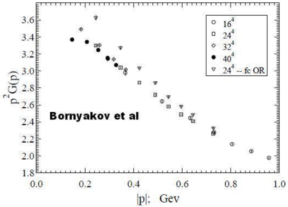
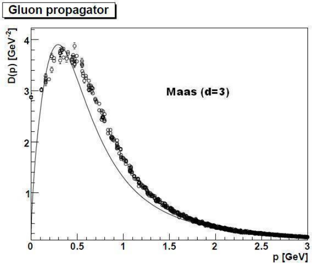


FIG. 5: The momentum dependence of the ghost dressing function $p^2 \cdot G(p)$ on the various lattices. For comparison results obtained with OR algorithm on 24^2 lattices are also shown.

Some plots for the gluon propagator



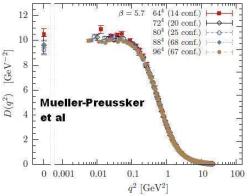
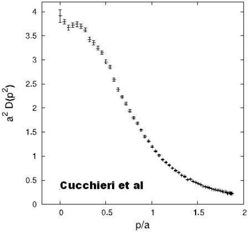


Figure 2: The bare lattice gluon propagator $D(q^2)$ versus q^2 for $\beta = 5.70$ and various lattice sizes. We also show data on $D(0)$ (left).



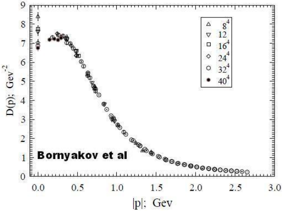


FIG. 2: The momentum dependence of the gluon propagator $D(p)$ on various lattice size. *bc* results are shown throughout.

GZ dynamics

Comparison GZ with lattice Landau gauge

- Many Schwinger-Dyson solutions are in agreement with such lattice data, see e.g. talks/work by Binosi, Papavassiliou, Natale, Aguilar, Pene, Maas, Fischer, Pawlowski, ...
- There seem to be a **discrepancy between GZ and lattice** data
- Working hypothesis: GZ can explain this data. Question is how?

GZ dynamics

A few observations

- Horizon condition is equivalent with a $d = 2$ condensate,

$$\left\langle g f^{abc} A^a_\mu (\varphi + \overline{\varphi}_\mu)^{bc} \right\rangle = 2\gamma^2 d(N^2 - 1) \neq 0$$

- There is a mass scale γ^2 present in GZ action and there is a (A, φ) coupling

$$S_h = \int d^4x \Big(\overline{\varphi}^{ac}_\mu \partial_\nu \left(\partial_\nu \varphi^{ac}_\mu + g f^{abm} A^b_\nu \varphi^{mc}_\mu \right) - \overline{\omega}^{ac}_\mu \partial_\nu \left(\partial_\nu \omega^{ac}_\mu + g f^{abm} A^b_\nu \omega^{mc}_\mu \right) - g \left(\partial_\nu \overline{\omega}^{ac}_\mu \right) f^{abm} \left(D_\nu c \right)^b \varphi^{mc}_\mu$$
$$-\gamma^2 g \left(f^{abc} A^a_\mu \varphi^{bc}_\mu + f^{abc} A^a_\mu \overline{\varphi}^{bc}_\mu + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \Big)$$

- **Might there be other ($d = 2$) condensates?**
  A few candidates: $\langle A^2 \rangle$, $\langle \varphi^2 \rangle$, $\langle \overline{\varphi}^2 \rangle$, $\langle \overline{\varphi}\varphi \rangle$, $\langle \overline{\omega}\omega \rangle$
  A nonvanishing VEV could be fueled by $\gamma^2 \neq 0$?

GZ dynamics

A study proposal

- Start from the GZ action
- Quantum dynamics of the GZ theory can give further nontrivial effects (e.g. nontrivial $d = 2$ condensates)
- Such condensates can further affect the propagator behaviour.

A study proposal: important statement

We do **not** alter the GZ action ad hoc. On the contrary, we wish to take into account what the GZ theory is telling us about its nontrivial quantum dynamics/vacuum structure.

GZ dynamics

A study proposal: a few requirements

- Renormalizability: the LCO's (=Local Composite Operator) should be at least renormalizable
- We need a tool to study the effective action Γ of LCO's

A study proposal: a few solutions

- Renormalizability of LCO's: this can be studied within the algebraic renormalization formalism
- Construction of $\Gamma(\mathrm{LCO})$: LCO formalism of Verschelde et al based on renormalization group (invariance)

GZ dynamics

Relevance of $d = 2$ operator(s) to define Refined GZ framework = RGZ

- RGZ results can give nice qualitative agreement with (most) lattice results:
  - $D(0) > 0$
    In fact, at tree level
    $$D(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + \lambda^4} = \frac{1}{p^2 + m^2(p^2)},$$
    it looks like an effective "gluon mass" $m^2(p^2) = \dfrac{\lambda^4}{p^2 + M^2}$
  - ghost $\sim \frac{1}{q^2}$ at low q^2
  - positivity violation

GZ dynamics

VEV of LCO in QFT

- Start from action $S(\phi)$
- We are e.g. interested in LCO $O = \phi^2$.
- Standard analysis
  1. introduce a source J to define O:

$$S(\phi) \to S(\phi) + JO$$

  2. introduce the functional $W(J)$

$$e^{-W(J)} = \int [D\phi] e^{-(S(\phi)+JO)}$$

GZ dynamics

VEV of LCO in QFT

3. $\langle O \rangle$?

$$\left.\frac{\delta W}{\delta J}\right|_{J=0} = \langle O \rangle$$

(this will be the perturbative value of $\langle O \rangle$)

4. More general: Legendre transform

$$\Gamma(\sigma) = W(J) - JO\,, \qquad \sigma = \frac{\delta W}{\delta J}$$

with

$$\left.\frac{\delta \Gamma}{\delta \sigma}\right|_{\sigma=\sigma_*} = 0(=J)\,, \qquad \sigma_* = \langle O \rangle\,.$$

(other solutions can emerge, with lower vacuum energy $\Gamma(\sigma_*)$.

- Of course, $\langle O \rangle$ is **not** a **free** parameter, but **dynamically fixed** using the original action! It's inherently part of the theory, not something you "add" by hand.

GZ dynamics

Application to GZ: dynamical derivation of RGZ

- We are currently investigating the following $d = 2$ operators within the GZ formalism

$$O_1 \equiv A_\mu^a A_\mu \,, \qquad O_2 \equiv \overline{\varphi}_\mu^{ab} \varphi_\mu^{ab} \,, \qquad O_3 \equiv \varphi_\mu^{ab} \varphi_\mu^{ab} \,, \qquad O_4 \equiv \overline{\varphi}_\mu^{ab} \overline{\varphi}_\mu^{ab} \,,$$

  each with associated field σ_i such that

$$\langle \sigma_i \rangle \propto O_i$$

- The $O_{2,3,4}$ are important for the RGZ formalism

GZ dynamics

Perturbative VEV's of $d = 2$ LCO's in GZ

- We for example find at 1-loop

$$\langle \overline{\varphi}\varphi \rangle = \left.\frac{\delta W}{\delta Q}\right|_{\text{sources}=0} = \frac{3(N^2-1)}{64\pi}\lambda^2 \neq 0 \qquad (\lambda^4 = 2g^2N\gamma^4)$$

- Analogously, one finds $\langle A^2 \rangle \propto \lambda^2$, $\langle \varphi^2 \rangle \propto \lambda^2$, $\langle \overline{\varphi}^2 \rangle \propto \lambda^2$. $(\langle \overline{\omega}\omega \rangle = 0)$
- Serves as extra motivation to investigate the $d = 2$ LCO's!

GZ dynamics

Application to GZ: dynamical derivation of RGZ

- there is the dynamical emergence of RGZ masses, if e.g. $\langle\sigma_2\rangle \neq 0$.
- Notice that due to $\gamma \neq 0$, it's even **not so easy to obtain**

$$\left.\frac{\partial\Gamma}{\partial\sigma_i}\right|_{\sigma_k=0} = 0$$

  or, it's **not so easy to find** $\langle\sigma_i\rangle = 0$ in GZ!
- $\Rightarrow$ $\boxed{\text{Hard to escape RGZ}}$
- Job to be done: explicit computations to confirm this and get estimates
- At the end, effective Lagrangian including the effects of other condensates
- (probably some a priori simplifications will be needed, as quite (too) complicated to do)

GZ dynamics

Crucial message

- **Every parameter in GZ is eventually fixed in terms of Λ_{QCD}.**
- No "choices" to be made. If you believe in GZ formalism as good starting point, it is telling you what to do. Eventually also if the RGZ dynamics is favoured (or not).
- If you believe GZ is wrong/incomplete, you can of course start playing around and looking for something else!
  - a "correct" implementation of the Faddeev-Popov trick incorporating signs etc, to average over all copies?
  - saying that the Gribov problem is irrelevant?
  - finding other implementations of the restriction?
  - you name it?

Overview

Summary

- We discussed differences between GZ and KO.
- (soft) BRST breaking in GZ spoils KO analysis.
- Imposing KO as boundary condition gives GZ partition function.
- GZ has a rich dynamical structure.
- Extra quantum effects can shift GZ predictions for propagators:
  - IR enhanced ghost $\rightarrow 1/p^2$ behaviour
  - IR vanishing gluon $\rightarrow$ IR finite gluon $\neq 0$
- Possibility to study analytically.