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Beyond renormalization in $D = 4$: an essay on nonlinear sigma model, massive YM and Electroweak Model

Ruggero Ferrari

CTP-MIT, Cambridge, MA
and
Università Milano and INFN, Sez. di Milano

Preamble: In power counting renormalizable theories there is a universally accepted rule, by which to every independent divergent one-particle-irreducible amplitude (1PI) one must associate a parameter in the tree-level action. This rule cannot be easily exported to any program of subtraction of infinities in nonrenormalizable theories. In fact, if this rule is used, the theory looses (in general) any predictivity and moreover the perturbative approach is unstable: for every new divergent 1PI amplitude emerging in the perturbative expansion, the whole series have to be updated from the beginning.

Work done in collaboration with D. Bettinelli and A. Quadri

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Introduction

A common structure is present in the nonlinear sigma model (NLSM), in the massive Yang-Mills (YM) and in the Higgs-less Electro-Weak model (EW). For $SU(2)$ one has the action structures: NLSM (Ref. [1]-[6])

$$S_{NLSM} = \Lambda^{D-4}\frac{M^2}{4}\int d^Dx\; Tr\{\partial^\mu\Omega^\dagger\partial_\mu\Omega\}$$

the Stückelberg mass for YM (Ref. [7]-[8])

$$S_{YM} \sim \Lambda^{D-4}M^2\int d^Dx\; Tr\{[A_\mu - i\Omega\partial_\mu\Omega^\dagger]^2\}$$

and EW (Ref. [9]-[11]) mass terms

$$S_{EW} \sim \Lambda^{D-4}M^2\int d^Dx(\,Tr\,\{(gA_\mu - \frac{g'}{2}\Omega\tau_3B_\mu\Omega^\dagger - i\Omega\partial_\mu\Omega^\dagger)^2\}$$
$$+\frac{\kappa}{2}[Tr\{gA_\mu - \frac{g'}{2}\Omega\tau_3B_\mu\Omega^\dagger - i\Omega\partial_\mu\Omega^\dagger\tau_3\}]^2).$$

The $2 \times 2 \in SU(2)$ matrix may be parametrized by the real fields

$$\Omega = \phi_0 + i\tau_i\phi_i, \qquad \phi_0 = \sqrt{1 - \vec{\phi}^2}.$$

The constraint is implemented in the path integral measure

$$\prod_x \mathcal{D}^4\phi(x)\theta(\phi_0)\delta(\vec{\phi}(x)^2 + \phi_0^2(x) - 1) = \prod_x \mathcal{D}^3\phi(x)\frac{2}{\sqrt{1 - \vec{\phi}^2}}.$$

The non trivial measure in the path integral is the source of very interesting facts.

Renormalization? No. What else?

The non polynomial interaction makes the theory nonrenormalizable

$$S_{NLSM} = \Lambda^{D-4}\frac{M^2}{2}\int d^Dx \,\{\partial^\mu\phi_0\partial_\mu\phi_0 + \partial^\mu\vec{\phi}\partial_\mu\vec{\phi}\}$$

$$= \Lambda^{D-4}\frac{M^2}{2}\int d^Dx \,\{\partial^\mu\vec{\phi}\partial_\mu\vec{\phi} + \frac{1}{\phi_0^2}\,\phi_a\partial^\mu\phi_a\,\phi_b\partial_\mu\phi_b\}.$$

Vertexes carry second power of momenta, therefore already at one loop there is an infinite number of independent divergent amplitudes. Moreover it has been shown in the seventies and in the eighties that some divergences break chiral invariance (the global) at the same order.

Strategy: Abandon Hamiltonian formalism and do perturbation theory directly on the effective action functional Γ.

The Local Functional Equation (LFE)

The measure is invariant under "local left multiplication" transformations $\Omega \to U(\omega(x))\Omega$

$$\delta\phi_0 = -\frac{\omega_a(x)}{2}\phi_a$$
$$\delta\phi_a = \frac{\omega_a(x)}{2}\phi_0 + \frac{\omega_c(x)}{2}\epsilon_{abc}\phi_b.$$

Technical work to do: (i) find the algebra of operators closed under local left multiplication transformations by starting from the classical action, (ii) associate to every composite operator an external classical source (for subtraction strategy), (iii) write the LFE which follows from the invariance of the path integral measure.

Step (i)

This is simple in the NLSM. Introduce the "gauge field"

$$F_\mu = \frac{\tau_a}{2} F_{a\mu} \equiv i\Omega\partial_\mu\Omega^\dagger.$$

Its field strength tensor is zero (it describes a scalar mode) and its transformation properties are those of a gauge field

$$F_\mu \to U F_\mu U^\dagger + iU\partial_\mu U^\dagger.$$

The classical action can be written

$$S_{NLSM} = \Lambda^{D-4}\frac{M^2}{4}\int d^D x\ Tr\{F_\mu F^\mu\}.$$

Thus the closed set of operator is $\{\vec{\phi}, \phi_0, \vec{F}_\mu\}$

Step (ii)

The complete effective action at the tree level is then

$$\Gamma^{(0)} = \Lambda^{D-4} \int d^D x \; (\frac{M^2}{8}\{F_{a\mu} - J_{a\mu}\}^2 + K_0\phi_0).$$

The effective action $\Gamma[\vec{\phi}, \vec{J}_\mu, K_0]$ **is obtained via Legendre transform of the logarithm of the path integral functional**

$$Z[\vec{K}, \vec{J}_\mu, K_0] \equiv \int \prod_x \frac{2}{\phi_0} \mathcal{D}^3\phi(x) \exp[\Gamma^{(0)} + \int d^D y \vec{K}\vec{\phi}].$$

Step (iii)

Now we exploit the invariance of the path integral measure under local left multiplication $(\delta\phi_a = \frac{\omega_a(x)}{2}\phi_0 + \frac{\omega_c(x)}{2}\epsilon_{abc}\phi_b)$**.**

We expand for small parameter $\vec{\omega}(x)$ and obtain the LFE ($\langle\cdots\rangle$ indicates the mean over the weighted paths)

$$\int d^Dx \left\langle (M_D^2(F-J)_{a\mu}(\epsilon_{abc}\omega_c F_b^\mu + \partial^\mu\omega_a) \right.$$
$$\left. -\Lambda^{D-4}K_0\frac{\omega_a}{2}\phi_a + \phi_0 K_a\frac{\omega_a}{2} + \epsilon_{abc}K_a\omega_c\phi_b)(x) \right\rangle = 0,$$

where

$$M_D^2 \equiv \Lambda^{D-4}M^2.$$

We will use the notation

$$\mathcal{D}[X]^\mu_{ab} = \delta_{ab}\partial_\mu - \epsilon_{abc}X_{c\mu}.$$

Thus for the effective action we get the local functional equation

$$-\partial^\mu\frac{\delta\Gamma}{\delta J_a^\mu} + \epsilon_{abc}J_c^\mu\frac{\delta\Gamma}{\delta J_b^\mu} + \frac{\Lambda^{D-4}}{2}\phi_a K_0 + \frac{1}{2\Lambda^{D-4}}\frac{\delta\Gamma}{\delta K_0}\frac{\delta\Gamma}{\delta\phi_a} + \frac{1}{2}\epsilon_{abc}\phi_c\frac{\delta\Gamma}{\delta\phi_b} = 0.$$

Hierarchy

The Spontaneous Breakdown of Symmetry is imposed by the condition

$$\left.\frac{\delta\Gamma}{\delta K_0}\right|_{\text{field \&sources}=0} = 1.$$

Then LFE naturally induces a strong hierarchy structure among the 1PI irreducible amplitudes: all amplitudes involving the $\vec{\phi}$ fields (descendant) are known in terms of the amplitudes involving only the (ancestor) sources $\vec{J}_\mu, K_0$. For instance, if we differentiate the LFE with respect to $J^\nu_{a'}(y)$ we get

$$\frac{M_D^2}{2}\partial^\mu \frac{\delta^2\Gamma}{\delta J^\mu_a(x)\delta J^\nu_{a'}(y)} + \frac{\delta^2\Gamma}{\delta\phi_a(x)\delta J^\nu_{a'}(y)} + 2\delta_{aa'}\partial_{x^\nu}\delta(x-y) = 0.$$

Weak Power Counting (WPC)

How many ancestor divergent amplitudes? The degree of divergence of a graph G for an ancestor amplitude is (n_L number of loops)

$$\delta(G) = D\ n_L - 2I + \sum_{j,k} j\ V_{jk} + N_F$$

$$n_L = I - \sum_{j,k} V_{jk} - N_F - N_{K_0} + 1$$

where I be the number of propagators, N_F the number of external F_μ sources and N_{K_0} those of K_0. V_{jk} denotes the number of vertexes with k ϕ-lines and j derivatives. The superficial degree of divergence $\delta(G)$ for a graph can be bounded by using standard arguments.

By removing I from these two equations one gets

$$\delta(G) = D\ n_L - 2n_L - \sum_{j,k}(2-j)\ V_{jk} - N_F - 2N_{K_0} + 2.$$

The classical action has vertexes with $j \leq 2$, therefore it can be stated that

$$\delta(G) \leq n_L(D-2) + 2 - N_F - 2N_{K_0}. \tag{1}$$

For instance at $n_L = 1$ the only ancestor divergent (independent) amplitudes are $(J-J)$, $(J-J-J)$, $(J-J-J-J)$, (K_0-J-J), (K_0-K_0). The one-loop divergences of graphs where the descendant field appears ($\vec{\phi}$) are expressible all in terms of the ancestor divergences.

Perturbative Expansion

This is an *Ansatz*. Consider the generic dimension D. Start with $\Gamma^{(0)}$, read from it the value of the vertexes and construct $\Gamma^{(n)}$ for $n > 0$. The connected amplitudes $W^{(n)}$ can then be obtained. Few questions are in order

1. **Does $\Gamma^{(0)}$ obey the LFE? Yes, by construction**

2. **Does $\Gamma^{(n)}$, $n > 0$ obey the linearized LFE?**

$$
\Big(-\partial^\mu \frac{\delta}{\delta J_a^\mu} + \epsilon_{abc} J_c^\mu \frac{\delta}{\delta J_b^\mu} + \frac{1}{2\Lambda^{D-4}} \frac{\delta \Gamma^{(0)}}{\delta \phi_a} \frac{\delta}{\delta K_0}
+ \frac{1}{2}\phi_0 \frac{\delta}{\delta \phi_a} + \frac{1}{2}\epsilon_{abc}\phi_c \frac{\delta}{\delta \phi_b}\Big)\Gamma^{(n)} = 0.
$$

3. **Assume that a *symmetric* subtraction procedure is given for the divergences in the limit $D = 4$, how is the breaking of the above equation?**

Answers

The answers to these questions are given in a compact form by the QAP

$$
\begin{aligned}
&\Big(-\partial^\mu \frac{\delta}{\delta J_a^\mu} + \epsilon_{abc} J_c^\mu \frac{\delta}{\delta J_b^\mu} \\
&-\frac{\Lambda^{D-4}}{2} K_0 \frac{\delta}{\delta K_a} + \frac{1}{2\Lambda^{D-4}} K_a \frac{\delta}{\delta K_0} + \epsilon_{acb} K_c \frac{\delta}{\delta K_b} \Big) Z \\
&= i \int \prod_x \frac{2}{\phi_0} \mathcal{D}^3\phi(x) [-\partial^\mu \frac{\delta \hat\Gamma}{\delta J_a^\mu} + \epsilon_{abc} J_c^\mu \frac{\delta \hat\Gamma}{\delta J_b^\mu} \\
&+\frac{\Lambda^{D-4}}{2} \phi_a K_0 + \frac{1}{2\Lambda^{D-4}} \frac{\delta \hat\Gamma}{\delta K_0} \frac{\delta \hat\Gamma}{\delta \phi_a} + \frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta \hat\Gamma}{\delta \phi_b}] \exp i [\hat\Gamma + \int d^D y \vec{K} \vec{\phi}],
\end{aligned}
$$

where $\hat\Gamma$ contains the counterterms $\hat\Gamma^{(j)}$

$$
\hat\Gamma = \Gamma^{(0)} + \sum_{j=1}^{\infty} \hat\Gamma^{(j)}.
$$

Subtraction Strategy

Thus if the counterterms at order n are missing, the linearized LFE is broken by the term

$$\Big(-\partial^\mu \frac{\delta}{\delta J_a^\mu} + \epsilon_{abc} J_c^\mu \frac{\delta}{\delta J_b^\mu} + \frac{1}{2\Lambda^{D-4}} \frac{\delta \Gamma^{(0)}}{\delta \phi_a} \frac{\delta}{\delta K_0}$$

$$+\frac{1}{2}\phi_0 \frac{\delta}{\delta \phi_a} + \frac{1}{2}\epsilon_{abc}\phi_c \frac{\delta}{\delta \phi_b}\Big)\Gamma^{(n)} = -\frac{1}{2\Lambda^{D-4}} \sum_{j=1}^{n-1} \frac{\delta \hat{\Gamma}^{(j)}}{\delta K_0} \frac{\delta \hat{\Gamma}^{(n-j)}}{\delta \phi_a}.$$

Notice that $\frac{1}{\Lambda^{D-4}}\frac{\delta\Gamma^{(0)}}{\delta\phi_a}$ is independent from Λ^{D-4}. Thus we use Laurent expansion on

$$\Lambda^{-D+4}\Gamma^{(n)}$$

to define the finite part and the counterterm $\Lambda^{-D+4}\hat{\Gamma}^{(n)} = -\Lambda^{-D+4}\Gamma^{(n)}|_{\rm poles}$.

Organization of the Divergences

The LFE is a power organizer of the divergences that WPC has classified. The full control can be obtained by finding the relevant local solutions of the linearized LFE

$$
\begin{aligned}
&(-\partial^\mu \frac{\delta}{\delta J_a^\mu} + \epsilon_{abc} J_c^\mu \frac{\delta}{\delta J_b^\mu} + \frac{1}{2\Lambda^{D-4}} \frac{\delta \Gamma^{(0)}}{\delta \phi_a} \frac{\delta}{\delta K_0} \\
&+\frac{1}{2}\phi_0 \frac{\delta}{\delta \phi_a} + \frac{1}{2}\epsilon_{abc}\phi_c \frac{\delta}{\delta \phi_b}) \Gamma^{(n)}[\vec{\phi}, \vec{J}_\mu, K_0] = 0.
\end{aligned}
$$

This can easily achieved by using the technique of *bleaching***. We shortly describe this procedure. The above equation naturally suggests the following infinitesimal transformations δ_0**

The Bleaching

The transformations

$$\delta_0 J_b^\mu = (\partial^\mu \delta_{ab} + \epsilon_{abc} J_c^\mu)\omega_a = \mathcal{D}[J]_{ba}^\mu \omega_a$$

$$\delta_0 F_a^\mu = \mathcal{D}[F]_{ab}^\mu \omega_b$$

$$\delta_0 K_0 = -\frac{\omega_a}{\Lambda^{D-4}} \frac{\delta \Gamma^{(0)}}{\delta \phi_a}$$

$$\delta_0 \left(-\frac{\delta \Gamma^{(0)}}{\delta \phi_a}\right) = \Lambda^{D-4} \frac{1}{2} \omega_a K_0 + \frac{1}{2} \epsilon_{abc} \omega_c \left(-\frac{\delta \Gamma^{(0)}}{\delta \phi_b}\right)$$

suggest the bleaching

$$\mathfrak{J}_\mu \equiv \Omega^\dagger (J_\mu - F_\mu) \Omega$$

$$\mathfrak{K}_0 \equiv \frac{K_0}{\phi_0} - \frac{M^2}{4} (J_b^\mu - F_b^\mu) \frac{\partial F_{b\mu}}{\partial \phi_a} \phi_a$$

Care in Bleaching

Few facts about bleaching. i) The relations are invertible, ii) In the case of $\mathfrak{J}_{a\mu}$, bleaching is a kind of gauge transformation where the parameters are the $\vec{\phi}$ fields

$$
\begin{aligned}
\mathfrak{J}_\mu &= \Omega^\dagger J_\mu \Omega + i\Omega\partial_\mu\Omega \\
\partial_\mu \mathfrak{J}_\nu &= \Omega^\dagger \left(\partial_\mu + \Omega\partial_\mu\Omega^\dagger\right)(J_\nu - F_\nu)\Omega = \Omega^\dagger \mathcal{D}_\mu[F](J_\nu - F_\nu)\Omega
\end{aligned}
$$

iii) the invariants can be constructed by using $\mathfrak{J}_\mu$ and $\mathfrak{K}_0$ and their space-time derivatives. Ancestor amplitudes do not depend explicitly from $\vec{\phi}$. We consider only those relevant for the one-loop divergences.

The one-loop Invariants

$$\mathcal{I}_1 = \int d^D x \, [D_\mu (F-J)_\nu]_a [D^\mu (F-J)^\nu]_a \, ,$$

$$\mathcal{I}_2 = \int d^D x \, [D_\mu (F-J)^\mu]_a [D_\nu (F-J)^\nu]_a \, ,$$

$$\mathcal{I}_3 = \int d^D x \, \epsilon_{abc} [D_\mu (F-J)_\nu]_a (F_b^\mu - J_b^\mu)(F_c^\nu - J_c^\nu) \, ,$$

$$\mathcal{I}_4 = \int d^D x \, \Big(\frac{K_0}{\phi_0} + \frac{M^2}{4} [F_b^\mu - J_b^\mu] \frac{\partial F_{b\mu}}{\partial \phi_a} \phi_a \Big)^2 \, ,$$

$$\mathcal{I}_5 = \int d^D x \, \Big(\frac{K_0}{\phi_0} + \frac{M^2}{4} [F_b^\mu - J_b^\mu] \frac{\partial F_{b\mu}}{\partial \phi_a} \phi_a \Big)(F_c^\mu - J_c^\mu)^2 \, ,$$

$$\mathcal{I}_6 = \int d^D x \, (F_a^\mu - J_a^\mu)^2 (F_b^\nu - J_b^\nu)^2 \, ,$$

$$\mathcal{I}_7 = \int d^D x \, (F_a^\mu - J_a^\mu)(F_a^\nu - J_a^\nu)(F_{b\mu} - J_{b\mu})(F_{b\nu} - J_{b\nu}) \, ,$$

where D_μ denotes the covariant derivative w.r.t $F_{a\mu}$:

$$D_{ab\mu} = \delta_{ab} \partial_\mu - \epsilon_{abc} F_{c\mu} \, .$$

The Counterterms

The counterterms are evaluated by extracting the pole parts from the relevant amplitudes given by the effective action functional normalized by $\Lambda^{-D+4}\Gamma.$ It is very important to care about the relation

$$2(\mathcal{I}_1 - \mathcal{I}_2) - 4\mathcal{I}_3 + (\mathcal{I}_6 - \mathcal{I}_7) = \int d^D x \mathcal{G}_{a\mu\nu}[\mathfrak{J}]\mathcal{G}_a^{\mu\nu}[\mathfrak{J}] = \int d^D x \mathcal{G}_{a\mu\nu}[J]\mathcal{G}_a^{\mu\nu}[J] =\sim 0.$$

The right hand term is sterile: no descendant terms are generated. Now the calculation gives

$$\Gamma^{(1)} = \frac{1}{D-4}\frac{\Lambda^{D-4}}{(4\pi)^2}[-\frac{1}{12}(\mathcal{I}_1 - \mathcal{I}_2 - \mathcal{I}_3) + \frac{1}{48}(\mathcal{I}_6 + 2\mathcal{I}_7)$$
$$+\frac{3}{2}\frac{1}{M^4}\mathcal{I}_4 + \frac{1}{2}\frac{1}{M^2}\mathcal{I}_5]$$

The massive Yang-Mills

Ω describes the Goldstone bosons, that are here unphysical modes. Then it is important to ensure that the Slavnov-Taylor Identity (STI) is valid in order to preserve unitarity. The LFE must be compatible with the STI. A suitable gauge-fixing term will help to achieve this result. The Landau gauge is the simplest, since the tadpole contributions can be neglected in most cases. The transformations to be considered are the local left $SU(2)_L$ and the global right $SU(2)_R$ on Ω, the gauge fields A_μ, the Faddeev-Popov fields $c, \bar{c}$. Few external sources are needed in order to describe the complete (under the $SU(2)_L \otimes SU(2)_R$) set of composite operators.

Landau Gauge-fixing

$$\Gamma^{(0)} = S_{YM} + \frac{\Lambda^{D-4}}{g^2}\int d^Dx\,(B_a(D^\mu[V](A_\mu - V_\mu))_a - \bar{c}_a(D^\mu[V]D_\mu[A]c)_a)$$
$$+\frac{\Lambda^{D-4}}{g^2}\int d^Dx\,\Theta_a^\mu\;(D_\mu[A]\bar{c})_a$$
$$+\int d^Dx\,(A^*_{a\mu}sA^\mu_a + \phi^*_0 s\phi_0 + \phi^*_a s\phi_a + c^*_a s c_a + K_0\phi_0)\,.$$

$$S_{YM} = \frac{\Lambda^{(D-4)}}{g^2}\int d^Dx\,\Big(-\frac{1}{4}G_{a\mu\nu}[A]G_a^{\mu\nu}[A] + \frac{M^2}{2}(A_{a\mu} - F_{a\mu})^2\Big)\,.$$

$$\Omega = \frac{1}{v}(\phi_0 + i\tau_a\phi_a), \qquad \phi_0^2 + \phi_a^2 = v^2 \tag{2}$$

where v is a parameter with dimension equal one. We stress that v is <u>not a parameter of the model</u>, because it can can be removed by a rescaling of the fields $\vec{\phi}, \phi_0$.

STI

The Slavnov-Taylor Identity necessary for the validity of physical unitarity: S-matrix satisfies the following equation at the perturbative level

$$\langle\alpha|\beta\rangle = \sum_{n\in\{\text{physical states}\}} \langle\alpha|S|n\rangle\langle n|S^\dagger|\beta\rangle$$

if both α, β are physical states. This in general is valid if the Slavnov-Taylor identity is valid.

$$\mathcal{S}(\Gamma) = \int d^D x \,(\frac{\delta\Gamma}{\delta A^*_{a\mu}}\frac{\delta\Gamma}{\delta A^\mu_a} + \frac{\delta\Gamma}{\delta\phi^*_a}\frac{\delta\Gamma}{\delta\phi_a} + \frac{\delta\Gamma}{\delta c^*_a}\frac{\delta\Gamma}{\delta c_a} + B_a\frac{\delta\Gamma}{\delta\bar{c}_a} + \Theta_{a\mu}\frac{\delta\Gamma}{\delta V_{a\mu}} - K_0\frac{\delta\Gamma}{\delta\phi^*_0}) = 0\,.$$

Massive YM: the Local Functional Equation

$$\mathcal{W}(\Gamma) \equiv \int d^D x \alpha_a^L(x)(-\partial_\mu \frac{\delta\Gamma}{\delta V_{a\mu}} + \epsilon_{abc} V_{c\mu} \frac{\delta\Gamma}{\delta V_{b\mu}} - \partial_\mu \frac{\delta\Gamma}{\delta A_{a\mu}}$$
$$+\epsilon_{abc} A_{c\mu} \frac{\delta\Gamma}{\delta A_{b\mu}} + \epsilon_{abc} B_c \frac{\delta\Gamma}{\delta B_b} + \frac{1}{2} K_0 \phi_a + \frac{1}{2} \frac{\delta\Gamma}{\delta K_0} \frac{\delta\Gamma}{\delta \phi_a}$$
$$+\frac{1}{2} \epsilon_{abc} \phi_c \frac{\delta\Gamma}{\delta \phi_b} + \epsilon_{abc} \bar{c}_c \frac{\delta\Gamma}{\delta \bar{c}_b} + \epsilon_{abc} c_c \frac{\delta\Gamma}{\delta c_b}$$
$$+\epsilon_{abc} \Theta_{c\mu} \frac{\delta\Gamma}{\delta \Theta_{b\mu}} + \epsilon_{abc} A^*_{c\mu} \frac{\delta\Gamma}{\delta A^*_{b\mu}} + \epsilon_{abc} c^*_c \frac{\delta\Gamma}{\delta c^*_b} + \frac{1}{2} \phi^*_0 \frac{\delta\Gamma}{\delta \phi^*_a}$$
$$+\frac{1}{2} \epsilon_{abc} \phi^*_c \frac{\delta\Gamma}{\delta \phi^*_b} - \frac{1}{2} \phi^*_a \frac{\delta\Gamma}{\delta \phi^*_0}) = 0\,.$$

Γ also obeys the Landau gauge equation

$$\frac{\delta\Gamma}{\delta B_a} = \frac{\Lambda^{D-4}}{g^2} D^\mu[V](A_\mu - V_\mu)_a \tag{3}$$

Linearized Equations and Induced Transformations

The structure of both STI and LFE is standard. Thus we can

1. **Establish the full hierarchy (only the Goldstone bosons are descendant fields)**
2. **Confirm the validity of the WPC**
3. **Introduce the linearized STI and LFE**
4. **Extract from the linearized STI and LFE the generators of the transformations on the effective action Γ**
5. **Check that the generators stemming from STI commute with those from LFE**

Subtraction procedure

With these tools we can construct the most general classical action compatible with the WPC and the invariance under the BRST transformations and the LFE induced symmetry. Surprisingly enough the resulting action is the standard YM field theory with a Stückelberg mass term.

The subtraction procedure of the divergences is then the same as in the NLSM: subtraction of the pure pole parts in the Laurent expansion around $D = 4$ of the normalized amplitudes $\Lambda^{-D+4}\Gamma$. This subtraction procedure has been implemented in the one-loop calculation of the gauge field two-point function.

Consistency of the Subtraction Procedure

The two-loop self-energy amplitude have been considered from the point of view of the consistency. It has been argued that the subtraction scheme is consistent: i) the counterterms are local ii) physical unitarity is satisfied iii) the STI and LFE induced symmetry on Γ is preserved. Ref. [8]

1) Explicit calculation of the gauge field two-point function.

2) Check that the counterterms are local at the two-loop level.

3) Validity of unitarity.

4) All divergences (infinite) at the one-loop level are subtracted by a finite number of counterterms.

Outlook and (some) open questions

- **Phenomenological applications (Andrea Quadri's talk)**
- **Running constant (dependence from Λ)**
- **How to proceed with a generic regularization tool?**
- **Well-defined strategy of minimal subtraction with anti-commuting γ_5.**
- **Extension to Grand Unified groups**

Conclusions

LFE $\Longrightarrow$

- **Hierarchy**
- **WPC**
- **Consistent subtraction procedure (symmetric and local)**
- **Finite number of physical parameters**

For massive YM, FTI and Landau gauge equation $\Longrightarrow$

- **Physical unitarity**
- **Consistency with LFE.**

't Hooft-Feynman gauge possible (many tadpole diagrams).

NOTE 1: LFE = Schwinger-Dyson equation?

The Schwinger-Dyson equation in presence of a non trivial path integral measure is

$$0 = \int \prod_z \mathcal{D}[\vec{\phi}(z)] \frac{\delta}{\delta \phi_a(x)} e^{iS_{NLSM}} \exp i \int d^D y \left[+i \log \phi_0(y) \ \right]$$

i.e.

$$\left\langle \ \frac{\delta S_{NLSM}}{\delta \phi_a(x)} - i \frac{\phi_a}{\phi_0^2} \right\rangle = 0.$$

Finally one gets

$$\left\langle \ \frac{M_D^2}{\phi_0} \left(-\frac{1}{2} \partial^\mu F_{a\mu} + \epsilon_{abc} \Box \phi_b \phi_c \ \right) - i \frac{\phi_a}{\phi_0^2} \ \right\rangle = 0.$$

The terms in red are not present in the LFE. Moreover the overall factor $1/\phi_0$ **is also absent in the LFE. In fact the Schwinger-Dyson equation might coincide with the LFE only for trivial path integral measure (i.e. invariant under shift of the fields** $\delta \phi_a = \alpha_a \in \Re$**).**

NOTE 2: LFE induced transformations

One can formally introduce external currents by using the Legendre transform

$$K_a \equiv -\frac{\delta}{\delta\phi_a}\Gamma^{(0)} = -\frac{\delta}{\delta\phi_a}S_0 + \Lambda^{D-4}\frac{K_0}{\phi_0}\phi_a, \qquad S_0 \equiv \Gamma^{(0)}|_{K_0=0}$$

Under the action

$$\delta_0 \equiv \int d^Dx\,[(\partial^\mu\omega_a - \epsilon_{aji}J_i^\mu\omega_j)\frac{\delta}{\delta J_a^\mu} + \frac{1}{2\Lambda^{D-4}}\omega_a\frac{\delta\Gamma^{(0)}}{\delta\phi_a}\frac{\delta}{\delta K_0}$$
$$+(\frac{1}{2}\omega_a\phi_0 + \frac{1}{2}\epsilon_{ajk}\phi_j\omega_k)\frac{\delta}{\delta\phi_a}]$$

we get

$$\delta_0\ K_{a'} = -[\delta_0,\ \frac{\delta}{\delta\phi_{a'}}]S_0 + \Lambda^{D-4}\delta_0\ \frac{K_0}{\phi_0}\phi_{a'}$$
$$= \Big(-\frac{1}{2}\omega_a\frac{\phi_{a'}}{\phi_0} + \frac{1}{2}\epsilon_{aa'k}\omega_k\Big)\frac{\delta}{\delta\phi_a}S_0 + \frac{1}{2}\omega_a\frac{\delta\Gamma^{(0)}}{\delta\phi_a}\frac{1}{\phi_0}\phi_{a'}$$

$$+\Lambda^{D-4}(\frac{1}{2}\omega_a\phi_0+\frac{1}{2}\epsilon_{ajk}\phi_j\omega_k)(\frac{K_0}{\phi_0^3}\phi_{a'}\phi_a+\frac{K_0}{\phi_0}\delta_{aa'})$$

$$=(\frac{1}{2}\epsilon_{aa'k}\omega_k)\frac{\delta}{\delta\phi_a}S_0+\Lambda^{D-4}(\frac{1}{2}\omega_a\phi_0+\frac{1}{2}\epsilon_{ajk}\phi_j\omega_k)(\frac{K_0}{\phi_0}\delta_{aa'})$$

$$=-\frac{1}{2}\epsilon_{a'ak}\omega_k K_a+\Lambda^{D-4}\frac{1}{2}\omega_{a'}K_0$$

and

$$\delta_0\ K_0=\frac{1}{2\Lambda^{D-4}}\omega_a\frac{\delta\Gamma^{(0)}}{\delta\phi_a}\frac{\delta}{\delta K_0}\ K_0=-\frac{1}{2\Lambda^{D-4}}\omega_a K_a.$$

Thus we have a bleached variable

$$\mathfrak{K}_0=K_0\phi_0+\frac{1}{\Lambda^{D-4}}K_a\phi_a$$

$$=\frac{K_0}{\phi_0}-\frac{1}{\Lambda^{D-4}}\phi_a\frac{\delta}{\delta\phi_a}S_0=\frac{K_0}{\phi_0}+\frac{M^2}{4}(F_b^\mu-J_b^\mu)\frac{\partial F_{b\mu}}{\partial\phi_a}\phi_a$$

NOTE 3: The Rosetta Stone

We write the mapping for the paper [2] and the present work notation (denoted by a hat)

$$
\begin{aligned}
&m_D^2 = m^{D-2}\\
&\hat{\phi}_0 = \frac{\phi_0}{m_D}\\
&\hat{\phi}_i = \frac{g}{m_D}\phi_i\\
&\hat{F}_{a\mu} = gF_{a\mu}\\
&\hat{\Lambda} = m\\
&\frac{\hat{\Lambda}}{\hat{M}} = g\\
&\hat{J}_{a\mu} = -4\frac{g}{m_D^2}J_{a\mu}
\end{aligned}
$$

With these mappings we get

$$\hat{\mathcal{I}}_1 = g^2\mathcal{I}_1, \quad \hat{\mathcal{I}}_2 = g^2\mathcal{I}_2, \quad \hat{\mathcal{I}}_3 = g^3\mathcal{I}_3, \quad \hat{\mathcal{I}}_4 = \frac{m_D^4}{m^4}\mathcal{I}_4, \quad \hat{\mathcal{I}}_5 = g^2\frac{m_D^2}{m^2}\mathcal{I}_5, \quad \hat{\mathcal{I}}_6 = g^4\mathcal{I}_6, \quad \hat{\mathcal{I}}_7 = g^4\mathcal{I}_7$$

Finally we get the counterterms

$$
\begin{aligned}
\hat{\Gamma}^{(1)} &= \frac{1}{D-4}\Big[-\frac{1}{12}\frac{g^2}{(4\pi)^2}\frac{m_D^2}{m^2}(\mathcal{I}_1-\mathcal{I}_2-g\mathcal{I}_3)+\frac{1}{(4\pi)^2}\frac{g^4}{48}\frac{m_D^2}{m^2}(\mathcal{I}_6+2\mathcal{I}_7)+\frac{1}{(4\pi)^2}\frac{3}{2}\frac{g^4}{m^2m_D^2}\mathcal{I}_4+\frac{1}{(4\pi)^2}\frac{1}{2}\frac{g^4}{m^2}\mathcal{I}_5\Big]\\
&= \frac{1}{D-4}\Big[-\frac{1}{12}\frac{1}{(4\pi)^2}\frac{m_D^2}{m^2}(\hat{\mathcal{I}}_1-\hat{\mathcal{I}}_2-\hat{\mathcal{I}}_3)+\frac{1}{(4\pi)^2}\frac{1}{48}\frac{m_D^2}{m^2}(\hat{\mathcal{I}}_6+2\hat{\mathcal{I}}_7)+\frac{1}{(4\pi)^2}\frac{3}{2}\frac{g^4m_D^2}{m^6}\hat{\mathcal{I}}_4+\frac{1}{(4\pi)^2}\frac{1}{2}\frac{g^2m_D^2}{m^4}\hat{\mathcal{I}}_5\Big]\\
&= \frac{1}{D-4}\frac{\Lambda^{D-4}}{(4\pi)^2}\Big[-\frac{1}{12}(\hat{\mathcal{I}}_1-\hat{\mathcal{I}}_2-\hat{\mathcal{I}}_3)+\frac{1}{48}(\hat{\mathcal{I}}_6+2\hat{\mathcal{I}}_7)\frac{3}{2}\frac{1}{M^4}\hat{\mathcal{I}}_4+\frac{1}{2}\frac{1}{M^2}\hat{\mathcal{I}}_5\Big]
\end{aligned}
$$