The static potential in the Gribov-Zwanziger Lagrangian

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Outline

- Brief review of Gribov problem in the Landau gauge
- Zwanziger's renormalizable extension of QCD to accommodate Gribov's observations
- One loop static potential in Gribov-Zwanziger Lagrangian
- Consequences such as large r behaviour and power corrections
- Enhancement of bosonic Zwanziger localizing ghost and its effect on the static potential
- Brief comment on decoupling solution in context of static potential

Background

- Yang-Mills action S is invariant under gauge transformations

$$A^a_\mu \;\rightarrow\; \tilde{A}^a_\mu \;=\; U^\dagger \partial_\mu U \;+\; U^\dagger A_\mu U$$

  where

$$S \;=\; -\,\frac{1}{4}\int d^4x\, G^a_{\mu\nu} G^{a\,\mu\nu}$$

  with $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$

- In a non-abelian gauge theory there is a problem fixing the gauge globally [Gribov]
- For a given A^a_μ satisfying the gauge condition $\partial^\mu A^a_\mu = 0$ there are (Gribov) copies $\tilde{A}^a_\mu$ obeying the same condition $\partial^\mu \tilde{A}^a_\mu = 0$
- The existence of Gribov copies is equivalent to the Faddeev-Popov operator having zero eigenvalues

$$\partial^\mu D_\mu \Lambda^a \;=\; 0$$

- To avoid the copy problem the integration region of the path integral must be restricted to the first domain bounded by the Gribov horizon which contains the origin, $A^a_\mu = 0$, and denoted by Ω

Consequences

- Restriction to Ω via the no pole condition on $\mathcal{M}^{ab}(A) = (\partial^\mu D_\mu)^{ab}$ means a natural mass parameter, γ, emerges
- It is central to the infrared or non-perturbative behaviour of the theory
- Gluon propagator propagator is

$$\langle A^a_\mu(p) A^b_\nu(-p) \rangle ~=~ - ~ \frac{\delta^{ab} D_A(p^2)}{p^2} P_{\mu\nu}(p)$$

  where the form factor is

$$D_A(p^2) ~=~ \frac{(p^2)^2}{[(p^2)^2 + C_A \gamma^4]}$$

  which has gluon suppression since $D_A(0) = 0$

- γ is not independent and satisfies the one loop $\overline{\text{MS}}$ gap equation [Gribov]

$$1 ~=~ C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a ~+~ O(a^2)$$

  where $a = g^2/(16\pi^2)$

- Using gap equation the Faddeev-Popov ghost propagator form factor is enhanced

$$D_c(p^2) \sim \frac{1}{p^2} \quad \text{as} \; p^2 \rightarrow 0$$

and the ghost propagator behaves as $1/(p^2)^2$ as $p^2 \rightarrow 0$

- Gribov's effective Lagrangian is non-local

$$L^\gamma = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \frac{C_A \gamma^4}{2} A^a_\mu \frac{1}{\partial^\nu D_\nu} A^{a\,\mu} - \frac{d N_A \gamma^4}{2 g^2}$$

- γ is defined by the horizon (or no-pole) condition which is equivalent to the Gribov gap equation

$$\left\langle A^a_\mu(x) \frac{1}{\partial^\nu D_\nu} A^{a\,\mu}(x) \right\rangle = \frac{d N_A}{C_A g^2}$$

- The key point is that the theory has no meaning as a gauge theory if γ is treated as independent and does not satisfy this condition

Gribov-Zwanziger Lagrangian

- With path integral restricted to Ω Zwanziger constructed a completely local Lagrangian L^Z
- Zwanziger's Lagrangian involved additional ghost fields $\{\phi^{ab}_\mu, \bar{\phi}^{ab}_\mu; \omega^{ab}_\mu, \bar{\omega}^{ab}_\mu\}$
- These localize the non-locality

$$\begin{aligned} L^Z &= -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2\alpha} (\partial^\mu A^a_\mu)^2 - \bar{c}^a \partial^\mu D_\mu c^a + i \bar{\psi}^{iI} \not{D} \psi^{iI} \\ &+ \bar{\phi}^{ab\,\mu} \partial^\nu \left(D_\nu \phi_\mu \right)^{ab} - \bar{\omega}^{ab\,\mu} \partial^\nu \left(D_\nu \omega_\mu \right)^{ab} \\ &- g f^{abc} \partial^\nu \bar{\omega}^{ae}_\mu \left(D_\nu c \right)^b \phi^{ec\,\mu} \\ &- \frac{\gamma^2}{\sqrt{2}} \left(f^{abc} A^{a\,\mu} \phi^{bc}_\mu - f^{abc} A^{a\,\mu} \bar{\phi}^{bc}_\mu \right) - \frac{d N_A \gamma^4}{2 g^2} \end{aligned}$$

- Fields ω^{ab}_μ and $\bar{\omega}^{ab}_\mu$ are anti-commuting
- Lagrangian is also *renormalizable*, [Zwanziger; Schaden, Maggiore; Sorella et al], so that it can be used to perform calculations
- ϕ^{ab}_μ and $\bar{\phi}^{ab}_\mu$ implement Gribov horizon condition

- Horizon condition equates to

$$f^{abc} \langle A^{a\,\mu}(x) \phi^{bc}_\mu(x) \rangle ~=~ \frac{d N_A \gamma^2}{\sqrt{2} g^2}$$

which defines γ through the gap equation from the equation of motion

$$\phi^{ab}_\mu ~=~ \frac{\gamma^2}{\sqrt{2}} f^{abc} \frac{1}{\partial^\nu D_\nu} A^c_\mu$$

- With non-zero γ gluon and localizing ghost propagators are

$$\begin{aligned}
\langle A^a_\mu(p) A^b_\nu(-p) \rangle &=& - \frac{\delta^{ab} p^2}{[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle A^a_\mu(p) \bar{\phi}^{bc}_\nu(-p) \rangle &=& - \frac{f^{abc} \gamma^2}{\sqrt{2}[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p) \\
\langle \phi^{ab}_\mu(p) \bar{\phi}^{cd}_\nu(-p) \rangle &=& - \frac{\delta^{ac} \delta^{bd}}{p^2} \eta_{\mu\nu} ~+~ \frac{f^{abe} f^{cde} \gamma^4}{p^2[(p^2)^2 + C_A \gamma^4]} P_{\mu\nu}(p)
\end{aligned}$$

- The presence of the non-zero γ leads to a gluon propagator which is suppressed in the infrared

- Reproduce one loop gap equation by integrating mixed propagator using dimensional regularization in $\overline{\text{MS}}$ scheme
- Two loop $\overline{\text{MS}}$ correction to gap equation (massless quarks) with $s_2 = (2\sqrt{3}/9)\text{Cl}_2(2\pi/3)$

$$
\begin{aligned}
1 \quad = \quad & C_A \left[\frac{5}{8} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right] a \\
& + \left[C_A^2 \left(\frac{2017}{768} - \frac{11097}{2048} s_2 + \frac{95}{256} \zeta(2) - \frac{65}{48} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right. \right. \\
& \qquad \left. + \frac{35}{128} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 + \frac{1137}{2560} \sqrt{5} \zeta(2) - \frac{205 \pi^2}{512} \right) \\
& \quad + C_A T_F N_f \left(- \frac{25}{24} - \zeta(2) + \frac{7}{12} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right. \\
& \quad \left. \left. - \frac{1}{8} \left(\ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right)^2 + \frac{\pi^2}{8} \right) \right] a^2 + O(a^3)
\end{aligned}
$$

- Two loop $\overline{\text{MS}}$ gap equation with massive quarks is also available [Ford & Gracey]
- Kugo-Ojima confinement criterion for Faddeev-Popov ghost form factor holds to two loops when gap equation is satisfied

Static potential

- One property of QCD is that gluons and quarks are confined and the potential between a pair of such fields rises linearly
- Can compute the potential between coloured sources using the Wilson loop either on the lattice or in perturbation theory
- For the latter this is known to three loops in $\overline{\text{MS}}$
- It involves a different type of propagator similar to that used in heavy quark effective theory
- Can extend the perturbative static potential in QCD to that for the Gribov-Zwanziger Lagrangian
- One loop calculation is complete

Definitions

- For the static potential the Wilson loop is defined to be a rectangle of spatial length r and temporal length t
- Due to non-abelian character of QCD the potential requires a path ordering, giving the coordinate space form

$$V(r) \;=\; - \lim_{t \to \infty} \frac{1}{it} \ln \left\langle 0 \left| \mathrm{Tr} \mathcal{P} \exp \left(ig \oint dx^\mu T^a A^a_\mu \right) \right| 0 \right\rangle$$

- Place two static sources at $\pm \frac{1}{2} \mathbf{r}$ then this is equivalent to a partition function with the source term $J^\mu A^a_\mu$ where

$$J^a_\mu(x) \;=\; g v_\mu T^a \left[\delta^{(3)} \left(\mathbf{x} + \frac{1}{2} \mathbf{r} \right) \;-\; \delta^{(3)} \left(\mathbf{x} - \frac{1}{2} \mathbf{r} \right) \right]$$

- The vector v_μ is defined as $v_\mu = \delta_{\mu 0}$
- Potential can be computed in momentum space and Fourier transformed to coordinate space

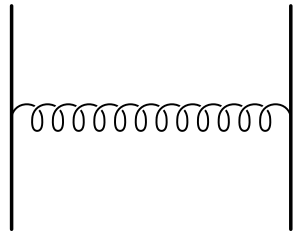
$$V(\mathbf{r}) \;=\; \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i \mathbf{k} . \mathbf{r}} V(\mathbf{k})$$

- Performing the angular integrations this equates to

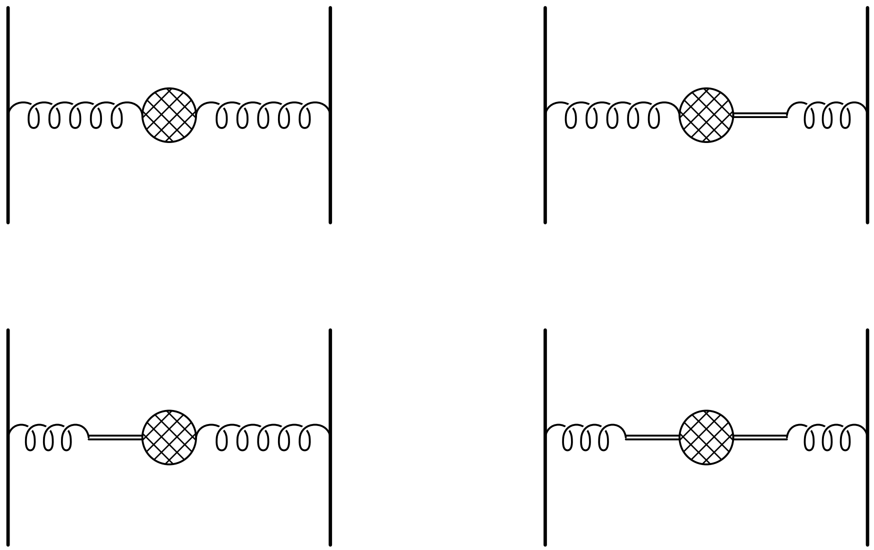
$$V(r) \;=\; \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 V(k) \frac{\sin(kr)}{kr}$$

- The presence of the (heavy) static colour sources modifies the Feynman rules
- The source acquires a propagator $\frac{i}{pv}$ where p is the momentum
- The gluon-source coupling is $igT^a v_\mu$ with all other Feynman rules unaltered
- Previously the canonical QCD Lagrangian has been considered. Now replace with the Gribov-Zwanziger Lagrangian
- The localizing Zwanziger ghost fields are regarded as completely internal and do not couple directly to the sources
- The static potential is a gauge independent object
- In perturbation theory the non-Gribov case has been computed in Feynman gauge originally [Susskind, Fischler, Peter] at various loops
- At two loops the arbitrary gauge result has been determined by Schröder
- As Gribov-Zwanziger is specifically Landau gauge the expression which emerges must agree with arbitary gauge result as $\gamma \to 0$

- There are 40 one loop graphs
- Computed using symbolic manipulation code written in FORM
- At leading order only simple exchange



- At one loop there are propagator corrections (vertex source corrections are also present)



- Full one loop expression is long

- For instance it has terms of the form

$$
\begin{aligned}
V(\mathbf{p}) \quad = \quad & - \frac{C_F \mathbf{p}^2 g^2}{[(\mathbf{p}^2)^2 + C_A \gamma^4]} \\
& - \left[\frac{\sqrt{2}}{\gamma^2} \left[\frac{\sqrt{C_A}}{768} \ln \left[\frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2} \right] \sqrt{ -1 + \sqrt{ 1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2} } } \right. \right. \\
& \qquad - \frac{\sqrt{C_A}}{768} \ln \left[1 + \sqrt{ 1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2} } \right] \sqrt{ -1 + \sqrt{ 1 + \frac{16 C_A \gamma^4}{(\mathbf{p}^2)^2} } } \\
& \qquad \left. \left. - \frac{\sqrt{C_A}}{48 \sqrt{2}} \tan^{-1} \left[\frac{\sqrt{C_A} \gamma^2}{\mathbf{p}^2} \right] + \ldots \right] + \ldots \right] \frac{C_F g^4}{16 \pi^2} \; + \; O(g^6)
\end{aligned}
$$

- Full expression agrees with usual perturbative expression

$$
\begin{aligned}
\lim_{\gamma \to 0} V(\mathbf{p}) \quad = \quad & - \frac{4 \pi C_F \alpha_s(\mu)}{\mathbf{p}^2} \left[1 \; + \; \left[\left[\frac{31}{9} - \frac{11}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] \right] C_A \right. \right. \\
& \left. \left. + \left[\frac{4}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] - \frac{20}{9} \right] T_F N_f \right] a \; + \; O(a^2) \right]
\end{aligned}
$$

- Full potential includes the following terms

$$V(\mathbf{p}) \; = \; + \left[\frac{\pi C_A^{3/2}\gamma^2}{384(\mathbf{p}^2)^2} \; - \; \frac{C_A^{3/2}\gamma^2}{192(\mathbf{p}^2)^2} \tan^{-1} \left[\frac{\sqrt{C_A}\gamma^2}{\mathbf{p}^2} \right] \right] \frac{C_F g^4}{16\pi^2} \; + \; \ldots$$

- Fourier transform of the first term leads to linear potential

$$V(r) \; = \; - \frac{C_F C_A^{3/2}\gamma^2 g^4}{49152\pi^2} r \; + \; \ldots$$

- However confining behaviour as $r \rightarrow 0$ is cancelled by Fourier transform of second term
- Full potential has threshold effect at $p^2 = 2\sqrt{C_A}\gamma^2$
- Similar point to that found by Zwanziger in the physical cut of the spectral density representation of the field strength correlation function at leading order
- Zwanziger suggested it was a potential candidate for a glueball mass

- Next define the V-scheme coupling constant via

$$V(\mathbf{p}) ~=~ - ~ \frac{4\pi C_F \alpha_V(\mathbf{p})}{\mathbf{p}^2}$$

- Since static potential is constructed from gauge invariant Wilson loop, this can be regarded as a gauge independent definition of the strong coupling constant
- At one loop the Gribov-Zwanziger static potential result would imply $\alpha_V(0) = 0$ since

$$\tilde{V}(\mathbf{p}) ~=~ - \frac{C_F \mathbf{p}^2 g^2}{C_A \gamma^4} - C_F \left[\frac{\pi\sqrt{C_A}}{32\gamma^2} + \left(\frac{13}{72} - \frac{3}{8} \ln \left(\frac{C_A \gamma^4}{\mu^4} \right) \right) \frac{\mathbf{p}^2}{\gamma^4} \right] \frac{g^4}{16\pi^2} + O((\mathbf{p}^2)^2; g^6)$$

Power corrections

- Can expand one loop static potential in powers of γ^2/p^2 which is effectively a short distance analysis

$$\tilde{V}(\mathbf{p}) = -\frac{4\pi C_F \alpha_s(\mu)}{\mathbf{p}^2} \times \left[\left[1 - \frac{C_A \gamma^4}{(\mathbf{p}^2)^2} + O\left(\frac{\gamma^8}{(\mathbf{p}^2)^4} \right) \right] + \left[\left[\frac{31}{9} - \frac{11}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] \right] C_A + \left[\frac{4}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] - \frac{20}{9} \right] T_F N_f - \frac{2\pi C_A^{3/2} \gamma^2}{\mathbf{p}^2} + O\left(\frac{\gamma^4}{(\mathbf{p}^2)^2} \right) \right] a + O(a^2) \right]$$

- Strong coupling in the V-scheme in the same short distance expansion gives

$$\alpha_V(\mathbf{p}) = \alpha_V^{\text{pert}}(\mathbf{p}) - \frac{C_A^{3/2} \gamma^2 \alpha_s^2(\mu)}{2\mathbf{p}^2} + O\left(\frac{\gamma^4}{(\mathbf{p}^2)^2} \right)$$

where

$$\alpha_V^{\rm pert}(\mathbf{p}) \;=\; \left[1 \,+\, \left[\left[\frac{31}{9} - \frac{11}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] \right] C_A \,+\, \left[\frac{4}{3} \ln \left[\frac{\mathbf{p}^2}{\mu^2} \right] - \frac{20}{9} \right] T_F N_f \right] a(\mu) \right.$$
$$\left. + \, O(a^2) \right] \alpha_s(\mu)$$

- Power series produces an effective dipole term in static potential, and a quadratic but not quartic correction to an effective strong coupling constant
- Similar correction observed in renormalization group invariant coupling based on the gluon Faddeev-Popov ghost vertex
- Full one loop static potential does *not* have a dipole term
- Seems the short distance expansion is not accessing a linear rising term in potential
- Can see this from analysing Yukawa potential and massive gluon example
- Ignoring gauge invariance issues, take gluon propagator form factor as $1/[p^2 + m^2]$
- Leading order momentum space potential is

$$V(\mathbf{p}) \;=\; - \, \frac{C_F g^2}{[\mathbf{p}^2 + m^2]}$$

- Fourier transform to coordinate space gives

$$V(r) \;=\; -\,\frac{C_F g^2}{4\pi r} e^{-mr}$$

- Expand in powers of $m^2/\mathbf{p}^2$ and mr

$$V(\mathbf{p}) \;=\; -\,\frac{C_F g^2}{\mathbf{p}^2}\left[1+\frac{m^2}{\mathbf{p}^2}+\ldots\right]$$

and

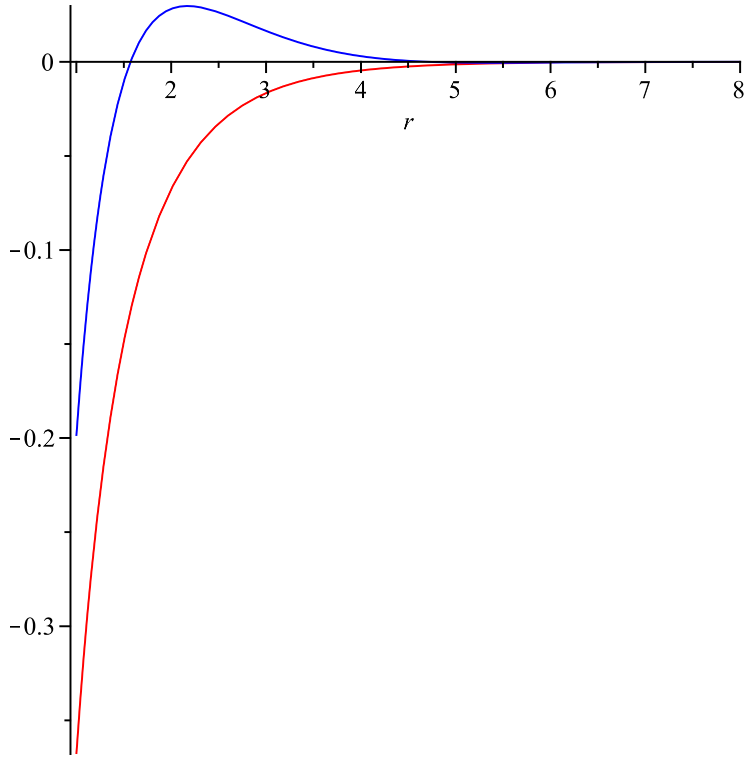
$$V(r) \;=\; -\,\frac{C_F g^2}{4\pi r}\left[1-mr+\frac{m^2 r^2}{2}+\ldots\right]$$

- Looks like a linear potential is present from Fourier transform of power correction to leading term of potential but *full* Yukawa potential does not have a confining piece at large r; one loop static potential in Gribov-Zwanziger Lagrangian has similar structure
- Appears consistent with Zakharov's recent observation that dimension two corrections are eventually subsumed with more accurate computations

- Examine Yukawa potential and leading order static potential in Gribov-Zwanziger Lagrangian schematically
- Fourier transform of single gluon exchange in latter is

$$V(r) \; = \; - \, \frac{C_F g^2}{4\pi r} \exp \left[- \frac{C_A^{1/4} \gamma r}{\sqrt{2}} \right] \cos \left(\frac{C_A^{1/4} \gamma r}{\sqrt{2}} \right)$$

- Graphically



ϕ^{ab}_μ enhancement

- The fact that the Kugo-Ojima criterion is satisfied suggests that the Gribov-Zwanziger Lagrangian describes a confined gluon since the Faddeev-Popov ghost is enhanced
- One loop static potential does not have a dipole due to the absence of the exchange of a single enhanced spin-1 coloured object
- Faddeev-Popov ghost is Grassmann and thus is excluded as an exchange particle
- Recently Zwanziger has analysed the ϕ^{ab}_μ propagator using a Dyson Schwinger approach and shown that ϕ^{ab}_μ enhances
- Also possible to see this in the perturbative set-up
- Requires study of the mixing matrix of 2-point functions in the $\{A^a_\mu, \phi^{ab}_\mu\}$ sector
- Clue in the ϕ^{ab}_μ sector since the Lagrangian kinetic term colour sector enhances

- Matrix of (transverse) 2-point functions in $\{A^a_\mu, \phi^{ab}_\mu\}$ sector to one loop is

$$\Lambda^{\{ab|cd\}} = \begin{pmatrix} \mathcal{X}\delta^{ac} & \mathcal{U}f^{acd} \\ \mathcal{M}f^{cab} & \mathcal{Q}\delta^{ac}\delta^{bd} + \mathcal{W}f^{ace}f^{bde} + \mathcal{R}f^{abe}f^{cde} + \mathcal{S}d_A^{abcd} \end{pmatrix}$$

where, for example, in the zero momentum limit

$$\begin{aligned} \langle \phi^{ab}_\mu(-p)\bar{\phi}^{cd}_\nu(p)\rangle &= \left[\delta^{ac}\delta^{bd}\left[1 - C_A\left(\frac{5}{8} - \frac{3}{8}\ln\left(\frac{C_A\gamma^4}{\mu^4}\right)\right)a\right]p^2 \right. \\ &\quad + \frac{3}{64}f^{ace}f^{bde}p^2 a + \frac{1}{24}f^{abe}f^{cde}p^2 a \\ &\quad \left. + \frac{9}{32}d_A^{abcd}\frac{p^2}{C_A}a + O(a^2)\right] P_{\mu\nu}(p) \\ &\quad + O\left((p^2)^2\right) \end{aligned}$$

and $d_A^{abcd} = \frac{1}{6}\mathrm{Tr}\left(T^a_A T^{(b}_A T^c_A T^{d)}_A\right)$ in terms of the adjoint colour generator, T^a_A

- The key piece is $\mathcal{Q}$ which is effectively the gap equation

- Inverting the full mixing matrix requires the solution of seven algebraic equations
- Have checked the equations reproduce the original propagators and the one loop corrections
- Applying enhancement prior to inversion similar to the Faddeev-Popov ghost case produces the enhanced ϕ^{ab}_μ propagator in the infrared limit

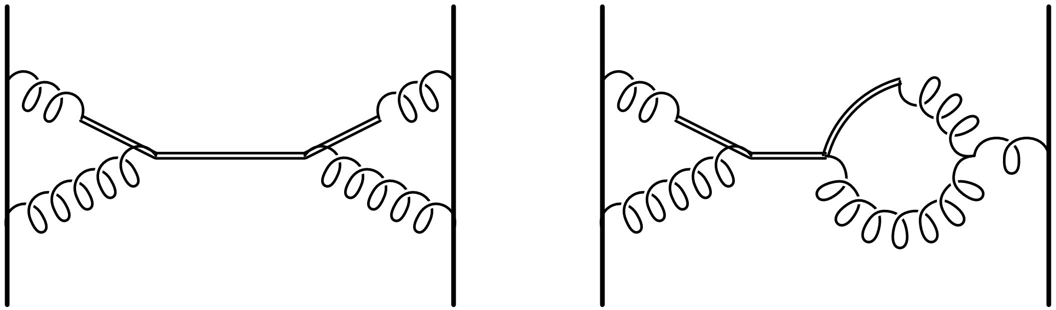
$$\langle \phi^{ab}_\mu(p) \bar{\phi}^{cd}_\nu(-p) \rangle \sim \left[\frac{8\gamma^2}{\pi\sqrt{C_A}(p^2)^2} \delta^{ac}\delta^{bd} - \frac{128\gamma^2}{25\pi C_A^{3/2}(p^2)^2} f^{abe} f^{cde} - \frac{3\gamma^4}{\pi^2 C_A (p^2)^3} f^{ace} f^{bde} - \frac{18\gamma^4}{\pi^2 C_A^2 (p^2)^3} d_A^{abcd} \right] \frac{P_{\mu\nu}(p)}{a}$$

as $p^2 \to 0$

- Propagator colour channels enhance similar to Faddeev-Popov and anti-commuting localizing ghosts
- Over-enhancement in other colour channels, in qualitative agreement with Zwanziger's Dyson Schwinger analysis

Enhancement and static potential

- Can examine the consequences of an enhanced bosonic spin-1 adjoint field in the static potential formalism
- However ϕ^{ab}_μ does not directly couple to coloured sources of formalism
- Indirect single exchange of ϕ^{ab}_μ via graphs of the form



- Second graph is a self-energy correction to mixed propagator
- First graph is indirect exchange of ϕ^{ab}_μ
- Examine the eight contributing Feynman diagrams in the $p^2 \to 0$ limit, with sources in the adjoint

- First with the usual ϕ^{ab}_μ propagator in the $p^2 \to 0$ limit

$$\tilde{V}^{2\text{ loop}}(\mathbf{p}) = \left[\frac{C_A^2 \mathbf{p}^2}{384\gamma^4} + O\left((\mathbf{p}^2)^2\right) \right] \frac{g^6}{(16\pi^2)^2}$$

  which vanishes in zero momentum limit

- Replace ϕ^{ab}_μ propagator with enhanced version to find, as $p^2 \to 0$,

$$\tilde{V}^{2\text{ loop}}(\mathbf{p}) \Big|_{\text{enhanced}} = - \left[\frac{C_A^2}{96\pi^2 \mathbf{p}^2} + O(1) \right] \frac{g^4}{16\pi^2}$$

- The enhancement drops the original expression by two powers of momentum but does not produce a dipole despite the over-enhancement
- There is a reordering of perturbation theory

Static potential and decoupling solution

- Currently decoupling solution appears to be favoured scenario
- Gluon is not suppressed and ghost is not enhanced
- Can access these features using local composite operator (LCO) formalism where the dimension two BRST invariant operator $\bar{\phi}^{ab}_\mu \phi^{ab\,\mu} - \bar{\omega}^{ab}_\mu \omega^{ab\,\mu}$ condenses
- LCO formalism requires coupling the operator to an external source
- Difficult to reconcile LCO approach with, say, role of sources in static potential formalism
- Irrespective of how this proceeds the expected lack of enhancement for ϕ^{ab}_μ would appear to be a problem in producing a dipole structure
- Quantitative example of enhanced ϕ^{ab}_μ in conformal solution, whilst not producing a dipole, does move in that direction

Conclusions

- One loop correction to static potential in Gribov-Zwanziger Lagrangian does not produce a dipole term
- Shape of potential does allow, in principle, for smooth matching onto linearly rising piece
- Alternatively can produce ϕ^{ab}_{μ} enhancement in perturbative approach consistent with Zwanziger's Dyson Schwinger analysis
- Appears to require over-enhancement for a net dipole term to produce a linear potential