Lattice Study of Dense Two Color Matter

Simon Hands (Swansea U.)

- Why two colors?
- Equation of state for $\mu \neq 0$
- Monopoles & Stars

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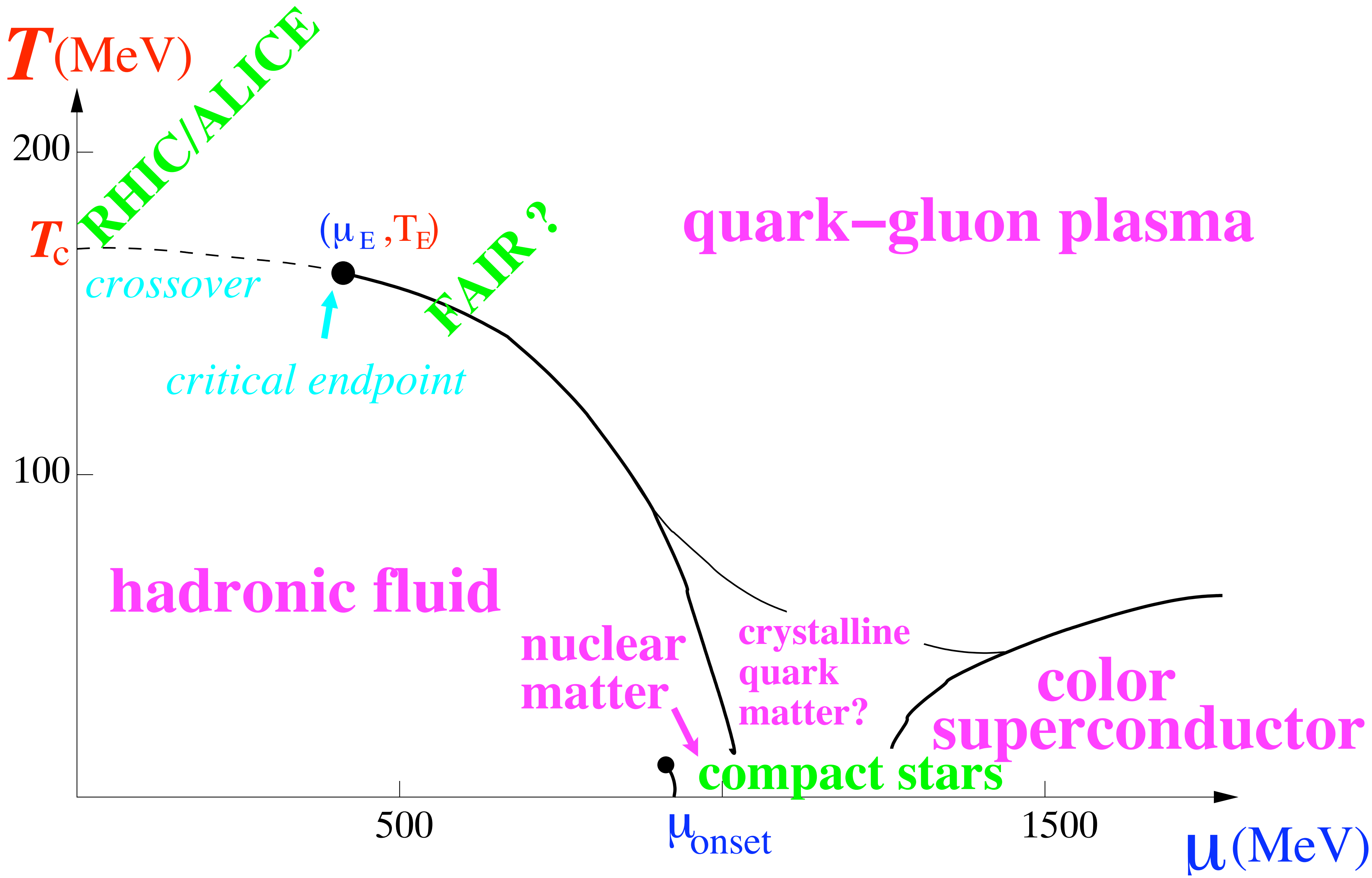
ECT* Trento 10th September 2009

The QCD Phase Diagram



The Sign Problem for $\mu \neq 0$

In Euclidean metric the QCD Lagrangian reads

$$\mathcal{L}_{QCD} = \bar{\psi}(M + m)\psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu}$$

with $M(\mu) = \not{D}[A] + \mu\gamma_0$

Straightforward to show

$$\gamma_5 M(\mu)\gamma_5 \equiv M^\dagger(-\mu) \quad \Rightarrow \quad \det M(\mu) = (\det M(-\mu))^*$$

ie. Path integral measure is not positive definite for $\mu \neq 0$

Fundamental reason is explicit breaking of time reversal symmetry

Monte Carlo importance sampling, the mainstay of lattice QCD, is ineffective

What goes wrong with the usual positive HMC measure?

$$\det M^\dagger M \begin{cases} M & \text{describes} \qquad\qquad\qquad \text{quarks } q \in 3 \\ M^\dagger & \text{describes conjugate quarks } q^c \in \bar{3} \end{cases}$$

In general $\exists\ qq^c$ gauge singlet bound states with $B > 0$

In QCD some qq^c states degenerate with the pion
$\Rightarrow$ unphysical onset of "nuclear matter" at $\mu_o \simeq \frac{1}{2}m_\pi$.

Goldstone baryons: bug for QCD, feature for QC$_2$D...

Calculations with the true complex measure $\det^2 M$ nullify effects of qq^c states for the vacuum with $T = 0$, $\frac{1}{2}m_\pi < \mu \lesssim \frac{1}{3}m_N$ by cancellations among configurations with different signs/phases

The *Silver Blaze* Problem...

QC$_2$D – the large N_c^{-1} limit

QCD with gauge group SU(2) and non-zero quark chemical potential μ has a real functional measure; it remains the *simplest* dense matter system with long-range interactions amenable to study with standard LGT methods.

Since q and $\bar{q}$ live in equivalent reps. of the color group, chiral multiplets contain both $q\bar{q}$ mesons and qq baryons. For $m_\pi \ll m_\rho$ the behaviour as μ is varied can be studied using chiral perturbation theory (χPT)

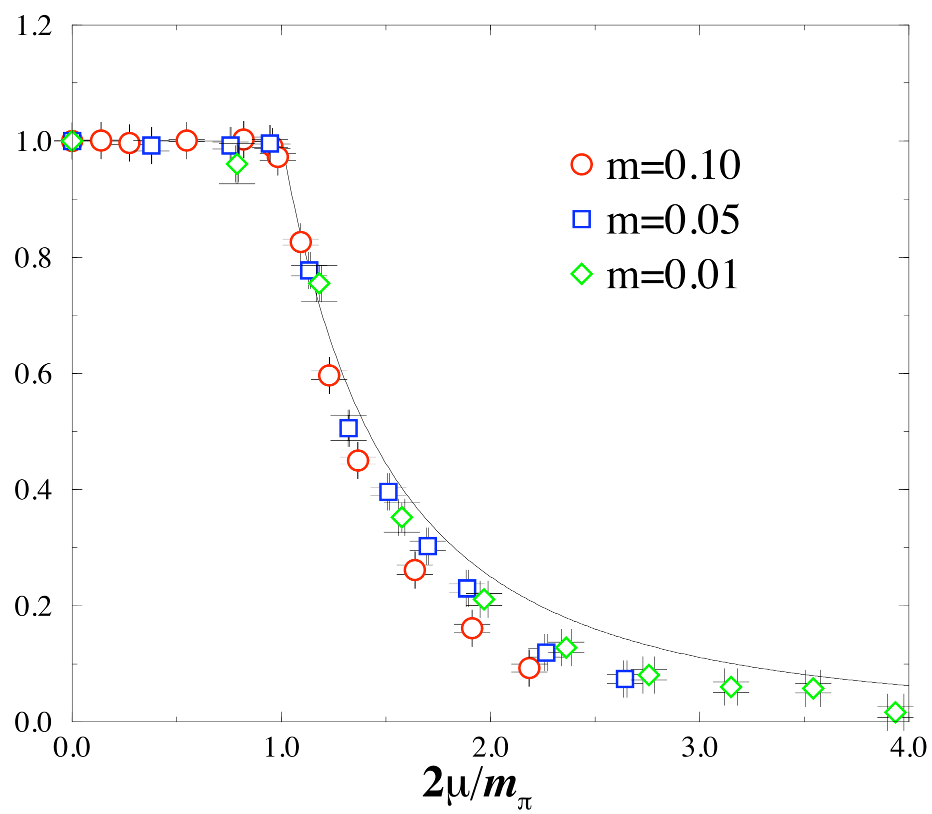
Key result: for $\mu \geq \mu_o = \frac{1}{2}m_\pi$ a baryon charge density develops, $n_q > 0$, along with a gauge invariant superfluid condensate $\langle qq \rangle \neq 0$. For $\mu \gtrsim \mu_o$, the system is a dilute Bose Einstein Condensate (BEC) consisting of weakly interacting scalar qq baryons.

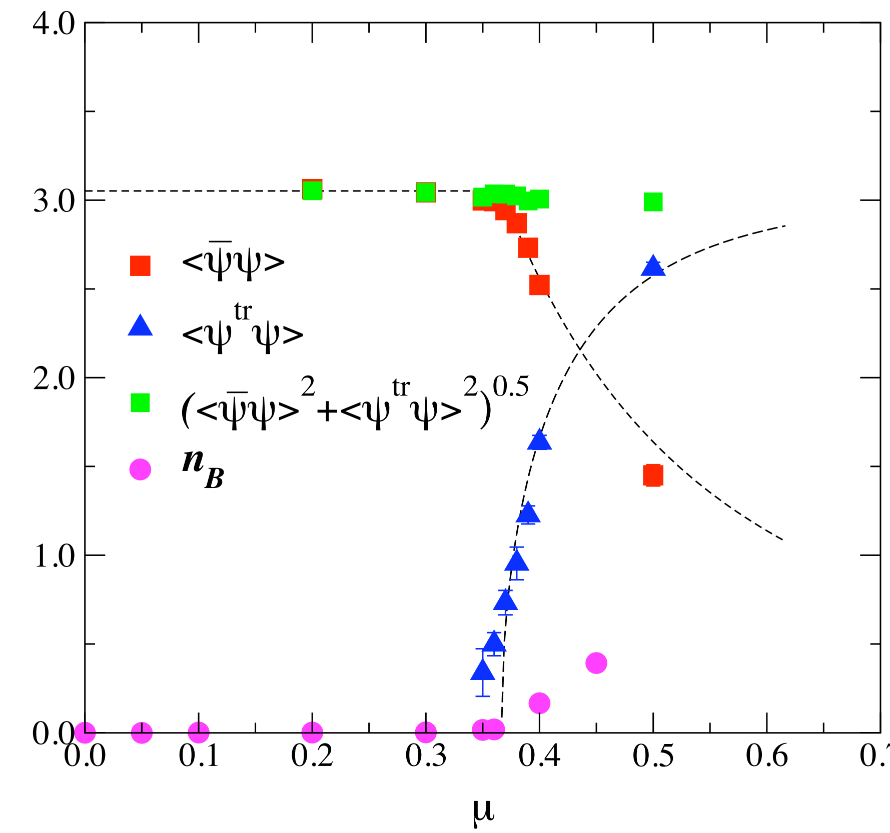
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

confirmed by QC_2D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

Thermodynamics at $T = 0$ from χPT

quark number density $n_{\chi PT} = 8 N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4 N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4 N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

conformal anomaly

$$(T_{\mu\mu})_{\chi PT} = \varepsilon - 3p = 8 N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1 - \frac{\mu_o^4}{\mu^4}}{1 + 3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$

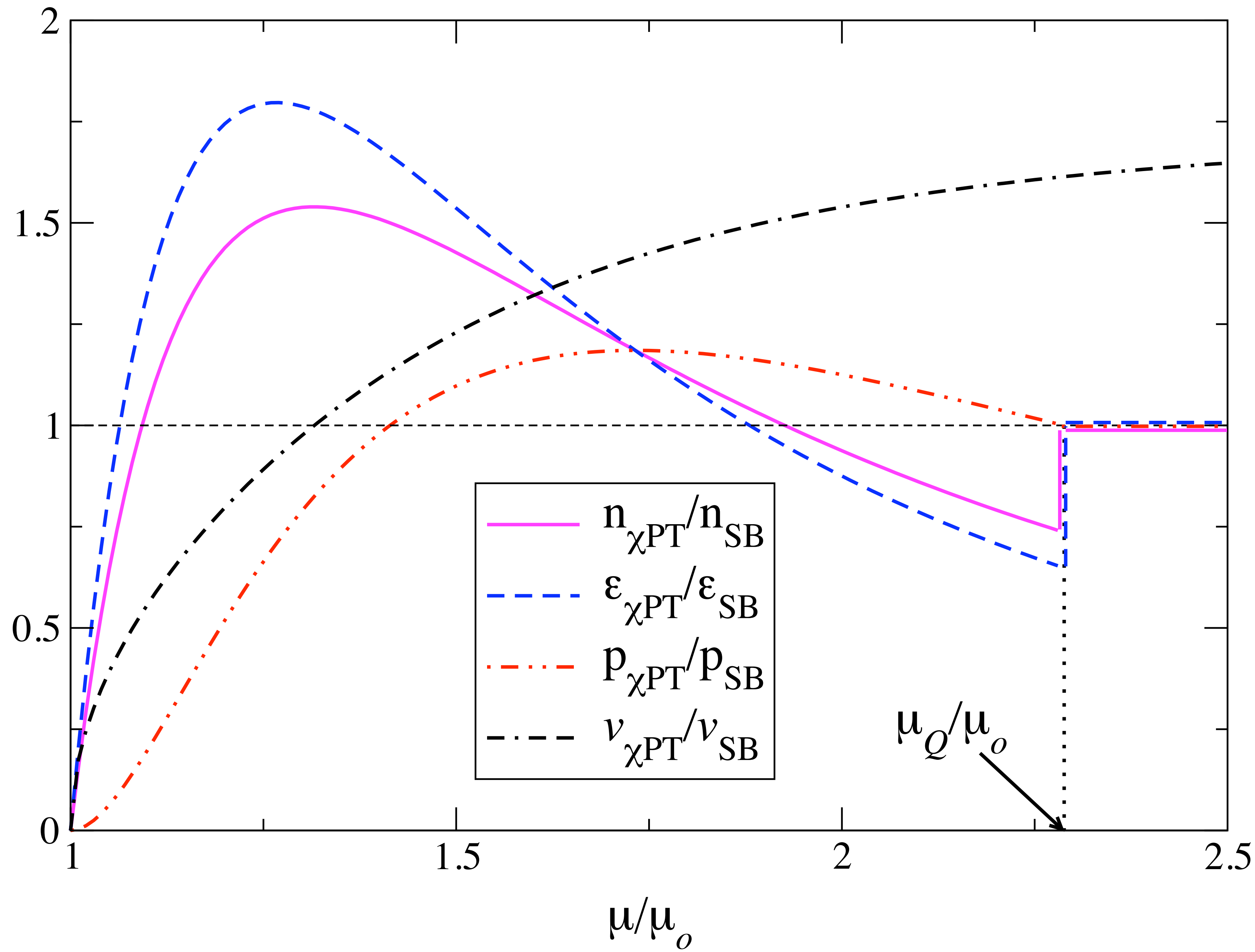
This is to be contrasted with another paradigm for cold dense matter, namely a degenerate system of weakly interacting (deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow \quad n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4;$$

$$\delta_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \quad \langle qq \rangle \propto \Delta\mu^2$$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter;
eg. for $f_\pi^2 = N_c/6\pi^2$, $\mu_d \approx 2.3\mu_o$.

Simulation Details ($N_f = 2$ Wilson flavors)

Initial runs used a $8^3 \times 16$ lattice with parameters $\beta = 1.7$, $\kappa = 0.1780$ (Wilson gauge action)

$\Rightarrow a = 0.23$ fm, $m_\pi a = 0.79(1)$, $m_\pi / m_\rho = 0.779(4)$

Now have data from a matched $12^3 \times 24$ lattice with $\beta = 1.9$, $\kappa = 0.1680$

$\Rightarrow a = 0.18$ fm, $m_\pi a = 0.68(1)$, $m_\pi / m_\rho = 0.80(1)$

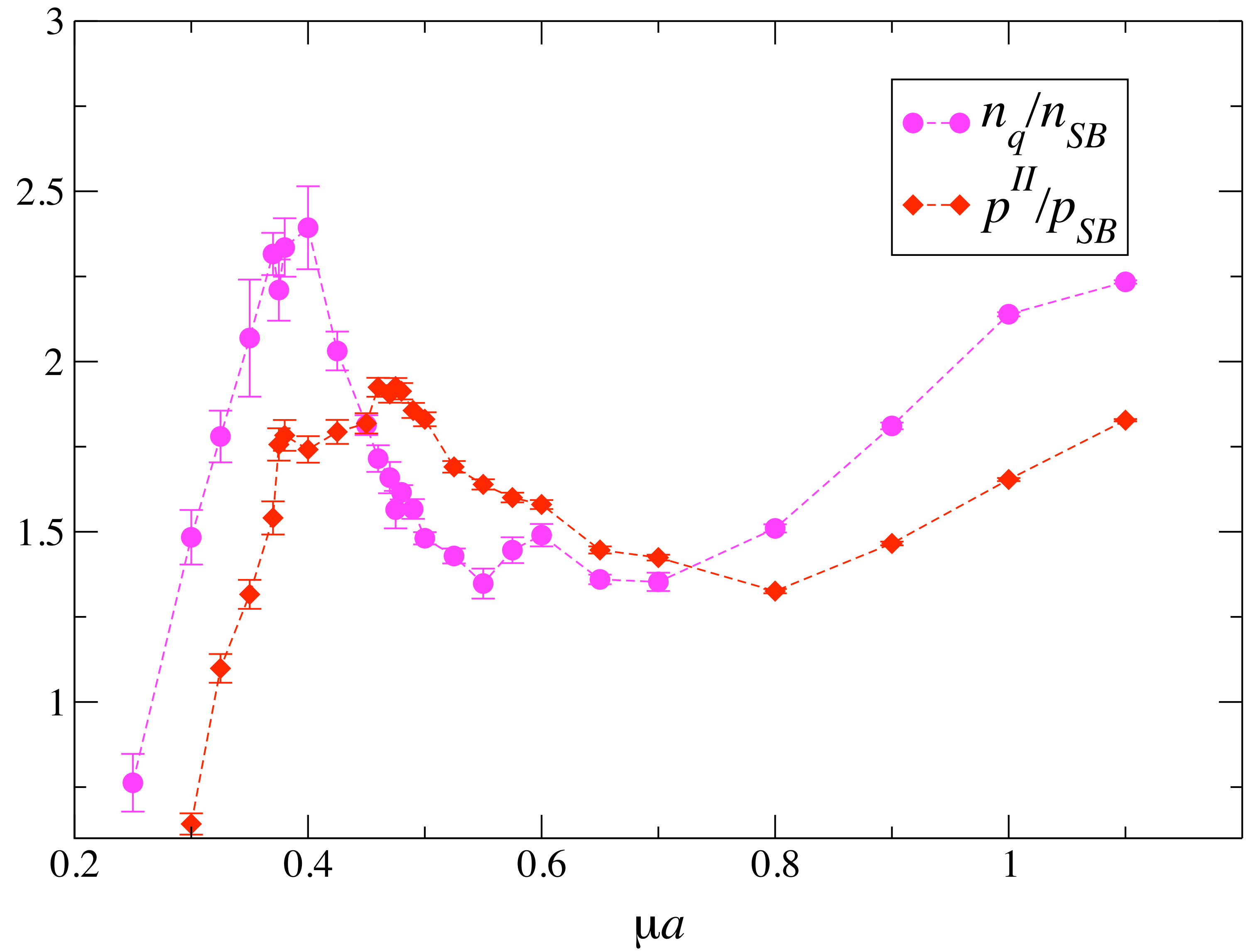
$\Rightarrow T \approx 50$MeV in both cases

To counter IR fluctuations and to maintain ergodicity, we introduce a diquark source $j\kappa(-\bar{\psi}_1 C\gamma_5\tau_2\bar{\psi}_2^{tr} + \psi_2^{tr} C\gamma_5\tau_2\psi_1)$

So far have accumulated roughly 300 trajectories of mean length 0.5 on $8^3 \times 16$ and 500 trajectories on $12^3 \times 24$

SJH, Kim, Skullerud, Eur.Phys. J. C48 (2006)193

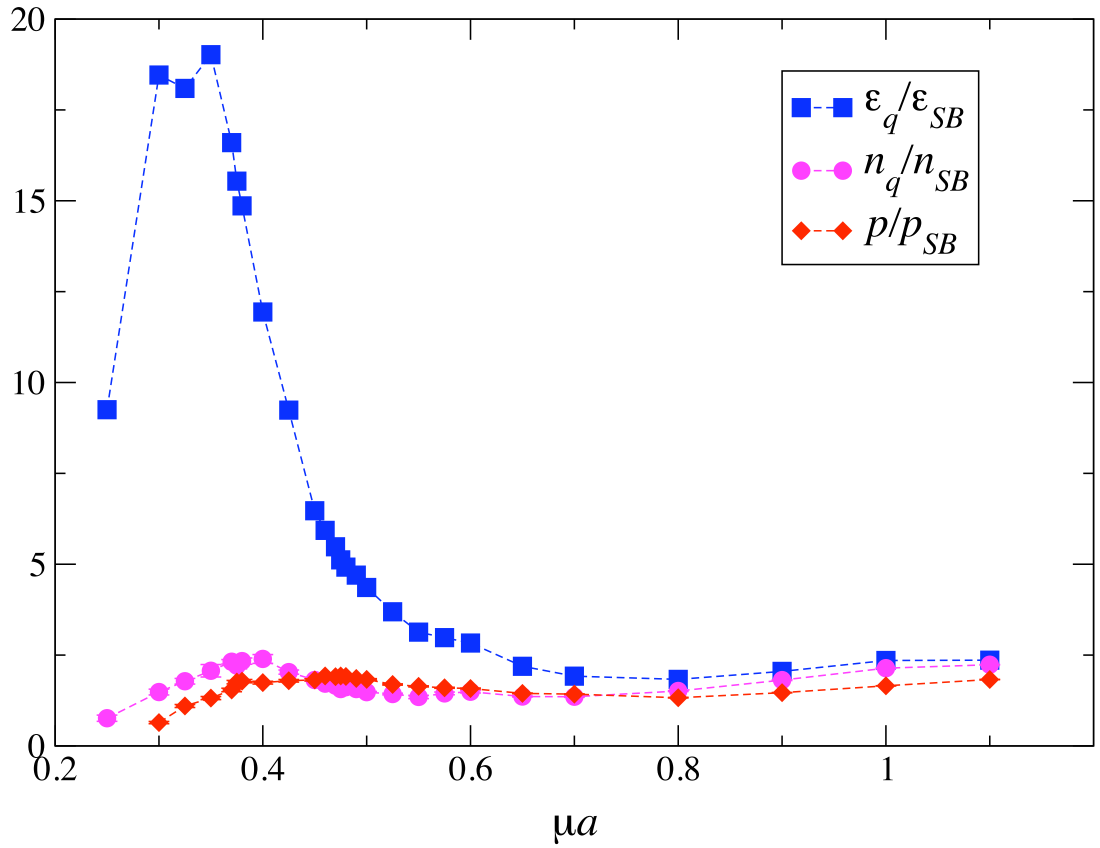
Towards the continuum limit on $12^3 \times 24$...

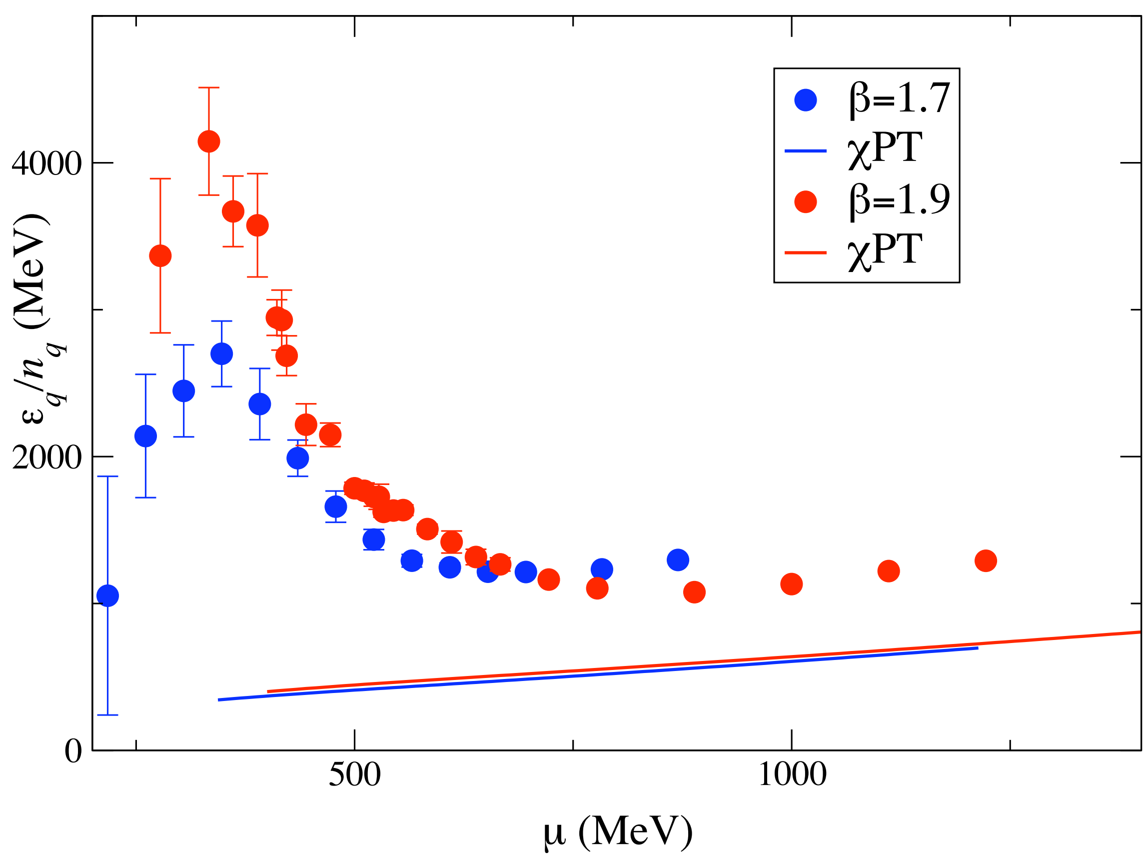


Identify onset transition at $\mu_o a \approx 0.32$ and a transition/crossover to "quark matter" at $\mu_Q a \approx 0.5$

i.e. with $\mu_q \approx 560$ MeV, $n_q \approx 6\,\mathrm{fm}^{-3}$

Quark Energy Density





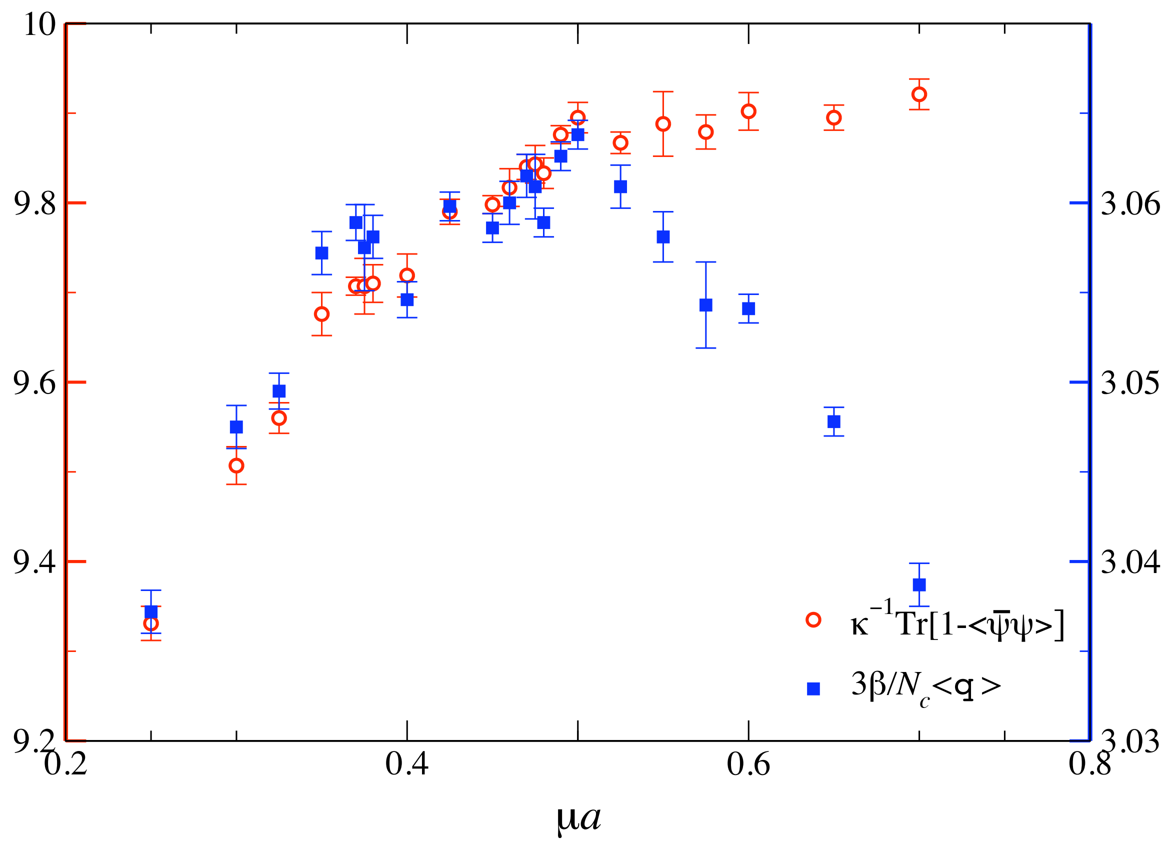
In contrast to χPT prediction, (unrenormalised) quark energy density ε_q greatly exceeds SB value as $\mu \searrow \mu_{o+}$

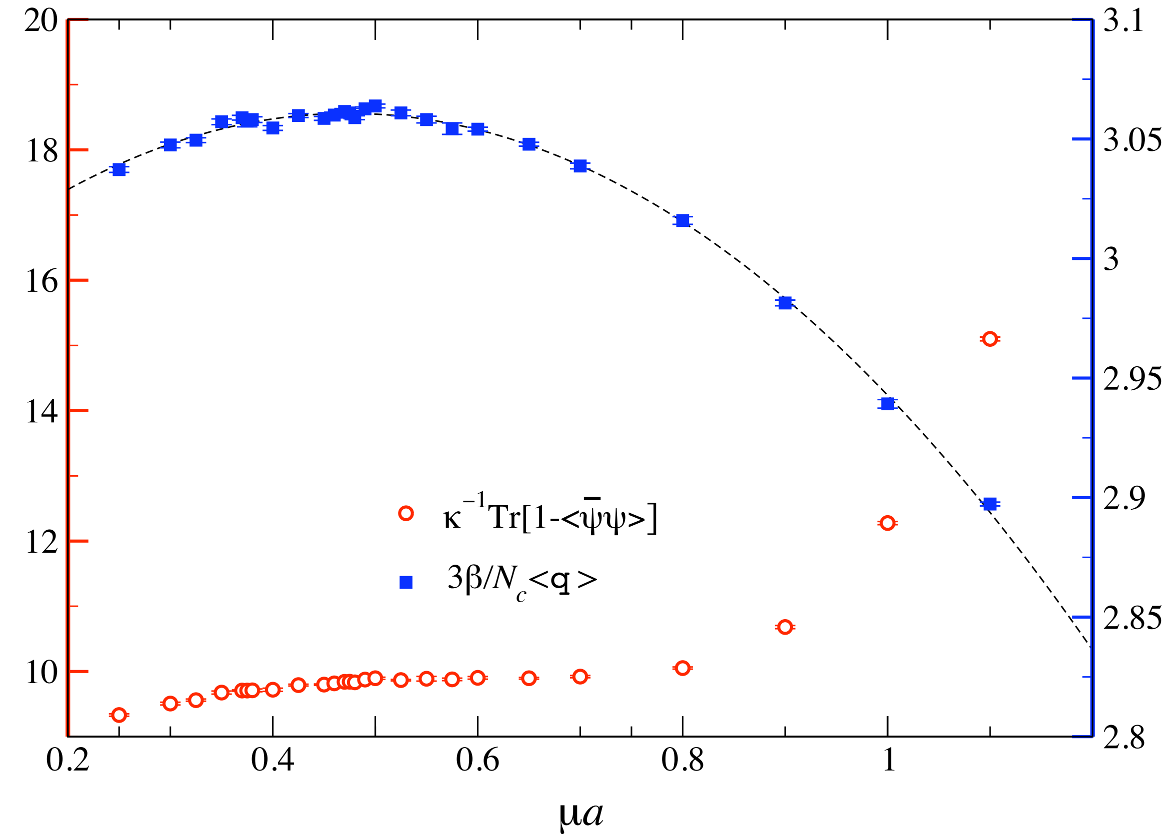
$\Rightarrow$

Energy per quark ε_q/n_q has shallow minimum for $\mu > \mu_Q$

NB in Grand Canonical Ensemble $p \neq 0$ at this minimum

Conformal Anomaly $T_{\mu\mu}$





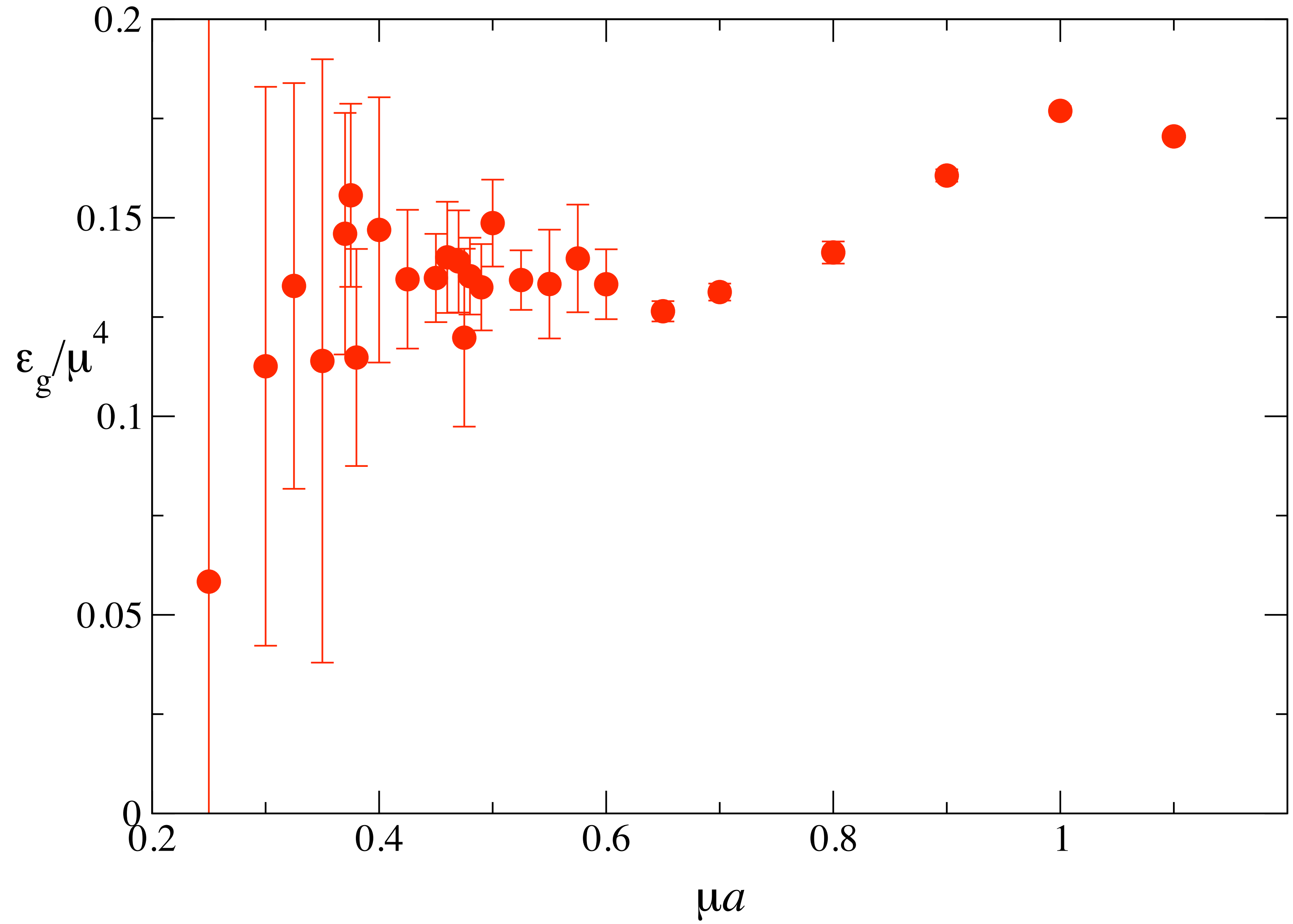
Quark and gluon contributions to $T_{\mu\mu}$ are qualitatively very similar below the transition at μ_Q, but differ above

Note the change in behaviour at $\mu_d a \simeq 0.8$

No recovery of conformal symmetry: $\varepsilon < 3p$ as $\mu \to \infty$

[Cf. Metlitski & Zhitnitsky, NPB731 309 (2005)]

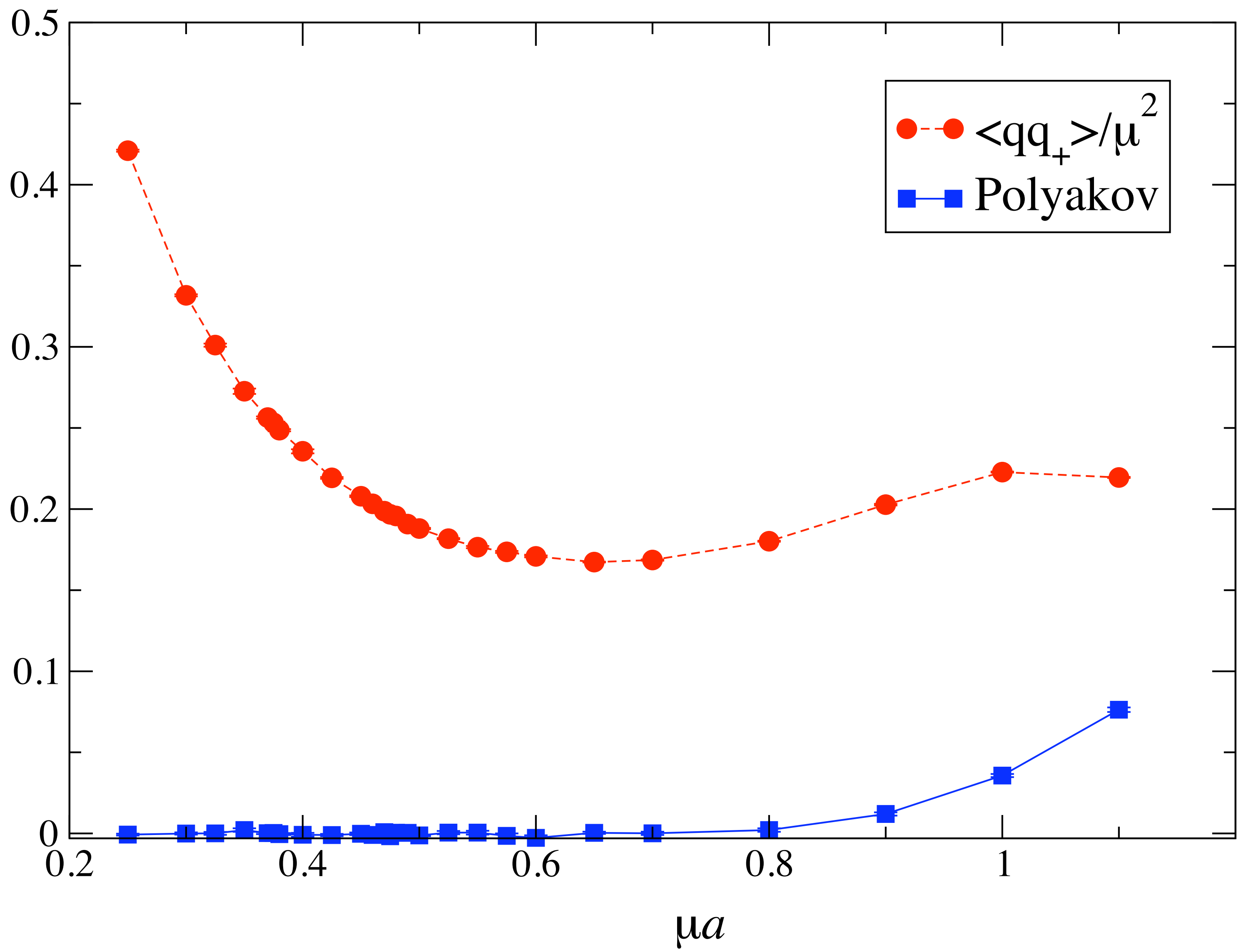
Gluon Energy Density



Gluon energy density $\propto \langle \Box_t - \Box_s \rangle$ scales according to dimensional analysis: ε_g/μ^4 constant over wide range of μ

No singular behaviour at $\mu = \mu_Q$, but perhaps at $\mu = \mu_d$?

Order Parameters



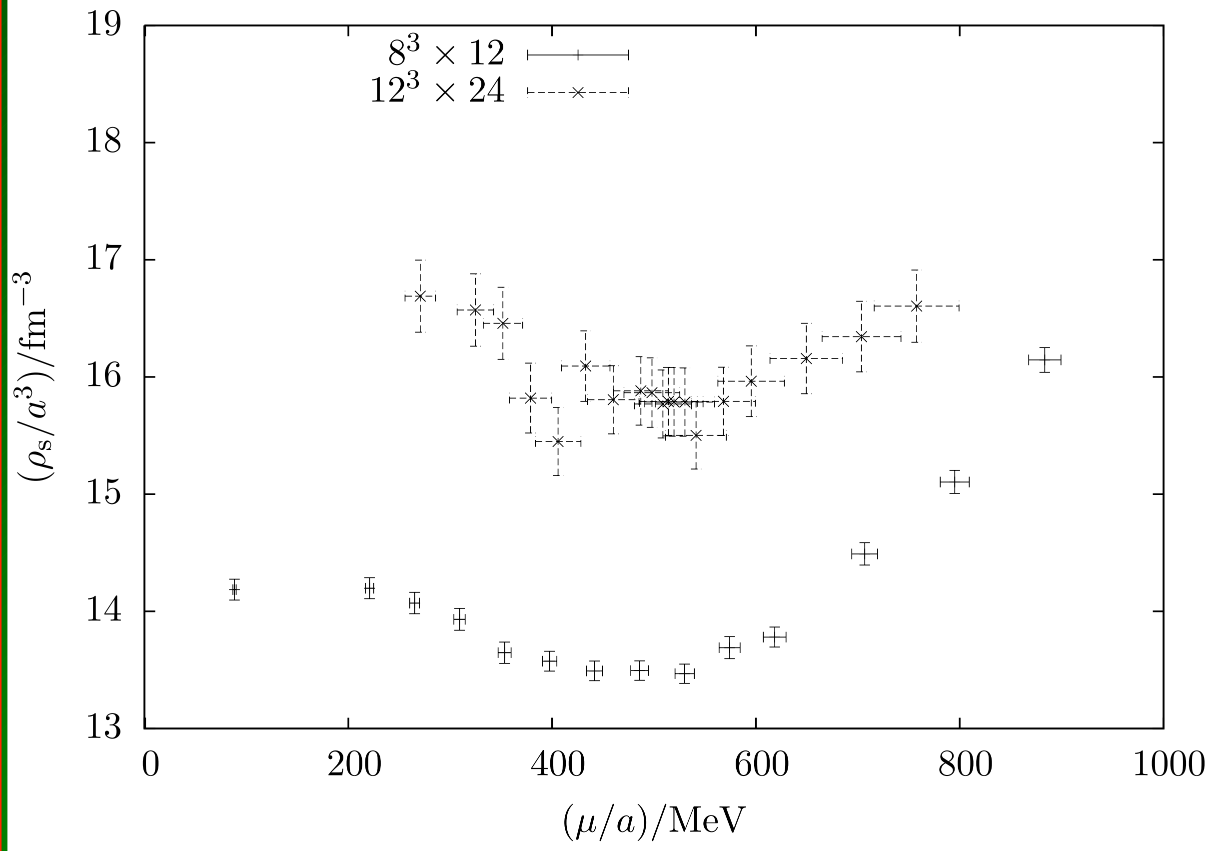
Superfluid condensate scaling ≈ BCS for $\mu_Q \lesssim \mu \lesssim \mu_d$

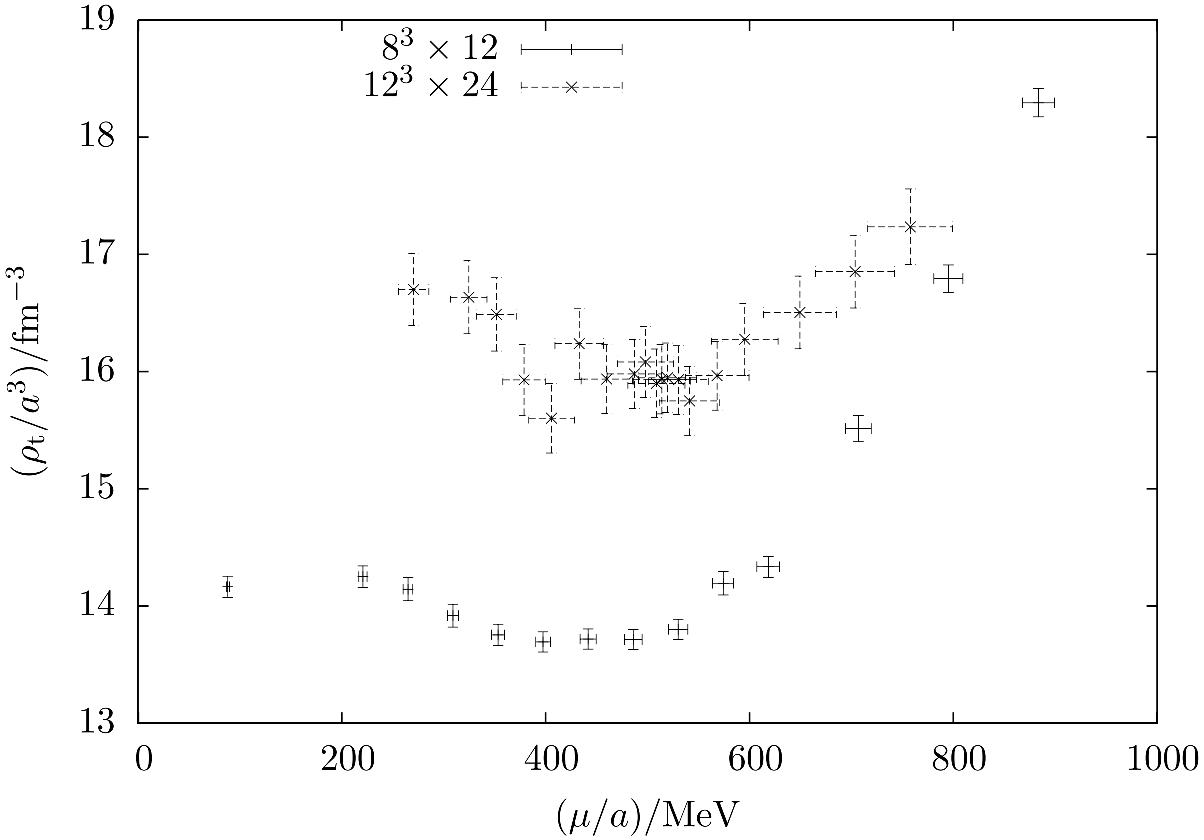
Polyakov loop ≈ 0 for $\mu < \mu_d$, but then rises from zero

⇒ Deconfinement at $\mu \approx 900$MeV, $n_q \approx 35\,\text{fm}^{-3}$

w/ E.M. Ilgenfritz, S. Schubert

Magnetic Monopoles in MAG Gauge

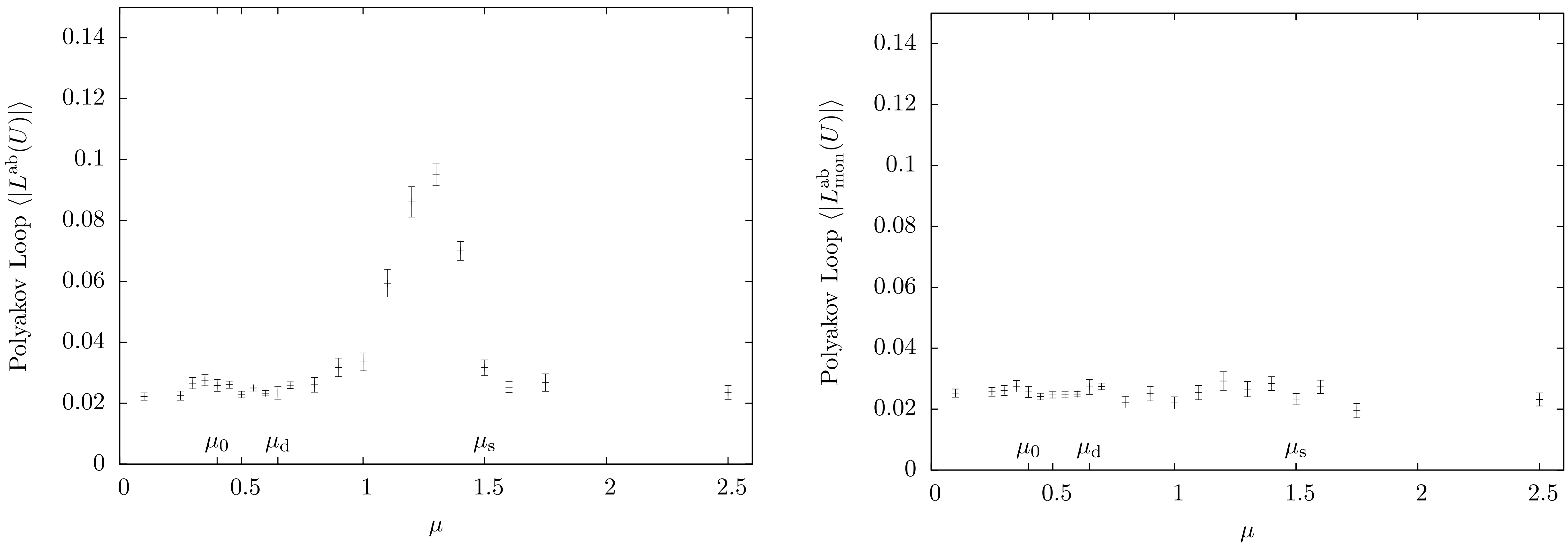




We have studied the distribution of magnetic monopoles found using the DeGrand-Toussaint procedure following abelian projection from the MAG gauge

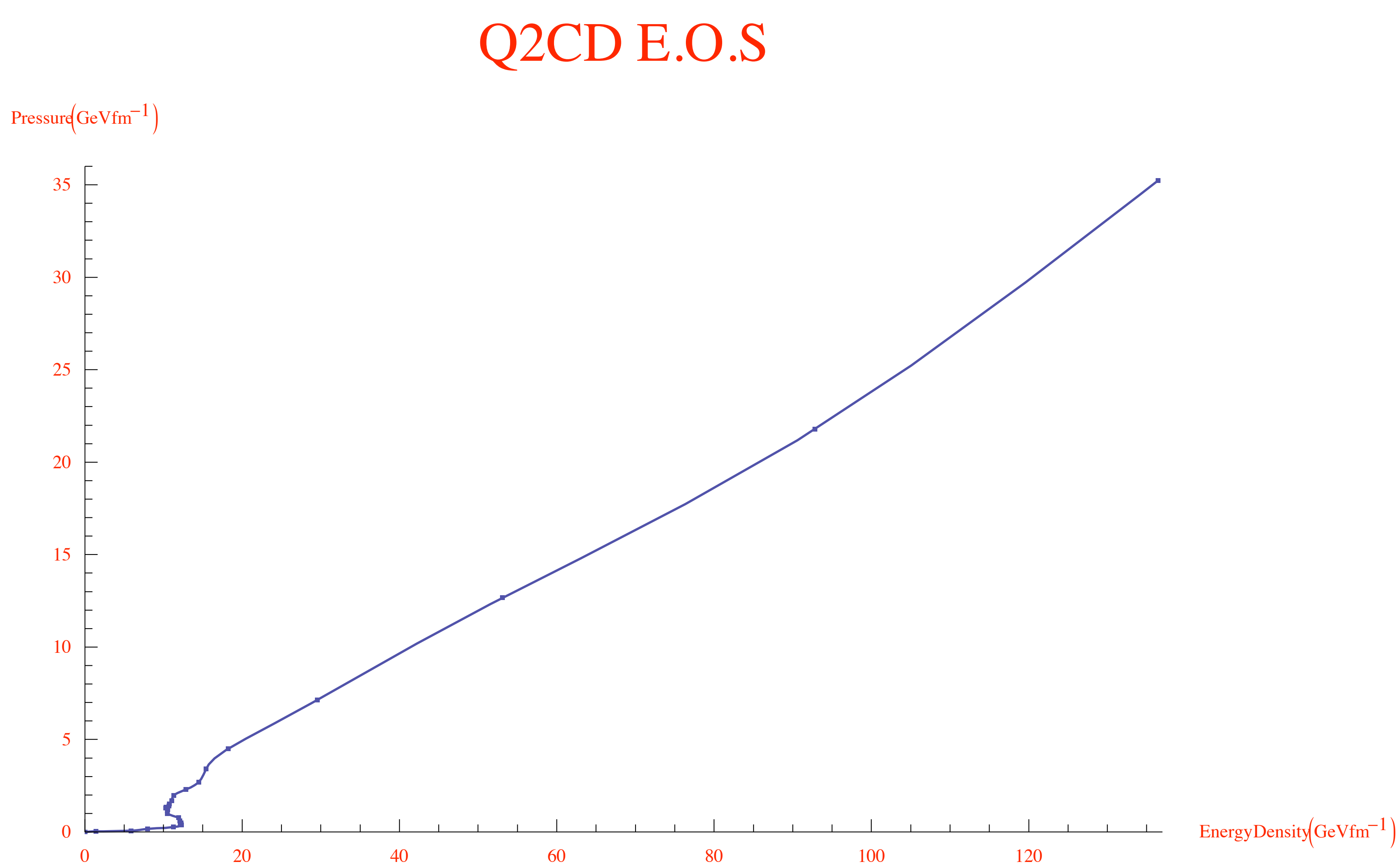
The monopole densities ρ_s and ρ_t are approximately equal, and show structure both at $\mu \approx \mu_o$ and $\mu \approx \mu_{QM}$

The Polyakov Loop is Non-Abelian?



After abelian projection we can decompose the abelian link fields as $\theta_\mu = \theta_\mu^{\rm mon} + \theta_\mu^{\rm phot}$ with $\theta_{\mu x}^{\rm mon} \equiv -2\pi \sum_y (\partial^+ \partial^-)^{-1}_{xy} m_{\mu y}$

The abelian Polyakov loop on $8^3 \times 16$ is about 25% of the full loop, and remarkably, the signal completely vanishes in the $\{\theta^{\rm mon}\}$ configuration, showing that "deconfinement" is NOT due to abelian monopoles in this case



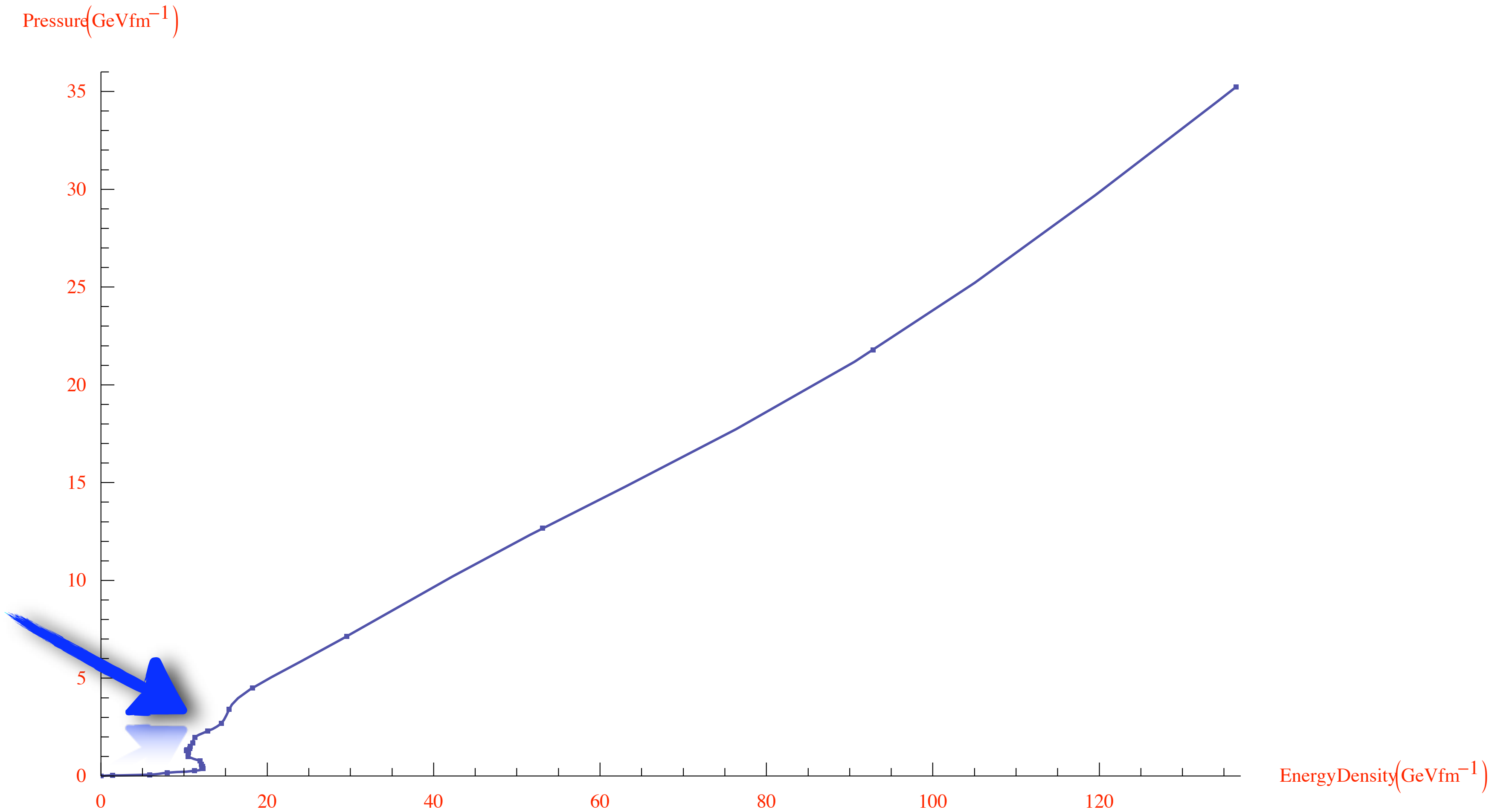
We can also use the EoS in the form ε(p) as input to the Tolman-Oppenheimer-Volkoff equations

$$\frac{dp}{dr} = -\frac{(p+\varepsilon(p))(M(r)+4\pi r^3 p)}{r(r-2M)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

G=c=1

Q2CD E.O.S



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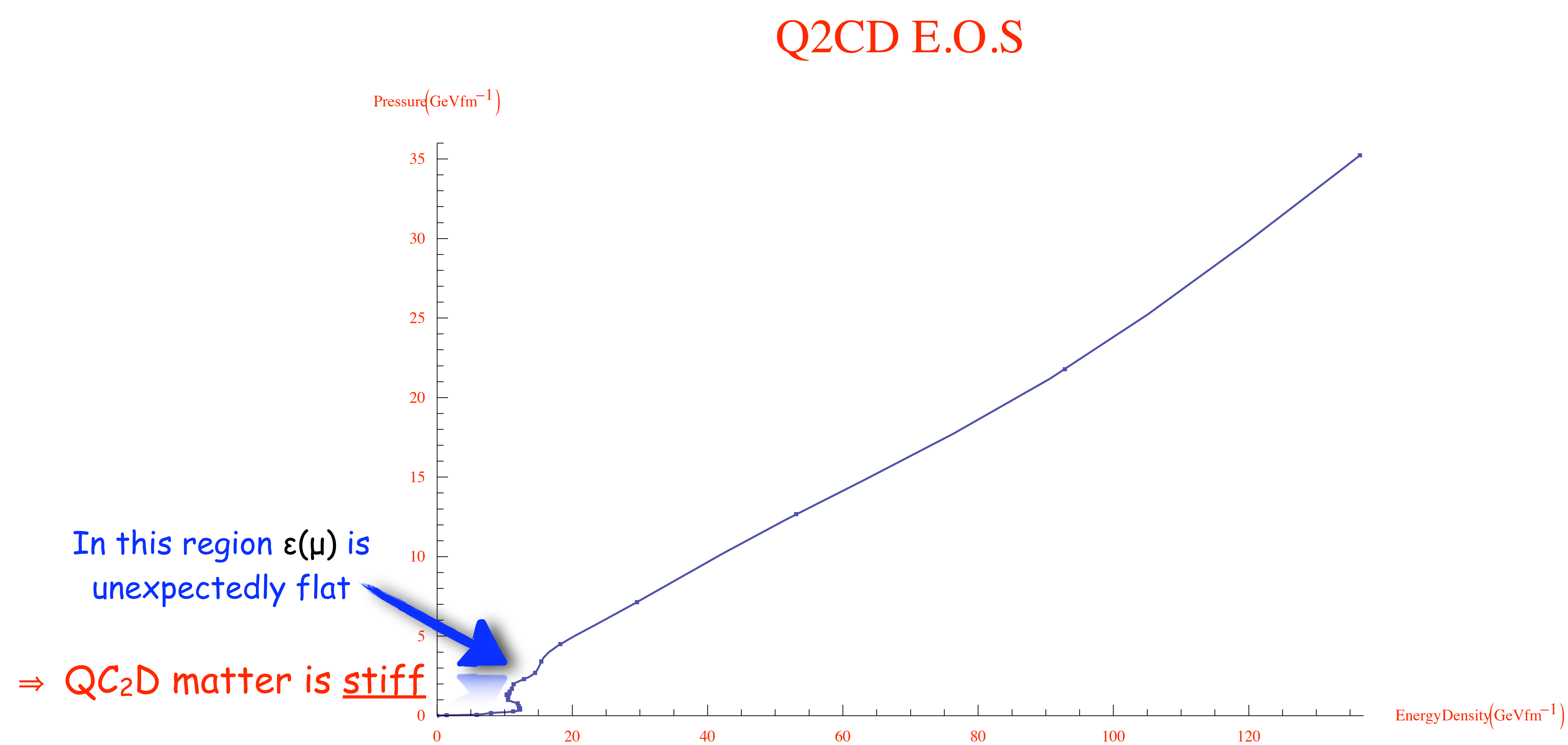

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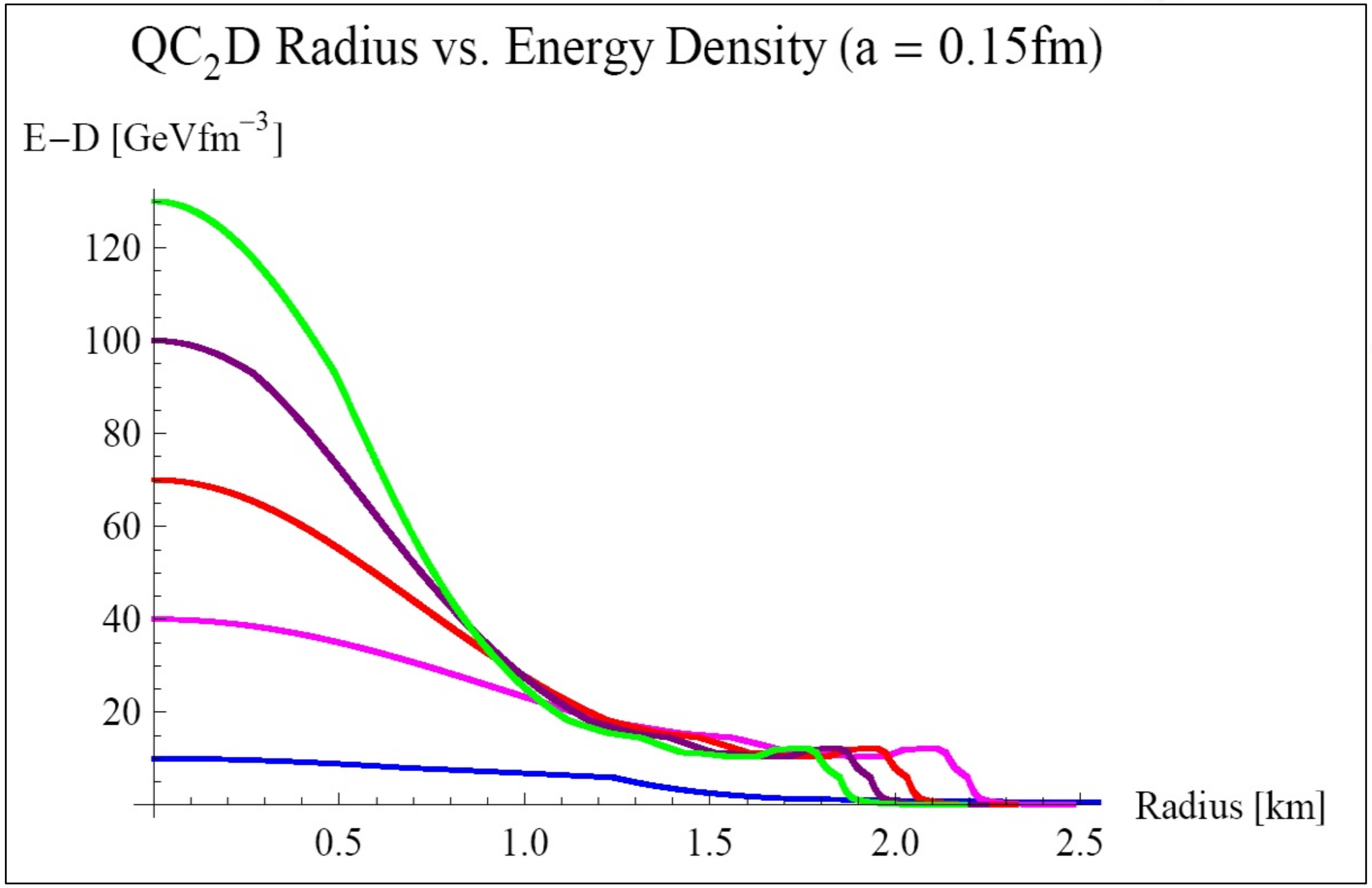
A Star is born?

w/ James Tonkin



Solution of TOV enables modelling of "neutron stars"

M_{max}=0.8M$_\odot$; R =3.3km;

ε_c=12GeVfm^{-3} (a=0.17fm)

need Karsch coefficient for definitive answer...

Summary

★ dense QC_2D has three distinct transitions/ crossovers at $\mu_o<\mu_Q<\mu_d$:

- ✳ Vacuum for $\mu<\mu_o$
- ✳ BEC for $\mu_o<\mu<\mu_Q$
- ✳ Small N_c "Quarkyonic" phase for $\mu_Q<\mu<\mu_d$
- ✳ Deconfined phase for $\mu>\mu_d$

★ It's deconfinement, Jim! but not as we know it....

Maybe a lattice artifact?

★ Time for `arXiv:astro-lat` ?