Numerical test of the Gribov-Zwanziger scenario in Landau gauge

Tereza Mendes

IFSC, University of São Paulo

Work in collaboration with Attilio Cucchieri

Summary

- Lattice study of IR gluon and ghost propagators is a crucial test of the Gribov-Zwanziger confinement scenario: suppressed gluon and enhanced ghost at small momenta. Note: finite-volume effects become an issue as the infrared limit is approached.
- We introduce rigorous upper and lower bounds to control the infinite-volume extrapolation. At a result, we gain a better understanding of the propagators in terms of more general quantities.
- We consider Landau gauge and SU(2) gauge group, using data from the largest lattices to date.
- We also consider the $\beta = 0$ case (strong coupling).

Pathways to Confinement

- How does linearly rising potential (seen in lattice QCD) come about?
- Theories of quark confinement include:
  dual superconductivity (electric flux tube connecting magnetic monopoles), condensation of center vortices, but also merons, calorons
- Proposal by Mandelstam (1979) linking linear potential to infrared behavior of gluon propagator as $1/p^4$
- Gribov-Zwanziger (similarly Kugo-Ojima) confinement scenario based on suppressed gluon propagator and enhanced ghost propagator in the infrared

Ghost Enhancement (I)

Ghost-enhanced scenario natural in Coulomb gauge.

Since $(\partial_i A_i)^a = 0$, the color-electric field is decomposed as $E_i^{tr} - \partial_i \phi(\vec{x}, t)$ and the classical (non-Abelian) Gauss's law

$$(D_i E_i)^a(\vec{x}, t) \;=\; \rho^a_{quark}(\vec{x}, t)$$

is written for a color-coulomb potential in terms of Faddeev-Popov operator: $\mathcal{M}\phi^a(\vec{x}, t) \;=\; \rho^a(\vec{x}, t)$, where $\mathcal{M} = -D_i \partial_i$. In "momentum space"

$$\phi^a(\vec{x}, t) \;\approx\; \int d^3p \int d^3y \; G(\vec{p}, t) \exp[i\vec{p} \cdot (\vec{x} - \vec{y})] \; \rho^a(\vec{y}, t)$$

IR divergence of ghost propagator as $1/p^4$ leads to linearly rising potential.

Ghost Enhancement (II)

Gribov's restriction beyond quantization using Faddeev-Popov (FP) method implies taking a minimal gauge, defined by a minimizing functional in terms of gauge fields and gauge transformation.

$\Rightarrow$ FP operator (second variation of functional) has non-negative eigenvalues. First Gribov horizon approached in infinite-volume limit, implying ghost enhancement.

Note: in principle no obvious connection with Kugo-Ojima confinement scenario; see Kondo, arXiv:09044897, Aguilar et al., arXiv:0907.0153.

IR gluon propagator and confinement

- Green's functions carry all information of a QFT's physical and mathematical structure.
- Gluon propagator (two-point function) as the most basic quantity of QCD.
- Confinement given by behavior at large distances (small momenta) $\Rightarrow$ nonperturbative study of IR gluon propagator.

Landau gluon propagator

$$
\begin{aligned}
D^{ab}_{\mu\nu}(p) &= \sum_x e^{-2i\pi k\cdot x} \langle A^a_\mu(x)\, A^b_\nu(0)\rangle \\
&= \delta^{ab}\left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2}\right) D(p^2)
\end{aligned}
$$

Early simulations: Mandula & Ogilvie, PLB 1987

IR ghost propagator and confinement

Ghost fields are introduced as one evaluates functional integrals by the Faddeev-Popov method, which restricts the space of configurations through a gauge-fixing condition. The ghosts are unphysical particles, since they correspond to anti-commuting fields with spin zero.
On the lattice, the (minimal) Landau gauge is imposed as a minimization problem and the ghost propagator is given by

$$G(p) \, = \, \frac{1}{N_c^2 - 1} \sum_{x,\, y,\, a} \frac{e^{-2\pi i \, k \cdot (x-y)}}{V} \, \langle \, \mathcal{M}^{-1}(a,x;a,y) \, \rangle \; ,$$

where the Faddeev-Popov (FP) matrix $\mathcal{M}$ is obtained from the second variation of the minimizing functional.

Early simulations: Suman & Schilling, PLB 1996; Cucchieri, NPB 1997

Gribov-Zwanziger Confinement Scenario

- The Gribov-Zwanziger confinement scenario in Landau gauge predicts a gluon propagator $D(p^2)$ suppressed in the IR limit.
- In particular, $D(0) = 0$ implying that reflection positivity is maximally violated.
- This result may be viewed as an indication of gluon confinement.
- Infinite volume favors configurations on the first Gribov horizon, where λ_{min} of $\mathcal{M}$ goes to zero.
- In turn, $G(p)$ should be IR enhanced, introducing long-range effects, related to the color-confinement mechanism.

Tests of Gribov-Zwanziger Scenario

Above results are also obtained by functional methods (e.g. solution of DSEs by Alkofer et al.)

$$D(p^2) \sim (p^2)^{2\kappa - 1} , \qquad G(p^2) \sim (p^2)^{-\kappa - 1}$$

Note: other solutions by Aguilar et al., Boucaud et al.

Tests of Gribov-Zwanziger Scenario

Above results are also obtained by functional methods (e.g. solution of DSEs by Alkofer et al.)

$$D(p^2) \sim (p^2)^{2\kappa - 1} \, , \qquad G(p^2) \sim (p^2)^{-\kappa - 1}$$

Note: other solutions by Aguilar et al., Boucaud et al.

What about lattice simulations?

Tests of Gribov-Zwanziger Scenario

Above results are also obtained by functional methods (e.g. solution of DSEs by Alkofer et al.)

$$D(p^2) \sim (p^2)^{2\kappa-1} , \qquad G(p^2) \sim (p^2)^{-\kappa-1}$$

Note: other solutions by Aguilar et al., Boucaud et al.

What about lattice simulations?

- Gluon propagator is suppressed in the limit $p \to 0$
- $\lambda_{min} \to 0$ with the volume
- On "small" lattices: could fit to $D(0) \to 0$, $G(p)$ showed enhancement.

Tests of Gribov-Zwanziger Scenario

Above results are also obtained by functional methods (e.g. solution of DSEs by Alkofer et al.)

$$D(p^2) \sim (p^2)^{2\kappa - 1} \, , \qquad G(p^2) \sim (p^2)^{-\kappa - 1}$$

Note: other solutions by Aguilar et al., Boucaud et al.

What about lattice simulations?

- Gluon propagator is suppressed in the limit $p \to 0$
- $\lambda_{min} \to 0$ with the volume
- On "small" lattices: could fit to $D(0) \to 0$, $G(p)$ showed enhancement.

After 2007:

- From data on very large lattices one sees that $D(0) > 0$
- On very large lattices $G(p)$ shows no enhancement in the IR

References

Studies on very large lattices presented by three groups at the Lattice 2007 Conference (PoS Lat2007)

- Bogolubsky et al. (Berlin): 80^4 lattices (13 fm), SU(3)
- Sternbeck et al. (Adelaide): 112^4 lattices (19 fm), SU(2)
- Cucchieri, T.M.: 128^4 lattices (27 fm), SU(2) plus 3d SU(2) case with 320^3 (85 fm)

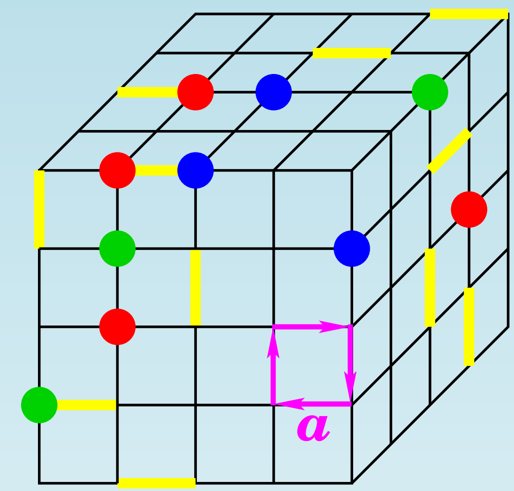
(possibly triggered by Fischer et al., Annals Phys. 2007)

Just before

- Scaling behavior seen on 2d lattice (A. Maas, Phys. Rev. D 2007)
- SU(2) & SU(3) are equivalent in the IR (Cucchieri et al., Phys. Rev. D 2007)

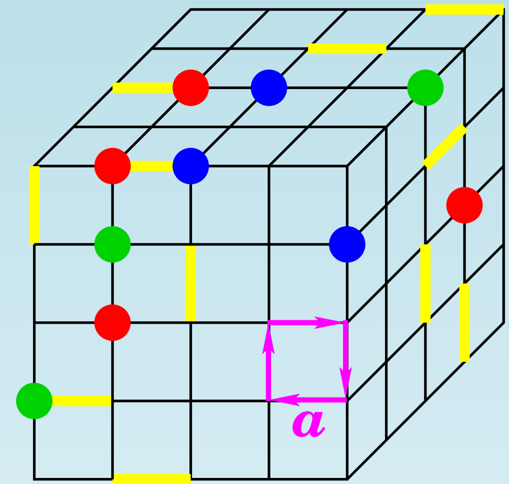
Latttice QCD

1. Quantization by path integrals $\Rightarrow$ sum over configurations with “weights” $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a$ $\Rightarrow$ regularization



Latttice QCD

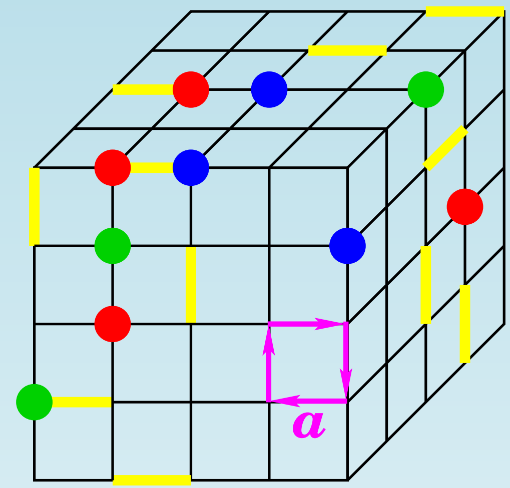
1. Quantization by path integrals $\Rightarrow$ sum over configurations with "weights" $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a$ $\Rightarrow$ regularization



Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

Latttice QCD

1. Quantization by path integrals $\Rightarrow$ sum over configurations with "weights" $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a$ $\Rightarrow$ regularization



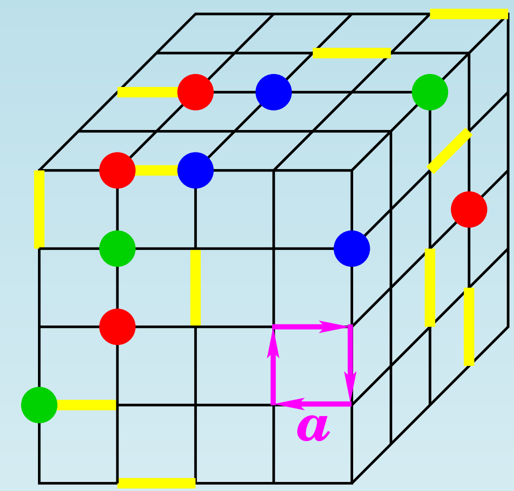
Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

The Wilson action

$$S \;=\; -\frac{\beta}{3}\sum_{\square} \mathrm{ReTr}\, U_{\square}\,, \quad U_{x,\mu} \;\equiv\; e^{i g_0 a A^b_\mu(x) T_b}\,, \quad \beta \;=\; 6/{g_0}^2$$

Latttice QCD

1. Quantization by path integrals $\Rightarrow$ sum over configurations with “weights” $e^{i\,S/\hbar}$
2. Euclidean formulation (analytic continuation to imaginary time) $\Rightarrow$ weight becomes $e^{-S/\hbar}$
3. Discrete space-time $\Rightarrow$ UV cut at momenta $p \lesssim 1/a \Rightarrow$ regularization



Also: finite-size lattices $\Rightarrow$ IR cut for small momenta $p \approx 1/L$

The Wilson action

$$S \;=\; -\frac{\beta}{3} \sum_{\square} \mathrm{ReTr}\, U_{\square}\,, \quad U_{x,\mu} \;\equiv\; e^{i g_0 a A^b_\mu(x) T_b}\,, \quad \beta \;=\; 6/{g_0}^2$$

$V(R) \sim \sigma R$ from small β expansion

In general: Monte Carlo simulations + ∞-vol limit + $a \to 0$

Lattice Features

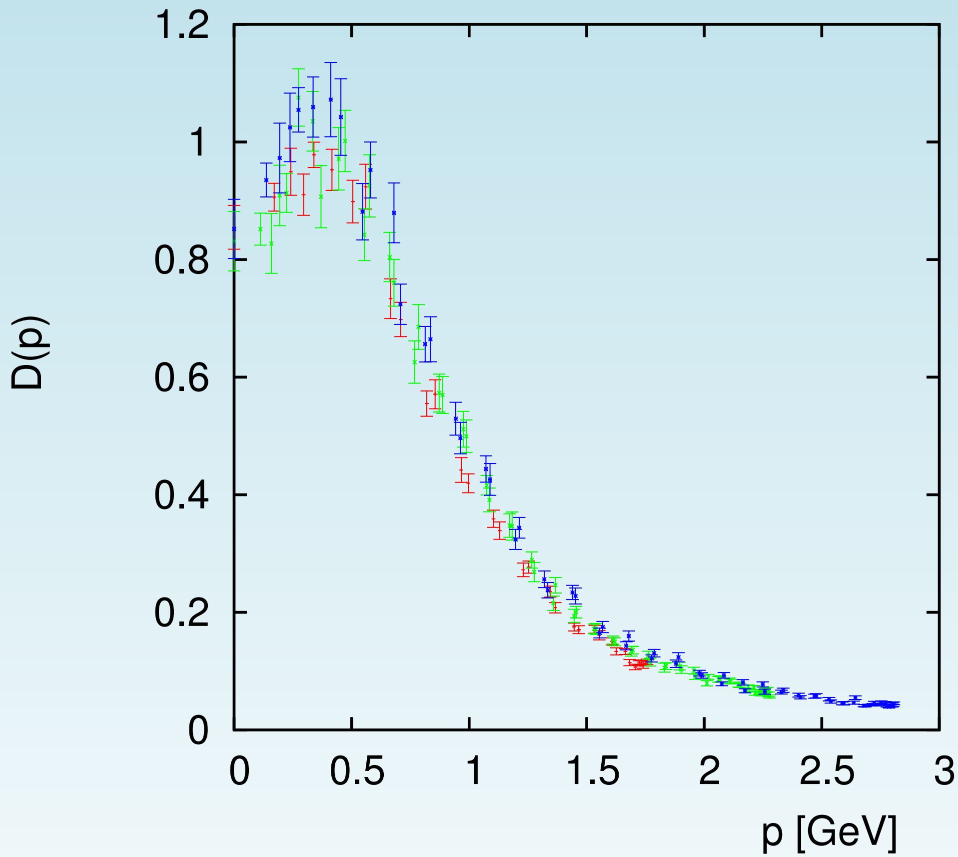
- Gauge action written in terms of oriented plaquettes formed by the link variables $U_{x,\mu}$, which are group elements
- under gauge transformations: $U_{x,\mu} \to g(x)\, U_{x,\mu}\, g^{\dagger}(x+\mu)$, where $g \in SU(3)$ $\Rightarrow$ closed loops are gauge-invariant quantities
- integration volume is finite: no need for gauge-fixing
- when gauge fixing, procedure is incorporated in the simulation, no need to consider FP matrix
- get FP matrix without considering ghost fields explicitly
- Lattice momenta given by $\hat{p}_\mu = 2\,\sin\left(\pi\, n_\mu/N\right)$ with $n_\mu = 0, 1, \ldots, N/2$ $\Leftrightarrow$ $p_{min} \sim 2\pi/(a\, N) = 2\pi/L$, $p_{max} = 4/a$ in physical units

Bounds and Results

for the Gluon Propagator

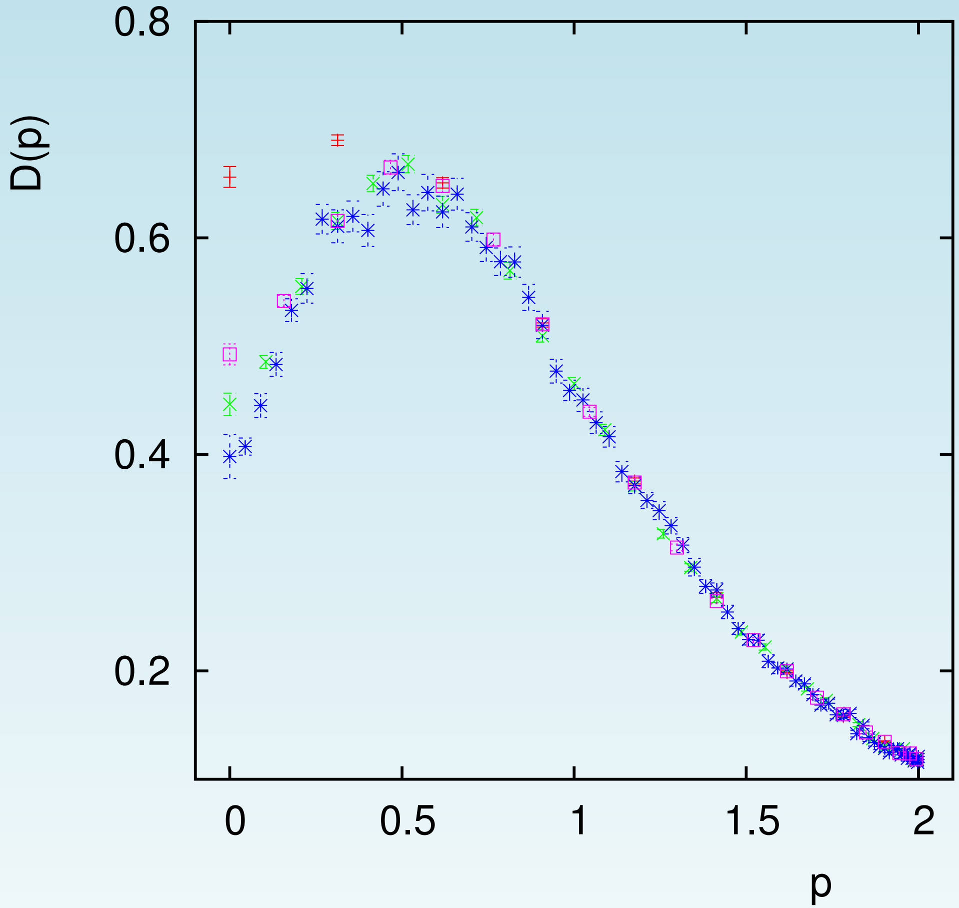
(A. Cucchieri, T.M., PoS LATTICE2007 and Phys. Rev. Lett. 2008)

Infinite-volume limit in 3d (I)



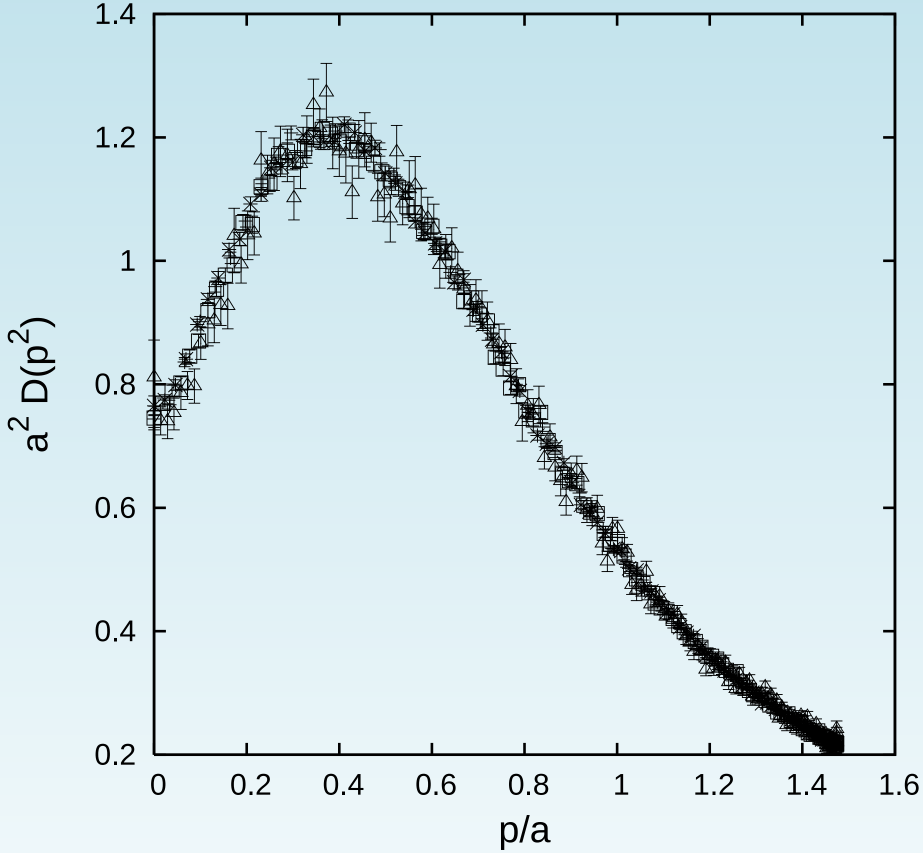
Gluon propagator as a function of the lattice momentum p for $\beta = 3.4$ and 32^3 (+), $\beta = 4.2$ and 64^3 ($\times$), $\beta = 5.0$ and 64^3 ($*$) (A. Cucchieri, Phys. Rev. D60 034508, 1999).

Infinite-volume limit in 3d (II)



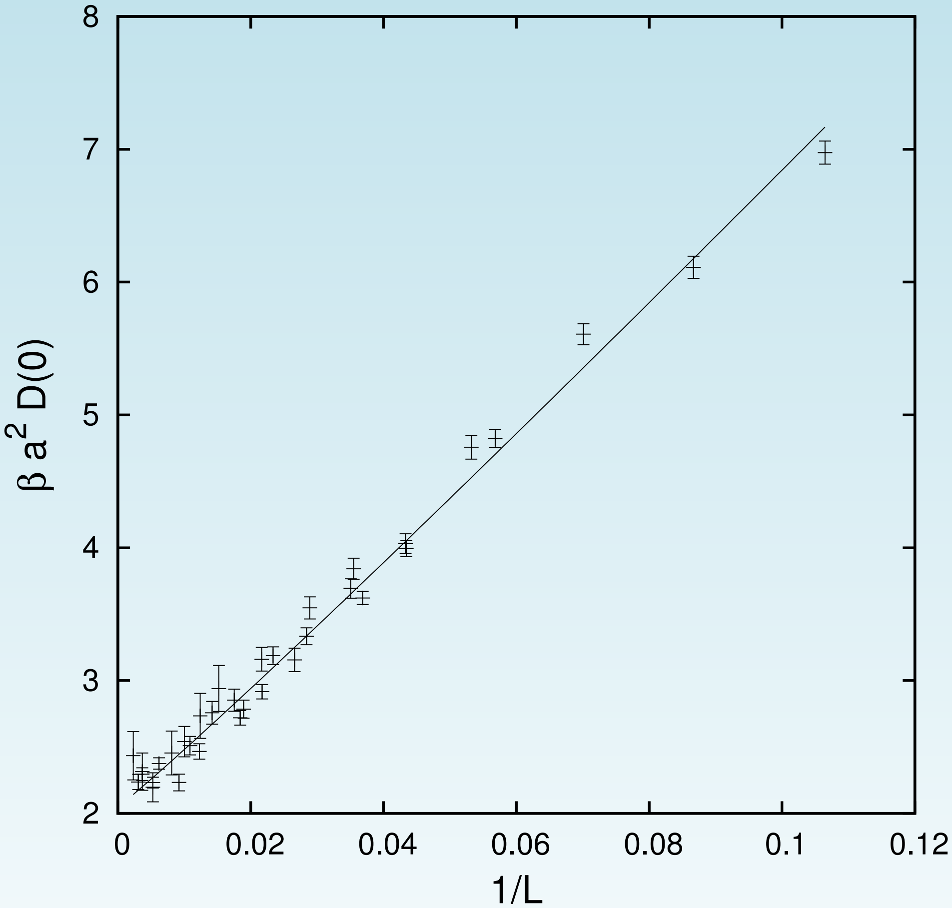
Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3, 60^3 and 140^3 at $\beta = 3.0$ (A. Cucchieri, T. M. and A. Taurines, Phys. Rev. D67 091502, 2003).

Infinite-volume limit in 3d (III)



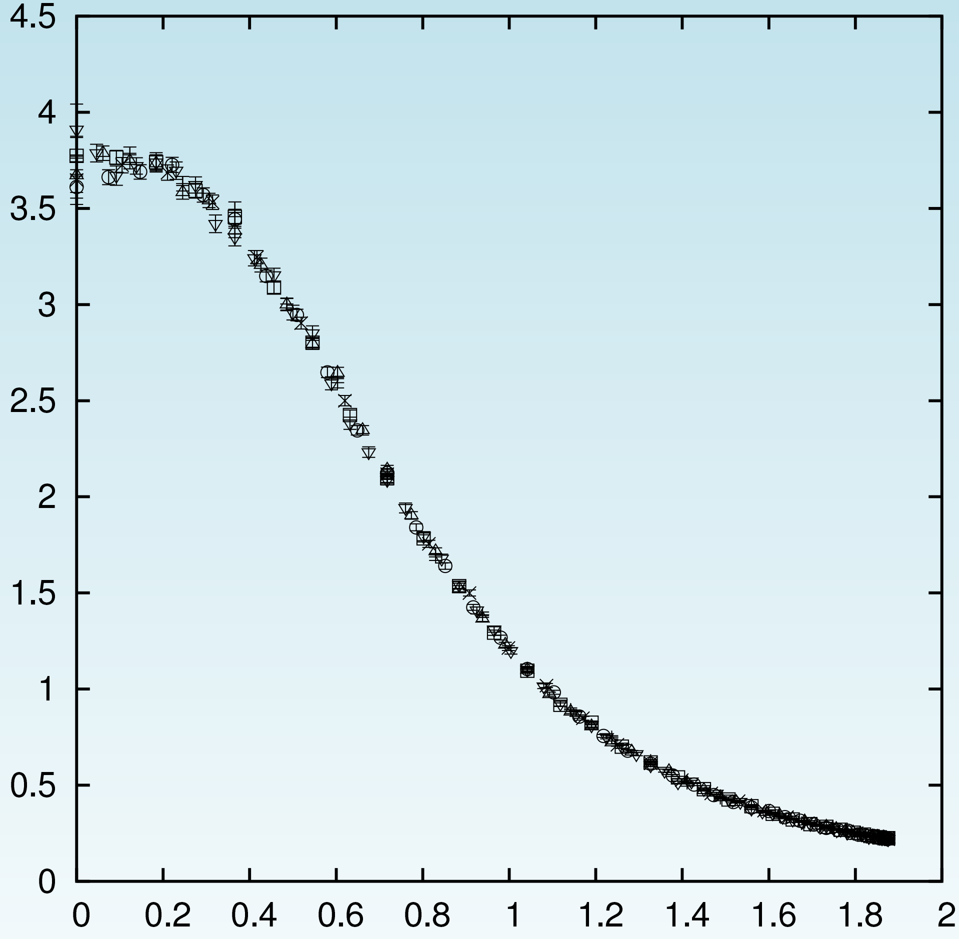
Gluon propagator as a function of the lattice momentum p including lattices up to 320^3 in the scaling region.

Infinite-volume limit in 3d: $D(0)$



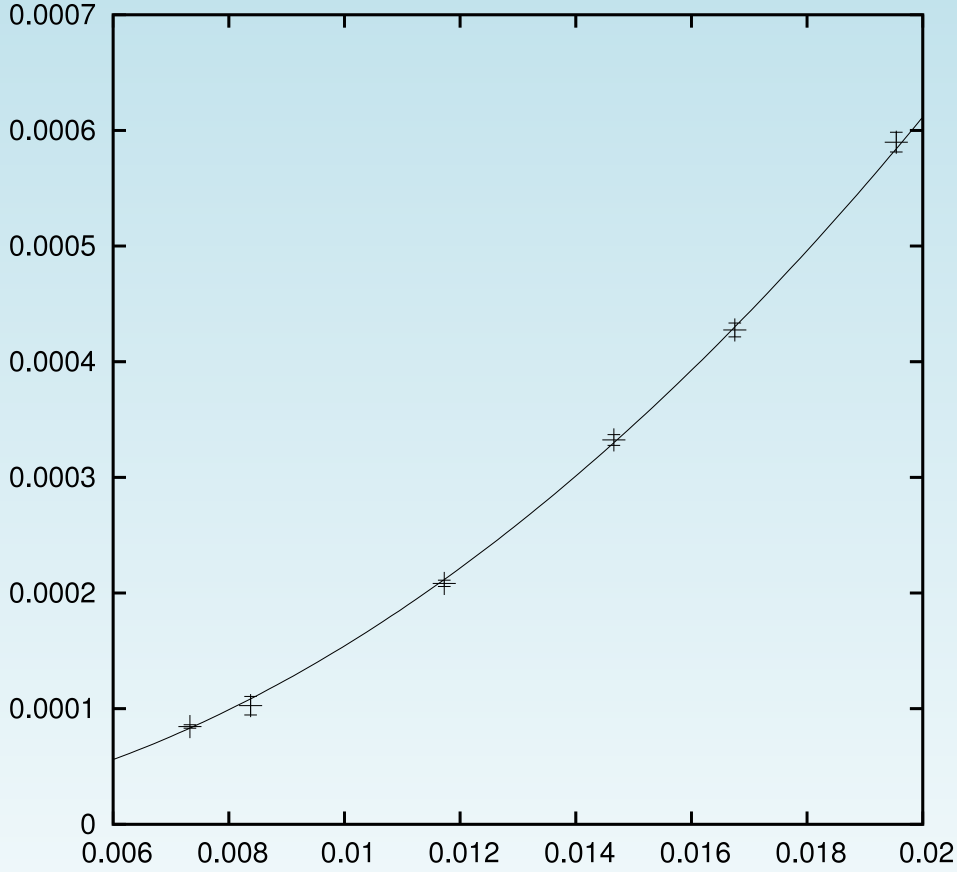
Gluon propagator at zero momentum as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. Data for lattice volumes up to 320^3 for $\beta = 3.0$.

Infinite-volume limit in 4d



Gluon propagator as a function of the lattice momentum p for lattice volume up to $V = 128^4$ at $\beta = 2.2$.

Extrapolation to infinite volume: a hint



Average absolute value of the gluon field at zero momentum $|\widetilde{A}^b_\mu(0)|$ (for $\beta = 2.2$) as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. Recall that $D(0) \propto V \sum_{\mu,b} |\widetilde{A}^b_\mu(0)|^2$. We also show the fit of the data using the Ansatz b/L^c (with $c = 1.99 \pm 0.02$).

Zwanziger proved that in Landau gauge this quantity should go to zero at least as fast as $1/L$.

Lower bound for $D(0)$

We can obtain a lower bound for the gluon propagator at zero momentum $D(0)$ by considering the quantity

$$\overline{M}(0) \; = \; \frac{1}{d(N_c^2-1)} \sum_{b,\mu} |\widetilde{A}^b_\mu(0)| \; .$$

Consider the Schwarz inequality $|\vec{X}\cdot\vec{Y}|^2 \leq \|\vec{X}\|^2 \|\vec{Y}\|^2$, a vector $\vec{Y}$ with all components equal to 1 and a vector $\vec{X}$ with components X_i, we find

$$\left(\frac{1}{m} \sum_{i=1}^{m} X_i \right)^2 \; \leq \; \frac{1}{m} \sum_{i=1}^{m} X_i^2 \; ,$$

where m is the number of components of the vectors $\vec{X}$ and $\vec{Y}$.

Lower bound for $D(0)$ (II)

We can now apply this inequality first to the vector with $m = d(N_c^2 - 1)$ components $\langle |\widetilde{A}_\mu^b(0)| \rangle$, where

$$\widetilde{A}_\mu^b(0) \; = \; \frac{1}{V} \sum_x A_\mu^b(x)$$

is the gluon field at zero momentum. This yields

$$\langle \overline{M}(0) \rangle^2 \; \leq \; \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |\widetilde{A}_\mu^b(0)| \rangle^2 \; .$$

Then, we can apply the same inequality to the Monte Carlo estimate of the average value

$$\langle |\widetilde{A}_\mu^b(0)| \rangle \; = \; \frac{1}{n} \sum_c |\widetilde{A}_{\mu,c}^b(0)| \; ,$$

where n is the number of configurations. In this case we obtain

$$\langle |\widetilde{A}_\mu^b(0)| \rangle^2 \; \leq \; \langle |\widetilde{A}_\mu^b(0)|^2 \rangle \; .$$

Lower bound for $D(0)$ (III)

Thus, by recalling that

$$D(0) \;=\; \frac{V}{d(N_c^2-1)} \sum_{b,\mu} \langle |\widetilde{A}_\mu^b(0)|^2 \rangle \; ,$$

we find

$$\left[V^{1/2} \langle \overline{M}(0) \rangle \right]^2 \;\leq\; D(0) \; .$$

We obtain that $\langle \overline{M}(0) \rangle$ goes to zero exactly as $1/V^{1/2}$!

This gives

- $D(0) \geq 0.5(1)$ (GeV^{-2}) in 3d
- $D(0) \geq 2.5(3)$ (GeV^{-2}) in 4d

Upper bound for $D(0)$

We can now consider the inequality

$$\langle \sum_{\mu,b} |\widetilde{A}^b_\mu(0)|^2 \rangle \, \leq \, \langle \Big\{ \sum_{\mu,b} |\widetilde{A}^b_\mu(0)| \Big\}^2 \rangle \; .$$

This implies

$$D(0) \, \leq \, V d(N_c^2 - 1) \, \langle \overline{M}(0)^2 \rangle \; .$$

Thus

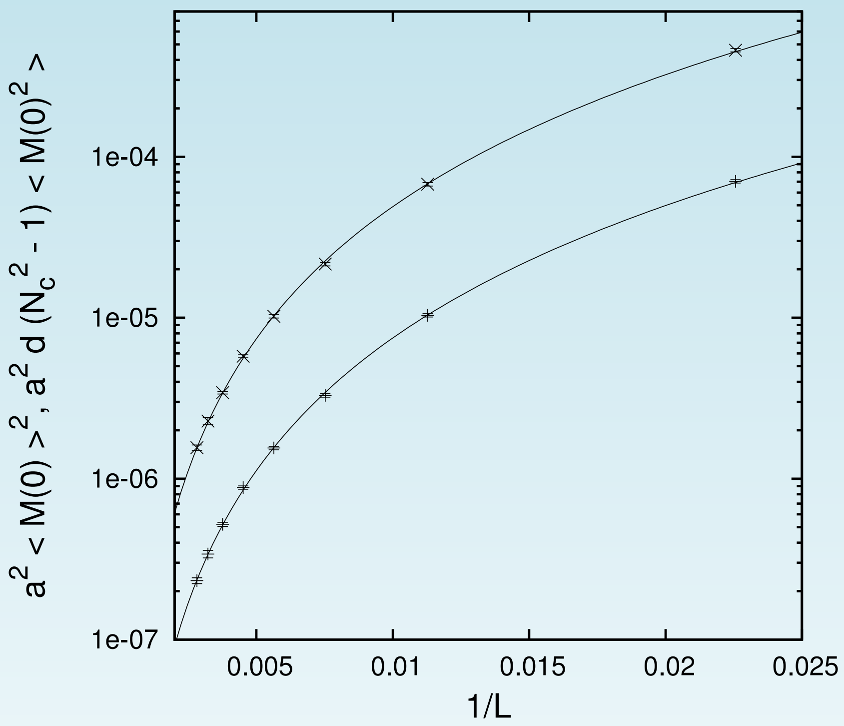
$$V \, \langle \overline{M}(0) \rangle^2 \, \leq \, D(0) \, \leq \, V d(N_c^2 - 1) \, \langle \overline{M}(0)^2 \rangle \; .$$

In summary, if $\overline{M}(0)$ goes to zero as $V^{-\alpha}$ we find that

$D(0) \to 0$, $\quad 0 < D(0) < +\infty \quad$ or $\quad D(0) \to +\infty$

respectively if α is larger than, equal to or smaller than 1/2.

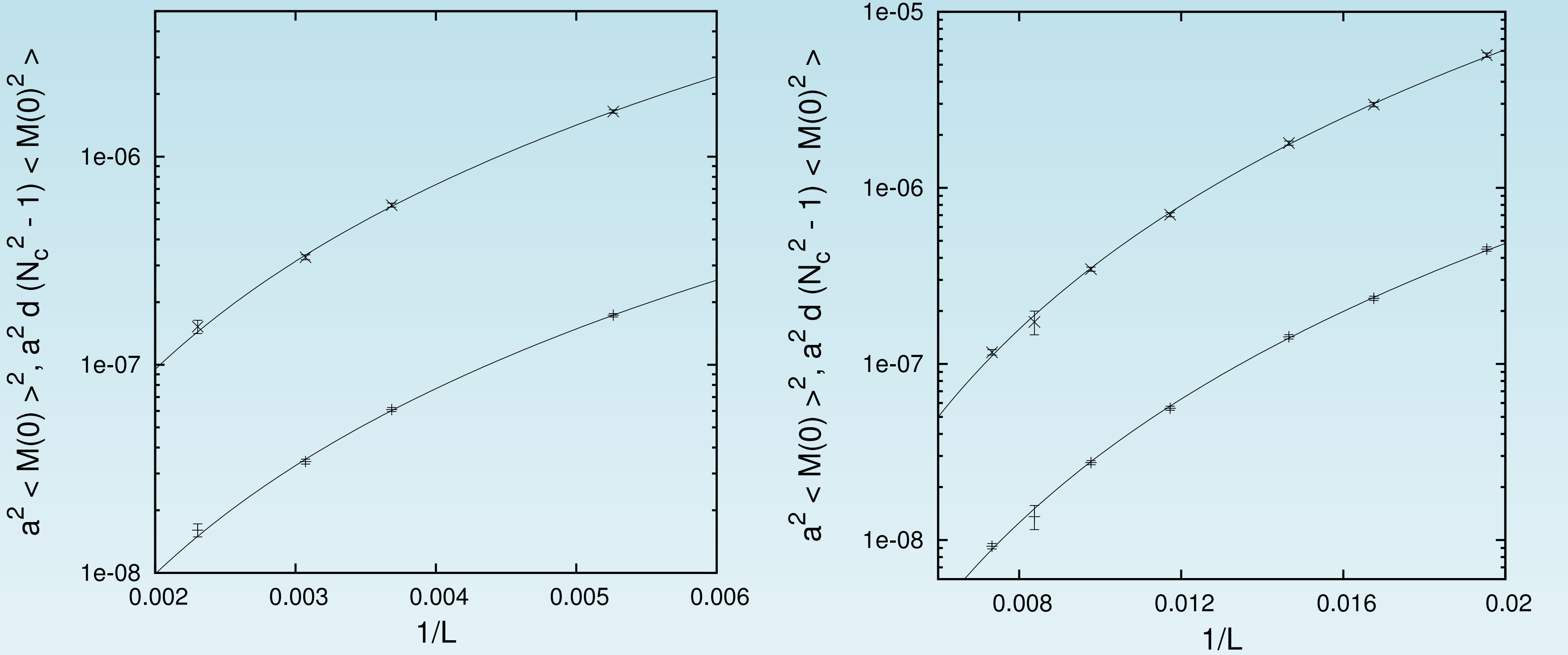
Upper and lower bounds for $D(0)$



Two-dimensional case: B_l/L^l (for $a\langle\overline{M}(0)\rangle$) and the Ansatz B_u/L^u (for $a^2\langle\,\overline{M}(0)^2\,\rangle$), with $B_l = 1.48(6)$, $l = 1.367(8)$ and $\chi/d.o.f. = 1.00$ and $B_u = 2.3(2)$, $u = 2.72(1)$ and $\chi/d.o.f. = 1.02$.

Upper and lower bounds extrapolate to zero faster than $1/V$, implying $D(0) = 0$.

Upper and lower bounds for $D(0)$ (II)

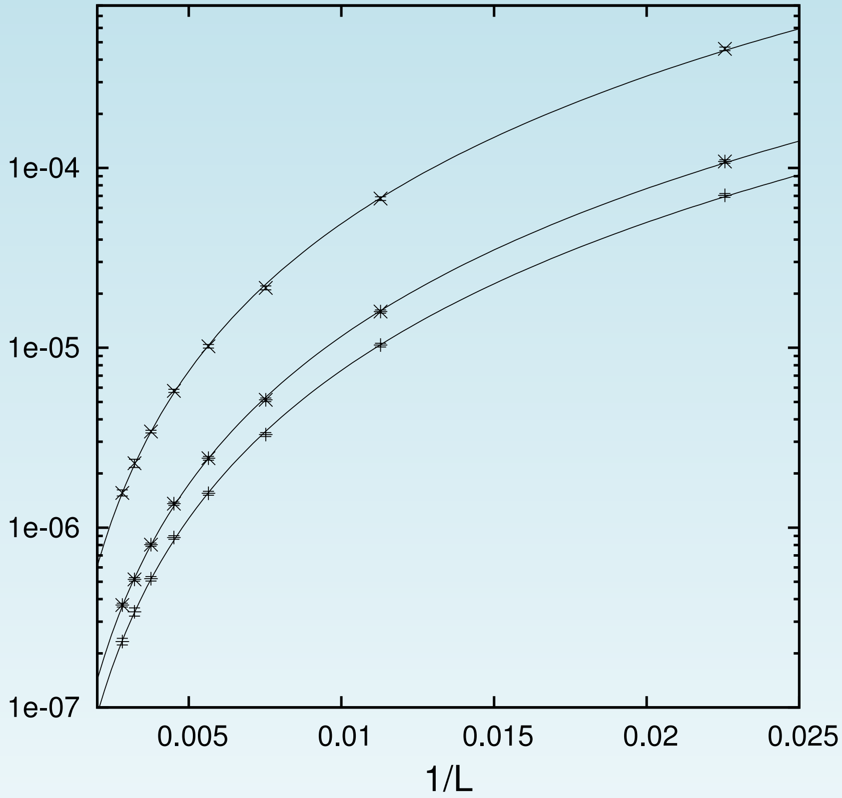


Similarly for 3d: $l = 1.48(3)$; $B_u = 1.0(3)$, $u = 2.95(5)$ and $\chi/d.o.f. = 0.95$.

Similarly for 4d: $l = 1.99(2)$; $B_u = 3.1(5)$, $u = 3.99(4)$ and $\chi/d.o.f. = 0.96$.

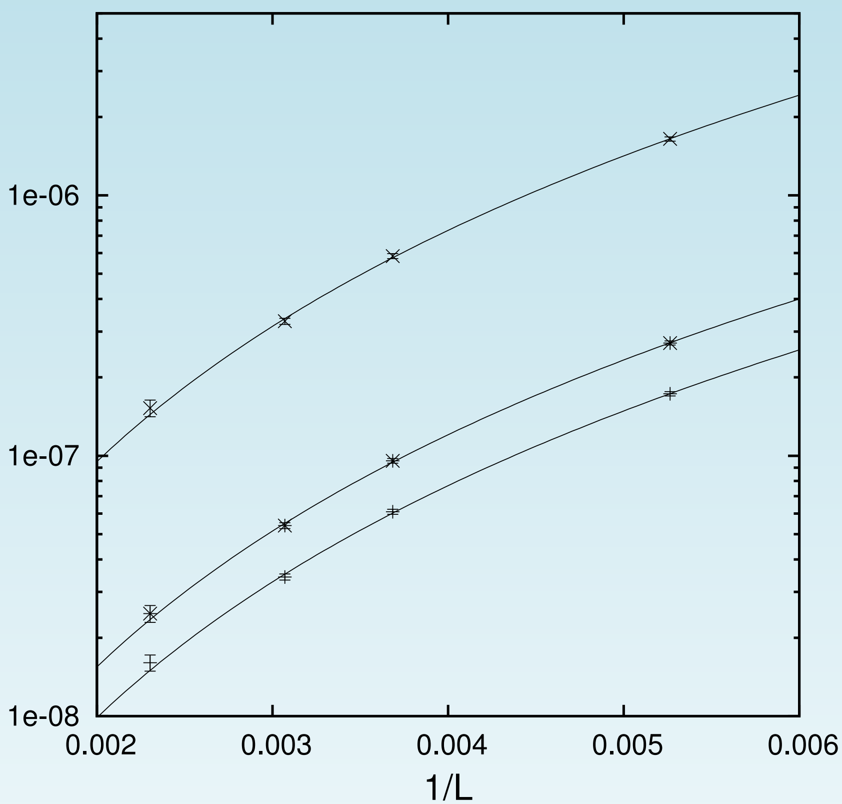
Upper / lower bounds extrapolate to zero as $1/V$, implying $D(0) > 0$.

Upper and lower bounds plus $D(0)/V$

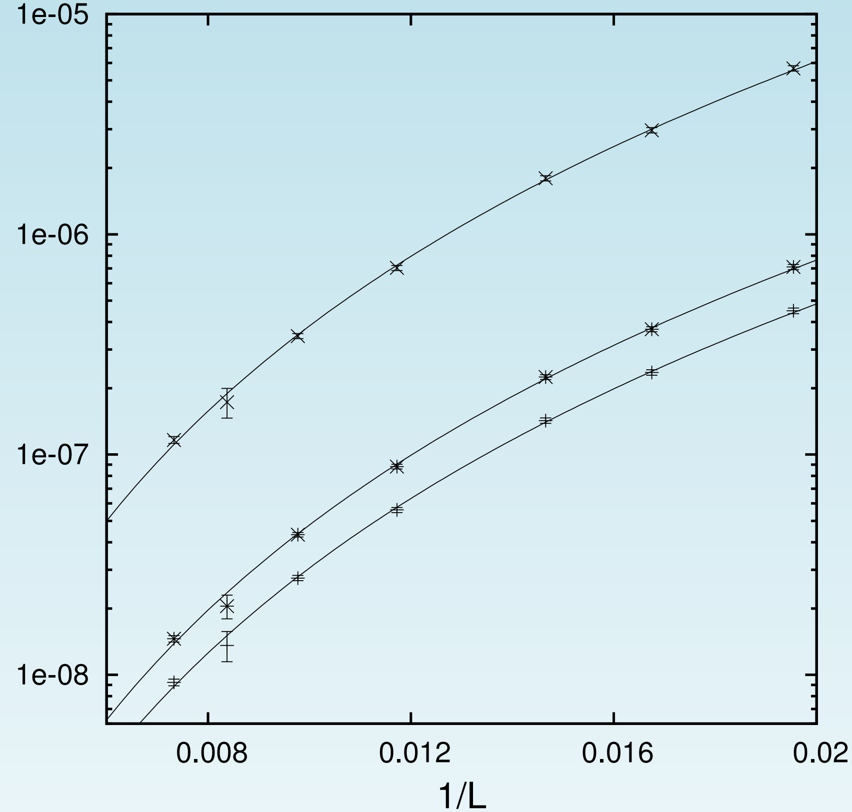


2d case

Upper and lower bounds plus $D(0)/V$ (II)



3d case



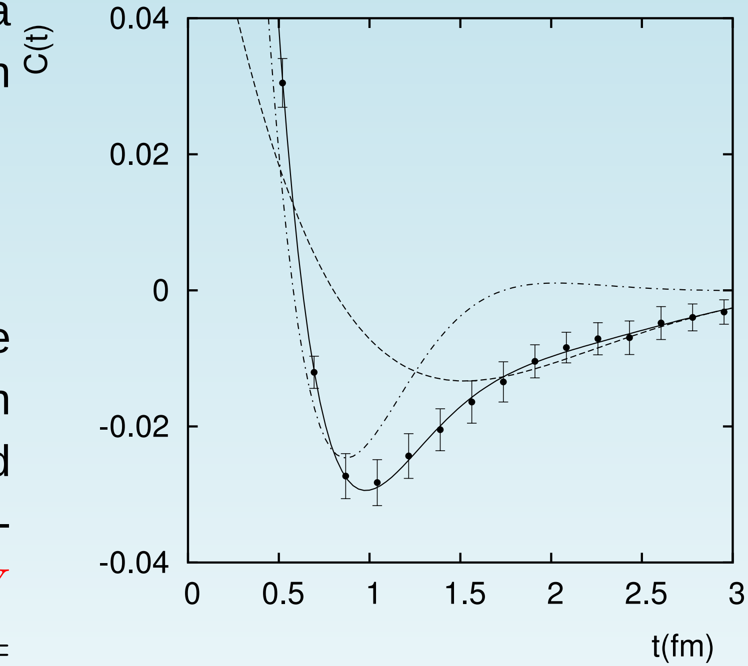
4d case

Gluon Propagator at Infinite Volume

- Gluon propagator in Landau gauge IR finite in 3d and 4d, as a consequence of "self-averaging" of a magnetization-like quantity [i.e. $M(0)$, without the absolute value].
- May think of $D(0)$ as a response function (susceptibility) of this observable ("magnetization"). In this case it is natural to expect $D(0) \sim const$ in the infinite-volume limit.
- 2d case is different, the magnetization is "over self-averaging", the susceptibility is zero.
- Question: why is the 2d case different? Possible solution from S. Sorella and collaborators.
- Note: violation of reflection positivity in 2d, 3d and in 4d.

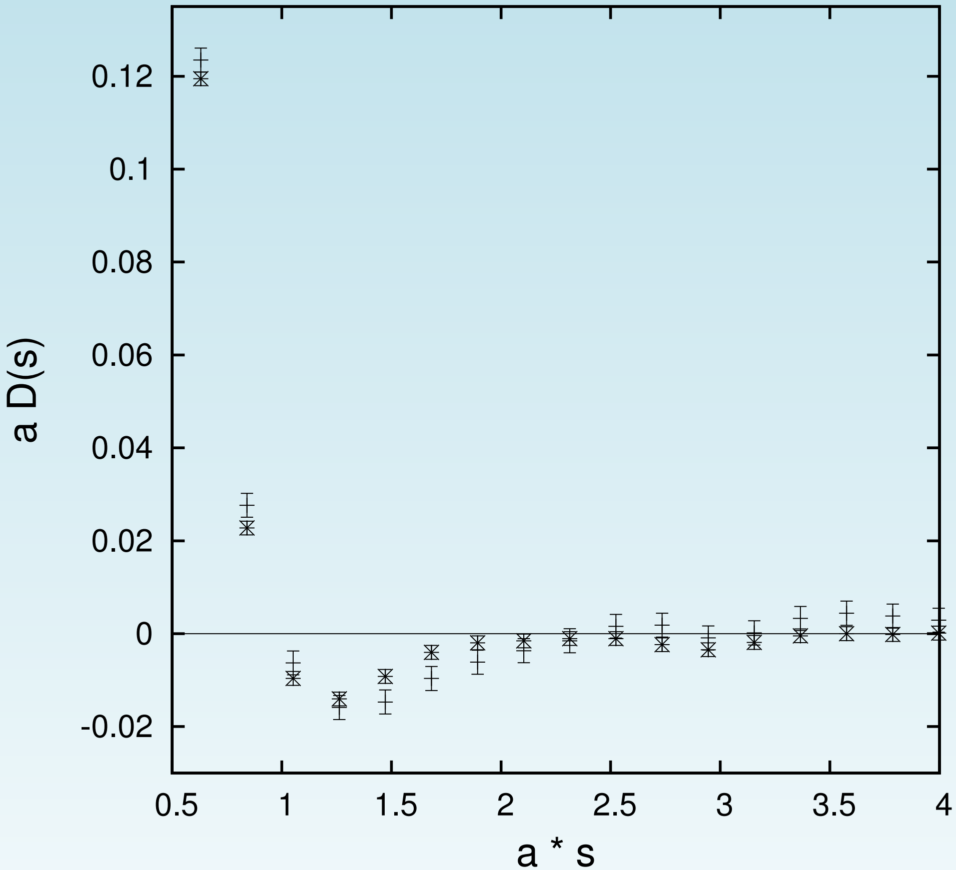
Violation of reflection positivity in 3d

- The transverse gluon propagator decreases in the IR limit for momenta smaller than p_{dec}, which corresponds to the mass scale λ in a Gribov-like propagator $p^2/(p^4 + \lambda^4)$. We can estimate $p_{dec} = 350^{+100}_{-50}$ MeV.



- Clear violation of reflection positivity: this is one of the manifestations of gluon confinement. In the scaling region, the data are well described by a sum of Gribov-like formulas, with a light-mass scale $M_1 \approx 0.74(1)\sqrt{\sigma} = 325(6)\,MeV$ and a second mass scale $M_2 \approx 1.69(1)\sqrt{\sigma} = 745(5)\,MeV$.

Violation of reflection positivity in 4d



Clear violation of reflection positivity for lattice volume $V = 128^4$ at $\beta = 2.2$.

Bounds and Results

for the Ghost Propagator

(A. Cucchieri, T.M., PoS LATTICE2007 and Phys. Rev. D 2008)

Upper and Lower Bounds for $G(p)$

Consider eigenvectors $\psi_i(a,x)$ and associated eigenvalues λ_i of the FP matrix $\mathcal{M}(a,x;b,y)$. The ψ's form a complete orthonormal set

$$\sum_{i=1}^{(N_c^2-1)V} \psi_i(a,x)\,\psi_i(b,y)^* \;=\; \delta_{ab}\delta_{xy} \quad \text{and} \quad \sum_{a,x} \psi_i(a,x)\,\psi_j(a,x)^* \;=\; \delta_{ij}\;.$$

If we now write

$$\mathcal{M}^{-1}(a,x;b,y) \;=\; \sum_{i,\lambda_i\neq 0} \frac{1}{\lambda_i} \psi_i(a,x)\,\psi_i(b,y)^*\;,$$

we get for $G(p)$ the expression

$$G(p) \;=\; \frac{1}{N_c^2-1} \sum_{i,\lambda_i\neq 0} \frac{1}{\lambda_i} \sum_a |\widetilde{\psi}_i(a,p)|^2 \;\;,$$

where

$$\widetilde{\psi}_i(a,p) \;=\; \frac{1}{\sqrt{V}} \sum_x \psi_i(a,x)\,e^{-2\pi i k\cdot x}\;.$$

Upper and Lower Bounds for $G(p)$ (II)

From the above expression we immediately get for $G(p)$ the lower bound

$$\frac{1}{N_c^2 - 1} \, \frac{1}{\lambda_{min}} \sum_a |\widetilde{\psi}_{min}(a,p)|^2 \, \leq \, G(p)$$

and the upper bound

$$G(p) \, \leq \, \frac{1}{N_c^2 - 1} \, \frac{1}{\lambda_{min}} \sum_{i, \lambda_i \neq 0} \sum_a |\widetilde{\psi}_i(a,p)\,|^2 \; .$$

Now by adding and subtracting the contribution from the null eigenvalue and using the completeness relation, the upper bound may be rewritten as

$$G(p) \, \leq \, \frac{1}{\lambda_{min}} \left[1 \, - \, \frac{1}{N_c^2 - 1} \sum_{j, \lambda_j = 0} \sum_a |\widetilde{\psi}_j(a,p)\,|^2 \right] \; .$$

Upper and Lower Bounds for $G(p)$ (III)

In Landau gauge the eigenvectors corresponding to null λ are constant modes. Thus for any nonzero p we have

$$\frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_a |\widetilde{\psi}_{min}(a, p)|^2 \leq G(p) \leq \frac{1}{\lambda_{min}} .$$

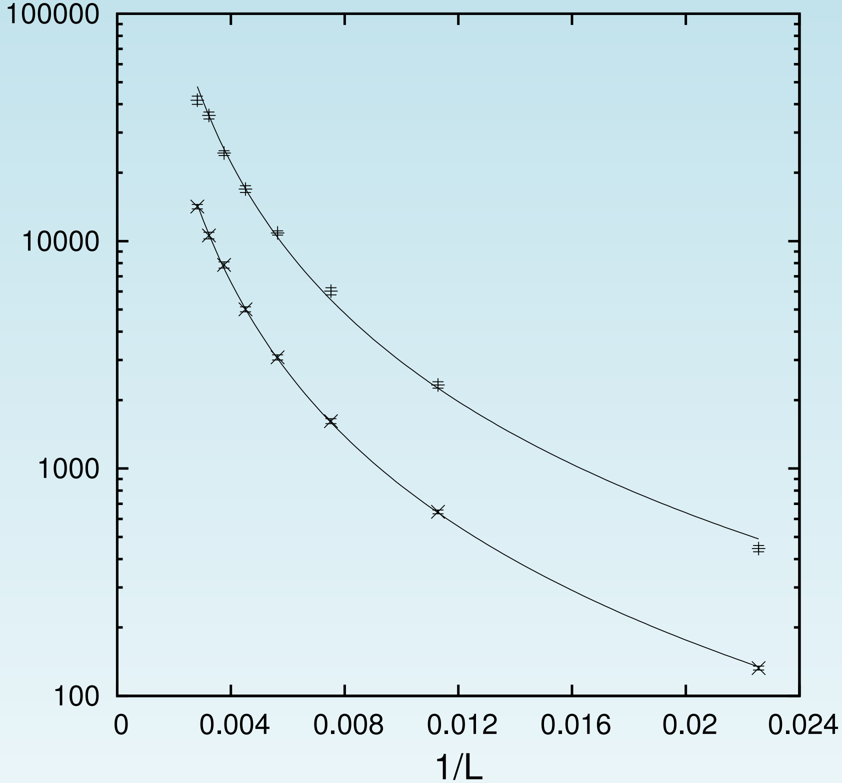
Now, assuming $\lambda_{min} \sim N^{-\alpha}$ and the power-law behavior $p^{-2-2\kappa}$ for the IR ghost propagator, we expect to have

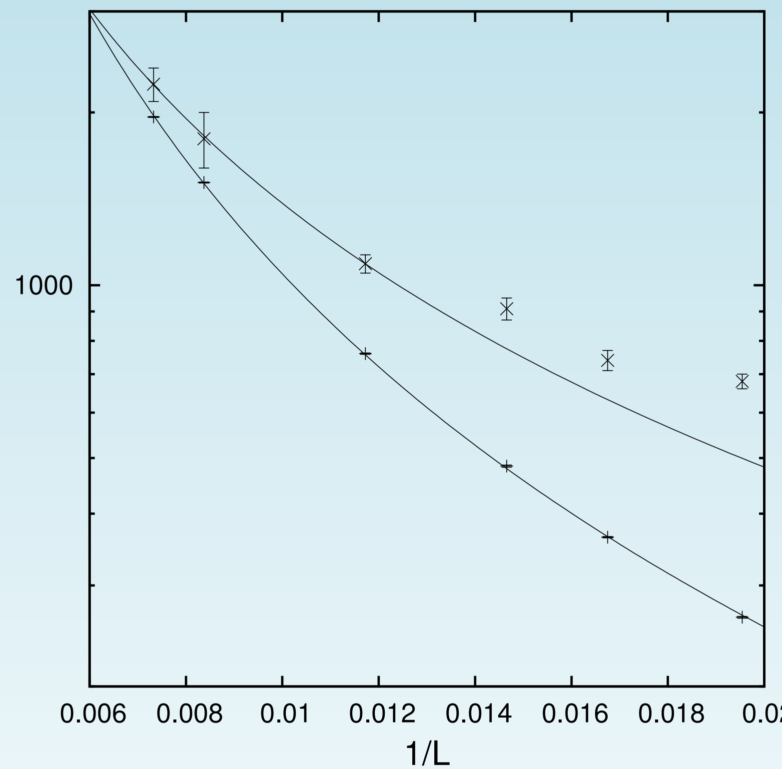
$$2 + 2\kappa \leq \alpha$$

and a necessary condition for IR enhancement of $G(p)$ is

$$\alpha > 2 .$$

Upper bound for $G(p_{min})$



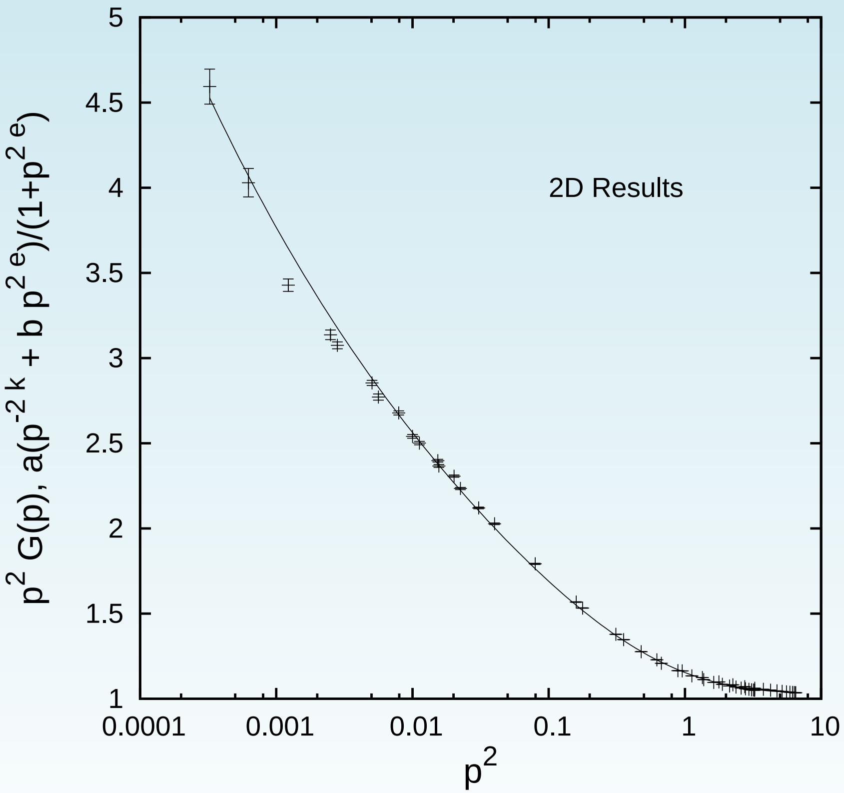


For 2d: $2\kappa = 0.251(9)$, $\alpha = 2.20(4)$.

For 4d: $2\kappa = 0.043(8)$, $\alpha = 1.53(2)$.

Ghost fits (I)

Fit of the ghost dressing function $p^2G(p^2)$ as a function of p^2 (in GeV) for the 2d case ($\beta = 10$ with volume 320^2). We find that $p^2G(p^2)$ is best fitted by the form $p^2G(p^2) = a(p^{-2k} + bp^{2e})/(1 + p^{2e})$, with

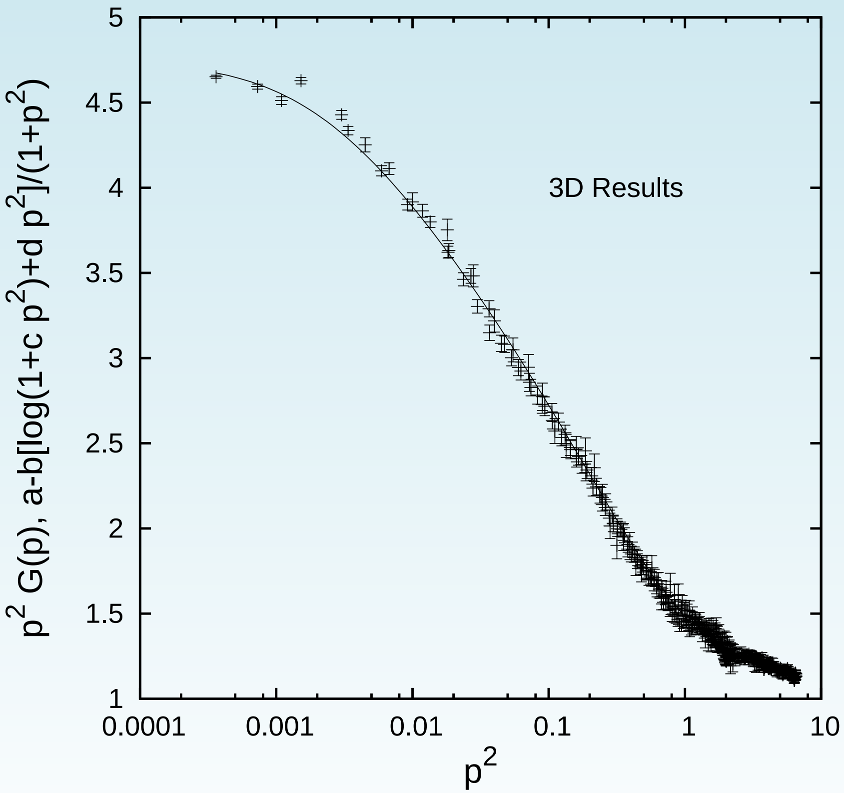


$$a = 1.24(3)\, GeV^{2(e+\kappa)},$$
$$\kappa = 0.16(2)\ ,$$
$$b = 0.86(3)\, GeV^{-2(e+\kappa)},$$
$$e = 0.75(15).$$

In the infrared limit $p^2G(p^2) \sim p^{-2k}$.

Ghost fits (II)

Fit of the ghost dressing function $p^2G(p^2)$ as a function of p^2 (in GeV) for the 3d case ($\beta = 3$ with volume 240^3). We find that $p^2G(p^2)$ is best fitted by the form $p^2G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with

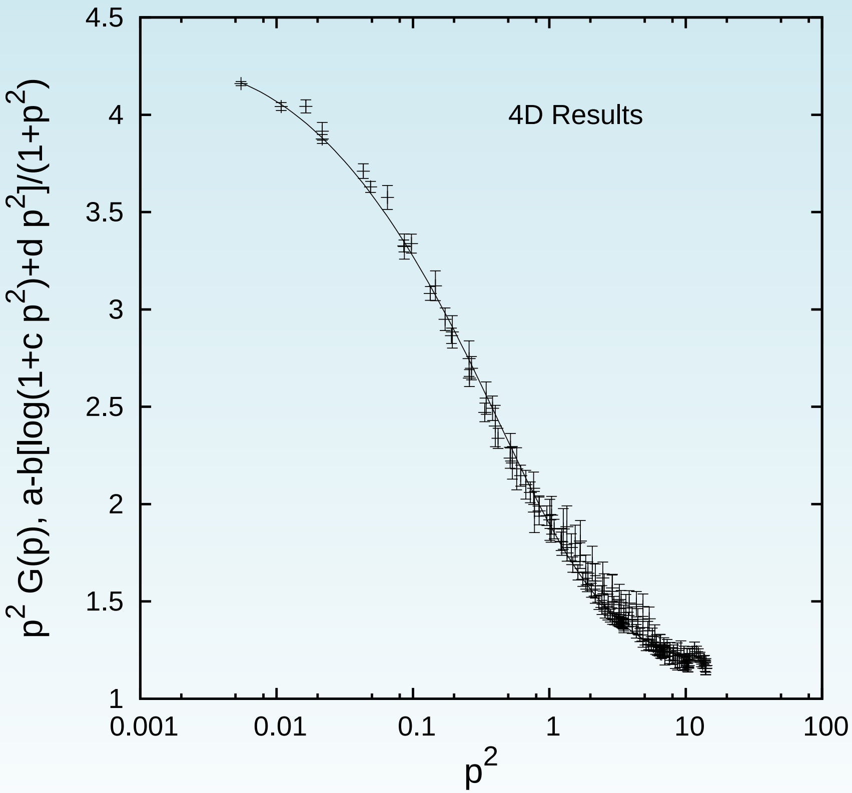


$a = 4.75(1)$,
$b = 0.491(5)\,GeV^2$,
$c = 450(30)\,GeV^{-2}$,
$d = 7.1(1)\,GeV^{-2}$.

In the infrared limit $p^2G(p^2) \sim a$.

Ghost fits (III)

Fit of the ghost dressing function $p^2G(p^2)$ as a function of p^2 (in GeV) for the 4d case ($\beta = 2.2$ with volume 80^4). We find that $p^2G(p^2)$ is best fitted by the form $p^2G(p^2) = a - b[\log(1 + cp^2) + dp^2]/(1 + p^2)$, with



$a = 4.32(2)$,
$b = 0.38(1)\, GeV^2$,
$c = 80(10)\, GeV^{-2}$,
$d = 8.2(3)\, GeV^{-2}$.

In the infrared limit $p^2G(p^2) \sim a$.

Ghost Propagator at Infinite Volume

- From present fits we have $\alpha > 2$ in 2d [implying IR enhancement of $G(p)$], but $\alpha < 2$ in 4d.
- On the other hand the expected relation $2 + 2\kappa \leq \alpha$ is not satisfied, although the upper bound is.
- Of course, we should get better data for λ_{min} in 2d, 3d and 4d.
- From fits of the ghost dressing function we find $p^2 G(p^2) \sim p^{-2k}$ in 2d and $p^2 G(p^2) \sim a$ in 3d and in 4d.

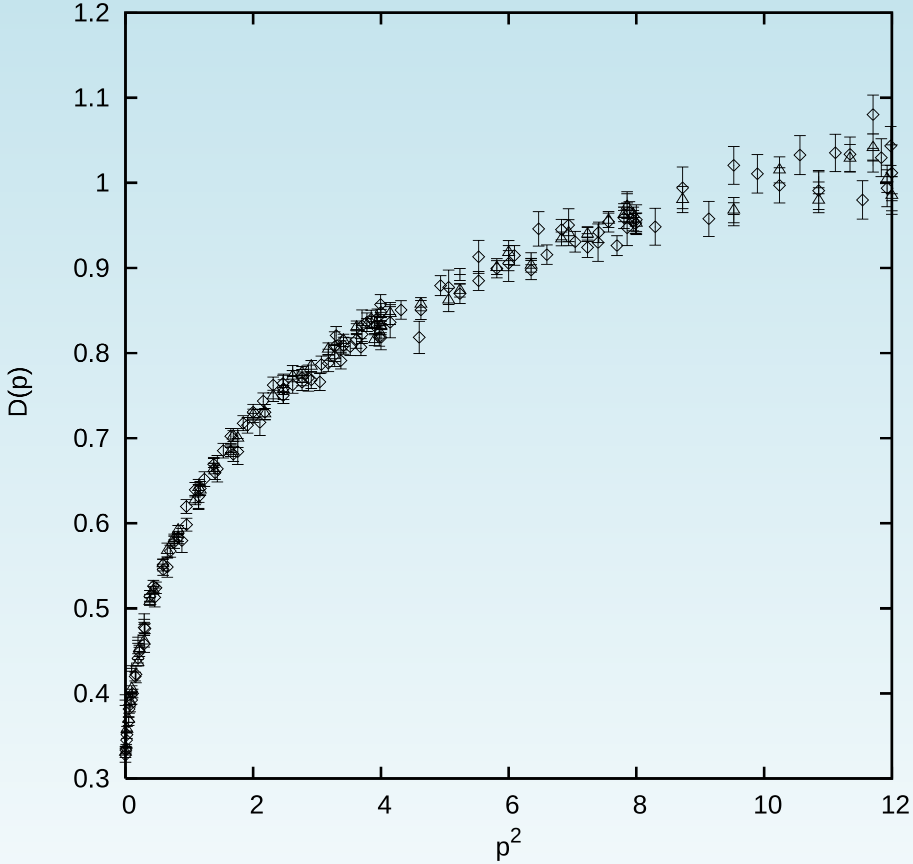
Simulations at $\beta = 0$

At $\beta = 0$ we have $a = \infty$. Of course, this is a completely unphysical limit. Why should we do it?

1. Since $a = \infty$, all volumes V are infinite in physical units.
2. At the same time, all momenta p are zero in physical units.
3. Since only the gauge fixing is left, we only probe its effects on the propagators.
4. Some results by Zwanziger do not depend on the value of β.

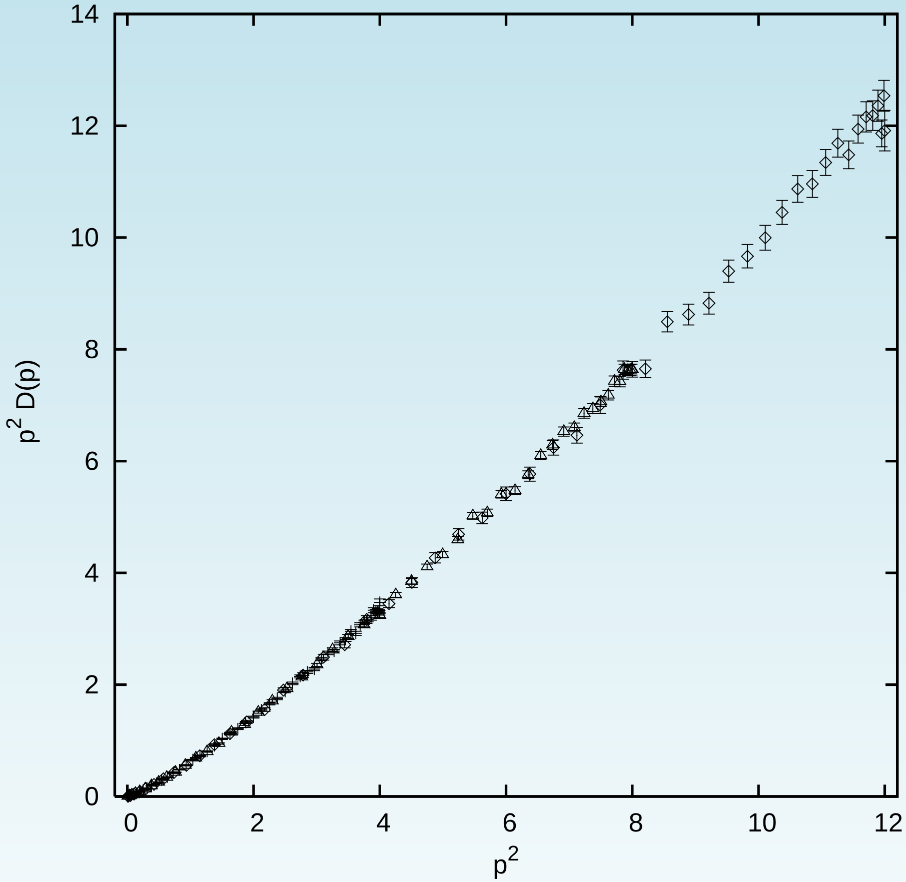
Thus, $\beta = 0$ is a nice laboratory to obtain qualitative results.

Is the Volume Infinite?



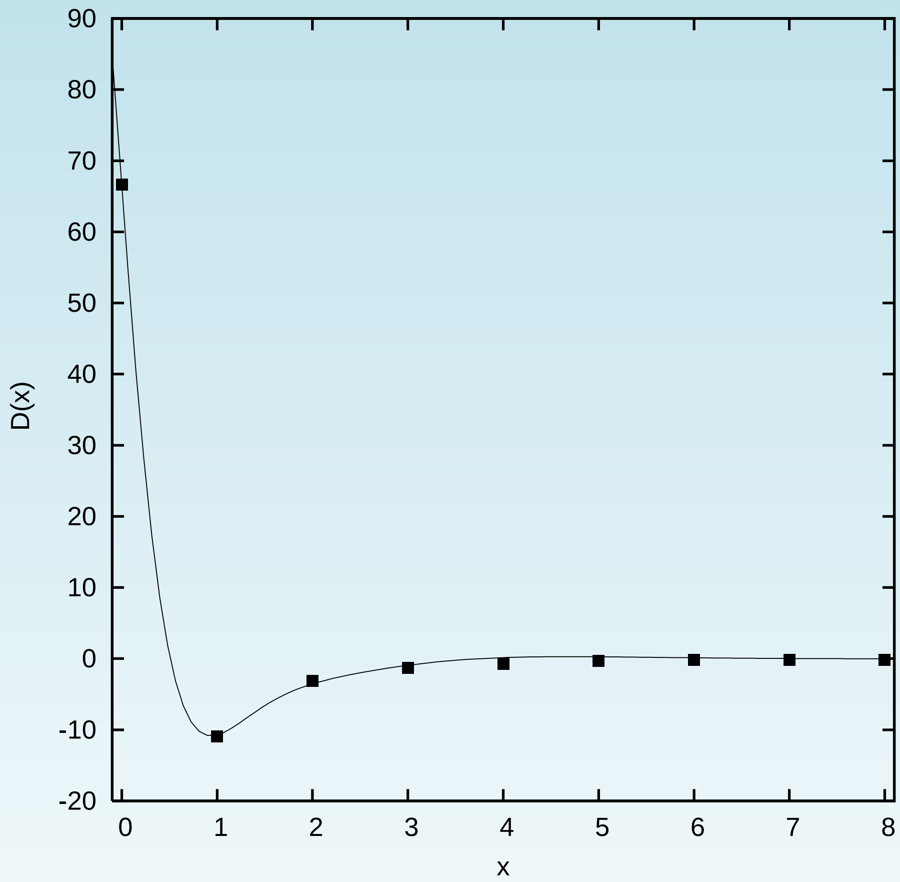
The gluon propagator $D(p)$ as a function of the lattice momenta p^2 for the lattice volumes $V = 20^3$ (symbol $+$), 40^3 (symbol $\triangle$) and 80^3 (symbol $\diamond$). No finite-size effects!

Are We Probing the IR Limit?



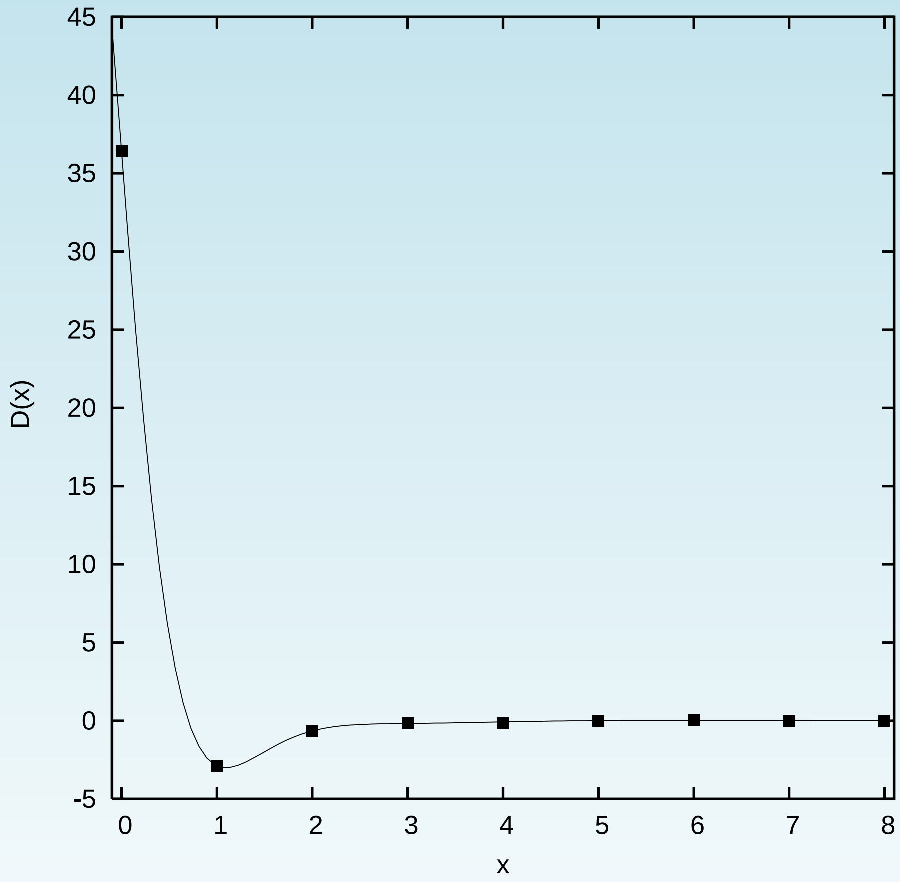
The gluon dressing function $p^2 D(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 100^3$. Symbols $+$, $\triangle$ and $\diamond$ represent data corresponding to momenta $(0, 0, q)$, $(0, q, q)$ and (q, q, q). No breaking of rotational invariance!

Violation of Reflection Positivity in 3d



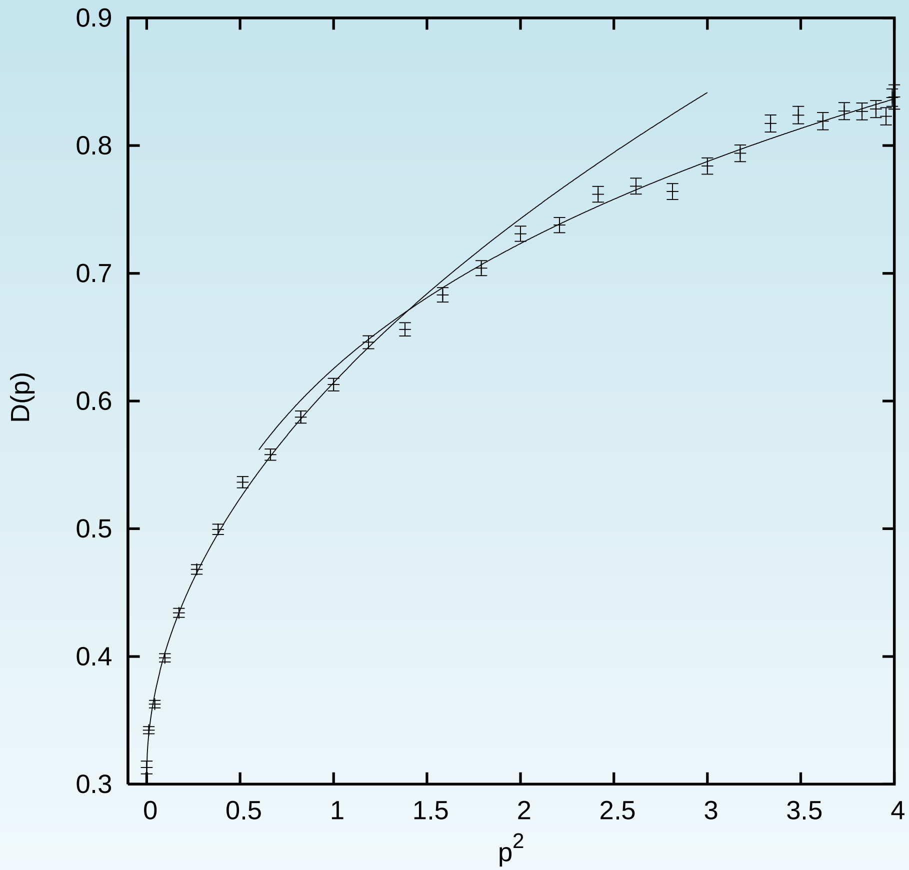
The gluon propagator $D(x)$ as a function of the space-time separation x for the lattice volume $V = 100^3$. The fitting function is $f(x) = f_1(x) + f_2(x)$, $f_i(x) = c_i \cos(b_i + \lambda_i x) e^{-\lambda_i x}$ with $c_1 = D(x = 0), b_1 = 0, b_2 = \pi/2, \lambda_2 = \lambda_1/3$, $c_2 = 20(2)$, $\lambda_1 = 2.5(2)$.

Violation of Reflection Positivity in 4d



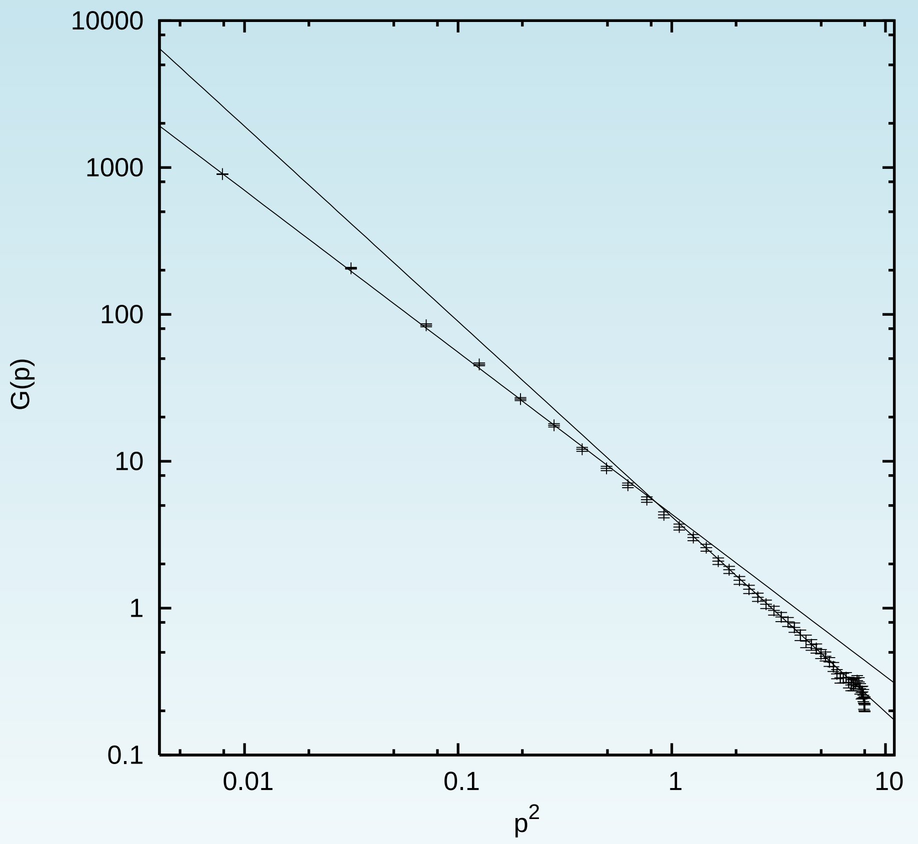
The gluon propagator $D(x)$ as a function of the space-time separation x for the lattice volume $V = 64^4$. The fitting function is $f(x) = f_1(x) + f_2(x)$, $f_i(x) = c_i \cos(b_i + \lambda_i x)\, e^{-\lambda_i x}$ with $c_1 = D(x = 0)$, $b_1 = 0$, $b_2 = \pi/2$, $\lambda_1 = 2.15(2), c_2 = 1.7(1)$, $\lambda_2 = 0.65(3)$.

Gluon Propagator in the IR Limit in 3d



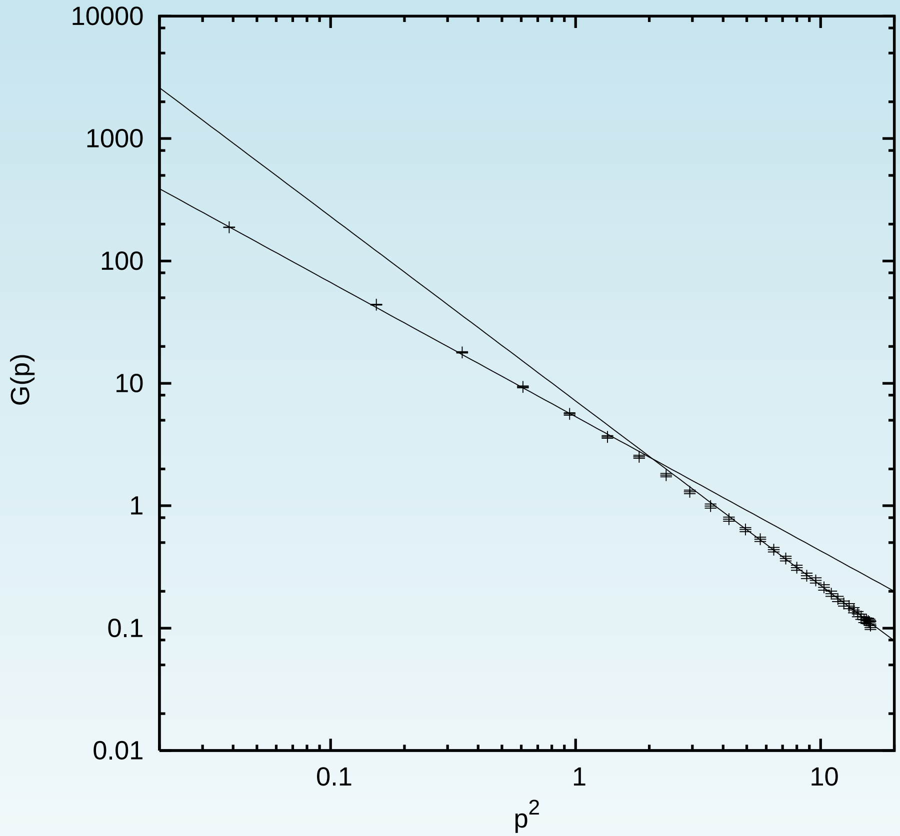
The gluon propagator $D(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 60^3$. The fit in the interval $p^2 \leq 1.5$ is $b + c(p^2)^{2\kappa - 0.5}$ with $\kappa = 0.50(1)$. The fit in the interval $p^2 > 1.5$ is $c(p^2)^{2\kappa-0.5}$ with $\kappa = 0.355(4)$.

Ghost Propagator in the IR Limit in 3d



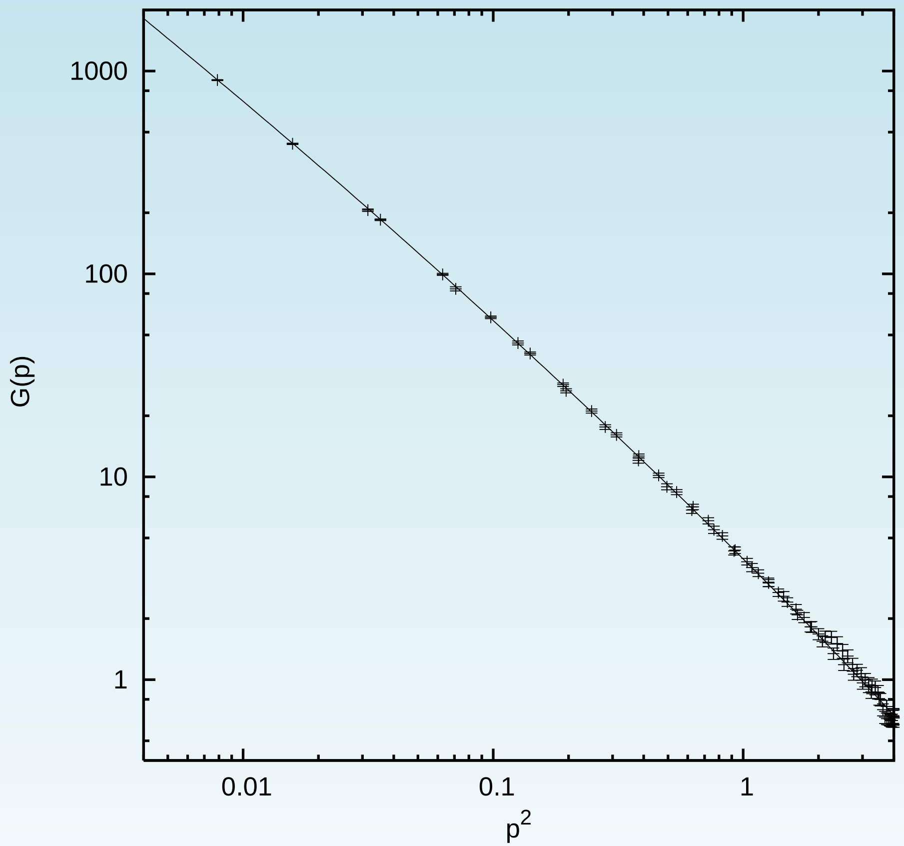
The ghost propagator $G(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 100^3$. The fit is $b(p^2)^{-\kappa-1}$, with $\kappa = 0.102(7)$ at small momenta and $\kappa = 0.33(3)$ at large momenta.

Ghost Propagator in the IR Limit in 4d



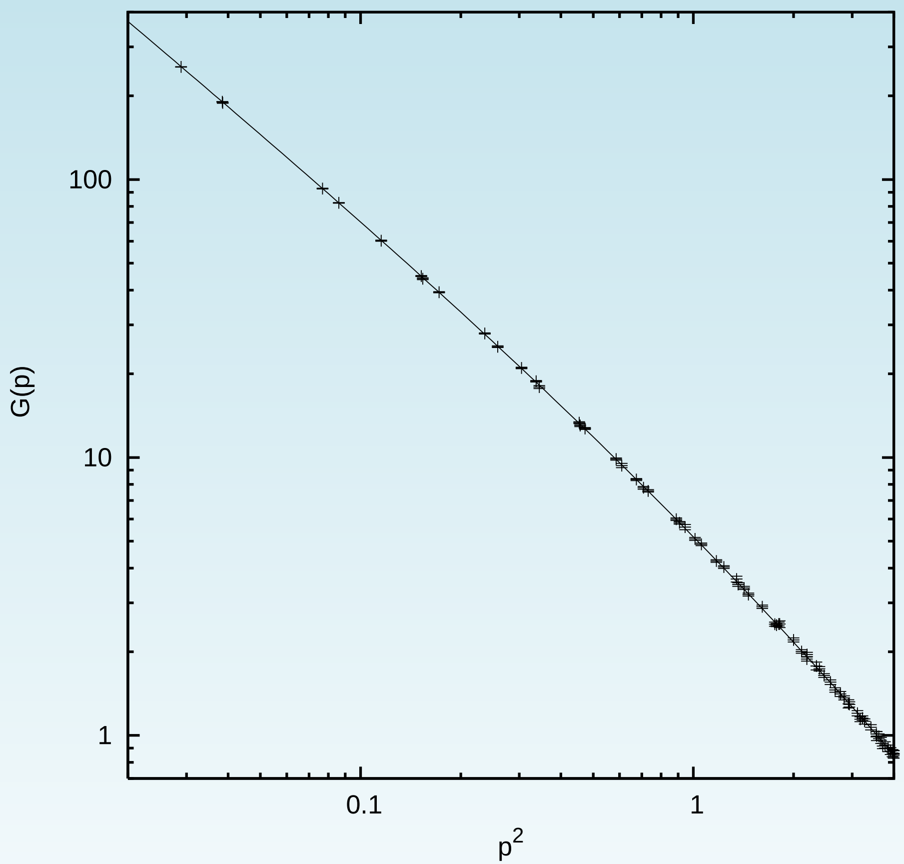
The ghost propagator $G(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 64^4$. The fit is $b(p^2)^{-\kappa-1}$, with $\kappa = 0.10(1)$ at small momenta and $\kappa = 0.51(4)$ at large momenta.

Fit for the Ghost Propagator in 3d



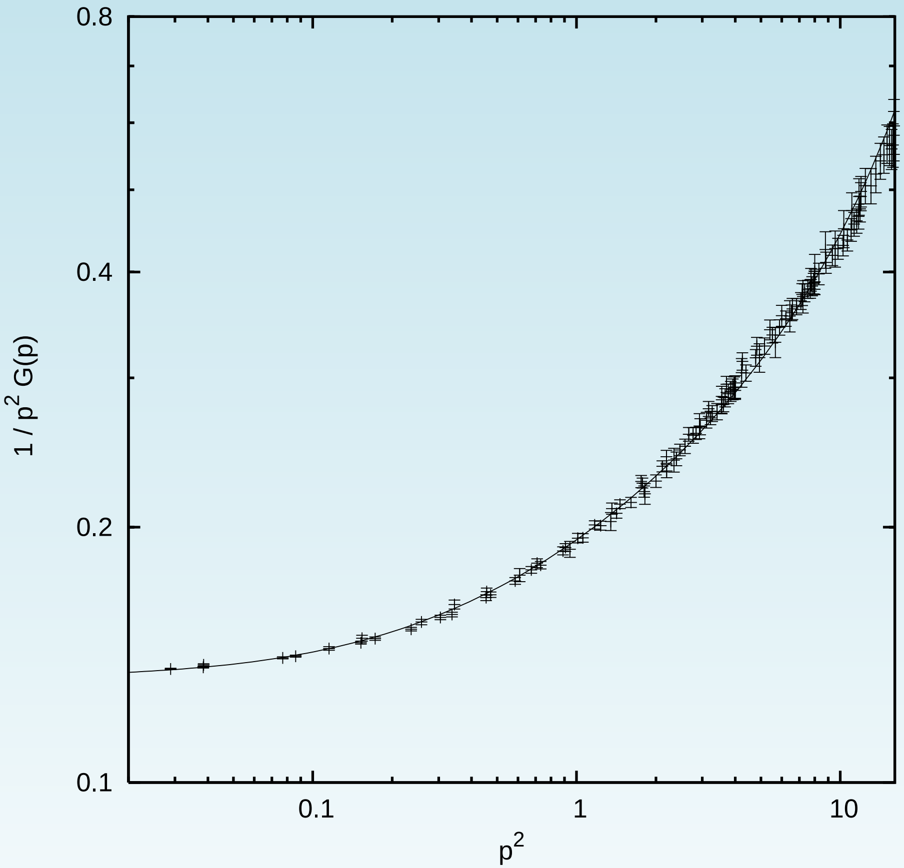
The ghost propagator $G(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 100^3$. The fit is $f(x) = \left[a - b \log\left(p^2 + c^2\right)\right] / p^2$ with $a = 3.95(2)$, $b = 0.94(1)$ and $c = 0.159(6)$.

Fit for the Ghost Propagator in 4d



The ghost propagator $G(p)$ as a function of the lattice momenta p^2 for the lattice volume $V = 64^4$. The fit is $f(x) = \left[a - b\log\left(p^2 + c^2\right)\right]/p^2$ with $a = 5.44(1)$, $b = 1.372(8)$ and $c = 0.466(5)$.

Ghost Dressing Function in 4d



The ghost dressing function as a function of the lattice momenta p^2 for the lattice volume $V = 64^4$. The fit is $f(x)\,p^2 = \left[a - b\log\left(p^2 + c^2\right)\right]$ with $a = 5.44(1)$, $b = 1.372(8)$ and $c = 0.466(5)$.

Propagators at $\beta = 0$

In agreement with the simulations at $\beta > 0$ we find (arXiv:0904.4033[hep-lat]) that

- the gluon propagator $D(p)$ violates reflection positivity,
- the gluon propagator at zero momentum $D(0)$ seems to be finite and nonzero,
- the ghost propagator $G(p)$ is not infrared enhanced in the deep infrared limit, but it is enhanced at larger momenta,
- a very good fit for the ghost propagator $G(p)$ in given by $f(x) = \left[a - b\log\left(p^2 + c^2\right)\right] / p^2$ (proposed by Aguilar et al.),
- see also recent work by A. Sternbeck and L. von Smekal.

Conclusions

- Simple properties of gluon and ghost propagators constrain (by upper and lower bounds) their IR behavior. For the gluon case we define a magnetization-like quantity, while for the ghost case we relate the propagator to λ_{min} of the FP matrix. These quantities are studied as a function of the lattice volume, to gain better control of the infinite-volume limit of IR propagators.
- For the gluon propagator, data & extrapolation (plus explanation as response function) support a finite value in the IR
- For the ghost case, enhancement seems unlikely...
- Questions: just considering large volumes is not enough? is the behavior of the propagators at $p = 0$ so crucial for confinement?
- Interesting to consider other gauges: MAG, λ-gauges, linear covariant gauge (collaboration with A. Maas, A. Mihara, E. Santos)