The role of **monopoles** in a gluon plasma

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In collaboration with Edward Shuryak, PRD80 (2009)

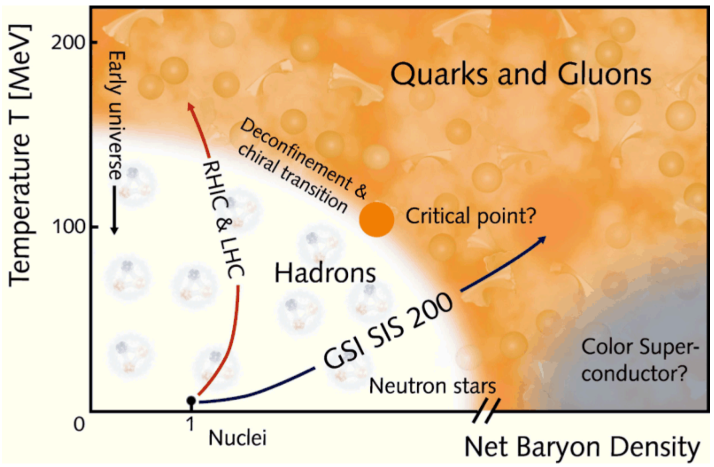
Some definitions

- In the following: electric and magnetic always stand for color-electric and color-magnetic
- Electric quasiparticles $\Longleftrightarrow$ quarks and gluons
- Magnetic quasiparticles $\Longleftrightarrow$ monopoles
- Magnetic monopoles $\Longleftrightarrow$ objects carrying a magnetic charge g which is the source of a static Coulomb-like magnetic field

$$\vec{B} = g\frac{\vec{r}}{r^3}$$

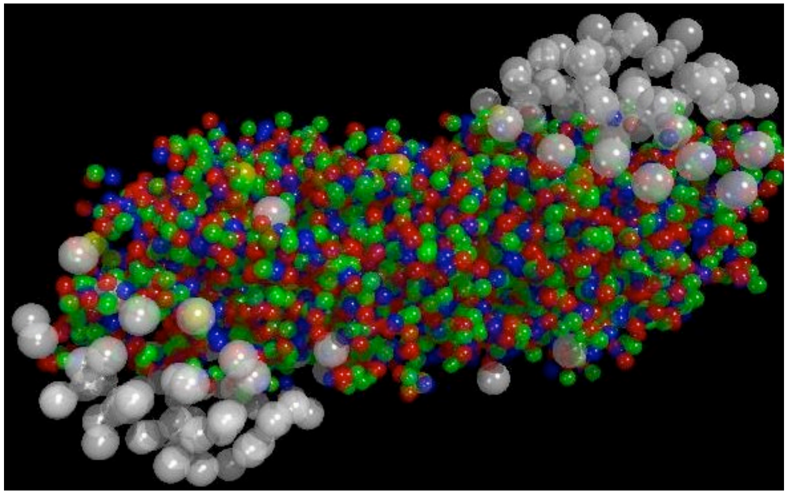
- Electric charge e: $\frac{e^2}{4\pi} = \alpha_s$
- Magnetic charge g: $\frac{g^2}{4\pi} = \alpha_M$

Introduction

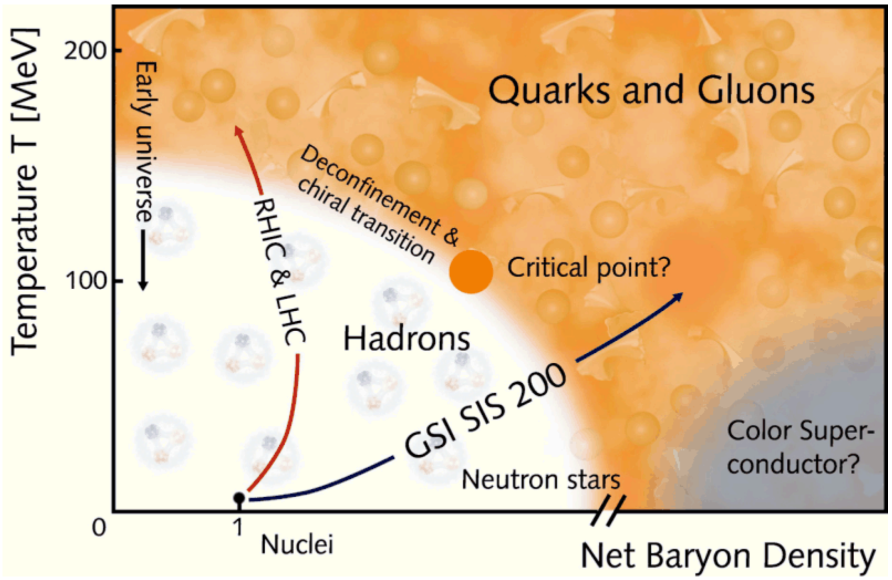


- QCD has a rich phase structure
- Many challenging items:
  - order of the phase transition
  - critical point
  - deconfinement and chiral symmetry
  - color superconductivity at high μ

- The deconfined phase of QCD is accessible in the lab (RHIC, Alice @ LHC, FAIR @ GSI)
- RHIC results show unambiguously that a new form of matter has been formed
- Quark-Gluon Plasma behaves like a nearly perfect fluid
  - very small shear viscosity
  - Microscopic origin of small viscosity?
  - How will QGP behave at the LHC?

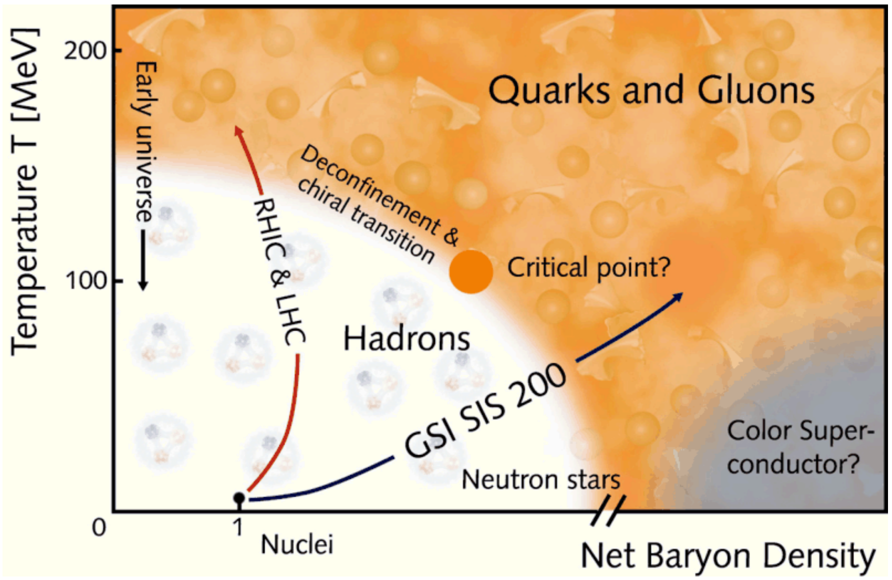


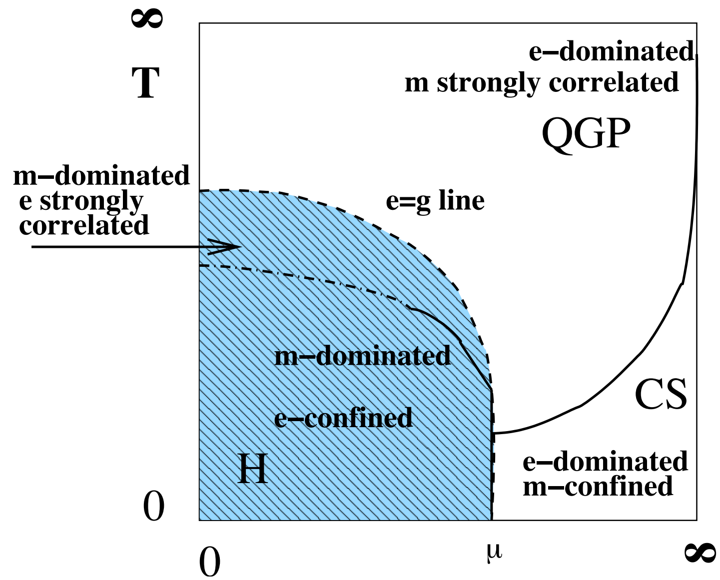
Scenario



- “Traditional” QCD phase diagram
  - Confinement at the center of the discussion
  - The (T, μ) plane is divided into:
    - Hadronic phase at small T and μ
    - Deconfined phase at large T, μ

Scenario





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- "New view" on the QCD phase diagram
  - Electric-magnetic competition
  - Dirac condition: $\frac{eg}{4\pi} = 1$

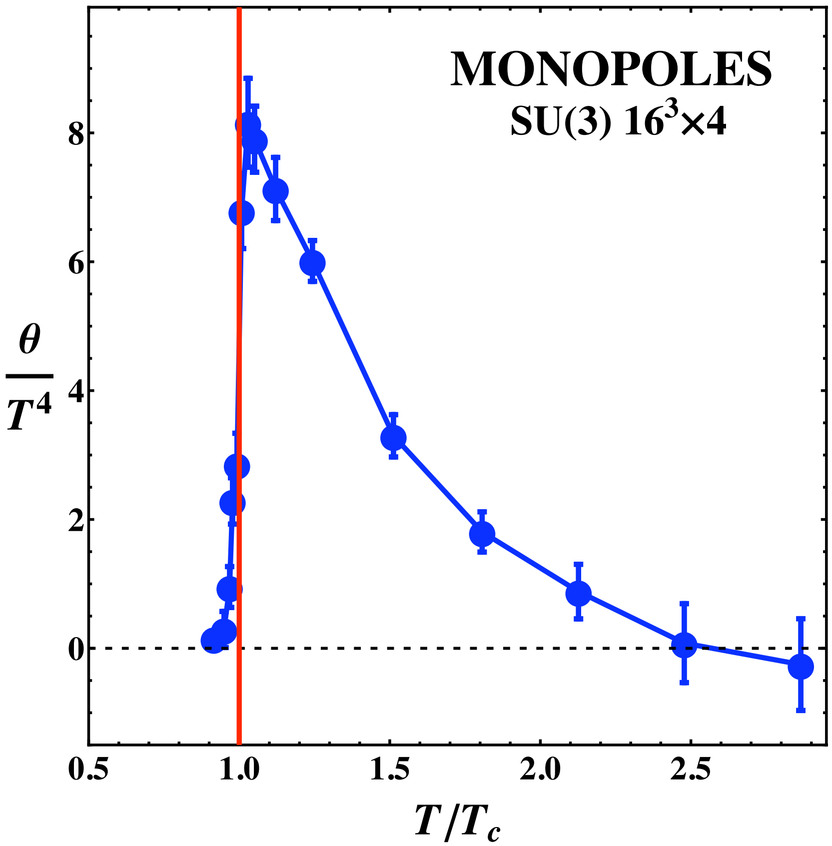
    (e: electric coupling, g: magnetic coupling)
    - At the $e = g$ line we have $\frac{e^2}{4\pi} = \frac{g^2}{4\pi}$

J. Liao and E. Shuryak, PRC75 (2007)

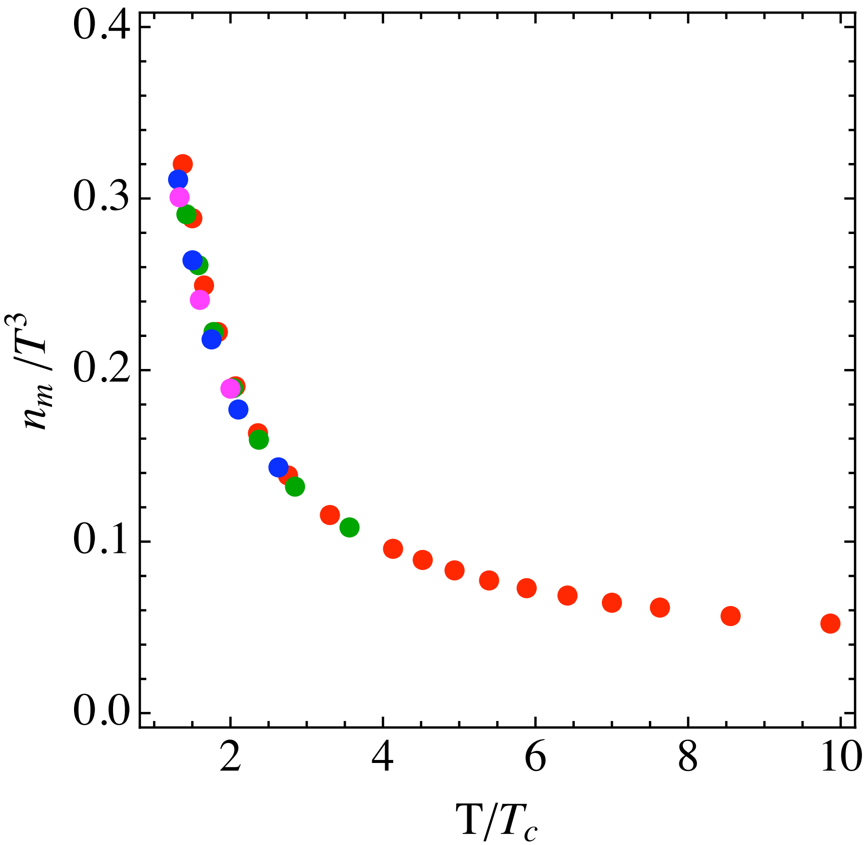
Monopoles on the lattice (I)

Interaction measure: $\frac{\theta}{T^4} = \frac{\epsilon - 3p}{T^4}$

Density: $\frac{n_m}{T^3} \sim \log(T)^{-3}$ for $T > 4T_c$

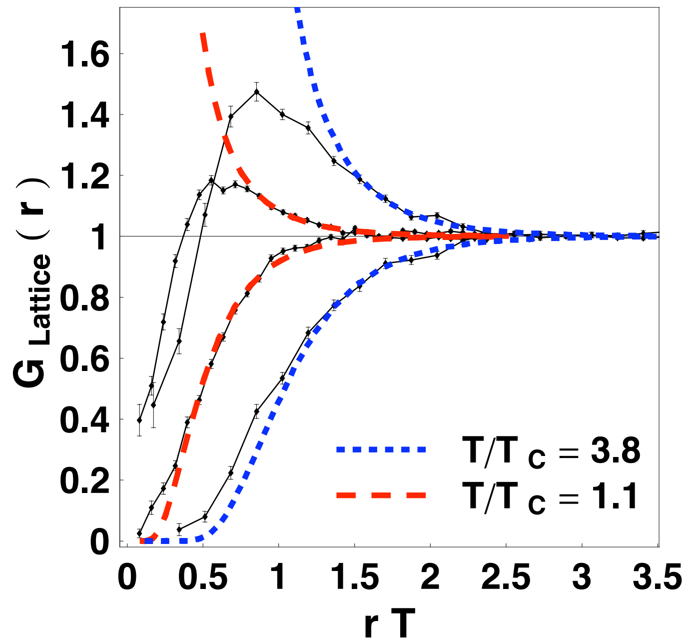


M. N. Chernodub *et al.*, 0710.2547.



A. D'Alessandro and M. D'Elia, NPB799 (2008).

Monopoles on the lattice (II)



- Monopole-(anti)monopole correlator

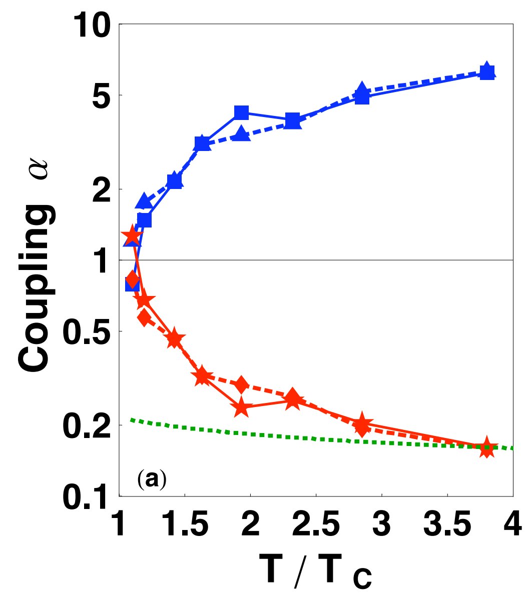
$$G(r) \equiv \frac{\langle \rho(0)\rho(r)\rangle}{\rho^2}$$

$$G(r) \sim \exp\left[\pm \frac{\alpha_M e^{-r/R_d}}{rT}\right]$$

- Well described by a picture of classical Coulomb plasma!

- Magnetic coupling growing with T

- Inverse of electric coupling e



Lattice results: A. D'Alessandro and M. D'Elia, NPB 799 (2008)

Curves: J. Liao and E. Shuryak, PRL 101 (2008)

Purpose of our work

- Study the effect of magnetic monopoles in the QGP
  - Thermodynamics
  - Transport Coefficients

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⇓

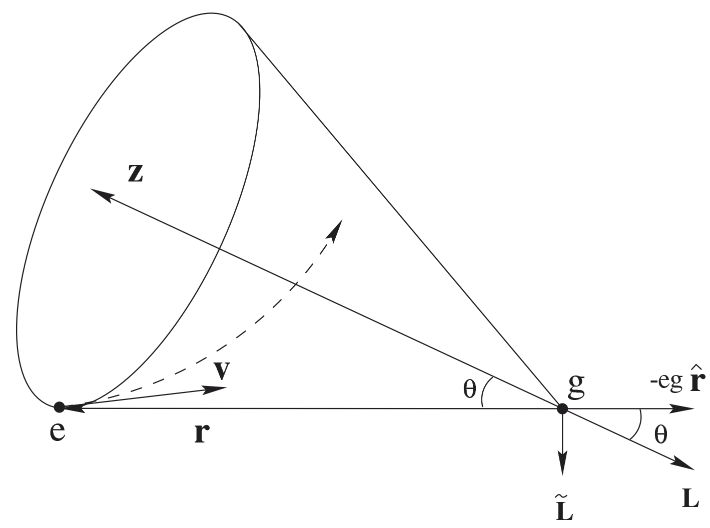
- Gluon scattering on monopoles
  - We work at $T \geq 2T_c$
  - Georgi-Glashow model
  - Input: lattice results

Non-Relativistic Scattering on a Magnetic Charge

- Equation of motion of an electrically-charged particle in a Coulomb-like magnetic field $\vec{B} = g\frac{\vec{r}}{r^3}$

$$m\frac{d^2\vec{r}}{dt^2} = \frac{eg}{r^3}\left(\frac{d\vec{r}}{dt} \times \vec{r}\right)$$

- The electric particle moves on the surface of a cone
- The magnitude of the angular momentum is conserved, but its direction is not
- There is no closed orbit in the charge-monopole system



Quantum Scattering on a Magnetic Charge: Georgi-Glashow model

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} \left(D^\mu \phi^a\right) \left(D_\mu \phi^a\right) - \frac{\lambda}{4} \left(\phi^a \phi^a - v^2\right)^2$$

- Classical solution

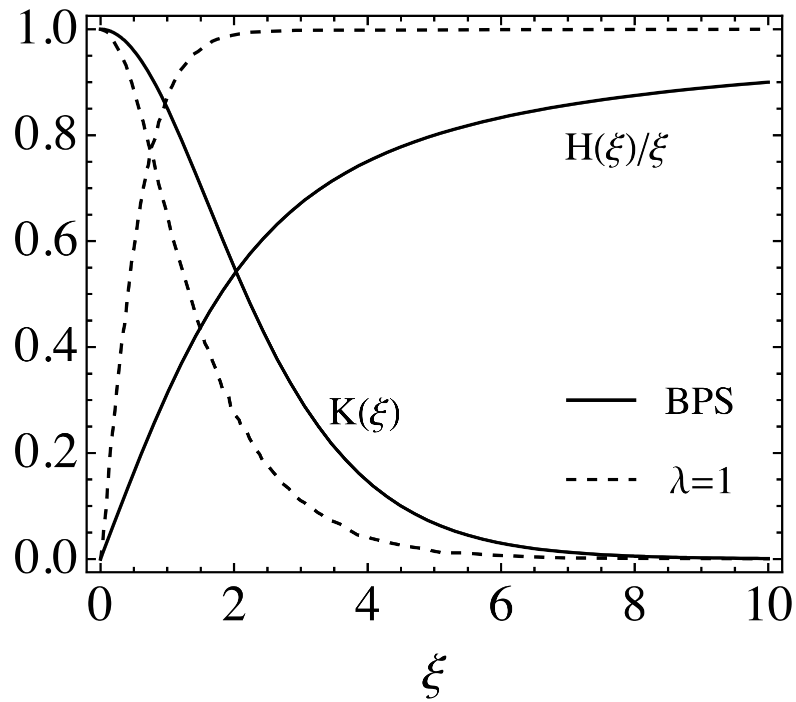
$$\Phi^a = \frac{r^a}{e r^2} H(\xi),$$

$$\mathcal{A}^a_n = \epsilon_{amn} \frac{r^m}{e r^2} \left[1 - K(\xi)\right]$$

where H and K are solutions of

$$\xi^2 \frac{d^2 K}{d\xi^2} = K H^2 + K \left(K^2 - 1\right),$$

$$\xi^2 \frac{d^2 H}{d\xi^2} = 2K^2 H + \frac{\lambda}{e^2} H \left(H^2 - \xi^2\right)$$



- Field equations for the quantum fluctuations ($A^a_\mu = \mathcal{A}^a_\mu + a^a_\mu, \quad \phi^a = \Phi^a + \chi^a$)

$$D_\nu F^{a\mu\nu} = -e\epsilon_{abc} \phi^b D^\mu \phi^c, \qquad D_\mu D^\mu \phi^a = -\lambda \phi^a \left(\phi^b \phi^b - v^2\right).$$

Vector fluctuations

- Gauge bosons have spin $\vec{S}$ and isospin $\vec{I}$ ($\vec{I} \cdot \vec{r}$ = charge)
- $\vec{J}$ is the total angular momentum:

$$\vec{J} = \vec{L} + \vec{I} + \vec{S}$$

- We can write vector fluctuations as

$$a(\vec{r})_{ai} = \sum_{\alpha=1}^{9} T_{j\alpha}(r) \Phi_{jn}^{m\sigma}(\theta, \varphi)_{ai} \quad \text{with} \quad \alpha = \alpha(\sigma, n)$$

- The angular functions must describe conical motion in the classical limit:

$$\left\{ \begin{array}{l} \vec{J}^2 \\ J_3 \\ (\hat{r} \cdot \vec{I}) \\ (\hat{r} \cdot \vec{S}) \end{array} \right\} \Phi_{j,n}^{m,\sigma}(\theta, \varphi)_{ai} = \left\{ \begin{array}{l} j(j+1) \\ m \\ n \\ \sigma \end{array} \right\} \Phi_{j,n}^{m,\sigma}(\theta, \varphi)_{ai}.$$

Radial equations (I)

$$T''_{j1} - \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi}T'_{j7} + \frac{K(\xi)}{\xi}T'_{j8} - \left[-\omega^2 + \frac{j(j+1) + 2K(\xi)^2}{2\xi^2}\right]T_{j1} + \sqrt{\frac{(j-1)(j+2)}{2}}\frac{K(\xi)}{\xi^2}T_{j2}$$
$$+\sqrt{\frac{j(j+1)}{2}}\frac{K(\xi)}{\xi^2}T_{j3} + \frac{j(j+1)}{2\xi^2}T_{j4} - \sqrt{2j(j+1)}\frac{K(\xi)}{\xi^2}T_{j5} + \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi^2}T_{j7} + \frac{2K'(\xi)\xi - K(\xi)}{\xi^2}T_{j8} = 0$$

$$T''_{j2} - \sqrt{\frac{(j-1)(j+2)}{2}}\frac{1}{\xi}T'_{j8} - \left[-\omega^2 + \frac{j(j+1) - 2K(\xi)^2 + 2H(\xi)^2}{2\xi^2}\right]T_{j2} + \sqrt{\frac{(j-1)(j+2)}{2}}\frac{K(\xi)}{\xi^2}T_{j1}$$
$$+\frac{\sqrt{(j-1)j(j+1)(j+2)}}{2\xi^2}T_{j5} + \sqrt{\frac{(j-1)(j+2)}{2}}\frac{1}{\xi^2}T_{j8} = 0$$

$$T''_{j3} + \frac{K(\xi)}{\xi}T_{j7}{}' - \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi}T'_{j9} - \left[-\omega^2 + \frac{j(j+1) - 2 + 4K(\xi)^2 + 2H(\xi)^2}{2\xi^2}\right]T_{j3} + \sqrt{\frac{j(j+1)}{2}}\frac{K(\xi)}{\xi^2}T_{j1}$$
$$-\sqrt{2j(j+1)}\frac{K(\xi)}{\xi^2}T_{j4} + \frac{K(\xi)^2}{\xi^2}T_{j5} + \frac{\sqrt{(j-1)j(j+1)(j+2)}}{2\xi^2}T_{j6} + \frac{2K'(\xi)\xi - K(\xi)}{\xi^2}T_{j7} + \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi^2}T_{j9} = 0$$

Radial equations (II)

$$T''_{j4} - \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi}T'_{j7} + \frac{K(\xi)}{\xi}T'_{j9} - \left[-\omega^2 + \frac{j(j+1) + 2K(\xi)^2}{2\xi^2}\right]T_{j4} + \frac{j(j+1)}{2\xi^2}T_{j1} - \sqrt{2j(j+1)}\frac{K(\xi)}{\xi^2}T_{j3}$$
$$+\sqrt{\frac{j(j+1)}{2}}\frac{K(\xi)}{\xi^2}T_{j5} + \sqrt{\frac{(j-1)(j+2)}{2}}\frac{K(\xi)}{\xi^2}T_{j6} + \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi^2}T_{j7} + \frac{2K'(\xi)\xi - K(\xi)}{\xi^2}T_{j9} = 0$$

$$T''_{j5} + \frac{K(\xi)}{\xi}T'_{j7} - \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi}T'_{j8} - \left[-\omega^2 + \frac{j(j+1) - 2 + 4K(\xi)^2 + 2H(\xi)^2}{2\xi^2}\right]T_{j5} - \sqrt{2j(j+1)}\frac{K(\xi)}{\xi^2}T_{j1}$$
$$+\frac{\sqrt{(j-1)j(j+1)(j+2)}}{2\xi^2}T_{j2} + \frac{K(\xi)^2}{\xi^2}T_{j3} + \sqrt{\frac{j(j+1)}{2}}\frac{K(\xi)}{\xi^2}T_{j4} + \frac{2K'(\xi)\xi - K(\xi)}{\xi^2}T_{j7} + \sqrt{\frac{j(j+1)}{2}}\frac{1}{\xi^2}T_{j8} = 0$$

$$T''_{j6} - \sqrt{\frac{(j-1)(j+2)}{2}}\frac{1}{\xi}T'_{j9} - \left[-\omega^2 + \frac{j(j+1) - 2K(\xi)^2 + 2H(\xi)^2}{2\xi^2}\right]T_{j6} + \frac{\sqrt{(j-1)j(j+1)(j+2)}}{2\xi^2}T_{j3}$$
$$+\sqrt{\frac{(j-1)(j+2)}{2}}\frac{K(\xi)}{\xi^2}T_{j4} + \sqrt{\frac{(j-1)(j+2)}{2}}\frac{1}{\xi^2}T_{j9} = 0$$

Radial equations (III)

$$\frac{T'_{j1}}{\xi}+\left(1-\frac{2K(\xi)^2}{j(j+1)}\right)\frac{T'_{j4}}{\xi}-\frac{\sqrt{2}K(\xi)}{\sqrt{j(j+1)}}\frac{T'_{j5}}{\xi}+\frac{\sqrt{2(j-1)(j+2)}K(\xi)}{j(j+1)}\frac{T'_{j6}}{\xi}+\frac{\sqrt{2}K'(\xi)}{\sqrt{j(j+1)}}\frac{T_{j3}}{\xi}+\frac{2K(\xi)K'(\xi)}{j(j+1)}\frac{T_{j4}}{\xi}$$

$$+\frac{\sqrt{2}K'(\xi)}{\sqrt{j(j+1)}}\frac{T_{j5}}{\xi}+\frac{\sqrt{2}}{\sqrt{j(j+1)}}\left(\omega^2-\frac{j(j+1)}{\xi^2}\right)T_{j7}+\frac{2K(\xi)}{\xi^2}T_{j8}-\frac{2K(\xi)\left(-1+H(\xi)^2+K(\xi)^2-\omega^2\xi^2\right)}{j(j+1)\xi^2}T_{j9}=0$$

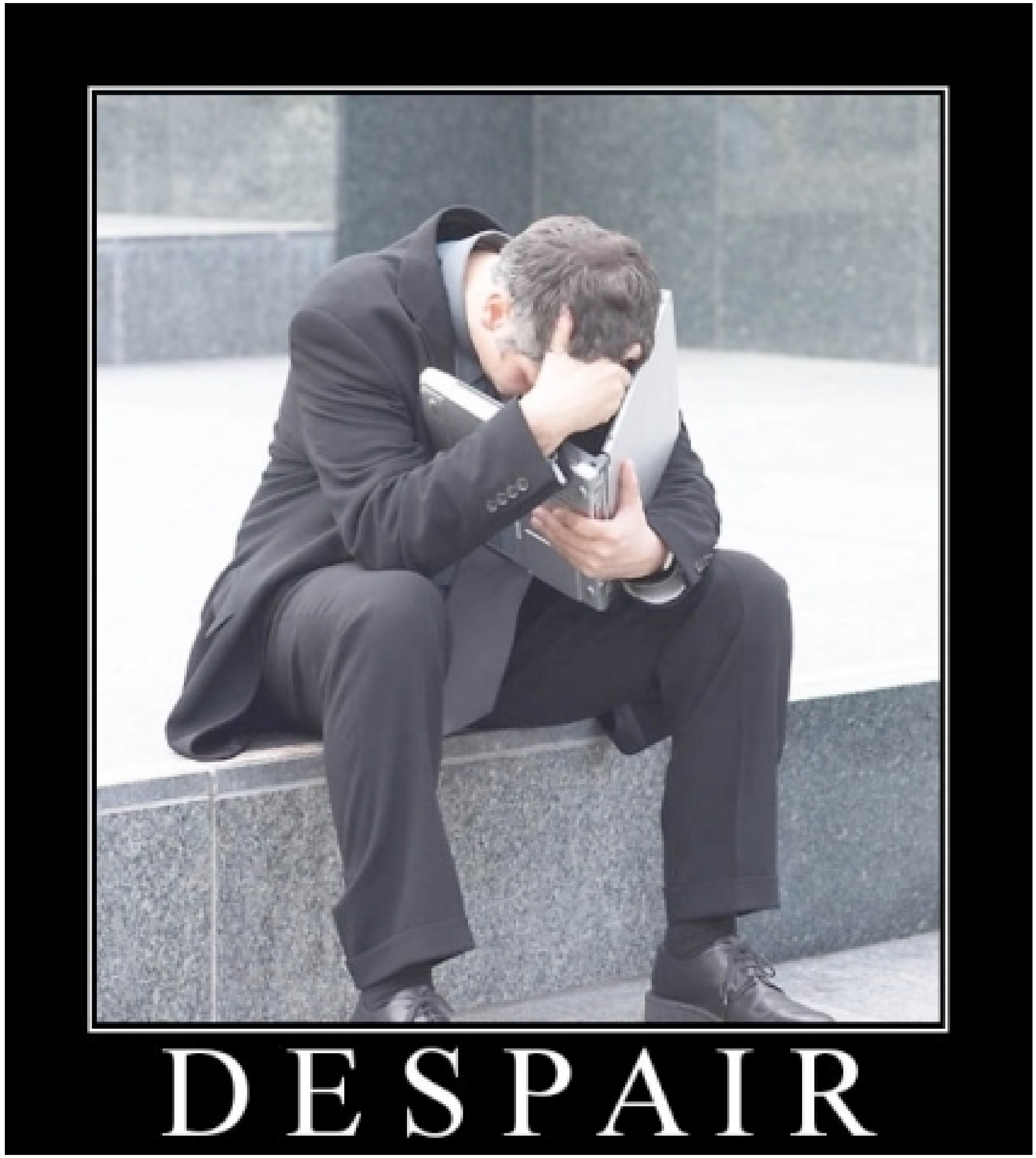
$$\frac{T'_{j3}}{\xi}-\frac{\sqrt{2}K(\xi)}{\sqrt{j(1+j)}}\frac{T'_{j4}}{\xi}+\sqrt{\frac{(j-1)(2+j)}{j(j+1)}}\frac{T'_{j6}}{\xi}+\frac{\sqrt{2}K'(\xi)}{\sqrt{j(1+j)}}\frac{T_{j4}}{\xi}$$

$$+\frac{2K(\xi)}{\xi^2}T_{j7}-\frac{\sqrt{2}\left(H(\xi)^2+j(j+1)-1+K(\xi)^2-\omega^2\xi^2\right)}{\sqrt{j(j+1)}\xi^2}T_{j9}=0$$

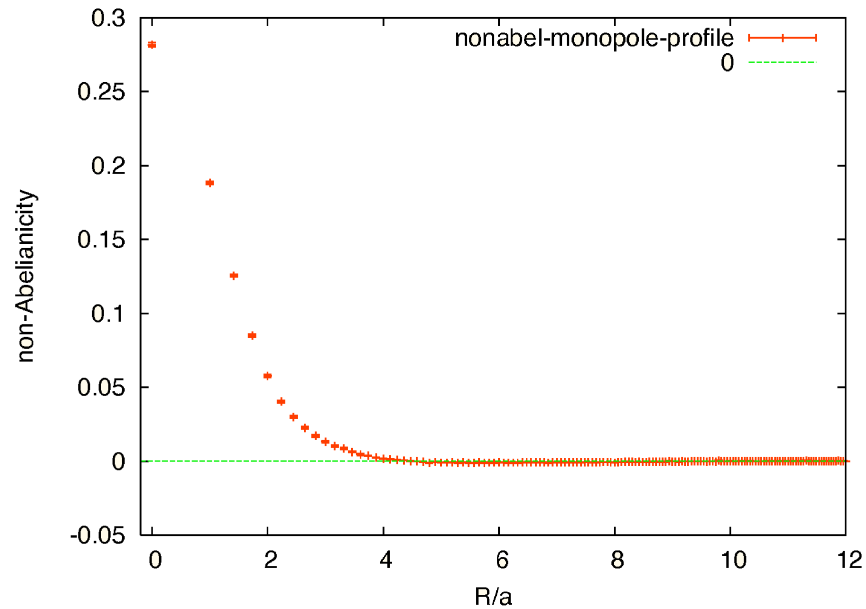
$$\frac{T'_{j2}}{\xi}+\frac{\sqrt{2}K(\xi)\left(j(j+1)-2K(\xi)^2\right)}{j(j+1)\sqrt{(j-1)(j+2)}}\frac{T'_{j4}}{\xi}+\frac{\left(j(j+1)-2K(\xi)^2\right)}{\sqrt{(j-1)j(j+1)(j+2)}}\frac{T'_{j5}}{\xi}+\frac{2K(\xi)^2}{j(j+1)}\frac{T'_{j6}}{\xi}+\frac{\sqrt{2}K'(\xi)}{\sqrt{(j-1)(j+2)}}\frac{T_{j1}}{\xi}$$

$$+\frac{2K(\xi)K'(\xi)}{\sqrt{(j-1)j(j+1)(j+2)}}\frac{T_{j3}}{\xi}+\frac{2\sqrt{2}K(\xi)^2K'(\xi)}{j(j+1)\sqrt{(j-1)(j+2)}}\frac{T_{j4}}{\xi}+\frac{2K(\xi)K'(\xi)}{\sqrt{(j-1)j(j+1)(j+2)}}\frac{T_{j5}}{\xi}-\frac{2\omega^2K(\xi)}{\sqrt{(j-1)j(j+1)(j+2)}}T_{j7}$$

$$-\frac{\sqrt{2}\left(H(\xi)^2+j(j+1)-1-K(\xi)^2-\omega^2\xi^2\right)}{\sqrt{(j-1)(j+2)}\xi^2}T_{j8}-\frac{2\sqrt{2}K(\xi)^2\left(H(\xi)^2-1+K(\xi)^2-\omega^2\xi^2\right)}{j(j+1)\sqrt{(j-1)(j+2)}}\frac{T_{j9}}{\xi^2}=0$$


DESPAIR

The lattice comes to our rescue



- We get information on the monopole size: $r_m \approx 1.5a \approx .15$ fm

Monopoles are small objects!

- In the radial equations we can neglect the monopole core

$$K(\xi) \to 0 \qquad H(\xi) \to \xi$$

Lattice results from: E. M. Ilgenfritz *et al.*, 0710.2607 [hep-lat]

Radial equations in the limit of pointlike monopoles

$$T''_{j1}(\xi) - \left[-\omega^2 + \frac{j\,(j+1)}{\xi^2}\right] T_{j1}(\xi) = 0$$

$$T''_{j4}(\xi) - \left[-\omega^2 + \frac{j\,(j+1)}{\xi^2}\right] T_{j4}(\xi) = 0$$

$$T''_{j2}(\xi) - \left[-\omega^2 + 1 + \frac{j\,(j+1) - n^2}{\xi^2}\right] T_{j2}(\xi) = 0$$

$$T''_{j5}(\xi) - \left[-\omega^2 + 1 + \frac{j\,(j+1) - n^2}{\xi^2}\right] T_{j5}(\xi) = 0$$

$$T''_{j3}(\xi) - \left[-\omega^2 + 1 + \frac{j\,(j+1) - n^2}{\xi^2}\right] T_{j3}(\xi) = 0$$

$$T''_{j6}(\xi) - \left[-\omega^2 + 1 + \frac{j\,(j+1) - n^2}{\xi^2}\right] T_{j6}(\xi) = 0.$$

✦ We get a system of Bessel equations with index $j' = -\frac{1}{2}[-1 + \sqrt{(2j+1)^2 - 4n^2}]$

$T_j(r) \rightarrow j_{j'}(kr)$ $\Rightarrow$ Scattering phase $\delta_{j'} = -j'\frac{\pi}{2}$ INDEPENDENT OF ENERGY

Contribution to thermodynamics

- Correction to partition function induced by scattering on monopoles

$$\delta M_m = -\frac{T}{\pi}\sum_j (2j+1)\int dk \frac{d\delta_j}{dk} f(k,T)$$

Constant scattering phase

NO CONTRIBUTION to thermodynamics

Scattering amplitude

- Consider a gauge boson entering from $z = -\infty$
- J_3 is fixed:

$$J_3 = -\left[\left(\vec{L}\cdot\vec{r}\right) + \left(\vec{I}\cdot\vec{r}\right) + \left(\vec{S}\cdot\vec{r}\right)\right] = -\left[\left(\vec{I}\cdot\vec{r}\right) + \left(\vec{S}\cdot\vec{r}\right)\right] = -\left[n+\sigma\right] = -\nu$$

- Our spherical harmonics are eigenstates of $\left(\vec{I}\cdot\vec{r}\right)$ and $\left(\vec{S}\cdot\vec{r}\right)$
- We decompose our solution as

$$\Psi^{(+)}(\vec{r}) = e^{-i\pi\nu}\sum_{j=|\nu|}^{j_{max}}(2j+1)e^{i\pi j}e^{-i\pi j'/2}\,j_{j'}(kr)e^{-2i\nu\varphi}\,d^{(j)}_{\nu,-\nu}(\theta)$$

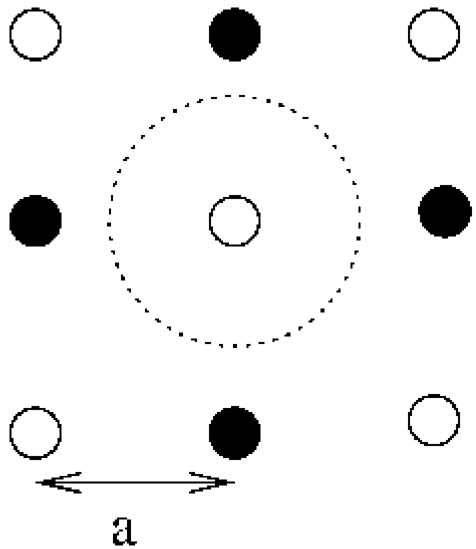
$$d^{j}_{n,-n}(z) = \frac{(-1)^{j-n}}{2^j(j-n)!}(1-z)^{-n}\left(\frac{d}{dz}\right)^{j-n}\left[(1-z)^{j+n}(1+z)^{j-n}\right]$$

for which we can write the asymptotic behavior

$$\Psi^{(+)}(\vec{r}) \sim e^{-2i\nu\varphi}\left[e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right]$$

Y. Kazama, C. N. Yang and A. S. Goldhaber, PRD15 (1977)

Transport cross section



$j_{max} \sim 5-6$

in our temperature regime

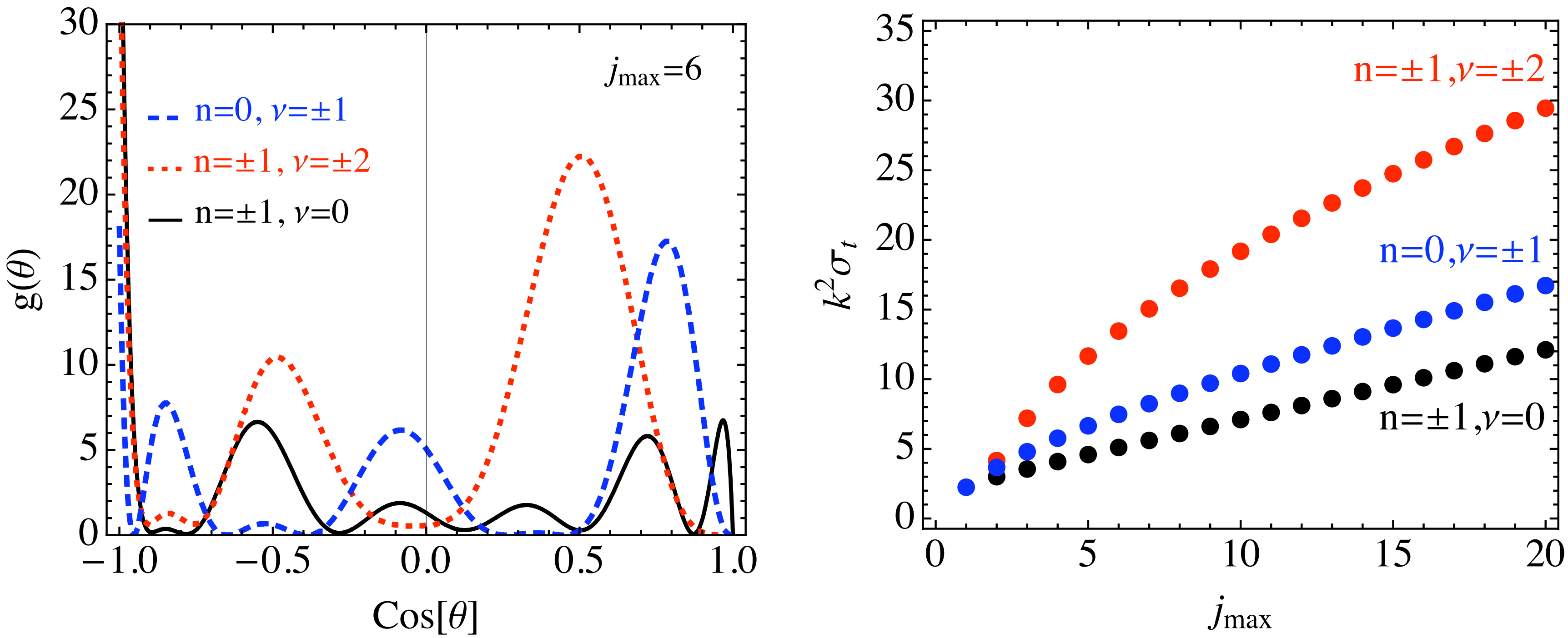
We have obtained the scattering amplitude

$$2ikf(\theta) = \sum_{j=\nu}^{j_{max}} (2j+1)e^{i\pi(j'-j)} d^j_{\nu,-\nu}(\cos\theta),$$

Now we can calculate the transport cross section:

$$\sigma_t = \int d\cos\theta(1-\cos\theta)|f(\theta)|^2$$

Transport cross section: results



$$g(\theta) = (1 - \cos\theta)|f(\theta)|^2$$

- Without cutoff j_{max}, $g(\theta)$ would be peaked in the forward direction
- Angular distribution dramatically changed by cutoff: strong backward enhancement
- Transport cross section insensitive to oscillations

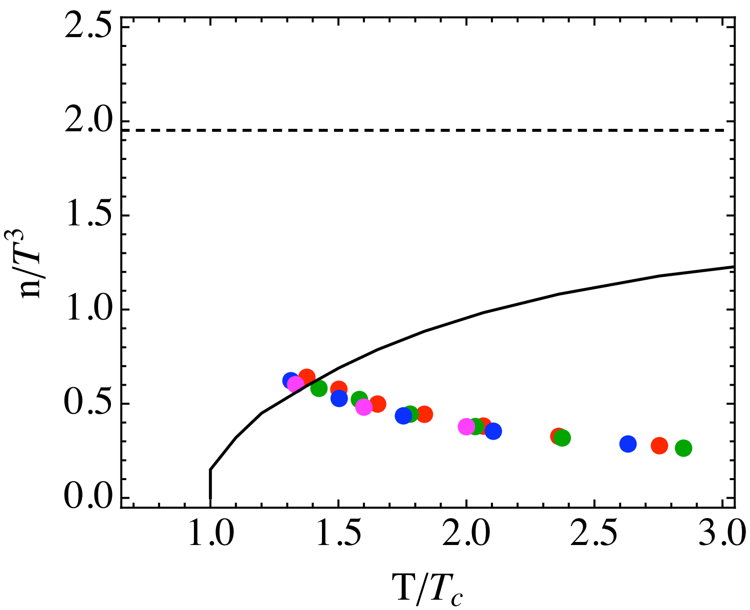
Scattering rate and viscosity

$$\frac{\dot{w}_{gm}}{T} = \frac{\langle n_m (\sigma_t)_{gm} \rangle}{T} \qquad \frac{\eta}{s} \approx \frac{T}{5\dot{w}}$$

$$\frac{\dot{w}_{gm}}{T} = \frac{n_m(T)}{n_g(T)T} \frac{4\pi}{(2\pi)^3} \int k^2 dk \, \sigma_t(k) \, \rho_g(k) \qquad \text{with} \qquad n_g(T) = \int \frac{d^3k}{(2\pi)^3} \rho_g(k)$$

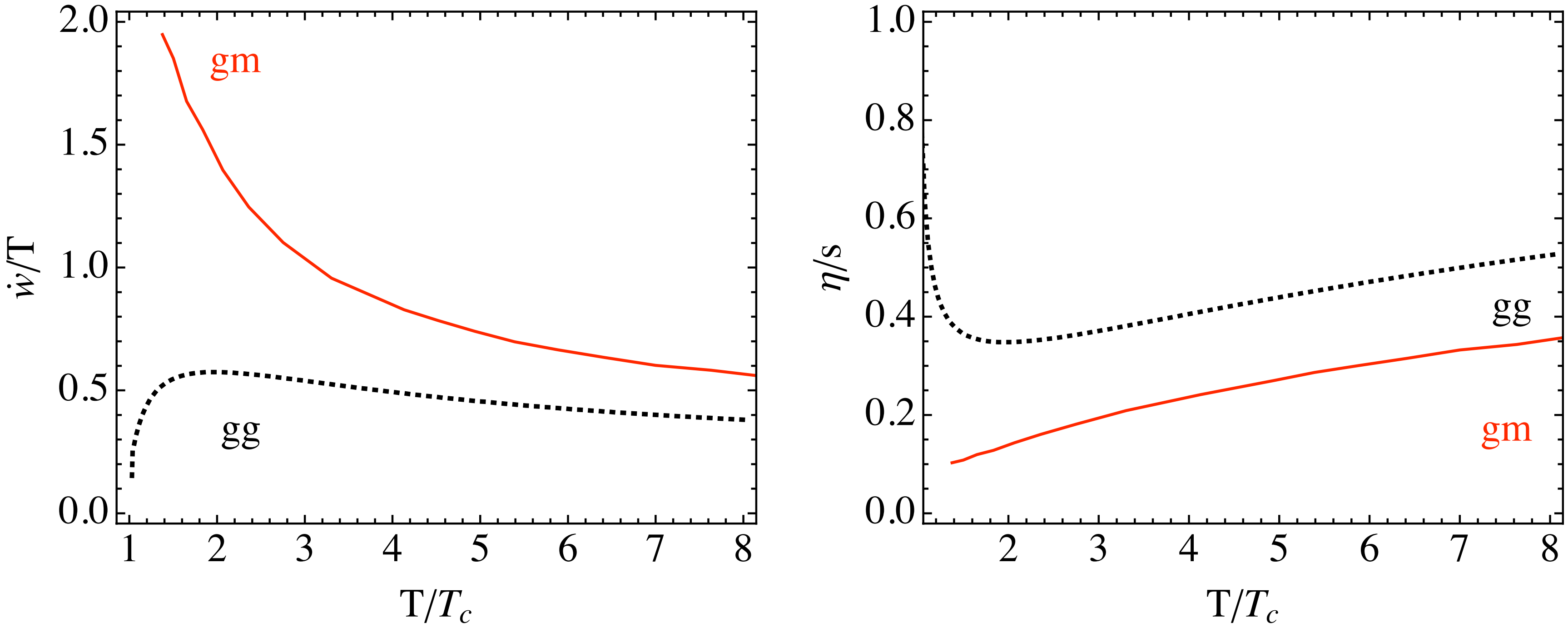
to be compared to the perturbative gluon-gluon scattering rate

$$\frac{\dot{w}_{gg}}{T} = \frac{1}{n_g(T)} \int \frac{4\pi k_1^2 dk_1}{(2\pi)^3} \int \frac{2\pi k_2^2 dk_2}{(2\pi)^3} \int_{-1}^{1} d\cos\theta \, \sigma^t_{gg}(k_1, k_2, \cos\theta) \, \rho_g(k_1, T) \, \rho_g(k_2, T)$$



- ✦ Gluon density decreasing close to T_c
  - ➠ Gluons become heavy: $m_g \sim 0.8$ GeV
  - ➠ Coupling to Polyakov loop gives further suppression

Scattering rate and viscosity: results



- Gluon-monopole scattering rate 4 times larger than gluon-gluon one close to T_c
- gluon-gluon scattering rate $\sim (\log T)^{-2}$ at large T
- gluon-monopole scattering rate $\sim (\log T)^{-3}$ at large T

Conclusions

- Study of gluon-monopole scattering
- LITTLE EFFECT on thermodynamics
- BIG ENHANCEMENT of transport cross section
  - ➟ VISCOSITY SUPPRESSION at moderate temperatures

Outlook

- Inclusion of quarks in the system
- Monopole contribution to thermodynamics close to T_c