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QCD phenomenology with infrared finite SDE solutions

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Plan of talk

- **SDE solutions: Infrared finite gluon propagator and coupling constant**
- **Phenomenology of IR finite gluon propagator and coupling constant: Tree level**
- **Phenomenology of IR finite gluon propagator and coupling constant: Loop level**
- **Concluding Remarks**

Infrared finite gluon propagator and coupling constant

Schwinger-Dyson equations (+ lattice QCD) $\longrightarrow$

1) gluon progagator is IR finite

2) coupling constant freezes as $q^2 \to 0$ (fixed-point)

Scenario to use it $\to$ **D**ynamical **P**erturbation **T**heory : (Pagels and Stokar, 1979)

Amplitudes that do not vanish to all orders in perturbation theory are given by their free field values, while amplitudes that vanish as $\lambda \propto e^{-1/g^2}$ are retained, and possibly dealt with in an expansion in $g^n \lambda$

($\to$...work with dressed quantities)

How are the solutions? (JMC,JP,ACA,DB,...)

$$\imath\Delta_{\mu\nu}(q) = P_{\mu\nu}\Delta(q) + \xi\frac{q_\mu q_\nu}{q^4} \; ; \; P_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \; ,$$

$$\Delta(Q^2) \propto \frac{1}{Q^2 + m_g^2(Q^2)}$$

dynamical gluon mass $(m_g^2(Q^2))$ - falls with the momentum as $1/Q^2$ (ACA & JP).

For simplicity, assume in actual calculations (ACA & AN) $\longrightarrow$

$$m_g^2(Q^2) = \frac{m_g^4}{Q^2 + m_g^2}$$

A simple fit for the coupling constant (JMC)

$$\bar{\alpha}_{sd}(q^2) = \frac{1}{4\pi b \ln[(4m_g^2 - q^2 - \imath\epsilon)/\Lambda^2]}$$

$b = (33 - 2n_f)/48\pi^2$ —— **The IR fixed-point :**

$$\bar{\alpha}_{sd}(0) \equiv \frac{1}{4\pi b \ln[(4m_g^2)/\Lambda^2]}$$

$m_g \approx \mathcal{O}(1.2 - 2)\Lambda$, with $\Lambda = \Lambda_{QCD} \approx 300$ MeV.

m_g **- a natural IR cutoff !**

Take it for granted ... next talks ...

Phenomenology of IR finite gluon propagator and coupling constant: Tree level

Examples: heavy meson decays, pion and proton form factor, Pomeron and QCD models for hadronic cross sections, pion transition form factor, survival probability for rapidity gaps, gluon structure functions at small-x,$\gamma - p$ and $\gamma - \gamma$ cross sections, gluon condensate, ... hep-ph/0803.0154;IJMPA19(2008)151; BJP37(2007)306; PRD73(2006)074019; PLB641(2006)171; PRD72(2005)034019; ....

1st. example: Pion form factor (AN, A.Mihara and A.C.Aguilar)

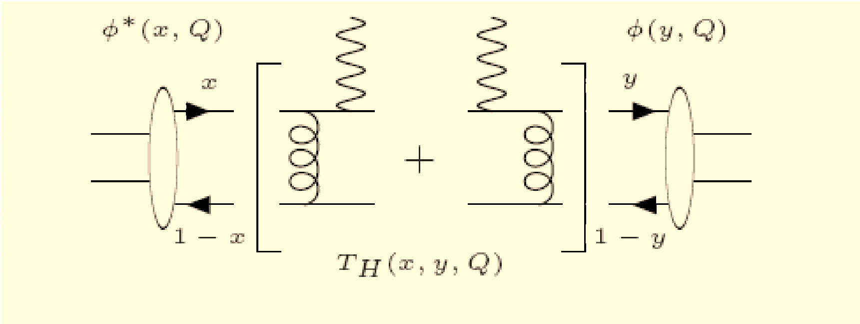

Figure 1: pion form factor - perturbative QCD.

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi^*(y, \tilde{Q}_y) T_H(x, y, Q^2) \phi(x, \tilde{Q}_x)$$

$$T_H(x, y, Q) = \frac{64\pi}{3Q^2} \left\{ \frac{2}{3} \frac{\alpha_s[(1-x)(1-y)Q^2]}{(1-x)(1-y)} + \frac{1}{3} \frac{\alpha_s(xyQ^2)}{xy} \right\}$$

DPT - (dressed quantities - $\alpha_{sd}(Q^2)$ and $D(Q^2)$)

$$T_H(x,y,Q^2) = \frac{64\pi}{3}\left[\frac{2}{3}\alpha_s(K^2)D(K^2) + \frac{1}{3}\alpha_s(P^2)D(P^2)\right]$$

where $K^2 = (1-x)(1-y)Q^2$ and $P^2 = xyQ^2$

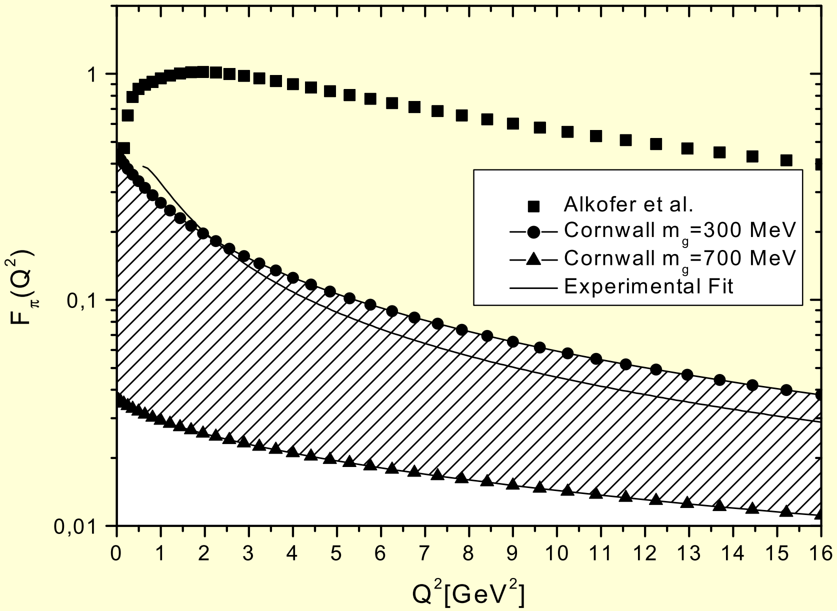

Figure 2: F_π **- DPT** $\times$ **experimental data**

2nd. example: Hadronic cross sections in a QCD-inspired model (AN and E.G.S.Luna)

Cross section for producing jets with $p_T > p_{T_{min}}$ (through the dominant process $gg \longrightarrow gg$)

$$\sigma_{jet}(s) = \int_{p^2_{T_{min}}} dp_T^2 \, \frac{d\hat{\sigma}_{gg}}{dp_T^2} \int_{x_1 x_2 > 4p_T^2/s} dx_1 dx_2 \, g(x_1, Q^2) \, g(x_2, Q^2)$$

$g(x, Q^2) \longrightarrow$ gluon flux, $p^2_{T_{min}} \longrightarrow$ min. p_T for pert. theory

Standard calculation $\longrightarrow$

$$\hat{\sigma}_{gg}(\hat{s}) \propto \frac{9\pi\alpha_0^2}{m_0^2} \, \theta(\hat{s} - m_0^2)$$

m_0 **and** α_0 (fitted parameters)

DPT $\longrightarrow$ compute elementary cross section, like $\hat{\sigma}_{gg}(\hat{s})$ with IR finite quantities

m_0, α_0 and $p^2_{T_{min}}$ substituted by m_g!

(detail: sum of gluon polarizations - 2 degrees...)

We fitted the pp and $p\bar{p}$ scattering data keeping m_g as a free parameter

Taking a 5% variation on the minimal χ^2/DOF value indicate $m_g \approx 400^{+350}_{-100}$ MeV

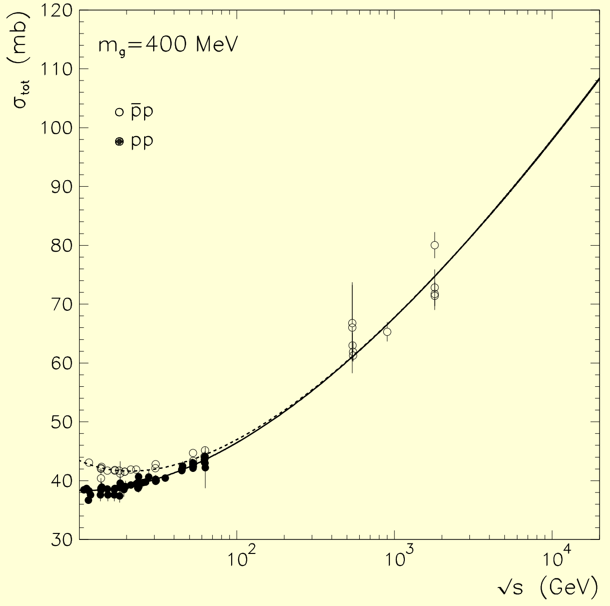

Figure 3: Total cross section for pp (solid curve) and $\bar{p}p$ (dashed curve) scattering.

3rd. example: Non-leptonic annihilation B mesons decays (AN and C.Zanetti)

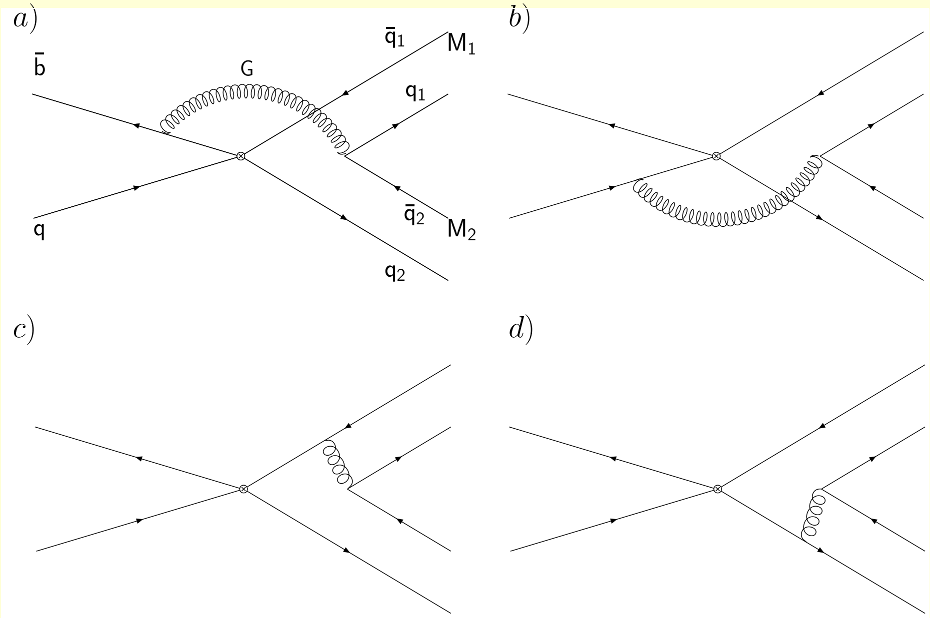

Figure 4: two-body non-leptonic annihilation B decays

(end-point divergences... arbitrary cutoff) $\int \frac{dx}{x} = \ln \frac{m_B}{\Lambda_h}(1 + \rho e^{i\phi}) \quad 0 \leq \rho \leq 1$

$\mathcal{B}r(B_d^0 \to K^+K^-) \times 10^8 \to \; 7.18$, $\mathcal{B}r(B_d^0 \to D_s^- K^+) \times 10^5 \to \; 1.98$, ...

(compatible with existent data - DPT → no end-point diverg. - $m_g = 500$MeV)

4th. example: A QCD-Pomeron model (AN, G.Krein and F.Halzen)

Amplitude for elastic proton-proton scattering in the Landshoff-Nachtmann Pomeron model

$$\frac{d\sigma}{dt} = \frac{|A(s,t)|^2}{16\pi s^2}$$

A(s,t)= $\imath s 8\alpha_s^2 \left[T_1 - T_2\right]$

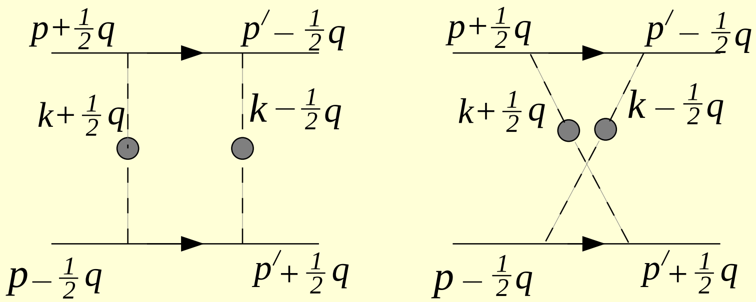

Figure 5: T_1 **and** T_2

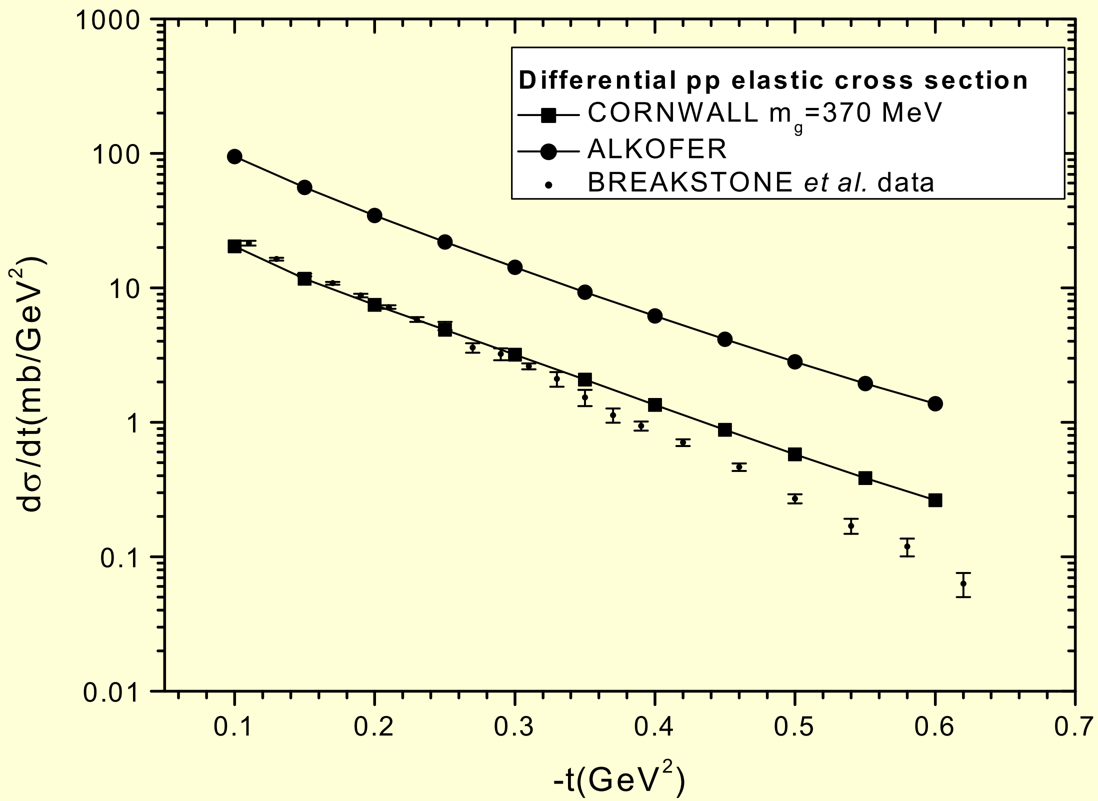

Figure 6: Differential pp elastic cross section at $\sqrt{(s)} = 53$ GeV computed within the Landshoff–Nachtmann model for the Pomeron, using different infrared couplings and gluon propagators obtained from DSE solutions.

Phenomenology of IR finite gluon propagator and coupling constant: Loop level

Pinch Technique is fundamental in order to apply DPT at loop level

Pinch Technique provides a skeleton expansion for QCD (JP et al., Brodsky et al.)

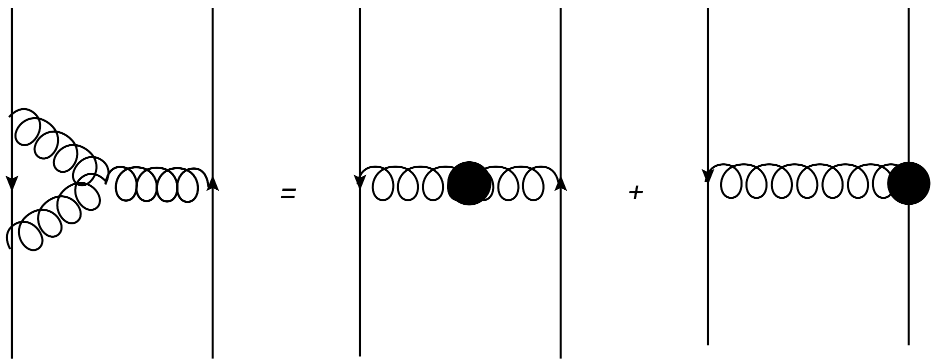

Figure 7: One of the several contributions for quark-quark scattering at one-loop where a typical non-Abelian contribution can be separated into contributions to the vertex and propagator. The black blob indicates the pinched parts that enter into the vertex and propagator at 1-loop level.

Example: Bjorken sum rule

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 dx \left[g_1^p(x, Q^2) - g_1^n(x, Q^2) \right]$$

$g_1^p (g_1^n)$: first spin structure function for the proton (neutron).

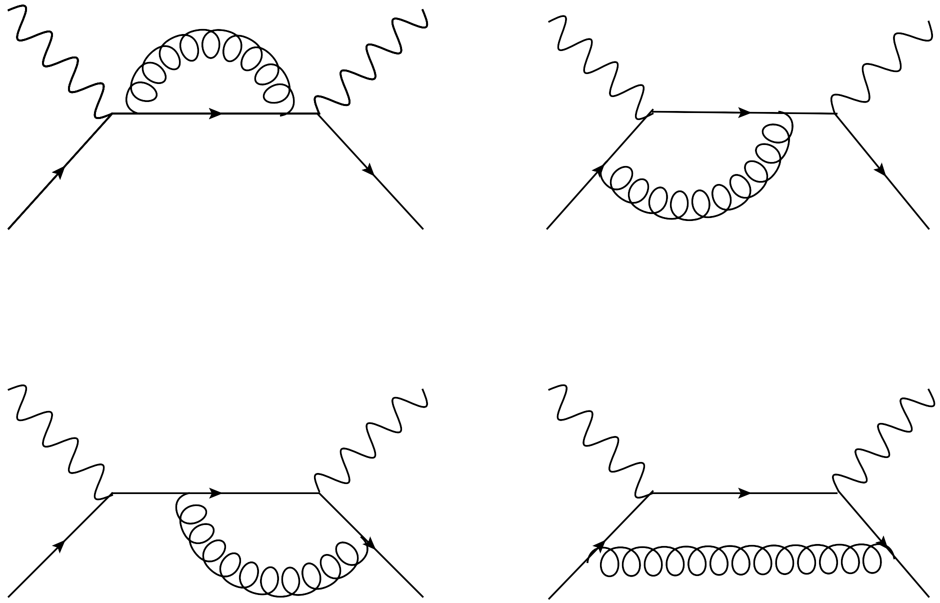

Figure 8: Perturbative corrections to the deep inelastic structure functions at 1-loop.

QCD correction up to third order in α_s ($n_f = 3$)

$$\Gamma_1^{p-n}(Q^2) = \frac{1}{6}\left|\frac{g_A}{g_V}\right|\left[1 - \frac{\alpha_s(Q^2)}{\pi} - 3.58\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 \ldots\right]$$

Pert. QCD → DPT dressed quantities...

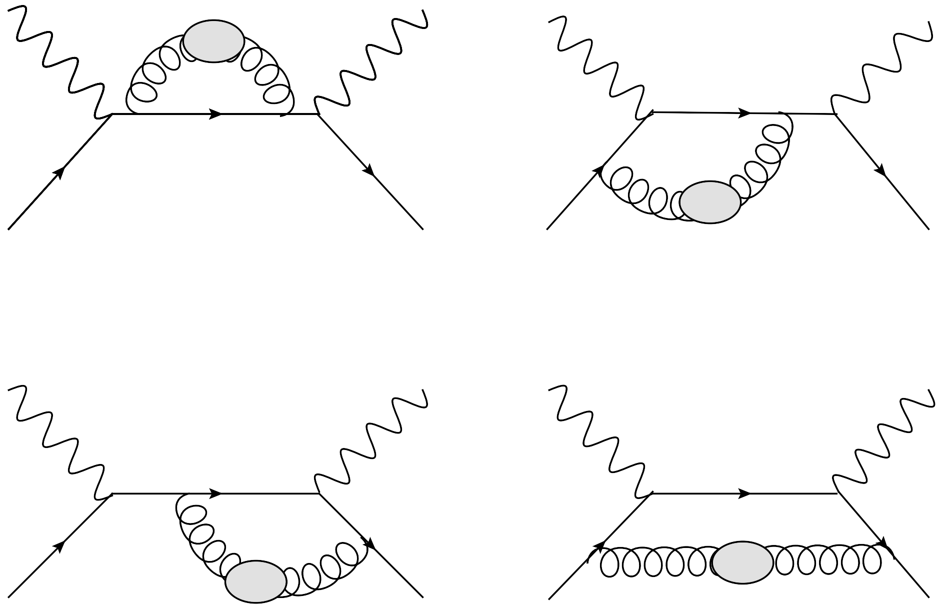

Figure 9: Leading one-loop correc. in DPT. Gray blobs: all-order sum of diagrams in the gluon polarization vector.

Jlab data of $\Gamma_1^{p-n}(Q^2)$ (up to order $\bar{\alpha}_{sd}^4$) was used to obtain m_g (work in progress - ACA,AN,JP)

The result is shown in Fig.(10), which is α_s fitted with $m_g = \mathcal{O}(335 - 400)$ MeV

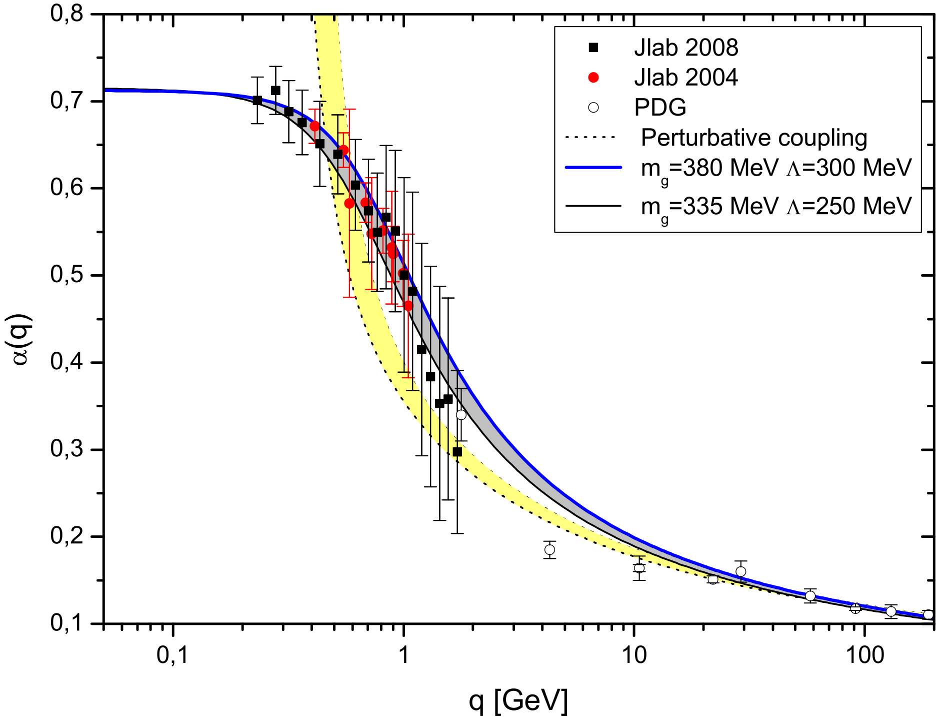

Figure 10: Effective coupling constant extracted from the experimental data of $\Gamma_1^{p-n}(Q^2)$.

missing: dynamical mass effects inside the loop (m_g and m_q)

Concluding Remarks

Tree Level

- IR finite gluon propagator and coupling constant: natural cutoff

- All tree level calculations: Fitted with dynamical gluon masses in the range - 370 - 600 MeV (higher order contributions....???)

Detail: Phase space - dictated by dynamical masses (...effect is small)

Sum of gluon polarization? Example: $g + g \rightarrow g + g$ (computed with 2 degrees of polarization) Necessary: useful rule!

Loop Level

- **Loop level calculations:** Pinch technique is necessary in order to apply DPT

- **Preliminary calculation:** Consistency with dynamical gluon mass in the range 350-500 MeV

- **IR fixed point:** Can we use it to fix the problem of renormalization scheme?

Detail: SDE solutions in MOM scheme $\leftrightarrow$ Pert. QCD calc. in $\overline{MS}$ scheme

The use of DPT may improve the behavior of QCD perturbative series