Glueballs within the Gribov-Zwanziger framework

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Overview

Overview

What are glueballs

QCD

- QCD is the theory of interactions between quarks and gluons
- High energies: asymptotically free
- Low energies: confining ⇒ still no good understanding, non-perturbative aspects play a role
- Therefore, study pure gluonic physical particles where confinement plays an important role:

glueballs = particle composed entirely of gluons

The experimental status

So far...

- 20 years of search towards glueball (*Crede and Meyer*)
- Many experiments:
    - ASTERIX, OBELIX, Crystal barrel at CERN,
    - BES-II (Bejing), CLEO (Cornell University), KLOE (DAPHNE, Italy)
- no definite answer: "has glueball been observed?"

The experimental status

The scalar sector: $J^{PC} = 0^{++}$

- The scalar sector is most complex and controversial sector
- $f_0(1500)$ is often considered as a candidate for the lightest glueball
- Many mixing models have been proposed for $f_0(1370)$, $f_0(1500)$, $f_0(1710)$

The pseudoscalar sector: $J^{PC} = 0^{-+}$

- Very unclear situation

The tensor sector: $J^{PC} = 2^{++}$

- Evidence for a tensor glueball is non-existent

The experimental status

Future experiments

- BES-III at BEPCII in Bejing (2008)
- The COMPASS Experiment at CERN (2002)
- The GlueX Experiment at Jefferson Laboratory in USA, VA (2014)
- The PANDA Experiment at GSI (part of FAIR) in Germany

Lattice QCD

Quenched approximation

- The fluctuation of a gluon into a quark-antiquark is left out
- The lightest three states: (*Morningstar*)
    1. scalar (0^{++}): 1.710 GeV/c^2
    2. tensor (2^{++}): 2.390 GeV/c^2
    3. pseudoscalar: (0^{+-}): 2.560 GeV/c^2

Different theoretical models/methods

Good review by *Mathieu, Kochelev and Vento*

- The MIT bag model (*Jaffe and Johnson*):
- Many other models/approaches: massive gluons, Effective hamiltonian, Ads/QCD, QCD sumrules, etc

Overview

1. Introduction
2. Yang-Mills and the Landau gauge fixing
    - The action
    - BRST
    - Physical operator
3. The Gribov-Zwanziger action
    - The action
    - BRST
    - Physical operator

The action

Faddeev-Popov

- The partition function:

$$Z_{FP} = \int [dA] \delta(\partial A) \det M^{ab} e^{-\frac{1}{4}\int \mathrm{d}^4 x F^a_{\mu\nu} F^a_{\mu\nu}}$$

with

$$M^{ab} = -\partial_\mu (\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu)$$

- Equivalently

$$Z_{FP} = \int [dA][dc][d\overline{c}][db] e^{-S_{FP}}$$

with

$$S_{FP} = \frac{1}{4} \int \mathrm{d}^4 x F^a_{\mu\nu} F^a_{\mu\nu} + \int \mathrm{d}^4 x \, \left(b^a \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

BRST

BRST invariance

- The action is invariant under BRST transformation:

$$sS_{FP} = 0 \qquad\qquad s^2 = 0$$

with

$$sA^a_\mu = -\left(D_\mu c\right)^a \ , \qquad sc^a = \frac{1}{2} g f^{abc} c^b c^c \ ,$$
$$s\bar{c}^a = b^a \ , \qquad sb^a = 0 \ ,$$

- BRST invariance is at the origin of the Slavnov-Taylor identity, which allows us to prove the renormalizability
- The BRST charge allows us to define the sub-space of the physical states and to establish the unitarity of the S matrix:

$$sO_{phys} = 0 \ , \text{modolo } s\text{-exact parts}$$

Physical operator

The glueball operator

- Try to construct a glueball operator which is renormalizable
- For this we add $\mathcal{F} = F_{\mu\nu}^2$ coupled to a source q to the action S_{FP}
- We renormalize the action

but...

The action $S_{YM} + \int \mathrm{d}^4 x q(x) F^2(x)$ will not be renormalizable...
we shall need to add extra operators

Physical operator

3 different classes of $D = 4$ operators

- $\mathcal{F}$: Gauge invariant operators (= BRST closed, but not exact ones), e.g. $F_{\mu\nu}^2$
- $\mathcal{E}$: Gauge invariant exact operators, e.g. $s(\overline{c}^a \partial_\mu A_\mu^a)$, with s the BRST variation
- $\mathcal{H}$: "Equation of motion terms", e.g. $A_\mu^a \frac{\delta S}{\delta A_\mu^a}$

Mixing

- These 3 different classes can mix!
- Mixing happens in a certain way

Physical operator

3 different classes of $D = 4$ operators

- $\mathcal{E}$ op. cannot mix with $\mathcal{F}$ op:

$$\begin{aligned} \langle \mathcal{E}_0 \rangle &= \langle s(...) \rangle = 0 \\ \langle \mathcal{E}_0 \rangle &= a \langle \mathcal{F} \rangle + b \underbrace{\langle \mathcal{E} \rangle}_{=0} + c \underbrace{\langle \mathcal{H} \rangle}_{=0} \end{aligned}$$

- $\mathcal{H}$ op. cannot mix with $\mathcal{F}$ and $\mathcal{E}$ as $\mathcal{H}$ will vanish upon using the equation of motion, while $\mathcal{F}$ and $\mathcal{E}$ will not.

Physical operator

3 different classes of $D = 4$ operators

This is equivalent with:

$$\begin{pmatrix} \mathcal{F}_0 \\ \mathcal{E}_0 \\ \mathcal{H}_0 \end{pmatrix} = \begin{pmatrix} Z_{\mathcal{F}\mathcal{F}} & Z_{\mathcal{F}\mathcal{E}} & Z_{\mathcal{F}\mathcal{E}} \\ 0 & Z_{\mathcal{E}\mathcal{E}} & Z_{\mathcal{E}\mathcal{H}} \\ 0 & 0 & Z_{\mathcal{H}\mathcal{H}} \end{pmatrix} \begin{pmatrix} \mathcal{F} \\ \mathcal{E} \\ \mathcal{H} \end{pmatrix}$$

Physical operator

Mixing in Yang-Mills

If we couple all the operators to a source

$$\begin{aligned} \mathcal{F}_0 &= \frac{1}{4}F_{\mu\nu}^2 \\ \mathcal{E}_0 &= s(\overline{c}^a\partial_\mu A_\mu^a) \\ \mathcal{H}_0 &= A_\mu^a\frac{\delta S_{FP}}{\delta A_\mu^a} \end{aligned}$$

and do the renormalization, we find:

$$\begin{pmatrix} \mathcal{F}_0 \\ \mathcal{E}_0 \\ \mathcal{H}_0 \end{pmatrix} = \begin{pmatrix} Z_{qq}^{-1} & -Z_{Jq}Z_{qq}^{-1} & -Z_{Jq}Z_{qq}^{-1} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{F} \\ \mathcal{E} \\ \mathcal{H} \end{pmatrix}$$

Physical operator

Determining the mixing matrix with the help of Green's function

- We start with the following most general $(n+2m+r)$-point functions:

$$\begin{aligned}\mathcal{G}^{n+2m+r} &= \langle A(x_1)\ldots A(x_n)c(y_1)\ldots c(y_m)\overline{c}(\hat{y}_1)\overline{c}(\hat{y}_m)b(z_1)\ldots b(z_r)\rangle \\ &= \int [d\Phi]A(x_1)\ldots A(x_n)c(y_1)\ldots c(y_m)\overline{c}(\hat{y}_1)\overline{c}(\hat{y}_m)b(z_1)\ldots b(z_r)\mathrm{e}^{-S}\end{aligned}$$

- From

$$\frac{d\mathcal{G}^{n+2m+r}}{dg^2} = \ldots \text{ must be finite}$$

we can determine the matrix to all orders!

Physical operator

Result: the mixing matrix

$$\begin{pmatrix} \mathcal{F}_0 \\ \mathcal{E}_0 \\ \mathcal{H}_0 \end{pmatrix} = \begin{pmatrix} 1 - \frac{\beta/g^2}{\epsilon} & -\frac{2\gamma_c}{\epsilon} & -\frac{2\gamma_c}{\epsilon} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{F} \\ \mathcal{E} \\ \mathcal{H} \end{pmatrix}$$

RGE

From this matrix we can find a RGE

$$\begin{aligned} O_{\text{phys}} = \mathcal{R} &= \frac{\beta(g^2)}{g^2}\mathcal{F} - 2\gamma_c\mathcal{E} - 2\gamma_c\mathcal{H} \\ &= \frac{\beta(g^2)}{g^2}F^2 + s(\ldots) \end{aligned}$$

=Trace anomaly

Physical operator

Physical operators

Due to the s-invariance of the action:

$$
\begin{aligned}
\langle O_{phys}(x) O_{phys}(y) \rangle &= \langle \mathcal{R}(x) \mathcal{R}(y) \rangle \\
&= \left\langle \left[\frac{\beta}{g^2} F^2 + s(\ldots) \right](x) \left[\frac{\beta}{g^2} F^2 + s(\ldots) \right](y) \right\rangle \\
&= \left\langle \left(\frac{\beta}{g^2} \right)^2 F^2(x) F^2(y) + s(\ldots) \right\rangle \\
&= \left(\frac{\beta}{g^2} \right)^2 \langle F^2(x) F^2(y) \rangle
\end{aligned}
$$

Overview

The action

- The Landau gauge is plagued by the existence of the Gribov copies:

$$\partial\widetilde{A} = \partial A = 0$$

with $\widetilde{A}_\mu$ a gauge transformation of A_μ

- We can get rid of part of these copies by restricting the domain of integration in the Feynman path integral to the Gribov region Ω

$$\Omega = \{A; \partial A = 0, -\partial D > 0\}$$

- The partition function

$$Z_{FP} = \int [dA]\delta(\partial A) \det M^{ab} e^{-\left(\frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu} F^a_{\mu\nu} + S_H\right)}$$

with

$$S_H = \gamma^4 g^2 \int f^{abc} A^b_\mu (M^{-1})^{ad} f^{dec} A^e_\mu$$

The action

- The local Gribov-Zwanziger action is given by

$$S_{GZ} = \underbrace{S_{\mathrm{YM}} + S_{gf}}_{S_{FP}} + S_0 + S_\gamma$$

with

$$\begin{aligned} S_0 &= \int \mathrm{d}^4x \left(\overline{\varphi}_\mu^{ac} M^{ab} \varphi_\mu^{bc} - \overline{\omega}_\mu^{ac} M^{ab} \omega_\mu^{bc} \right) \\ S_\gamma &= -\gamma^2 g \int \mathrm{d}^4x \left(f^{abc} (\varphi_\mu^{bc} + \overline{\varphi}_\mu^{bc}) A_\mu^a + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \end{aligned}$$

- The parameter γ is called the Gribov parameter, has dimension of a mass and is not free

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0$$

with Γ the quantum action defined as $\mathrm{e}^{-\Gamma} = \int [D\Phi] \mathrm{e}^{-S}$

BRST

Breaking of BRST

- s is given by

$$sA^a_\mu = -\left(D_\mu c\right)^a \ , \quad sc^a = \frac{1}{2} g f^{abc} c^b c^c \ , \quad s\bar{c}^a = b^a \ , \quad sb^a = 0 \ ,$$
$$s\varphi^a_i = \omega^a_i \ , \quad s\omega^a_i = 0 \ , \quad s\overline{\omega}^a_i = \overline{\varphi}^a_i \ , \quad s\overline{\varphi}^a_i = 0 \ ,$$

- S_{GZ} is not invariant under the BRST s:

$$sS_{\mathrm{GZ}} = s\left(S_{\mathrm{YM}} + S_{gf} + S_0 + S_\gamma\right) = s\left(S_\gamma\right) \sim \gamma^2 \neq 0$$

BRST

Breaking of BRST

- Despite breaking, GZ action is renormalizable, due to rich set of Ward identities
- Only two renormalization constants are needed: Z_A and Z_g
- Gluons get confined by the horizon:

$$\left\langle A^a_\mu A^b_\nu \right\rangle_k = \delta^{ab} \frac{k^2}{k^4 + \gamma^4} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$$

- complex poles $\longrightarrow$ cannot describe physical exitations
- Fourier transform shows positivity violation

BRST

Breaking of BRST

- How to define physical operators?

BRST

Restoring the BRST

- We can embed S_{GZ} in "larger" action:

$$\begin{aligned} S_{\mathrm{GZ}} &= S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_0 + S_\gamma \\ &\updownarrow \\ \Sigma_{\mathrm{GZ}} &= S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_0 + S_s \end{aligned}$$

  whereby

$$S_s = s\left(\int \mathrm{d}^4x f(\text{fields, sources})\right)$$

- In $f(\text{fields, sources})$, we have introduced two new doublets: (U, M) and (V, N)
- In the end, we set sources to values so that $S_s|_{\mathrm{phys}} = S_\gamma$
  $\Rightarrow$ We didn't change the theory

Physical operator

The glueball operator

- Perhaps this breaking is a good thing
- With Σ_{GZ} we can construct a glueball operator which is renormalizable completely analogous as in the Yang-Mills case
- We find the following renormalization group invariant

$$O_{\mathrm{phys}} = \mathcal{R} \quad = \quad \frac{\beta(g^2)}{g^2}\mathcal{F} - 2\gamma_c\mathcal{E} - 2\gamma_c\mathcal{H}$$

with $\mathcal{E} = s(\ldots) + \gamma^2 D_\mu^{ab}\left(\varphi_\mu^{ba} + \overline{\varphi}_\mu^{ba}\right) + d(N^2-1)\gamma^4$

The counterterm

We find

$$
\begin{aligned}
\Sigma^c \quad = \quad & a_0 S_{\mathrm{YM}} + b_0 \widehat{S}_{\mathrm{YM}} + \mathcal{B}_\Sigma \int \mathrm{d}^d x \Bigg\{ \Bigg[a_1 (K^a_\mu + \partial_\mu \overline{c}^a) A^a_\mu + a_2\, L^a c^a + a_3 U^a_{\mu i}\, \partial_\mu \varphi^a_i + a_4\, V^a_{\mu i}\, \partial_\mu \overline{\omega}^a_i + a_5\, \overline{\omega}^a_i \partial^2 \varphi^a_i \\
& + a_6\, U^a_{\mu i} V^a_{\mu i} + a_7\, g f^{abc} U^a_{\mu i}\, \varphi^b_i A^c_\mu + a_8\, g f^{abc} V^a_{\mu i}\, \overline{\omega}^b_i A^c_\mu + a_9\, g f^{abc} \overline{\omega}^a_i A^c_\mu\, \partial_\mu \varphi^b_i + a_{10}\, g f^{abc} \overline{\omega}^a_i (\partial_\mu A^c_\mu) \varphi^b_i \\
& + a_{11} X^i \overline{\omega}^a_i \partial A^a_\mu + a_{12} X^i \partial \overline{\omega}^a_i A^a_\mu + a_{13} X^i \overline{\varphi}^a_i \overline{c}^a + a_{14} g f_{abc} X^i \omega^a_i \overline{\omega}^b_j \overline{\omega}^c_j + a'_{14} g f_{abc} X^i \omega^a_j \overline{\omega}^b_i \overline{\omega}^c_j \\
& + a_{15} X^i \overline{\omega}^a_i b^a + a_{16} X^i U^{ia}_\mu A^a_\mu + a_{17} g f_{abc} X^i \overline{\omega}^a_i \varphi^b_j \overline{\varphi}^c_j + a'_{17} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_i \overline{\varphi}^c_j + + a''_{17} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_j \overline{\varphi}^c_i \\
& + a_{18} g f_{abc} X^i \overline{\omega}^a_i \overline{c}^b c^c + a_{19} X^i X^i \overline{\varphi}^a_j \overline{\omega}^a_j + a'_{19} X^i X^j \overline{\varphi}^a_i \overline{\omega}^a_j + a_{20} X^i Y^j \overline{\omega}^i_a \overline{\omega}^j_a + a_{21} g f_{abc} Y^i \overline{\omega}^a_i \overline{\omega}^b_j \varphi^c_j \\
& + a'_{21} g f_{abc} Y^i \overline{\omega}^a_j \overline{\omega}^b_j \varphi^c_i + a_{22} Y^i \overline{\omega}^a_i \overline{c}^a \Bigg] \\
& + q \Bigg[b_1 (K^a_\mu + \partial_\mu \overline{c}^a) A^a_\mu + c_1 \overline{c}^a \partial_\mu A^a_\mu + b_2 L^a c^a + b_3 U^a_{\mu i}\, \partial_\mu \varphi^a_i + c_3 \partial_\mu U^a_{\mu i} \varphi^a_i + b_4 V^a_{\mu i}\, \partial_\mu \overline{\omega}^a_i + c_4 \partial_\mu V^a_{\mu i}\, \overline{\omega}^a_i \\
& + b_5 \overline{\omega}^a_i \partial^2 \varphi^a_i + c_5 \partial_\mu \overline{\omega}^a_i \partial_\mu \varphi^a_i + d_5 \partial^2 \overline{\omega}^a_i \varphi^a_i + b_6\, U^a_{\mu i} V^a_{\mu i} + b_7 g f^{abc} U^a_{\mu i}\, \varphi^b_i A^c_\mu + b_8 g f^{abc} V^a_{\mu i}\, \overline{\omega}^b_i A^c_\mu \\
& + b_9 g f^{abc} \overline{\omega}^a_i A^c_\mu\, \partial_\mu \varphi^b_i + c_9 g f^{abc} \overline{\omega}^a_i (\partial_\mu A^c_\mu) \varphi^b_i + d_9 g f^{abc} \partial_\mu \overline{\omega}^a_i A^c_\mu \varphi^b_i + b_{10} X^i \overline{\omega}^a_i \partial A^a_\mu + c_{10} X^i \partial \overline{\omega}^a_i A^a_\mu \\
& + d_{10} \partial X^i \overline{\omega}^a_i A^a_\mu + b_{11} X^i \overline{\varphi}^a_i \overline{c}^a + b_{12} g f_{abc} X^i \omega^a_i \overline{\omega}^b_j \overline{\omega}^c_j + b'_{12} g f_{abc} X^i \omega^a_j \overline{\omega}^b_i \overline{\omega}^c_j + b_{13} X^i \overline{\omega}^a_i b^a + b_{14} X^i U^{ia}_\mu A^a_\mu \\
& + b_{15} g f_{abc} X^i \overline{\omega}^a_i \varphi^b_j \overline{\varphi}^c_j + b'_{15} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_i \overline{\varphi}^c_j + b''_{15} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_j \overline{\varphi}^c_i + b_{16} g f_{abc} X^i \overline{\omega}^a_i \overline{c}^b c^c + b_{17} X^i X^i \overline{\varphi}^a_j \overline{\omega}^a_j \\
& + b'_{17} X^i X^j \overline{\varphi}^a_i \overline{\omega}^a_j + b_{18} X^i Y^j \overline{\omega}^i_a \overline{\omega}^j_a + b_{19} g f_{abc} Y^i \overline{\omega}^a_i \overline{\omega}^b_j \varphi^c_j + b'_{19} g f_{abc} Y^i \overline{\omega}^a_j \overline{\omega}^b_j \varphi^c_i + b_{20} Y^i \overline{\omega}^a_i \overline{c}^a \Bigg]
\end{aligned}
$$

The counterterm

We find

$$
\begin{aligned}
&+\eta\bigg[e_1 K^a_\mu A^a_\mu + e'_1 \partial_\mu \overline{c}^a A^a_\mu + f_1 \overline{c}^a \partial_\mu A^a_\mu + e_2 L^a c^a + e_3 U^a_{\mu i}\, \partial_\mu \varphi^a_i + f_3 \partial_\mu U^a_{\mu i} \varphi^a_i + e_4 V^a_{\mu i}\, \partial_\mu \overline{\omega}^a_i + f_4 \partial_\mu V^a_{\mu i}\, \overline{\omega}^a_i \\
&+e_5\, \overline{\omega}^a_i \partial^2 \varphi^a_i + f_5 \partial_\mu \overline{\omega}^a_i \partial_\mu \varphi^a_i + g_5 \partial^2 \overline{\omega}^a_i \varphi^a_i + e_6\, U^a_{\mu i} V^a_{\mu i} + e_7 g f^{abc}\, U^a_{\mu i}\, \varphi^b_i A^c_\mu + e_8 g f^{abc}\, V^a_{\mu i}\, \overline{\omega}^b_i A^c_\mu \\
&+e_9 g f^{abc} \overline{\omega}^a_i A^c_\mu\, \partial_\mu \varphi^b_i + f_9 g f^{abc} \overline{\omega}^a_i (\partial_\mu A^c_\mu) \varphi^b_i + g_9 g f^{abc} \partial_\mu \overline{\omega}^a_i A^c_\mu \varphi^b_i + e_{10} X^i \overline{\omega}^a_i \partial A^a_\mu + f_{10} X^i \partial \overline{\omega}^a_i A^a_\mu \\
&+g_{10} \partial X^i \overline{\omega}^a_i A^a_\mu + e_{11} X^i \overline{\varphi}^a_i \overline{c}^a + e_{12} g f_{abc} X^i \omega^a_i \overline{\omega}^b_j \overline{\omega}^c_j + e'_{12} g f_{abc} X^i \omega^a_j \overline{\omega}^b_i \overline{\omega}^c_j + e_{13} X^i \overline{\omega}^a_i b^a + e_{14} X^i U^{ia}_\mu A^a_\mu \\
&+e_{15} g f_{abc} X^i \overline{\omega}^a_i \varphi^j_b \overline{\varphi}^j_c + e'_{15} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_i \overline{\varphi}^c_j + e''_{15} g f_{abc} X^i \overline{\omega}^a_j \varphi^b_j \overline{\varphi}^c_i + e_{16} g f_{abc} X^i \overline{\omega}^a_i \overline{c}^b c^c + e_{17} X^i X^i \overline{\varphi}^a_j \overline{\omega}^a_j \\
&+e'_{17} X^i X^j \overline{\varphi}^a_i \overline{\omega}^a_j + e_{18} X^i Y^j \overline{\omega}^i_a \overline{\omega}^j_a + e_{19} g f_{abc}\, Y^i \overline{\omega}^a_i \overline{\omega}^b_j \varphi^c_j + e'_{19} g f_{abc}\, Y^i \overline{\omega}^a_j \overline{\omega}^b_j \varphi^c_i + e_{20}\, Y^i \overline{\omega}^a_i \overline{c}^a \bigg] \\
&\lambda\bigg[h_1 g f_{abc} X^i \varphi^{aj} \overline{\omega}^b_i \overline{\omega}^c_j + h'_1 g f_{abc} X^i \varphi^{ai} \overline{\omega}^b_j \overline{\omega}^c_j + h_2 X^i \overline{c}^a \overline{\omega}^a_i + h_3 \overline{\omega}^a_i \overline{\omega}^b_j \varphi^a_i \varphi^b_j + (\text{variants of } h_3)\bigg]\bigg\}\,.
\end{aligned}
$$

Physical operator

The glueball operator

- *D. Zwanziger(1989)* has calculated the pole structure of

$$\int \mathrm{d}^4 x \left\langle F^2(x) F^2(0) \right\rangle e^{ipx} = G_{phys}(p) + G_{unphys}(p)$$

  - G_{phys} displays a physical cut at $p^2 = -2\gamma^2$
  - G_{unphys} displays unphysical cuts at $p^2 = \pm 4i\gamma^2$

- The correlator

$$\begin{aligned}
\langle O_{phys}(x) O_{phys}(y) \rangle &= \langle \mathcal{R}(x)\mathcal{R}(y) \rangle \\
&= \left\langle \left[\frac{\beta}{g^2} F^2 - 2\gamma_c \mathcal{E} \right](x) \left[\frac{\beta}{g^2} F^2 - 2\gamma_c \mathcal{E} \right](y) \right\rangle \\
&\neq \left(\frac{\beta}{g^2} \right)^2 \langle F^2(x) F^2(y) \rangle
\end{aligned}$$

  has extra parts in comparison with Zwanziger

- We have to study the pole structure of this object (future work)

Conclusion

Yang-Mills: S_{FP}

- BRST
- Physical operator is given by

$$\mathcal{R} = \frac{\beta(g^2)}{g^2}\mathcal{F} - 2\gamma_c\mathcal{E}$$

with

$$\mathcal{E} = s(\bar{c}^a\partial_\mu A^a_\mu)$$

- s-exact part vanishes when taking expectation value:

$$\langle\mathcal{R}(x)\mathcal{R}(y)\rangle = \left(\frac{\beta}{g^2}\right)^2\left\langle F^2(x)F^2(y)\right\rangle$$

Gribov-Zwanziger: S_{GZ}

- BRST is softly broken → Σ_{GZ}
- Physical operator is given by

$$\mathcal{R} = \frac{\beta(g^2)}{g^2}\mathcal{F} - 2\gamma_c\mathcal{E}$$

with

$$\begin{aligned}\mathcal{E} &= s(\bar{c}^a\partial_\mu A^a_\mu + \partial_\mu\bar{\varphi}^{ac}_\nu D^{ab}_\mu\varphi^{bc}_\nu) \\ &\quad +\gamma^2 D^{ab}_\mu(\varphi^{ba}_\mu + \bar{\varphi}^{ba}_\mu) + d\left(N^2-1\right)\gamma^4\end{aligned}$$

- s-exact part does not vanish when taking expectation value:

$$\langle\mathcal{R}(x)\mathcal{R}(y)\rangle = \left(\frac{\beta}{g^2}\right)^2\left\langle F^2(x)F^2(y)\right\rangle - 2\gamma_c\frac{\beta}{g^2}\left\langle F^2(x)\mathcal{E}(y)\right\rangle - 2\gamma_c\frac{\beta}{g^2}\left\langle\mathcal{E}(x)F^2(y)\right\rangle + 4\gamma_c^2\langle\mathcal{E}(x)\mathcal{E}(y)\rangle$$

Conclusion

- The issue of the Gribov copies leads to a modification of the Faddeev-Popov formula
- The BRST invariance of the Faddeev-Popov action turns out to be softly broken. The understanding of the physical consequences of this breaking represents a big challenge
- The possibility that, somehow, such a breaking might be relevant in order to construct the local physical operators in the presence of the Gribov horizon looks very attractive

$$O_{\text{phys}} = \mathcal{R} = \frac{\beta(g^2)}{g^2}\mathcal{F} - 2\gamma_c\mathcal{E}$$

- Also some freedom, $\mathcal{E}$ is also a renormalization group invariant, so $O'_{\text{phys}} = \mathcal{R} + c\mathcal{E}$

The End

Thank you for your attention!

Questions?