Clustering in AMD

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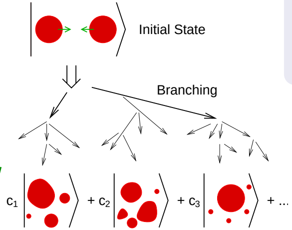
- AMD framework (including some details)
- Cluster correlations in AMD
- Skyrme force with AMD

Homework calculations - AMD

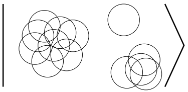
- Gogny force (finite range, density-dependent; $\rho_0 = 0.166\ \text{fm}^{-3}$, $K = 228$ MeV)
- The width parameter $\nu = (2.5\ \text{fm})^{-2}$
- $\Rightarrow$ Ground state of the Au nucleus (B.E. = 1537 MeV, $\langle r^2 \rangle^{1/2} = 5.56$ fm)
- $\sigma_{\text{NN}} = 40$ mb with isotropic angular distribution. ($\sigma_{\text{NN}} = 0$ if $p_{\text{rel}} < 50$ MeV/c.)
- $b = 0$ and $b = 8.376$ fm. Initial distance = 15 fm.
- A two-nucleon collision is a scattering of the centroids of two wave packets which have momentum widths.
  As the momenta (or $\sqrt{s}$) of colliding nucleons,
  1. the momentum centroids are written out (1st result of HW1).
  2. the *test particle momenta* taken randomly from the Gaussian distribution $f_{\text{W}}(\mathbf{r}, \mathbf{p})$ are written out (updated result of HW1, and HW2).
- The state is represented by $\{\mathbf{Z}_k, S_{k,ab};\ k = 1, \ldots, A\}$ in AMD.
  $100A$ test particles for output are generated randomly based on $f_{\text{S}}(\mathbf{r}, \mathbf{p})$.
  - Good for the momenta of emitted nucleons.
  - Not good for the internal wave function of nuclei.

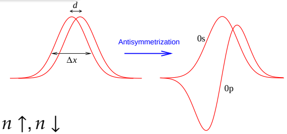
Antisymmetrized Molecular Dynamics



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$





$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \text{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Stochastic equation of motion for the wave packet centroids Z:

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \Delta\mathbf{Z}_i(t) + (\text{NN collisions})$$

$$\mathcal{H}(Z^*, Z) = \frac{\langle\Phi(Z)|(T+V)|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} - \frac{3\hbar^2\nu}{2M}A + T_0\big(A - \mathcal{N}_{\text{F}}(Z^*, Z)\big)$$

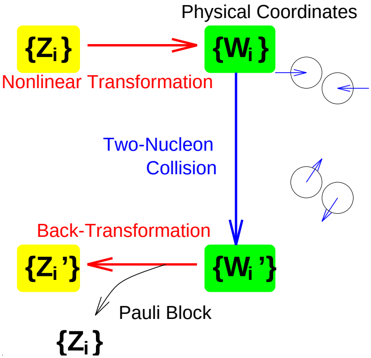
- Mean field (Time evolution of single-particle wave functions)
- Nucleon-nucleon collisions (as the residual interaction)

Stochastic Two-Nucleon Collisions

Physical coordinates

$$\mathbf{W}_k = \sum_j \left(\sqrt{Q(Z)}\right)_{kj} \mathbf{Z}_j = \sqrt{\nu}\mathbf{R}_k + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{P}_k, \qquad f_{\mathrm{W}}(\mathbf{r},\mathbf{p}) = 8\sum_{k=1}^{A} e^{-2\nu(\mathbf{r}-\mathbf{R}_k)^2-(\mathbf{p}-\mathbf{P}_k)^2/2\hbar^2\nu}$$

Collision between 1 and 2 : $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3, \ldots, \mathbf{W}_A \Rightarrow \mathbf{W}'_1, \mathbf{W}'_2, \mathbf{W}_3, \ldots, \mathbf{W}_A$



$$\mathbf{P}_1 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} + \mathbf{p}_{\mathrm{rel}}$$
$$\mathbf{P}_2 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} - \mathbf{p}_{\mathrm{rel}}$$

$$\Downarrow\ d\sigma_{NN}/d\Omega$$

$$\mathbf{P}'_1 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} + p'_{\mathrm{rel}}\hat{\mathbf{n}}$$
$$\mathbf{P}'_2 = \tfrac{1}{2}\mathbf{p}_{\mathrm{tot}} - p'_{\mathrm{rel}}\hat{\mathbf{n}}$$

When do two nucleons collide?

All the nucleon pairs are checked at every time step dt.

Assumptions

1. Collision attempt rate is proportional to the density overlap and the relative velocity.

$$P(\mathbf{r})|d\mathbf{r}| = \alpha e^{-\nu \mathbf{r}^2}|d\mathbf{r}|, \qquad \mathbf{r} = \mathbf{R}_1 - \mathbf{R}_2$$

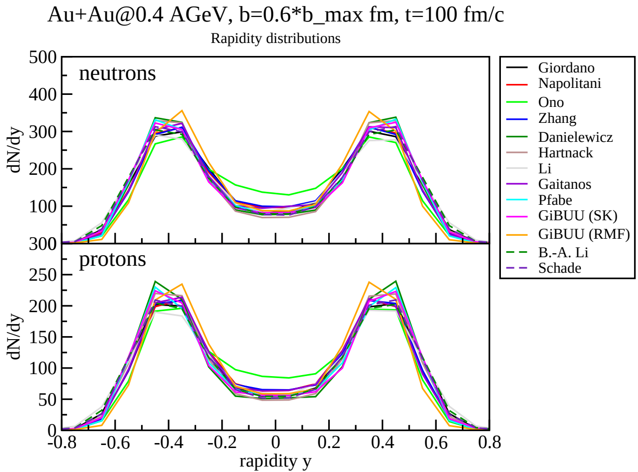
$$d\mathbf{r} = \mathbf{r}(t) - \mathbf{r}(t-dt) = (\mathbf{v}_1 - \mathbf{v}_2)dt$$

2. The two nucleons move on straight lines until they collide, and the total cross section is σ_{NN}.

$$\int_0^\infty 2\pi b db \left[1 - e^{-\int_{-\infty}^{\infty} P(\mathbf{b}+\mathbf{z})dz}\right] = \sigma_{\mathrm{NN}}$$

$\Rightarrow$ The coefficient α is given as a function of σ_{NN} and ν.

$$y = \frac{\nu \sigma_{\mathrm{NN}}}{\pi}, \quad x = \alpha\sqrt{\frac{\pi}{\nu}}, \qquad y = -\sum_{n=1}^{\infty} \frac{(-x)^n}{n(n!)}$$



The [197]Au nuclei (represented by $f_W(\mathbf{r},\mathbf{p})$) are too large?

Representations of the state

1. Original AMD coordinates: $Z = \{\mathbf{Z}_k\}_{k=1,\dots,A}$ $\qquad \mathbf{u} = \sqrt{\nu}\mathbf{r} + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{p}$

$$|\Phi(Z)\rangle = \det_{ij}[e^{-2(\sqrt{\nu}\mathbf{r}_i - \mathbf{Z}_j)^2}] \quad \Rightarrow \quad f(\mathbf{r},\mathbf{p}) = 8\sum_{ij} e^{-2(\mathbf{u}-\mathbf{Z}_i)^*\cdot(\mathbf{u}-\mathbf{Z}_j)} B_{ij} B^{-1}_{ji}$$

Used for expectation values, the deterministic term $\{Z, \mathcal{H}\}_{\text{PB}}$ etc.

2. Physical coordinates: $\{\mathbf{W}_k\} = \{\sqrt{\nu}\mathbf{R}_k + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{P}_k\}_{k=1,\dots,A}$

$$\mathbf{W}_k = \sum_j \left(\sqrt{Q(Z)}\right)_{kj} \mathbf{Z}_j \quad \Rightarrow \quad f_{\text{W}}(\mathbf{r},\mathbf{p}) = 8\sum_{k=1}^{A} e^{-2\nu(\mathbf{r}-\mathbf{R}_k)^2 - (\mathbf{p}-\mathbf{P}_k)^2/2\hbar^2\nu}$$

Used in two-nucleon collisions.

3. Deformed Gaussian parameters in phase space: $\{X_{k,a}, S_{k,ab}\}$ $\qquad \{X_{k,a}\}_{a=1,\dots,6} = \mathbf{W}_k$

$$f_{\text{S}}(\mathbf{r},\mathbf{p}) = 8\sum_{k=1}^{A} \exp\Big[-\tfrac{1}{2}\sum_{ab}(x_a - X_a)S^{-1}_{k,ab}(x_b - X_b)\Big], \qquad \{x_a\}_{a=1,\dots,6} = \{\sqrt{\nu}\mathbf{r}, \frac{1}{2\hbar\sqrt{\nu}}\mathbf{p}\}$$

Vlasov eq. is solved to evaluate the fluctuation ΔZ and $\frac{d}{dt}S_{k,ab}$.

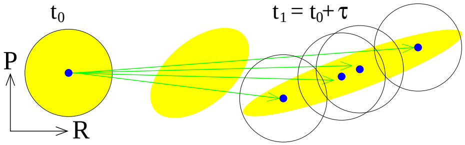
The momentum width is respected in all the representations.

At each time t_0, for each wave packet k, ...

Mean field propagation $t_0 \to t_0 + \tau$ + Branching at $t_0 + \tau$ τ: Coherence time

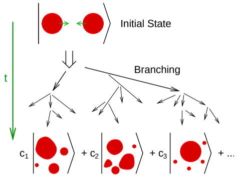
$$t = t_0 \qquad\qquad t = t_0 + \tau$$

$$|\mathbf{Z}_k\rangle\langle\mathbf{Z}_k| \xrightarrow[\text{Mean field}]{} |\psi_k\rangle\langle\psi_k| \xrightarrow[\text{Branching}]{} \int |\mathbf{z}\rangle\langle\mathbf{z}|\, w_k(\mathbf{z})d\mathbf{z} \qquad \text{for } k = 1,\dots,A$$



$$i\hbar\frac{d}{dt}|\psi_k(t)\rangle = h^{\text{HF}}|\psi_k(t)\rangle \quad \text{or} \quad \frac{\partial f_k}{\partial t} = -\frac{\partial h^{\text{HF}}}{\partial \mathbf{p}}\cdot\frac{\partial f_k}{\partial \mathbf{r}} + \frac{\partial h^{\text{HF}}}{\partial \mathbf{r}}\cdot\frac{\partial f_k}{\partial \mathbf{p}}$$

- $\tau \to 0$ (Strongest branching)
- $\tau = \tau(\rho)$ (Density-dependent)
- $\tau = \tau_{\text{NN-coll}}$ (Decoherence at NN collisions)



Equation of motion for the wave packet centroids

$$\begin{aligned}\frac{d}{dt}\mathbf{Z}_i &= \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} && \text{Mean field}\\ &+ \Delta\mathbf{Z}_i(t) && \text{Mean field \& Branching}\\ &+ \mu(\mathbf{Z}_i, \mathcal{H}') && \text{Dissipation}\\ &+ \text{NN-Collision}\end{aligned}$$

If $\mathbf{Z}_i$ were canonical variables for simplicity,

$$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} = \frac{1}{i\hbar}\frac{\partial\mathcal{H}}{\partial\mathbf{Z}_i^*}$$

$$\overline{\Delta Z_{ia}(t)} = 0, \qquad \overline{\Delta Z_{ia}(t)\Delta Z_{jb}(t)} = D_{iab}(t)\delta_{ij}\delta(t-t')$$

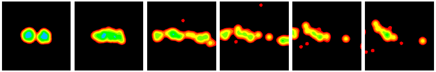
$$(\mathbf{Z}_i, \mathcal{H}') = \frac{1}{\hbar}\frac{\partial\mathcal{H}'}{\partial\mathbf{Z}_i^*}, \qquad \mathcal{H}' = \mathcal{H} + \sum_m \beta_m Q_m$$

- μ is determined by the total energy conservation.
- Lagrange multipliers β_m are determined so that Q_m are not changed by the $(\mathbf{Z}_i, \mathcal{H}')$ term.

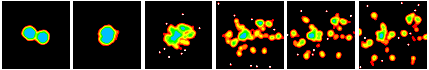
$$\{Q_m\} = \Big\{\Big\langle\sum_i \mathbf{r}_i\Big\rangle, \Big\langle\sum_i \mathbf{p}_i\Big\rangle, \Big\langle\sum_i \mathbf{r}_i\times\mathbf{p}_i\Big\rangle, \Big\langle\sum_i r_{i\sigma}r_{i\tau}\Big\rangle, \Big\langle\sum_i p_{i\sigma}p_{i\tau}\Big\rangle\Big\} \qquad \sigma,\tau = x,y,z$$

AMD results for multifragmentation (central collisions)

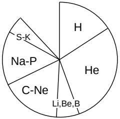
^{40}Ca + ^{40}Ca at 35 MeV/u, $b = 0$



Xe + Sn at 50 MeV/u, $0 \leq b \leq 4$ fm



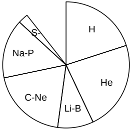
Experiment



Hagel et al.

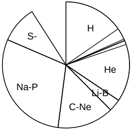
PRC50(1994)2017

AMD



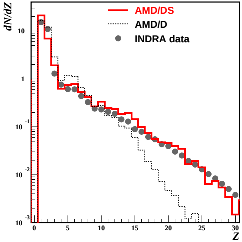
$\tau(\rho)$

AMD



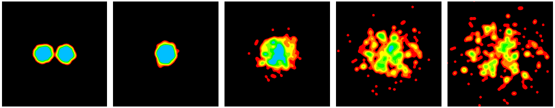
$\tau_{\text{NN-coll}}$

Charge distribution

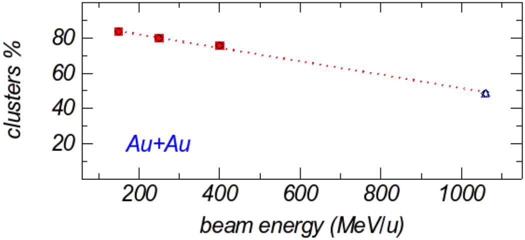


- AMD ($\tau \to 0$)
- AMD ($\tau_{\text{NN-coll}}$)

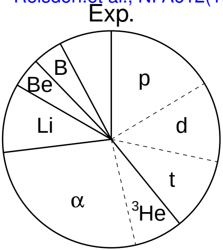
Can we reproduce different data with the same model of branching?
(Cluster correlations?)

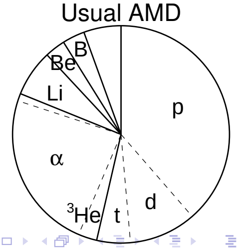


^{197}Au + ^{197}Au at 150 MeV/u



Reisdorf et al., NPA612(1997)493.





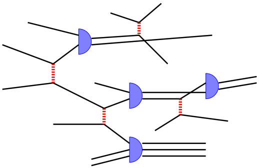
Example: Four nucleons in the gas at $T = 10$ MeV

- Uncorrelated: $\langle E\rangle = \frac{3}{2}T \times 4 = 60$ MeV
- α cluster: $\langle E\rangle = \frac{3}{2}T \times 1 - 28.3\,\text{MeV} = -13.3$ MeV

Clusters in AMD

During the time evolution of AMD,

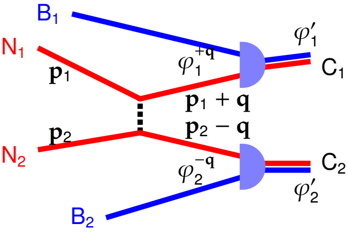
- Cluster formation
- Propagation /definition of cluster
- Breakup



$\boxed{\mathrm{N}_1 + \mathrm{B}_1 \;+\; \mathrm{N}_2 + \mathrm{B}_2 \;\rightarrow \mathrm{C}_1 + \mathrm{C}_2}$

- N_1, N_2 : Colliding nucleons
- B_1, B_2 : Spectator nucleons/clusters
- C_1, C_2 : N, $(2N)$, $(3N)$, $(4N)$

$$\frac{d\sigma}{d\Omega} = F_{\mathrm{kin}}\, |\langle\varphi_1'|\varphi_1^{+\mathbf{q}}\rangle|^2\, |\langle\varphi_2'|\varphi_2^{-\mathbf{q}}\rangle|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{NN}}$$

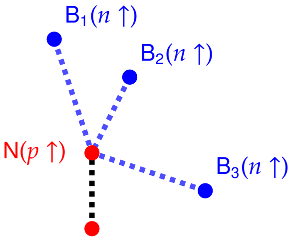


c.f. Danielewicz et al., NPA533 (1991) 712.

How much is the probability that the scattered nucleon N_1 is a part of a cluster after the momentum transfer $\mathbf{q}$?

Probability of forming a cluster

The probability for a scattered nucleon N to form a cluster is not as simple as $|\langle\varphi'|\varphi^{\mathrm{q}}\rangle|^2$, when there are many possible combinations of cluster formation.



$|\Phi^{\mathrm{q}}\rangle$: The state of the left figure.

$|\Phi'_{\mathrm{B}}\rangle$: The state when N and B are placed at the same phase space point.

$|\langle\Phi'_{\mathrm{B}}|\Phi^{\mathrm{q}}\rangle|^2$ is not the probability because of the non-orthogonality: $N_{\mathrm{BB'}} \equiv \langle\Phi'_{\mathrm{B}}|\Phi'_{\mathrm{B'}}\rangle \neq \delta_{\mathrm{BB'}}$

The probability that N forms a cluster with one of B's:

$$P = \langle\Phi^{\mathrm{q}}|\hat{X}|\Phi^{\mathrm{q}}\rangle = \sum_{\mathrm{B}} |\langle\tilde{\Phi}'_{\mathrm{B}}|\Phi^{\mathrm{q}}\rangle|^2$$

$$\hat{X} = \sum_{\mathrm{BB'}} |\Phi'_{\mathrm{B}}\rangle N^{-1}_{\mathrm{BB'}} \langle\Phi'_{\mathrm{B'}}| \qquad |\tilde{\Phi}'_{\mathrm{B}}\rangle = (N^{-1/2})_{\mathrm{BB'}}|\Phi_{\mathrm{B'}}\rangle$$

$|\langle\tilde{\Phi}'_{\mathrm{B}}|\Phi^{\mathrm{q}}\rangle|^2$ is regarded as the probability that N forms a cluster with B.

G. Röpke and H. Schulz, NPA477 (1988) 472.

. . . we introduce creation and annihilation operators for bound states

$$a^\dagger_{A\alpha}(\mathbf{p}) = \sum_{1\ldots A} \psi^*_{A\alpha,\mathbf{p}}(1\ldots A)\, a^\dagger(1)\cdots a^\dagger(A).$$

Here $\psi_{A\alpha,\mathbf{p}}(1\ldots A)$ denotes the wave function of the A-nucleon cluster state with total momentum $\mathbf{p}$ and internal quantum number α . . .

. . . the operator

$$n_{A\alpha}(\mathbf{p},\mathbf{r}) = \int \frac{d^3k}{(2\pi\hbar)^3}\, e^{i\mathbf{k}\cdot\mathbf{r}/\hbar}\, a^\dagger_{A\alpha}(\mathbf{p}-\tfrac{1}{2}\mathbf{k})\, a_{A\alpha}(\mathbf{p}+\tfrac{1}{2}\mathbf{k})$$

is the Wigner transform of the density associated with a cluster of mass number A. The mean values

$$f_{A\alpha}(\mathbf{p},\mathbf{r},t) = \langle n_{A\alpha}(\mathbf{p},\mathbf{r})\rangle^t$$

represent the Wigner distribution functions of the corresponding clusters. In this way, clusters are considered as new species like in a chemical picture.

Clustering probability (?)

Is it true that the number of clusters is obtained by

$$N_{A\alpha}(t) = \int \frac{d^3p\, d^3r}{(2\pi\hbar)^3}\, f_{A\alpha}(\mathbf{p},\mathbf{r},t) = \int \frac{d^3p}{(2\pi\hbar)^3}\, \langle a^\dagger_{A\alpha}(\mathbf{p})\, a_{A\alpha}(\mathbf{p})\rangle^t \qquad ?$$

Consider $(A\alpha)$ = deuteron, and choose $\psi_{\mathrm{d},\mathbf{p}}(1,2) = e^{i\mathbf{p}\cdot(\mathbf{r}_1+\mathbf{r}_2)/2\hbar}\left(\frac{\nu}{\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{2}\nu(\mathbf{r}_1-\mathbf{r}_2)^2}$.

For an AMD wave function $|\Phi(Z)\rangle$ (of nucleons with the same spin), N_d is calculated as

$$N_\mathrm{d} = \sum_{ik}^{\text{proton}} \sum_{jl}^{\text{neutron}} e^{-\frac{1}{2}(\mathbf{Z}_i^*-\mathbf{Z}_j^*)\cdot(\mathbf{Z}_k-\mathbf{Z}_l)} B_{ik}B_{jl}B^{-1}_{ki}B^{-1}_{lj}$$

In the case of a single proton (at $\mathbf{Z}_\mathrm{p}$) and many neutrons,

$$N_\mathrm{d} = \sum_{jl}^{\text{neutron}} e^{-\frac{1}{2}(\mathbf{Z}_\mathrm{p}^*-\mathbf{Z}_j^*)\cdot(\mathbf{Z}_\mathrm{p}-\mathbf{Z}_l)} B_{jl}B^{-1}_{lj} \quad \neq \quad F(\mathbf{Z}_\mathrm{p}) = \sum_{jl}^{\text{neutron}} e^{-(\mathbf{Z}_\mathrm{p}^*-\mathbf{Z}_j^*)\cdot(\mathbf{Z}_\mathrm{p}-\mathbf{Z}_l)} B_{jl}B^{-1}_{lj}$$

$F(\mathbf{r},\mathbf{p})$ is the Husimi function satisfying $0 \le F(\mathbf{r},\mathbf{p}) \le 1$, but N_d easily exceeds 1.

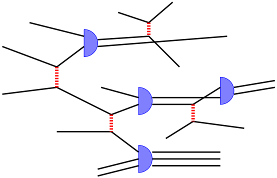
Conclusion: This N_d is not what we want!

The details of cluster correlations

- **Formation**
  - $(d\sigma/d\Omega)_{\mathrm{NN}} \Rightarrow$ Cluster formation cross section
  - Clusters: N, $2N$, $3N$, $4N = (0s)^n$
  - Pauli-blocking factor: $\prod_{i\in \mathrm{C}}(1-f_i)$
  - Avoid double countings of final states
  - Take care of the non-orthogonality of final states
- **Propagation**

  Nucleons i in a cluster C are propagated as usual, except that the internal fluctuations are turned off:

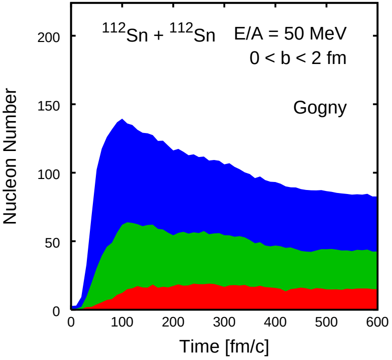
$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + \Delta\mathbf{Z}_i(t), \quad \Delta\mathbf{Z}_i(t) := \frac{1}{C}\sum_{j\in \mathrm{C}} \Delta\mathbf{Z}_j(t)$$



- **Breakup**

  A cluster C is broken when a nucleon in C collides with another nucleon.

Number of nucleons in correlated clusters



Non-clustered

(2N)

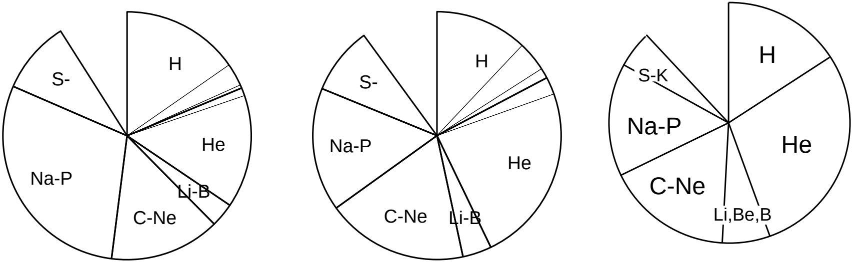
(3N)

(4N)

Effects of cluster correlations

^{40}Ca + ^{40}Ca, $E/A = 35$ MeV, filtered violent collisions

w/o cluster correlations | with cluster correlations | experiment



	w/o cluster correlations		with cluster correlations
p	6.7	p	4.4
d	1.5	d	1.8
t	0.3	t	0.5
^{3}He	0.3	^{3}He	0.6
α	2.7	α	5.0

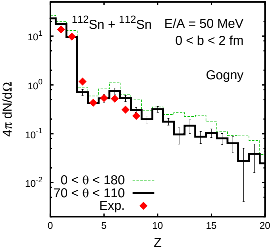
Effects of clusters

- M_p $\downarrow$
- M_α $\uparrow$
- $\sum_{\text{IMF}} Z$ $\downarrow$

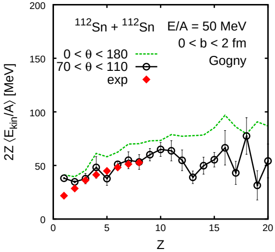
Coherence time: $\tau_{\text{NN-coll}}$

^{112}Sn + ^{112}Sn at E/A = 50 MeV/nucleon, $0 < b < 2$ fm

With cluster correlations $\Sigma Z(70^\circ < \theta < 110^\circ) = 22.6$



n	27.3
p	10.4
d	6.4
t	3.0
^{3}He	1.2
α	12.2



Xe+Sn, INDRA data

p	8.4
d	4.4
t	3.3
^{3}He	0.9
α	10.1

multiplicities of detected particles

Effective interactions in AMD

- So far AMD calculations are done with the Gogny force.

$$v_{ij} = \sum_{k=1,2} (W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P_\tau) e^{-(\mathbf{r}_i - \mathbf{r}_j)^2 / a_k^2} + \ t_\rho (1 + P_\sigma) \rho(\mathbf{r}_i)^\sigma \delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$\langle V \rangle = \frac{1}{2} \sum_{i=1}^{A} \sum_{j=1}^{A} \sum_{k=1}^{A} \sum_{l=1}^{A} \langle ij|v|kl - lk \rangle B_{ki}^{-1} B_{lj}^{-1} \qquad \sim A^4$$

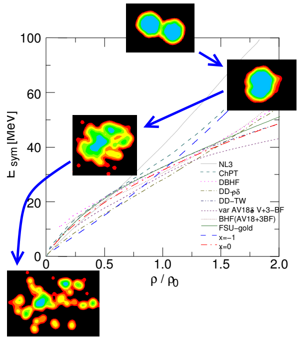
- Skyrme force is applicable in principle.
  (May be advantageous for heavy systems.)

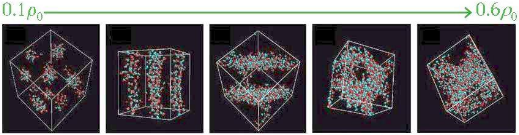
$$v_{ij} = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \tfrac{1}{2} t_1 (1 + x_1 P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2 + \mathbf{k}^2 \delta(\mathbf{r})] \qquad \mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$$

$$+ t_2 (1 + x_2 P_\sigma) \mathbf{k} \cdot \delta(\mathbf{r}) \mathbf{k} + t_3 (1 + x_3 P_\sigma) [\rho(\mathbf{r}_i)]^\alpha \delta(\mathbf{r}) \qquad \mathbf{k} = \tfrac{1}{2\hbar}(\mathbf{p}_i - \mathbf{p}_j)$$

$$\langle V \rangle = \int \mathcal{V}\Big(\rho(\mathbf{r}), \tau(\mathbf{r}), \triangle\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})\Big) d\mathbf{r} \qquad \sim A^2 V \quad (+\epsilon A^3)$$

$$\rho(\mathbf{r}) = \left(\frac{2\nu}{\pi}\right)^{3/2} \sum_{i=1}^{A} \sum_{j=1}^{A} e^{-2\nu(\mathbf{r} - \mathbf{R}_{ij})^2} B_{ij} B_{ji}^{-1}, \qquad \mathbf{R}_{ij} = \frac{1}{2\sqrt{\nu}} (\mathbf{Z}_i^* + \mathbf{Z}_j)$$





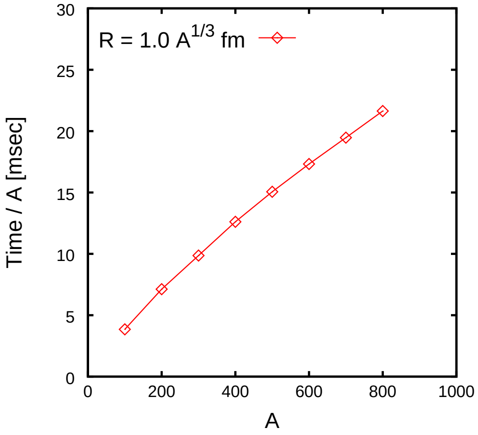
QMD simulation of nuclear pasta. G. Watanabe et al.

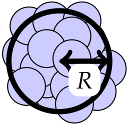
- Inhomogeneous. (fragments, clusters, pasta)
- Excited. (finite temperature and/or dynamic)
- Liquid-gas phase transition.

with Hasnaoui, Furuta, Gulminelli,Chomaz

Application to heavy systems with Skyrme force

System size dependence of the CPU time for an evaluation of $\left\{\frac{\partial}{\partial \mathbf{Z}_k^*}\langle V\rangle;\ k = 1, 2, \ldots A\right\}$





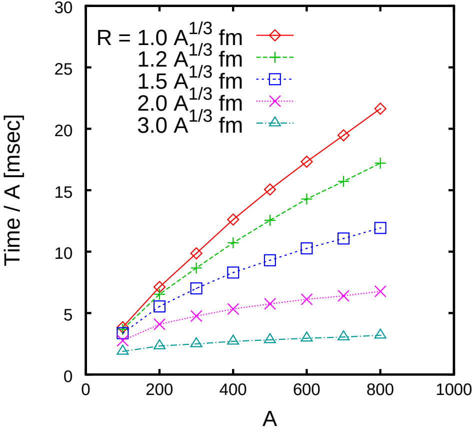
$\Leftrightarrow$ Naive expectation $\sim A^2 V$

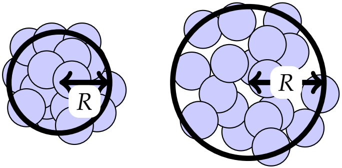
$$\langle V\rangle = \int d\mathbf{r}\, \mathcal{V}\Big(\rho(\mathbf{r}), \tau(\mathbf{r}), \triangle\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})\Big)$$

$$\rho(\mathbf{r}) = \left(\frac{2\nu}{\pi}\right)^{3/2} \sum_{i=1}^{A}\sum_{j=1}^{A} e^{-(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B^{-1}_{ji}$$

- Mesh size $\Delta r = 0.75$ fm, $\quad Z_\uparrow = Z_\downarrow = N_\uparrow = N_\downarrow$
- Xeon E5430 Harpertown 2.66 GHz, Using 1 of 8 cores, Almost no load by other processes

System size dependence of the CPU time for an evaluation of $\left\{\frac{\partial}{\partial \mathbf{Z}_k^*}\langle V\rangle;\ k=1,2,\dots A\right\}$





$\Leftrightarrow$ Naive expectation $\sim A^2V$

$$\langle V\rangle = \int d\mathbf{r}\ \mathcal{V}\Big(\rho(\mathbf{r}), \tau(\mathbf{r}), \triangle\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})\Big)$$

$$\rho(\mathbf{r}) = \left(\frac{2\nu}{\pi}\right)^{3/2} \sum_{i=1}^{A}\sum_{j=1}^{A} e^{-(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij}B^{-1}_{ji}$$

- Mesh size $\Delta r = 0.75$ fm, $\quad Z_\uparrow = Z_\downarrow = N_\uparrow = N_\downarrow$
- Xeon E5430 Harpertown 2.66 GHz, Using 1 of 8 cores, Almost no load by other processes

Summary

Cluster correlations in AMD

- $\mathrm{N}_1 + \mathrm{B}_1 \ + \ \mathrm{N}_2 + \mathrm{B}_2 \ \rightarrow \mathrm{C}_1 + \mathrm{C}_2,$ based on $(d\sigma/d\Omega)_{NN}$
- A problem how to count the number of clusters.
- Cluster correlations have systematic effects on M_p, M_α, and $\sum_{\mathrm{IMF}} Z$.
- Consistent reproduction of various multifragmentation data may be improved.

AMD with Skyrme force

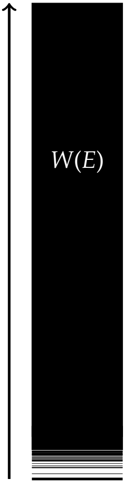
- In progress.
- May be applicable to neutron star problems.

Reaction and Equilibrium — A unified study with AMD

- Equivalence for fragment observables
  at each reaction time [$80 \lesssim t < (300+)$ fm/c]
- Some dynamical effects

Excited low-density system

E^*



8A MeV

10 MeV

0 MeV

^{36}Ar

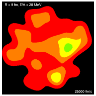
Gas

(nucleons + clusters)

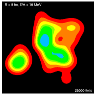
Liquid-gas

phase transition

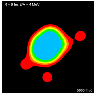
$W(E) \approx e^{2\sqrt{aE^*}}$



$E^* = 28A$ MeV



$E^* = 10A$ MeV

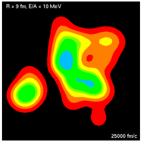


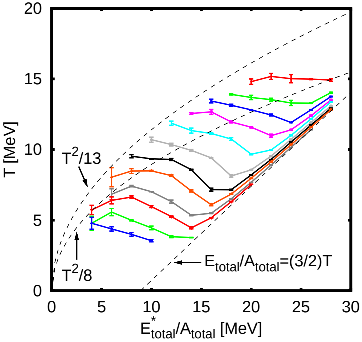
$E^* = 4A$ MeV

Volume $V = \frac{4}{3}\pi(9\text{ fm})^3$

Can molecular dynamics describe excited nuclear many-body systems in equilibrium?

- Ono & Horiuchi
- Ohnishi & Randrup
- Schnack & Feldmeier
- Sugawa & Horiuchi
- Furuta & Ono
- Hasnaoui et al.





Furuta and Ono, PRC79 (2009) 014608; PRC74 (2006) 014612.