

**In-medium heavy-quark and quarkonium
propagators:
the static and finite-mass cases**

Andrea Beraudo
University of Torino

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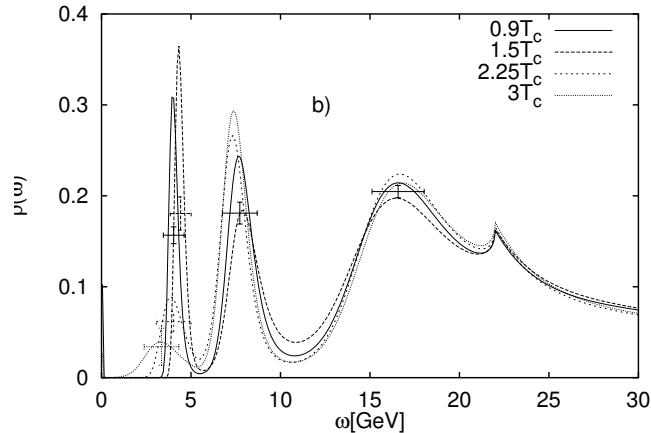
*Work in progress in collaboration with J.P. Blaizot (CEA-Saclay),
G. Garberoglio and P. Faccioli (University of Trento)*

Outline

- The physical motivations: study of medium effects on the spectral densities of HQ and $Q\bar{Q}$ correlators;
- The static ($M = \infty$) case: how to get a real-time in-medium $Q\bar{Q}$ potential;
- Beyond the static approximation:
 - Easy extension to address the finite-mass case in terms of a QM path-integral;
 - Some physical insight: the HQ spectral function from a resummed one-loop calculation;
- Preliminary numerical results of the MC simulations;
- Conclusions and future developments.

The physical motivations

- QGP effects on HQs and quarkonia can be studied through the modifications of the corresponding spectral functions: broadening, shift or disappearance of peaks, development of new peaks and/or non-vanishing strength at low-energy;
- Most of the present spectral studies are based on IQCD calculations of euclidean correlators like $G_M(\tau) \equiv \langle J_M(\tau) J_M^\dagger(0) \rangle$



S. Datta, F. Karsch, P. Petreczky and I. Wetzorke,
Phys. Rev. D 69, 094507 (2004)

$$G_M(\tau) = \int_0^\infty d\omega \underbrace{\sigma_M(\omega)}_{MSF} \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\beta\omega/2)}$$

Our goal

We wish to perform *a study resulting*

- numerically *less expensive than lattice calculations* (hence allowing a *more robust reconstruction of the spectral function*);
- capable to *get a deeper physical insight* on the processes involved.

The basic object of our study

$$G^>(t) \equiv \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle$$

- \mathcal{O}^\dagger creates a Q or a $Q\bar{Q}$ pair;
- Spectral decomposition

$$\begin{aligned} G^>(t) &= Z^{-1} \sum_n e^{-\beta E_n} \sum_m \langle n | \mathcal{O}(t) | m \rangle \langle m | \mathcal{O}^\dagger(0) | n \rangle \\ &= Z^{-1} \sum_n e^{-\beta E_n} \sum_m e^{i(E_n - E_m)t} |\langle m | \mathcal{O}^\dagger(0) | n \rangle|^2, \end{aligned}$$

- $G^>(t)$ is an **analytic function** in the strip $-\beta < \text{Im}t < 0 \implies$
unified description of real and imaginary-time propagation;
- HQs: *external probe placed in a hot/dense medium of light particles* $\implies \{|n\rangle\}$ do not contain heavy quarks.

Getting the in-medium spectral function...

- In the general case the spectral density of a correlator would be given by

$$\sigma(\omega) \equiv G^>(\omega) \mp G^<(\omega);$$

- Dealing with the propagation of an **external probe** one has $G^< \equiv 0$, so that

$$\sigma(\omega) = G^>(\omega) \implies G^>(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sigma(\omega);$$

- The standard procedure to get $\sigma(\omega)$ is then, **exploiting the analyticity of $G^>$** :

$$\underbrace{G^>(t = -i\tau)}_{\text{evaluated}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\omega\tau} \underbrace{\sigma(\omega)}_{\text{reconstructed}} .$$

The static ($M = \infty$) case:
the heavy quarks frozen to their positions

The hot-QED case:

A $Q\bar{Q}$ pair in a plasma of
photons, electrons and positrons

$$\mathcal{L}_{\text{QED}}^{M=\infty} = \mathcal{L}_{\text{em}} + \mathcal{L}_{\text{light}} + \underbrace{\psi^\dagger i(\partial_0 - igA_0)\psi}_{\text{heavy } Q} + \underbrace{\chi^\dagger i(\partial_0 + igA_0)\chi}_{\text{heavy } \bar{Q}}$$

It represents a **simple abelian model for the QGP**, nevertheless
sufficient to study important medium effects^a.

^aA.B., J.P. Blaizot and C. Ratti, Nucl. Phys. A **806**, 312 (2008).
ArXiv: 0712.4394

The strategy

- Consider the $Q\bar{Q}$ propagation in a given background configuration of the gauge-field A_μ

$$G_A(t, \mathbf{r}_1; t, \mathbf{r}_2 | 0, \mathbf{r}'_1; 0, \mathbf{r}'_2) = \delta(\mathbf{r}_1 - \mathbf{r}'_1) \delta(\mathbf{r}_2 - \mathbf{r}'_2) \times \\ \times \exp\left(ig \int_0^t dt' A_0(\mathbf{r}_1, t')\right) \exp\left(-ig \int_0^t dt' A_0(\mathbf{r}_2, t')\right)$$

- Average over the gauge-field configuration with an action accounting for thermal effects

$$G^>(t, \mathbf{r}_1; t, \mathbf{r}_2 | 0, \mathbf{r}'_1; 0, \mathbf{r}'_2) = Z^{-1} \int [\mathcal{D}A] G_A(t, \mathbf{r}_1; t, \mathbf{r}_2 | 0, \mathbf{r}'_1; 0, \mathbf{r}'_2) e^{iS[A]} \\ \equiv \delta(\mathbf{r}_1 - \mathbf{r}'_1) \delta(\mathbf{r}_2 - \mathbf{r}'_2) \bar{G}(t, \mathbf{r}_1 - \mathbf{r}_2)$$

Which is the action to employ to weight the field configurations?

The HTL effective action I

⇒ Momentum scales in a *relativistic (weakly coupled) plasma*:

- **Hard** (*plasma particles*):

$$E \sim T^4 \quad N \sim T^3 \quad \Longrightarrow \quad K \sim T;$$

- **Soft** (*collective modes*): $K \sim gT$.

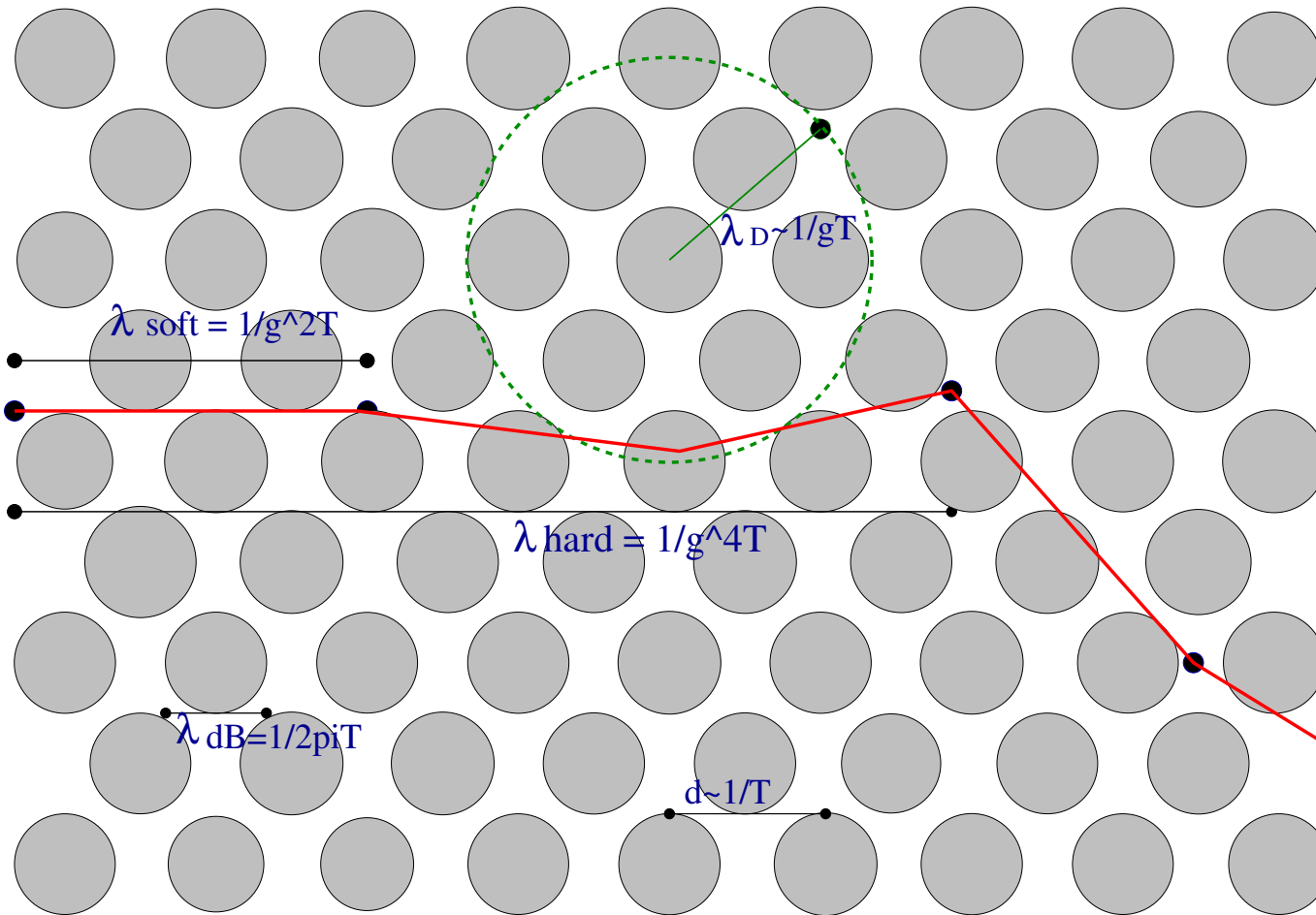
⇒ **Mean Free Path** of a plasma particle:

- For hard momentum exchange: $\lambda_{mfp}^{hard} \sim 1/g^4 T$,
- For soft momentum exchange: $\lambda_{mfp}^{soft} \sim 1/g^2 T$.

For weak coupling one has $\lambda_{mfp}^{soft} \ll \lambda_{mfp}^{hard}$, i.e.

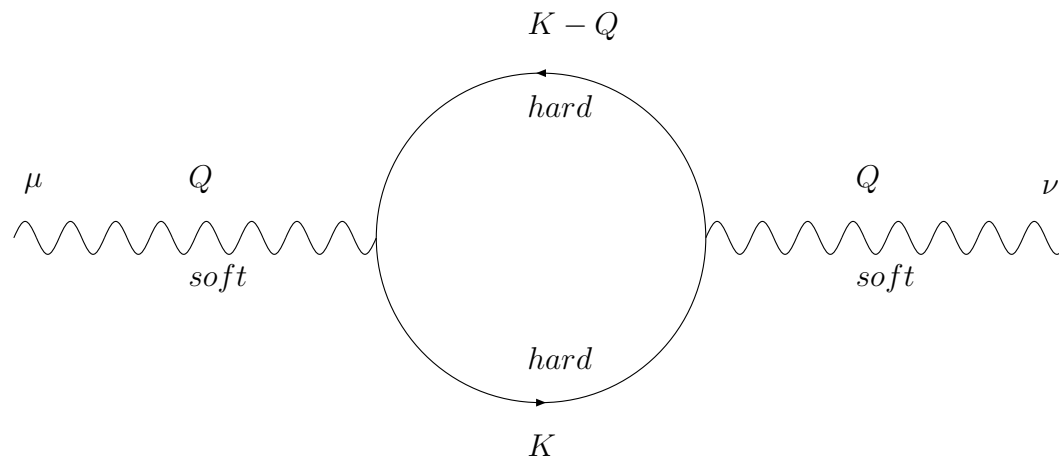
most of the scattering processes involve small momentum transfer.

A cartoon...

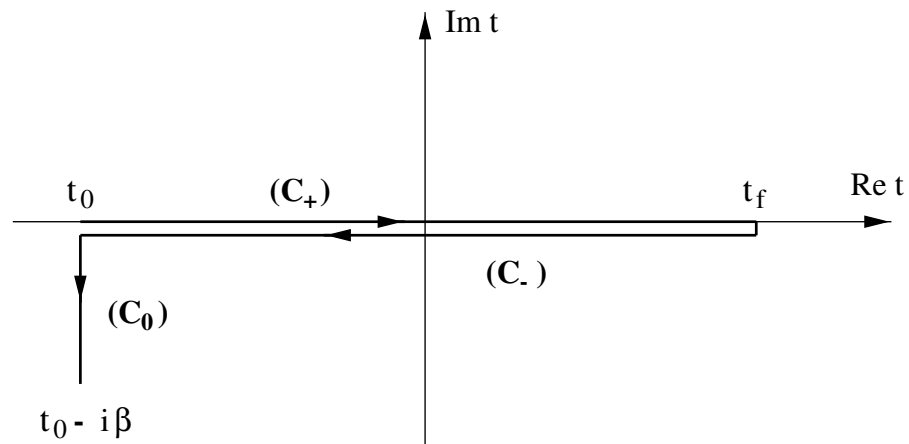


The HTL effective action II

- *Most of the interactions* are mediated by the exchange of **soft gauge-bosons** ($Q \sim gT \ll T$)
- The propagation of soft (long wave-length) gauge-bosons is *dressed by the interactions with the light plasma-particle* which are **hard** ($K \sim T$)



The HTL effective action III



The HTL effective action will be expressed in terms of the **gauge-boson propagator** in the *complex-time plane*:

$$iD_{\mu\nu}(x - y) \equiv \theta_C(x^0 - y^0) \langle A_\mu(x) A_\nu(y) \rangle + \theta_C(y^0 - x^0) \langle A_\nu(y) A_\mu(x) \rangle$$

- Along C_+ it coincides with the **time-ordered propagator**;
- Along C_0 it coincides with the **Matsubara propagator**.

The HTL effective action IV

- The HTL effective action (*gaussian for a QED plasma!*):

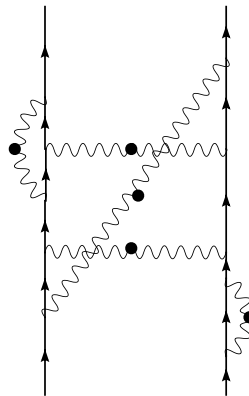
$$S_C^{HTL}[A] = \frac{1}{2} \int_C d^4x \int_C d^4y A^\mu(x) (D^{-1})_{\mu\nu}^{HTL}(x-y) A^\nu(y).$$

- The functional integral *can be performed exactly*

$$\bar{G}(t, \mathbf{r}_1 - \mathbf{r}_2) = \exp \left[-\frac{i}{2} \int_C d^4x \int_C d^4y J^\mu(x) D_{\mu\nu}^{HTL}(x-y) J^\nu(y) \right]$$

with $J^\mu(x)$ the $Q\bar{Q}$ current.

- In terms of Feynman diagrams:



Real-time $Q\bar{Q}$ propagator

- $Q\bar{Q}$ current non-vanishing along C_+ :

$$\bar{G}(t, \mathbf{r}_1 - \mathbf{r}_2) = \exp \left[-\frac{i}{2} \int_{C_+} d^4x \int_{C_+} d^4y J^\mu(x) D_{\mu\nu}(x-y) J^\nu(y) \right]$$

$$J^\mu(z) = \delta^{\mu 0} \theta(z^0) \theta(t - z^0) [-g\delta(z - \mathbf{r}_1) + g\delta(z - \mathbf{r}_2)]$$

- Large time behavior: $\bar{G}(t, \mathbf{r}_1 - \mathbf{r}_2) \underset{t \rightarrow \infty}{\sim} \exp[-iV_{\text{eff}}(\mathbf{r}_1 - \mathbf{r}_2)t]$, with

$$\begin{aligned} V_{\text{eff}}(\mathbf{r}_1 - \mathbf{r}_2) &\equiv g^2 \int \frac{d\mathbf{q}}{(2\pi)^3} \left(1 - e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}\right) D_{00}(\omega=0, \mathbf{q}) \\ \text{effective potential} & \\ &= g^2 \int \frac{d\mathbf{q}}{(2\pi)^3} \left(1 - e^{i\mathbf{q} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}\right) \left[\underbrace{\frac{1}{\mathbf{q}^2 + m_D^2}}_{\text{screening}} - i \underbrace{\frac{\pi m_D^2 T}{|\mathbf{q}|(\mathbf{q}^2 + m_D^2)^2}}_{\text{collisions}} \right] \\ &= -\frac{g^2}{4\pi} \left[m_D + \frac{e^{-m_D r}}{r} \right] - i \frac{g^2 T}{4\pi} \phi(m_D r) \end{aligned}$$

The $Q\bar{Q}$ effective potential: real part

With a consistent treatment of screened *self-energy* and *interaction*^a

$$V_{\text{eff}}(r) = -\alpha m_D - \frac{\alpha}{r} e^{-m_D r}$$
$$\underset{r \rightarrow 0}{\sim} -\alpha m_D - \frac{\alpha}{r} + \alpha m_D = -\frac{\alpha}{r}$$

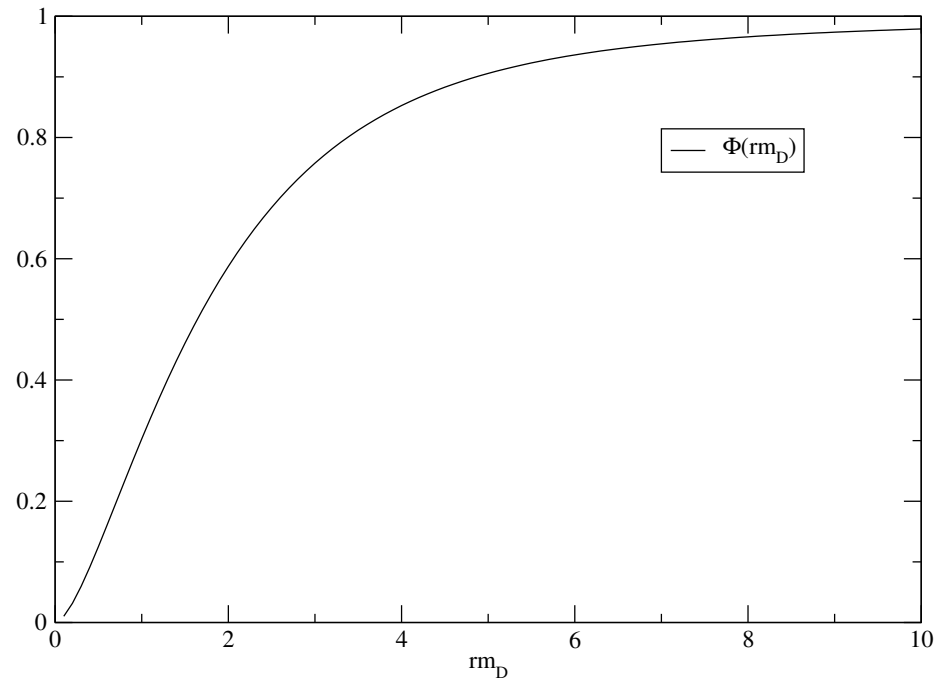
for bound states of *very small size* *medium effects cancel!*

An analogous problem in solid-state physics...

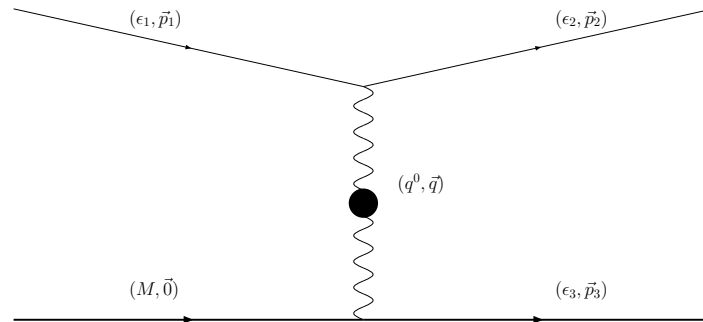
$V_{\text{eff}}(r)$ turns out to coincide with the *Ecker-Weitzel potential* used to study *excitons* (*e-h bound states*) in semiconductors.

^aSee also R. Rapp, D. Blaschke and P. Crochet, arXiv:0807.2470

The $Q\bar{Q}$ effective potential: imaginary part



- For small separation the $Q\bar{Q}$ pair is seen as a *neutral object* and it does not interact with the particles of the medium;
- For large separation the HQs suffer *uncorrelated scatterings* with the plasma particles.



Interpretation of the damping as the **interaction rate of a heavy fermion** in the thermal bath

$$\Gamma(M) = 2 \frac{1}{2M} \int_{p_1} \int_{p_2} \int_{p_3} (2\pi)^4 \delta^{(4)}(P + P_1 - P_2 - P_3) \times \\ \times [n_1(1 - n_2)(1 - n_3) + (1 - n_1)n_2n_3] \overline{|\mathcal{M}|^2}$$

In the $M \rightarrow \infty$ limit:

$$\Gamma(\infty) = g^2 T \int \frac{dq}{(2\pi)^3} \frac{\pi m_D^2}{(q^2 + m_D^2)^2 q}$$

NB The resulting width in $G^>(\omega)$ should be interpreted as a *collisional broadening* of the state.

The finite-mass case:

the heavy quarks free to move in the medium

The general idea

Treat the heavy fermion propagating in a thermal bath as a point-like particle in Quantum-Mechanics. Hence:

- Sum over all the possible trajectories in a given background field:

$$\langle \mathbf{x}_f \tau_f | \mathbf{x}_i \tau_i \rangle = \int_{\mathbf{x}(\tau_i) = \mathbf{x}_i}^{\mathbf{x}(\tau_f) = \mathbf{x}_f} [\mathcal{D}\mathbf{x}(\tau')] \exp \left[- \int_{\tau_i}^{\tau_f} d\tau' \left(\frac{1}{2} M \dot{\mathbf{x}}^2 + V(\mathbf{x}) \right) \right],$$

where $V(\mathbf{x}) \equiv g\Phi(\mathbf{x})$ (scalar interaction) and $\dot{\mathbf{x}} \equiv d\mathbf{x}/d\tau'$.

- Average over all the possible field configurations (the action accounting for medium effects)

$$G^>(-i\tau, \mathbf{r}_1 | 0, \mathbf{r}'_1) = Z^{-1} \int_{\mathbf{z}_1(0) = \mathbf{r}'_1}^{\mathbf{z}_1(\tau) = \mathbf{r}_1} [\mathcal{D}\mathbf{z}_1] \int [\mathcal{D}\Phi] \exp \left[- \int_0^\tau d\tau' \frac{1}{2} M \dot{\mathbf{z}}_1^2 \right] \times \\ \times \exp \left[-g \int_0^\tau d\tau' \Phi(\tau', \mathbf{z}_1(\tau')) \right] e^{-S_E^{\text{eff}}[\Phi]}$$

For a gaussian effective action...

Also on the finite-mass case if the action is gaussian

$$S_E^{\text{eff}}[\Phi] = \frac{1}{2} \int d^4 x_E \int d^4 y_E \Phi(x) \Delta^{-1}(x - y) \Phi(y),$$

the integration over the field configurations can be performed exactly:

$$G^>(-i\tau, \mathbf{r}_1 | 0, \mathbf{r}'_1) = \int_{\mathbf{z}(0)=\mathbf{r}'_1}^{\mathbf{z}(\tau)=\mathbf{r}_1} [\mathcal{D}\mathbf{z}] \exp \left[- \int_0^\tau d\tau' \frac{1}{2} M \dot{\mathbf{z}}^2 \right] \times \\ \times \exp \left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta(\tau' - \tau'', \mathbf{z}(\tau') - \mathbf{z}(\tau'')) \right],$$

with $\Delta(\tau, \mathbf{x})$ the Matsubara propagator of the exchanged meson.

A heavy “quark” in hot-QED

We perform Monte Carlo simulations for

$$G^>(-i\tau, \mathbf{r}_1 | 0, \mathbf{r}'_1) = \int_{z(0)=\mathbf{r}'_1}^{z(\tau)=\mathbf{r}_1} [\mathcal{D}z] \exp \left[- \int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{z}^2 \right) \right] \times \\ \times \exp \left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', z(\tau') - z(\tau'')) \right]$$

where

$$\Delta_L(\tau, \mathbf{q}) \equiv \Delta_L^{vac}(\tau, \mathbf{q}) + \Delta_L^T(\tau, \mathbf{q}) \\ = \frac{-1}{q^2} \delta(\tau) + \int_{-\infty}^{+\infty} \frac{dq_0}{2\pi} e^{-q_0\tau} \rho_L(q_0, \mathbf{q}) [\theta(\tau) + N(q^0)]$$

is expressed in terms of the HTL spectral function

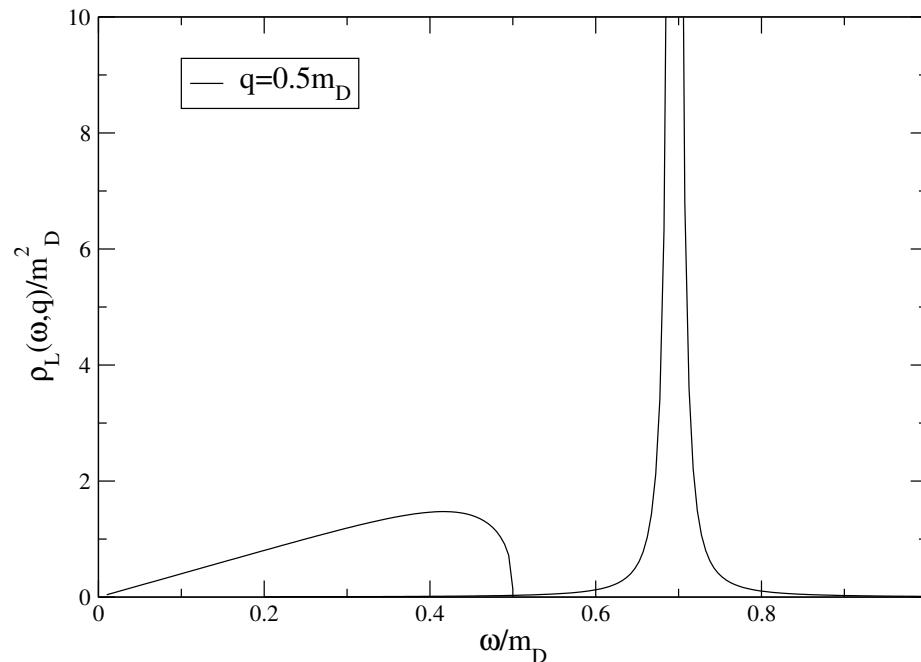
$$\rho_L(\omega > 0, q) \equiv 2\pi \left[\underbrace{Z_L(q) \delta(\omega - \omega_L(q))}_{\text{plasmon pole}} + \underbrace{\theta(q^2 - \omega^2) \beta_L(\omega, q)}_{\text{Landau damping}} \right]$$

HTL longitudinal spectral function

$$\rho_L(\omega) \equiv 2 \operatorname{Im} D_L^{\text{ret}}(\omega) = 2 \operatorname{Im} \Delta_L(\omega + i\eta),$$

where:

$$\Delta_L(q^0, q) = \frac{-1}{q^2 + m_D^2 \left(1 - \frac{q^0}{2q} \ln \frac{q^0 + q}{q^0 - q} \right)}$$



Pole + **Continuum**. The width is put by hand!

Our long term goal...

...would be to address the $Q\bar{Q}$ case within the same approach:

$$\begin{aligned} G^>(-i\tau; \mathbf{r}_1, \mathbf{r}_2 | 0; \mathbf{r}'_1, \mathbf{r}'_2) &= e^{-(M_1+M_2)\tau} \int_{\mathbf{r}'_1}^{\mathbf{r}_1} [\mathcal{D}\mathbf{z}_1] \int_{\mathbf{r}'_2}^{\mathbf{r}_2} [\mathcal{D}\mathbf{z}_2] \times \\ &\times \exp\left[-\int_0^\tau d\tau' \left(\frac{1}{2}M_1\dot{\mathbf{z}}_1^2 - \frac{g^2}{2}\int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', \mathbf{z}_1(\tau') - \mathbf{z}_1(\tau''))\right)\right] \times \\ &\times \exp\left[-\int_0^\tau d\tau' \left(\frac{1}{2}M_2\dot{\mathbf{z}}_2^2 - \frac{g^2}{2}\int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', \mathbf{z}_2(\tau') - \mathbf{z}_2(\tau''))\right)\right] \times \\ &\times \exp\left[-g^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L(\tau' - \tau'', \mathbf{z}_1(\tau') - \mathbf{z}_2(\tau''))\right] \end{aligned}$$

*Before facing the numerical outcomes of the
simulations....*

...some physical insight from (weak-coupling)
thermal field theory calculations

General setup

- Analytic non-relativistic HQ propagator

$$G(z) = \frac{-1}{z - E_p - \Sigma(z, \mathbf{p})},$$

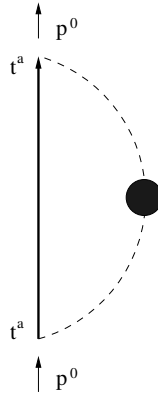
where $E_p = M + p^2/2M$ and setting $z = \omega + i\eta$ corresponds to *retarded boundary conditions*;

- HQ spectral function:

$$\sigma(\omega) \equiv 2\text{Im} G^R(\omega) = \frac{\Gamma(\omega)}{[\omega - E_p - \text{Re} \Sigma(\omega)]^2 + \Gamma^2(\omega)/4},$$

with $\Gamma(\omega) \equiv -2\text{Im} \Sigma^R(\omega) \implies$ *HQ spectral function non-vanishing only for energies for which the self-energy develops an imaginary-part.*

HQ self-energy: resummed one-loop result



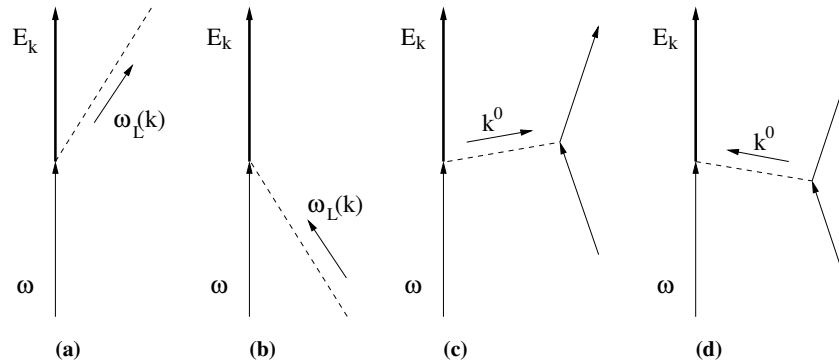
The zero-momentum HQ self-energy reads:

$$\Sigma(p^0) = g^2 C_F \int \frac{d\mathbf{k}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0, k) \frac{1 + N(k^0) - n_F(E_k)}{p^0 - E_k - k^0}$$

Test-particle limit recovered setting $n_F(E_k) = 0$, which arises naturally in the regime $T/M \ll 1$

$$\Sigma^{\text{test}}(p^0) = g^2 C_F \int \frac{d\mathbf{k}}{(2\pi)^3} \int_0^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0, k) \left[\frac{1 + N(k^0)}{p^0 - E_k - k^0} + \frac{N(k^0)}{p^0 - E_k + k^0} \right]$$

HQ self-energy: imaginary-part

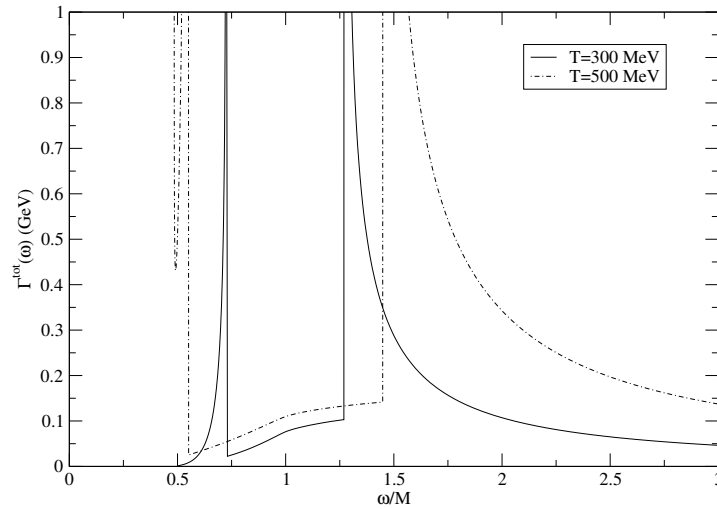
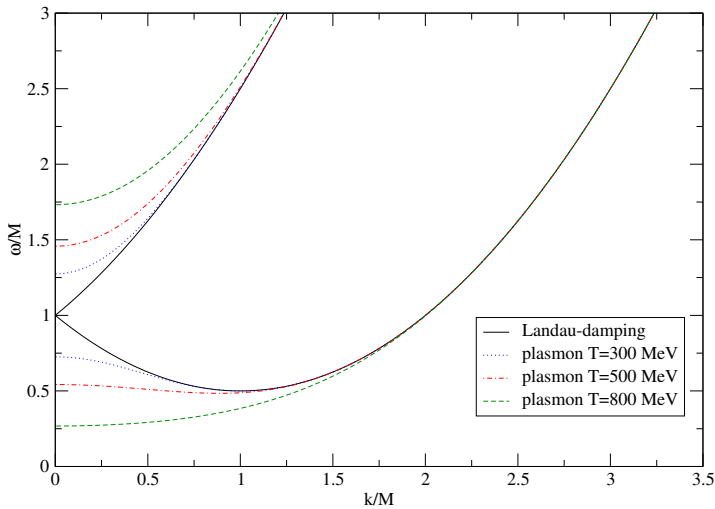


- **Plasmon-pole** contribution (a and b)

$$\Gamma^{\text{pole}}(\omega) = g^2 C_F \int \frac{d\mathbf{k}}{(2\pi)^3} (2\pi) Z_L(k) \times \\ \times [(1 + N(\omega_L(k))) \delta(\omega - E_k - \omega_L(k)) + N(\omega_L(k)) \delta(\omega - E_k + \omega_L(k))]$$

- **Continuum** contribution (c and d)

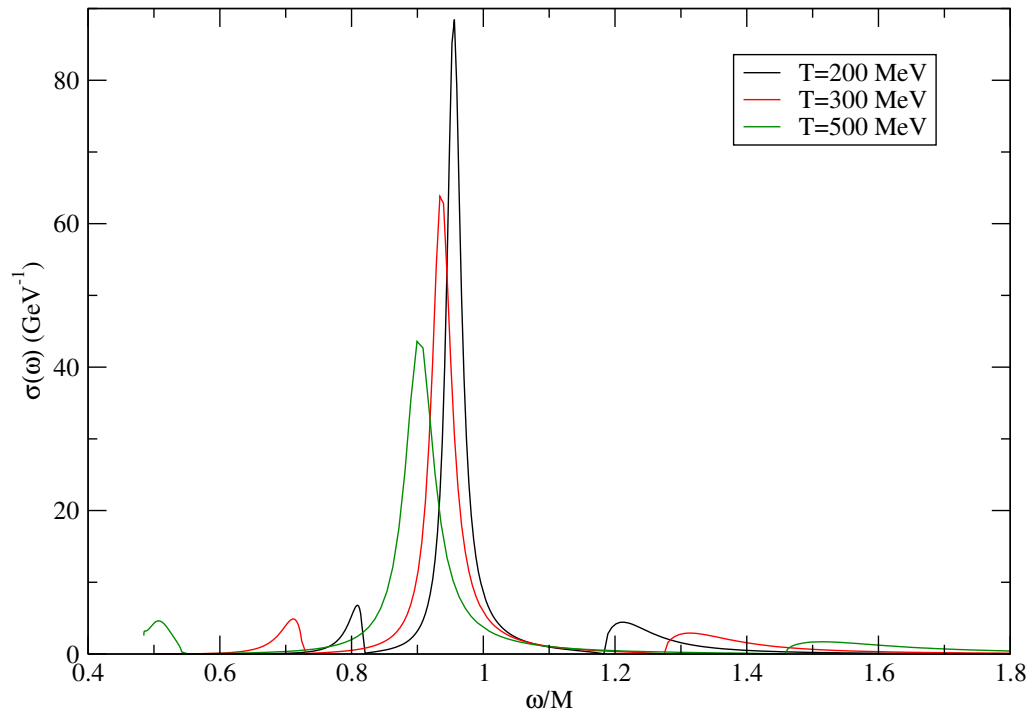
$$\Gamma^{\text{cont}}(\omega) = g^2 C_F \int \frac{d\mathbf{k}}{(2\pi)^3} \int_0^k dk^0 \beta_L(k^0, k) \times \\ \times (2\pi) \{ [1 + N(k^0)] \delta(\omega - E_k - k^0) + N(k^0) \delta(\omega - E_k + k^0) \}$$



- Spectrum displaying a **threshold close to $M/2$** ;
- Very narrow *peaks arising from a divergence in the density of states (Van-Hove singularities)*. Defining $\omega \equiv E_{k_{1/2}} \pm \omega_L(k_{1/2})$

$$\Gamma^{\text{pole}}(\omega) = \frac{g^2 C_F}{\pi} \left\{ \frac{k_1^2}{|E'_{k_1} + \omega'_L(k_1)|} Z_L(k_1) [1 + N(\omega_L(k_1))] + \right. \\ \left. + \sum_{k_2} \frac{k_2^2}{|E'_{k_2} - \omega'_L(k_2)|} Z_L(k_2) N(\omega_L(k_2)) \right\}$$

HQ spectral-function



- **Negative shift** and **broadening** of the principal peak;
- Appearance of secondary peaks at energies corresponding to a large density of states for *plasmon absorption/emission processes*

Numerical results

from the MC simulations for the
path-integral

$$\underbrace{G^>(t=-i\tau)}_{\text{evaluated}} \equiv G(\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-\omega\tau} \underbrace{\sigma(\omega)}_{\text{reconstructed}} .$$

$G(\tau)$ obtained after averaging over at least 10^6 paths!

Evaluation of the path-integral I

We can reduce

$$\begin{aligned} G(\tau, r) &= \int [\mathcal{D}z] \exp \left[- \int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{z}^2 \right) \right] \times \\ &\quad \times \exp \left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', z(\tau') - z(\tau'')) \right] \\ &\equiv \int [\mathcal{D}z] \exp[-S[z]] \end{aligned}$$

to the evaluation of an *expectation value*, by *rescaling the coupling*
 $g^2 \rightarrow \alpha g^2$

$$G_\alpha(\tau, r) \equiv \int [\mathcal{D}z] \exp[-S_\alpha[z]],$$

so that

$$\frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \left\langle \frac{g^2}{2} \int d\tau' \int d\tau'' \Delta_L^T(\tau' - \tau'', z(\tau') - z(\tau'')) \right\rangle_\alpha$$

Evaluation of the path-integral II

- For a given α the *expectation value* is evaluated by generating paths distributed according to

$$W_\alpha[z] = \frac{1}{G_\alpha} \exp(-S_\alpha[z])$$

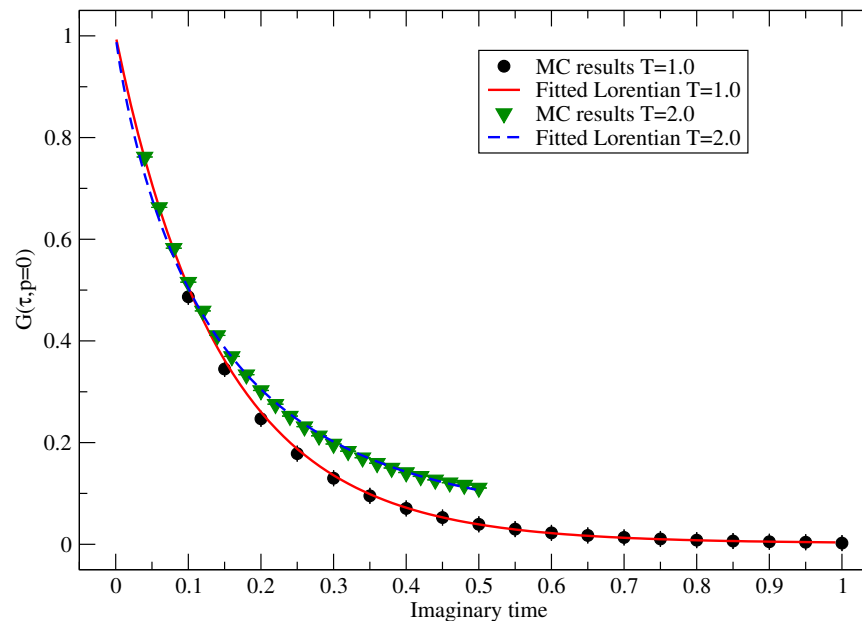
- By integrating over the parameter α one gets:

$$\int_0^1 d\alpha \frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \ln \left(\frac{G(\tau, r)}{G_{\text{free}}(\tau, r)} \right) = \int_0^1 d\alpha \langle \Delta \rangle_\alpha,$$

where

$$G_{\text{free}} = [M/(2\pi\tau)]^{3/2} \exp[-Mr^2/(2\tau)].$$

HQ euclidean propagator: the $p=0$ case



Information on the spectral density obtained through

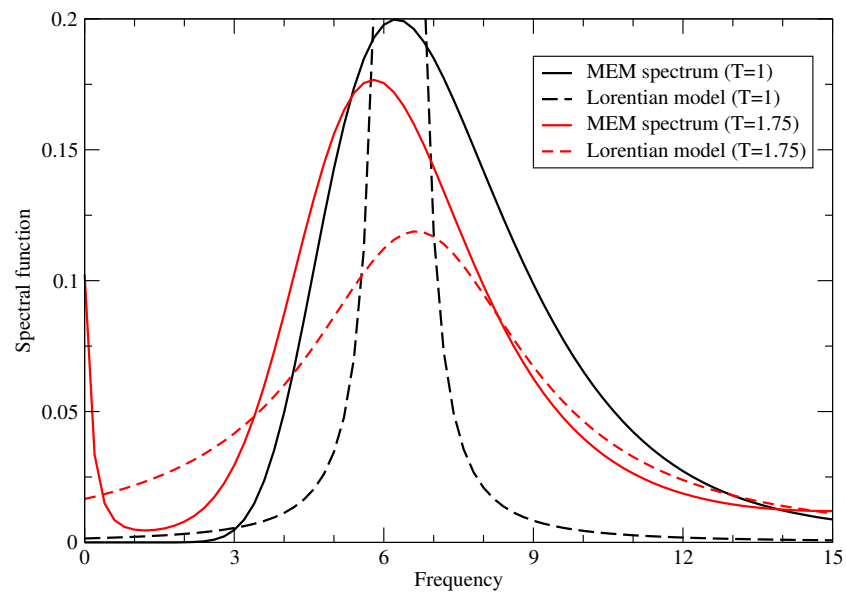
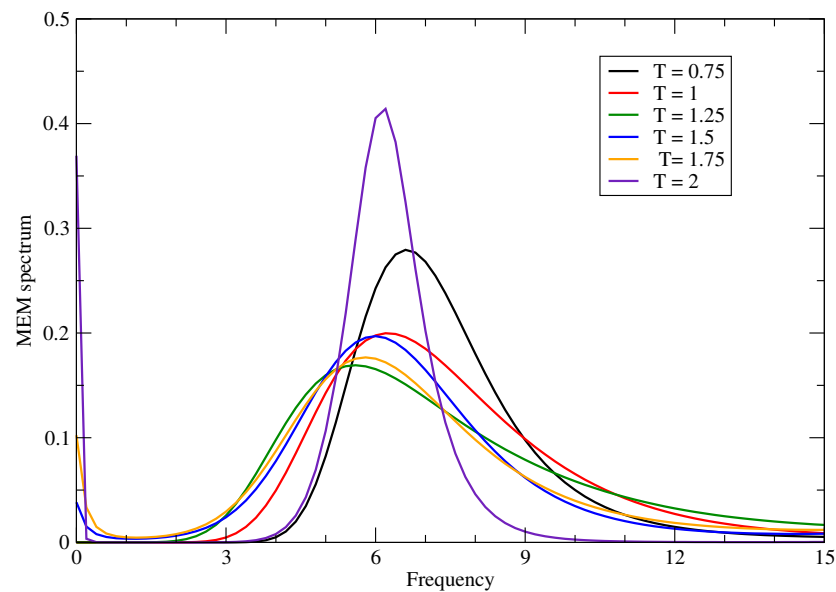
- a best-fit procedure with the simple ansatz:

$$\sigma(\omega) \sim \frac{2\gamma}{(\omega - M_0)^2 + \gamma^2}$$

- MEM reconstruction.

Results for the HQ spectral function I

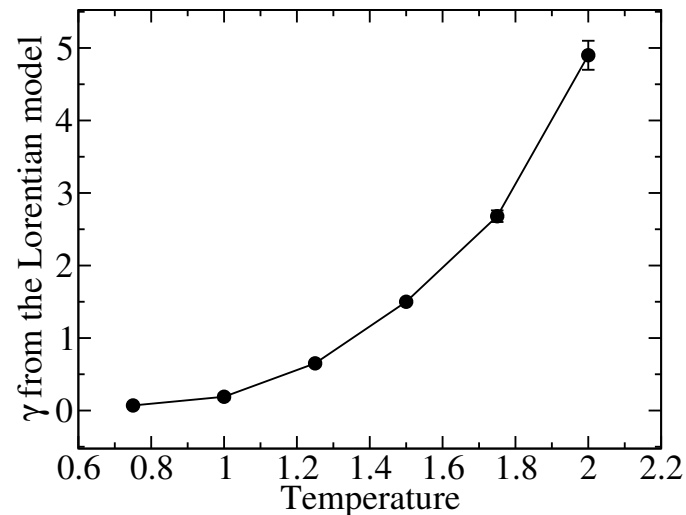
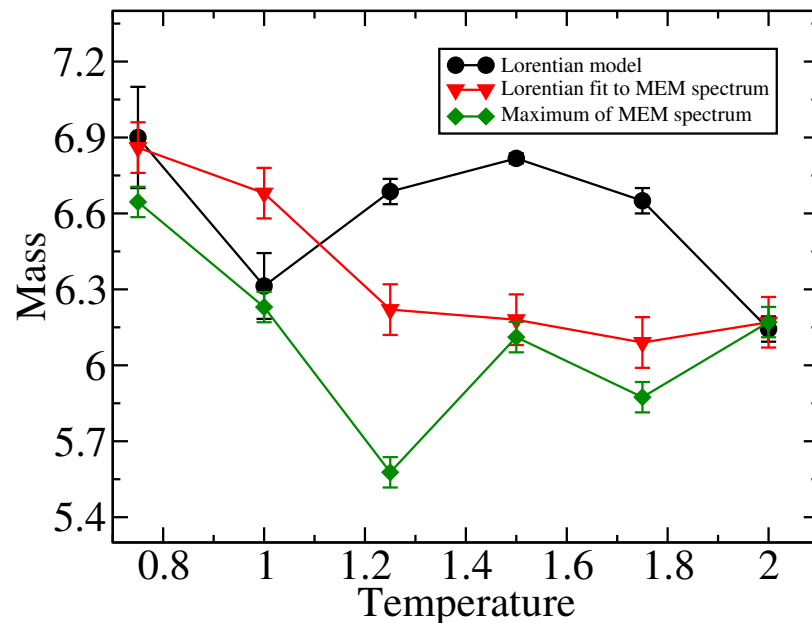
Starting from $M^{\text{vac}} = 7.5\dots$ (energies in units of 200 MeV)



Origin of MEM peak close to zero-energy probably fake!

Results for the HQ spectral function II

Starting from $M^{\text{vac}} = 7.5\dots$



Huge (negative) mass-shift and width, occurring already at moderate temperatures, reflect a *sizable spectral strength at low-energy!*

Summary

- The **effective-action approach**, introduced to derive a real-time *static potential*, results very convenient **to address also the finite-mass case**: *QFT problem reduced to a QM problem!*
- Numerical results for $G(\tau)$ indicates an **important spectral strength at low-energy**;
- **Resummed one-loop calculation** of interest to shed light on possible **processes responsible for such a strength**.

Future developments

- Better control of the MEM procedure;
- *Addressing the $Q\bar{Q}$ case.*

Back-up slides

Renormalization of the coupling in the path-integral

The link

g cont \longleftrightarrow $g_0(a_\tau)$ path – integral

is obtained by *matching* the calculations of $\overline{G}(\tau, \mathbf{0})$ for a static ($M = \infty$) quark in both schemes.

$$\overline{G}^{\text{ren}}(\tau, \mathbf{r} = \mathbf{0}) = \exp \left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \int \frac{d\mathbf{q}}{(2\pi)^3} \Delta_{00}^T(\tau' - \tau'', \mathbf{q}) \right]$$

$$\overline{G}_{\text{lat}}^{\text{reg}}(\tau, \mathbf{r} = \mathbf{0}) = \exp \left[\frac{g_0^2(a_\tau)}{2} a_\tau^2 \sum_{i \neq j} \int \frac{d\mathbf{q}}{(2\pi)^3} \Delta_{00}^T(\tau_i - \tau_j, \mathbf{q}) \right]$$