Effective field theories for quarkonium at finite temperature

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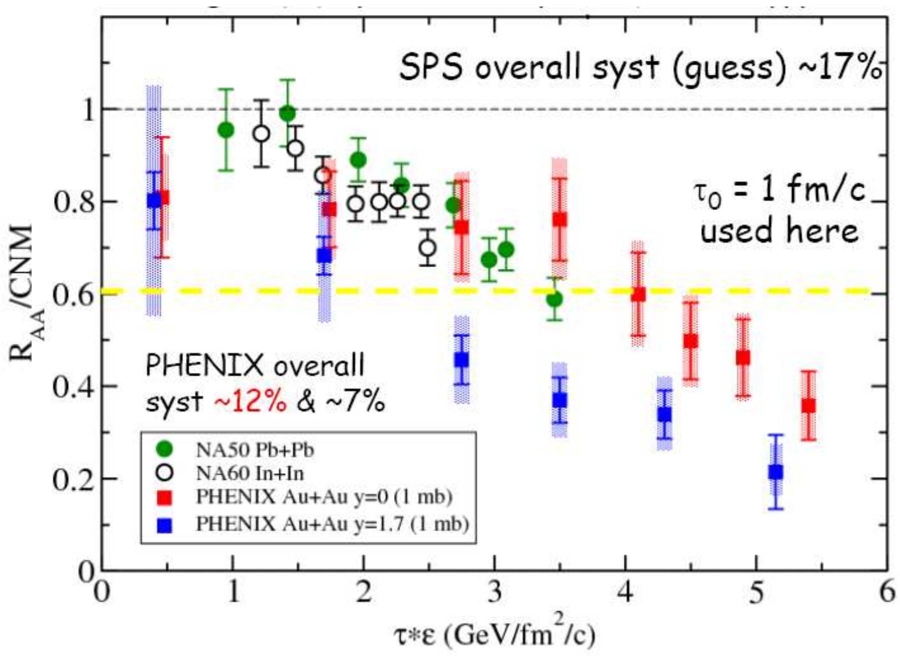
Motivations

- Thermal medium induces color screening.
- Color screening induces hadron dissociation in the Quark Gluon Plasma. Quarkonium dissociation as a clear signature of QGP formation; sequential dissociation works as thermometer.

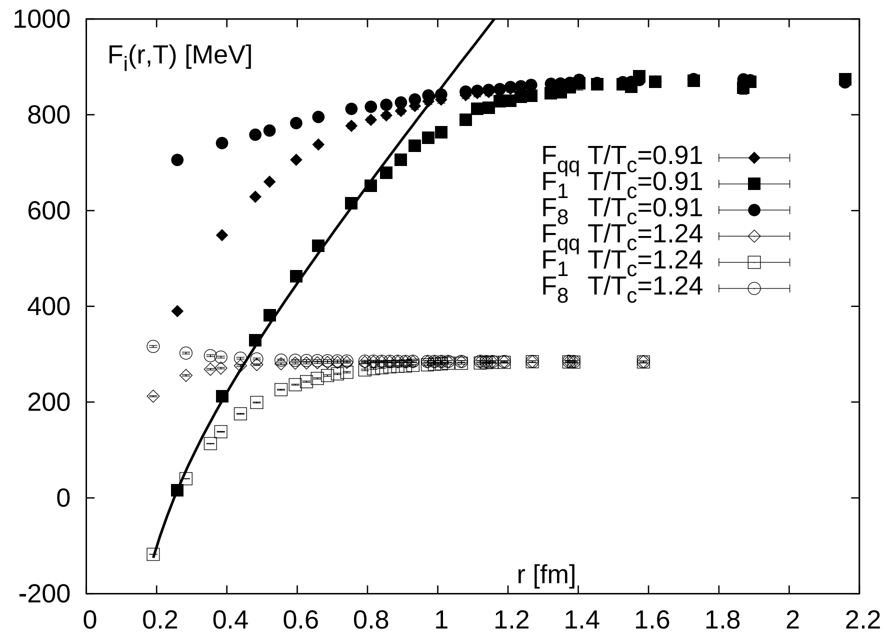
  - Matsui Satz PLB 178(86)416



  - Leitch 08

Free energy vs potential

- Either phenomenological potentials have been used so far or the free energy calculated on the lattice.
- The free energy is not the static potential: the average free energy ($\sim \langle \mathrm{Tr}\, L^\dagger(r) \mathrm{Tr}\, L(0) \rangle$) is an overlap of singlet and octet quark-antiquark states, what is called the singlet ($\sim \langle \mathrm{Tr}\, L^\dagger(r)\, L(0) \rangle$) and the octet ($\sim \langle \mathrm{Tr}\, L^\dagger(r) \mathrm{Tr}\, L(0) \rangle$ $-1/3\, \langle \mathrm{Tr}\, L^\dagger(r)\, L(0) \rangle$) free energy are gauge dependent;



○ Kaczmarek Zantow PRD 71 (2005) 114510

The quarkonium potential at finite T

In order to study quarkonium properties in a thermal bath at temperature T, the quantity to be determined is the quarkonium potential, which describes the real-time evolution of a $Q\bar{Q}$ pair through the Schrödinger equation

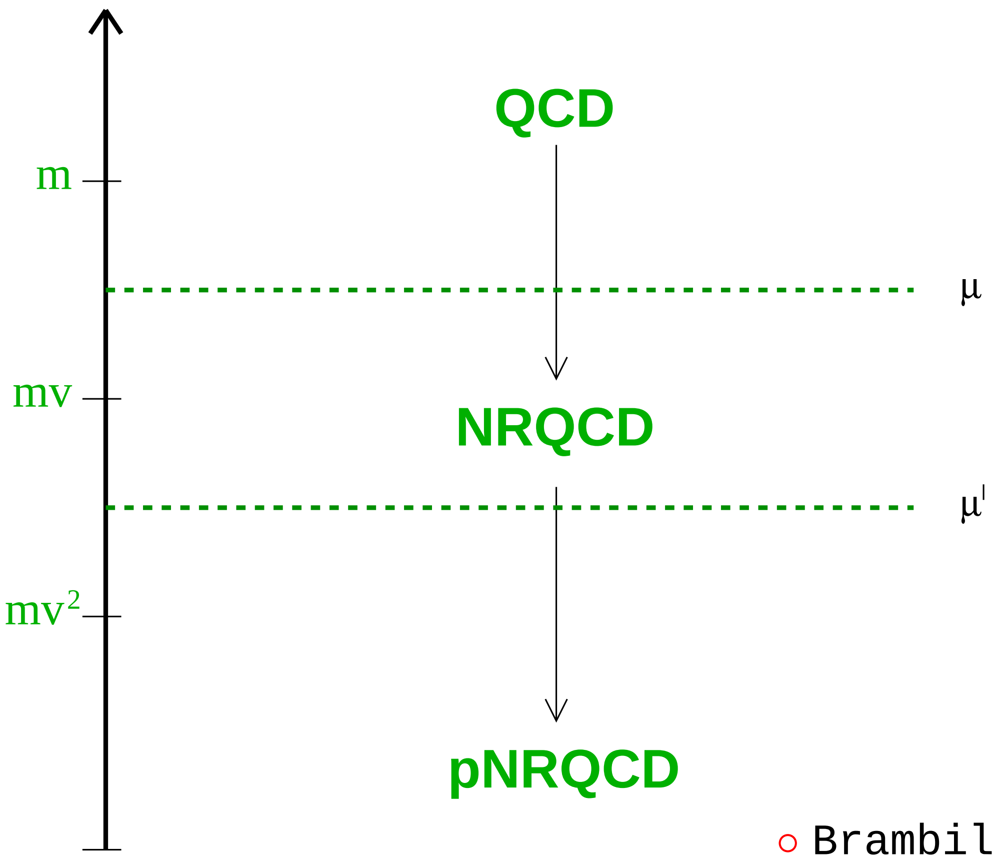
$$E\,\Phi = \left(\frac{p^2}{m} + V(r,T)\right)\,\Phi$$

- The free energy is a thermodynamical quantity, which has inspired many potential models for quarkonium at finite T, but is NOT, in general, the potential $V(r,T)$.
- In the full theory, $V(r,T)$ must come from a systematic expansion
  - in $1/m$ (non-relativistic expansion), the leading term being the static potential;
  - in the energy E (ultrasoft expansion).

  It will encode all contributions from scales larger than E and smaller than m.
- One may exploit these expansions by constructing a suitable hierarchy of EFTs as it has been done at $T = 0$.
- EFTs will account for thermal effects both of the potential and/or of the non-potential type.

The $T = 0$ case

Scales and EFTs



Non-relativistic bound states are characterized by the energy and momentum scales:

- m (mass)
- mv (momentum transfer, inverse distance)
- mv^2 (kinetic energy, binding energy, potential V)

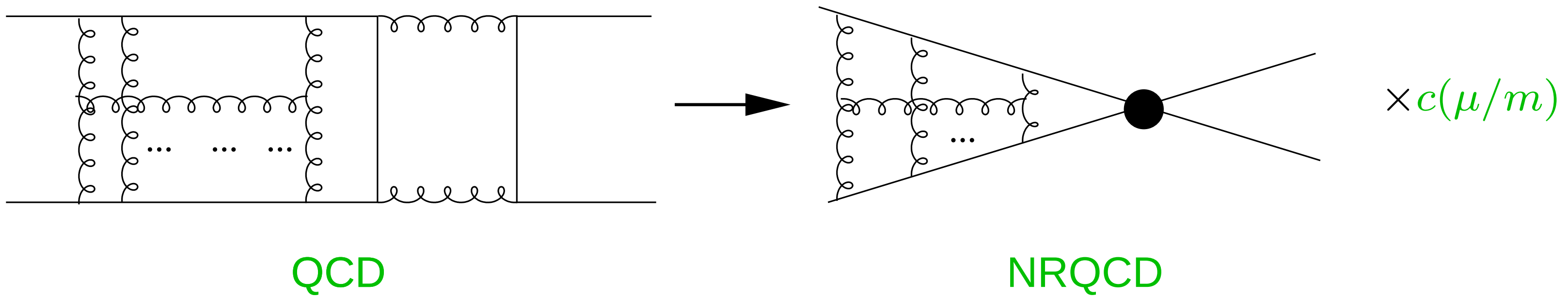
v is the relative heavy-quark velocity

○ `Brambilla Pineda Soto Vairo RMP 77(05)1423`

We may expand physical observables in the ratios of the scales. If we separate/factorize explicitly the contributions from the different scales at the Lagrangian level this amounts to substituting QCD with a hierarchy of EFTs, which are equivalent to QCD order by order in the expansion parameters.

NRQCD

NRQCD is obtained by integrating out modes associated with the scale m

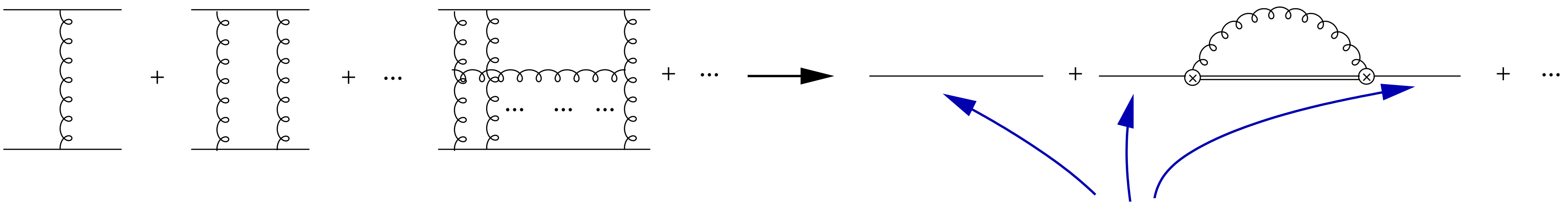


- The matching is perturbative.
- The Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

$$\mathcal{L} = \sum_n c(\alpha_s(m/\mu)) \times O_n(\mu, \lambda)/m^n$$

pNRQCD

pNRQCD is obtained by integrating out modes associated with the scale $\frac{1}{r} \sim mv$



$$\frac{1}{E - p^2/m - V(r)}$$

NRQCD pNRQCD

- The Lagrangian is organized as an expansion in $1/m$, r, and $\alpha_s(m)$:

$$\mathcal{L} = \sum_k \sum_n \frac{1}{m^k} \times c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

pNRQCD in the weak-coupling regime

- Degrees of freedom:
  - Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
    $\Rightarrow$ (i) singlet $\rm S$ (ii) octet $\rm O$
  - Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Power counting:

  $p \sim \frac{1}{r} \sim mv$;
  all gauge fields are multipole expanded: $A(R,r,t) = A(R,t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R,t) + \dots$
  and scale like $(\Lambda_{\rm QCD}$ or $mv^2)^{\rm dimension}$.

  Non-analytic behaviour in $r \to$ matching coefficients V

pNRQCD in the weak-coupling regime

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} q_i + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

- LO in r

$\theta(T)\, e^{-iTH_s}$

$\theta(T)\, e^{-iTH_o} \left(e^{-i\int dt\, A^{\mathrm{adj}}} \right)$

pNRQCD in the weak-coupling regime

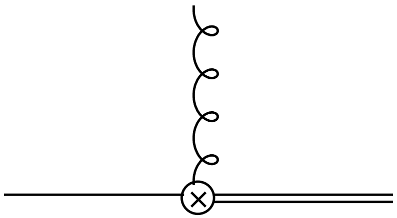
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\,a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

- LO in r

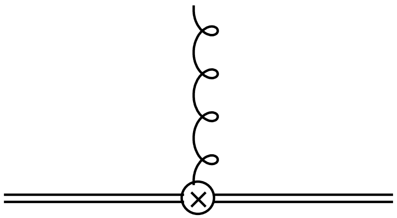
$\theta(T)\, e^{-iTH_s}$ $\qquad$ $\theta(T)\, e^{-iTH_o} \left(e^{-i\int dt\, A^{\mathrm{adj}}} \right)$

$$+V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

- NLO in r



$O^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$

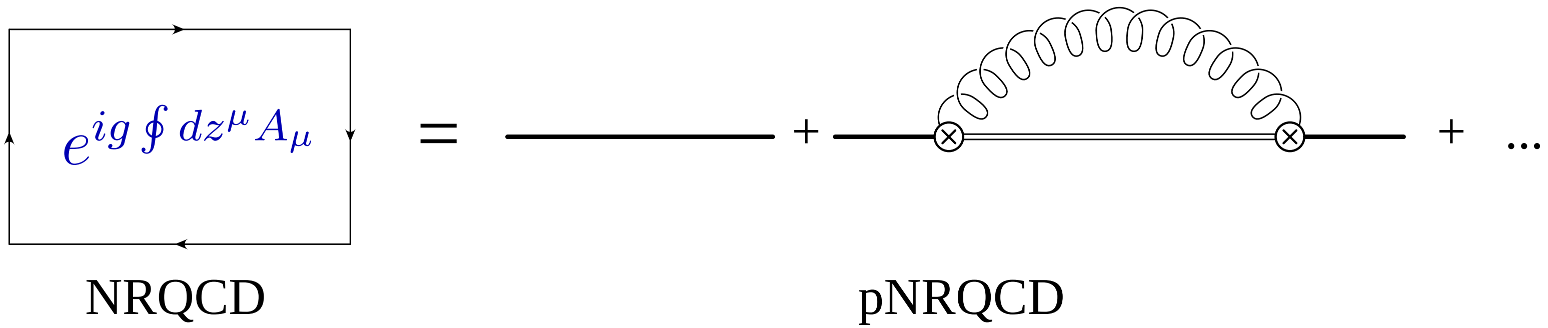


$O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$

$$+ \cdots$$

- The Schrödinger equation and the potential appear once all scales above the binding energy have been integrated out.
- The potential is the Wilson coefficient of the EFT multiplying the four-fermion operator. In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations.
- The EFT accounts for non-potential terms as well. They provide loop corrections to the leading potential picture.

The Static Potential



$$\lim_{T\to\infty} \frac{i}{T} \ln \langle \Box \rangle = V_s(r,\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \, \langle \mathrm{Tr}(r\cdot E(t)\, r\cdot E(0))\rangle(\mu) + \ldots$$

The μ dependence cancels between the two terms in the right-hand side:

- $V_s \sim \ln r\mu, \ln^2 r\mu$, ...
- ultrasoft contribution $\sim \ln(V_o - V_s)/\mu, \ln^2(V_o - V_s)/\mu, \ldots \ln r\mu, \ln^2 r\mu, \ldots$

Static singlet potential

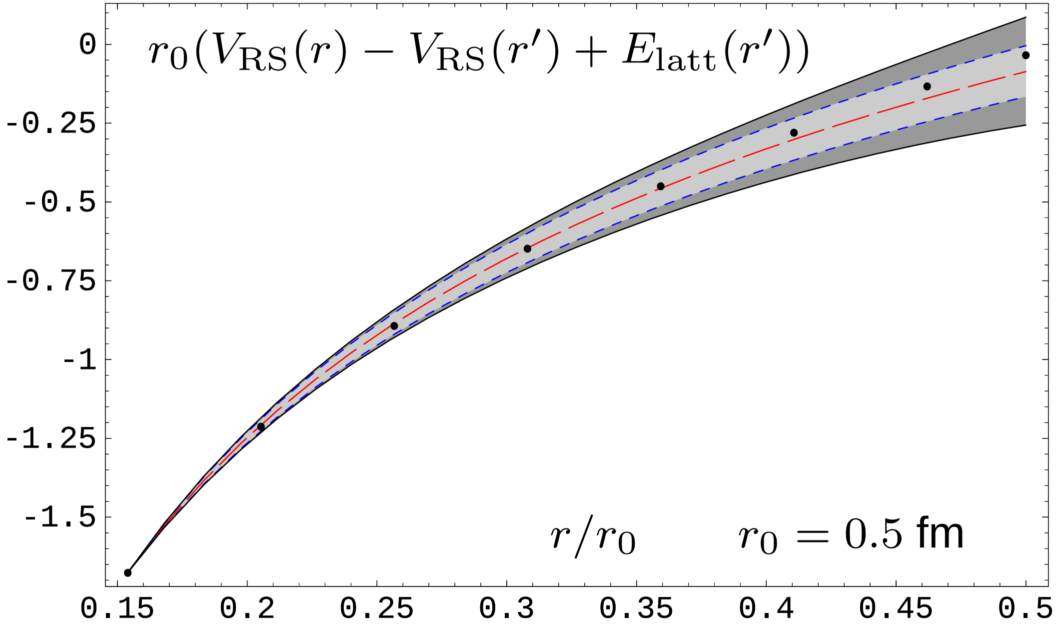
$$
\begin{aligned}
V_s(r,\mu) \quad = \quad & -C_F\,\frac{\alpha_{\rm s}(1/r)}{r}\Bigg\{1+\frac{\alpha_{\rm s}(1/r)}{4\pi}\,[a_1+2\gamma_E\beta_0] \\
& +\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^2\left[a_2+\left(\frac{\pi^2}{3}+4\gamma_E^2\right)\beta_0^2+\gamma_E\,(4a_1\beta_0+2\beta_1)\right] \\
& +\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^3\left[\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu+a_3\right] \\
& +\left(\frac{\alpha_{\rm s}(1/r)}{4\pi}\right)^4\left[a_4^{L2}\ln^2 r\mu+\left(a_4^L+\frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu+a_4\right] \\
& + \qquad \cdots \qquad \Bigg\}
\end{aligned}
$$

Static energy

$$
\begin{aligned}
E_0(r) \quad = \quad & -\frac{C_F\alpha_s(1/r)}{r}\Bigg\{1+\frac{\alpha_s(1/r)}{4\pi}\left[a_1+2\gamma_E\beta_0\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2\left[a_2+\left(\frac{\pi^2}{3}+4\gamma_E^2\right)\beta_0^2+\gamma_E\left(4a_1\beta_0+2\beta_1\right)\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3\left[\frac{16\pi^2}{3}C_A^3\ln\frac{C_A\alpha_s(1/r)}{2}+\tilde{a}_3\right] \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\left[a_4^{L2}\ln^2\frac{C_A\alpha_s(1/r)}{2}+a_4^L\ln\frac{C_A\alpha_s(1/r)}{2}+\tilde{a}_4\right] \\
& + \qquad \cdots \qquad \Bigg\}
\end{aligned}
$$

- Brambilla et al PRD 60(99)1502, PLB 647(07)185

Static energy vs lattice QCD



○ Pineda JPhysG 29(03)371

The $T \neq 0$ case: framework

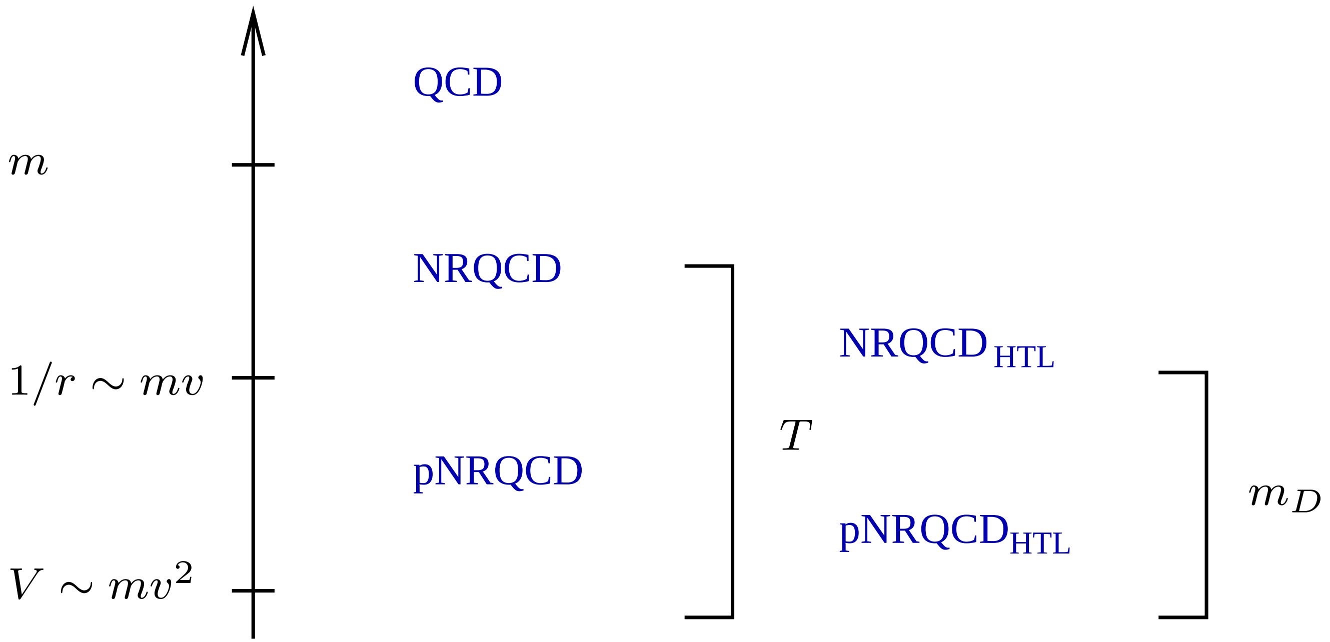
Scales

Quarkonium in a medium is characterized by different energy and momentum scales:

- the scales of the bound state (v is the relative heavy-quark velocity):
  - m (mass),
  - mv (momentum transfer, inverse distance),
  - mv^2 (kinetic energy, binding energy, potential V), ...
- the thermodynamical scales:
  - T (temperature),
  - m_D (Debye mass, i.e. screening of the chromoelectric interactions), ...

If these scales are hierarchically ordered (if the bound state is non relativistic: $v \ll 1$; in the weak coupling regime $T \gg m_D$) then we may expand physical observables in the ratio of the scales. If we separate/factorize explicitly the contributions from the different scales at the Lagrangian level this amounts to substituting QCD with a hierarchy of EFTs, which are equivalent to QCD order by order in the expansion parameters.

Effective Field Theories



∘ Brambilla Pineda Soto Vairo RMP 77(05)1423

We assume that bound states exist for

- $T \ll m$
- $1/r \sim mv \gtrsim m_D$

We neglect smaller thermodynamical scales.

Weak coupling

In the weak coupling regime:

- $v \sim \alpha_{\rm s} \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ, ...
- $T \gg gT \sim m_D$.

Effects due to the scale $\Lambda_{\rm QCD}$ will not be considered.

Static limit of QCD/NRQCD

We assume $m \gg$ any other scale.

- This allows to integrate out m first and organize the EFTs as expansions in $1/m$: the first EFT is NRQCD.
- The leading order term corresponds to the static limit of QCD (or NRQCD):

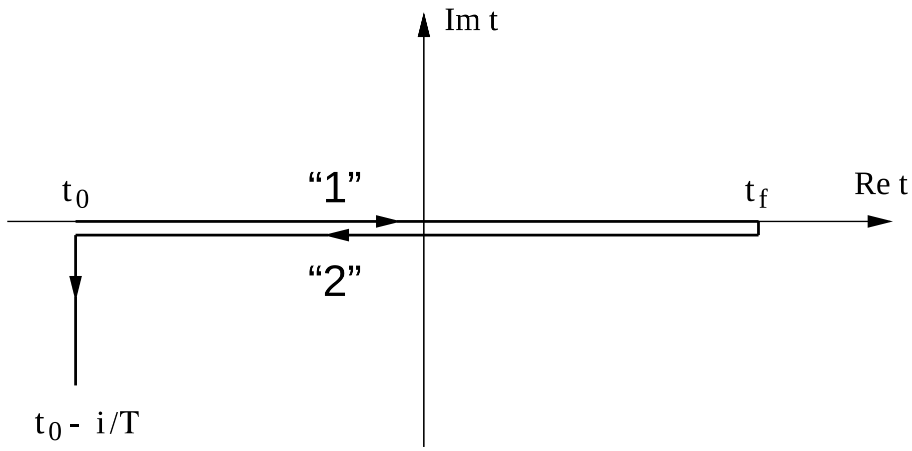
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \psi^\dagger i D_0 \psi + \chi^\dagger i D_0 \chi$$

  ψ (χ) is the field that annihilates (creates) the (anti)fermion.
  Only longitudinal gluons couple to static quarks.

- The relevant scales in static QCD/NRQCD are: $1/r$, V, ... T, m_D, ...

Real time

The contour of the partition function is modified to allow for real time:



In real time, the degrees of freedom double ("1" and "2"), however, the advantages are

- the framework becomes very close to the one for $T = 0$ EFTs;
- in the static quark sector, the second degrees of freedom, labeled "2", decouple from the physical degrees of freedom, labeled "1".

This leads to a simpler treatment with respect to alternative calculations in imaginary time formalism + analytical continuation in real time.

Real-time gluon propagator

- Free gluon propagator in Coulomb gauge:

$$\mathbf{D}_{00}^{(0)}(\vec{k}) = \frac{i}{\vec{k}^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathbf{D}_{ij}^{(0)}(k) = \left(\delta_{ij} - \frac{k^i k^j}{\vec{k}^2}\right)\left\{\begin{pmatrix} \frac{i}{k^2+i\epsilon} & \theta(-k^0)\,2\pi\delta(k^2) \\ \theta(k^0)\,2\pi\delta(k^2) & -\frac{i}{k^2-i\epsilon} \end{pmatrix} + 2\pi\delta(k^2)\,n_{\mathrm{B}}(|k^0|)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right\}$$

where

$$n_{\mathrm{B}}(k^0) = \frac{1}{e^{k^0/T}-1}$$

In Coulomb gauge, only transverse gluons carry a thermal part.

Real-time static quark propagator

- Free static quark propagator:

$$\mathbf{S}_Q^{(0)}(p) = \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 2\pi\delta(p^0) & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix}$$

Since $[\mathbf{S}_Q^{(0)}(p)]_{12} = 0$, the static quark fields labeled "2" never enter in any physical amplitude, i.e. any amplitude that has the physical fields, labeled "1", as initial and final states.

Real-time static quark-antiquark propagator

- Free static quark-antiquark propagator:

$$\mathbf{S}^{(0)}_{\bar{Q}Q}(p) = \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 2\pi\delta(p^0) & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix} = \mathbf{U}^{(0)} \begin{pmatrix} \dfrac{i}{p^0 + i\epsilon} & 0 \\ 0 & \dfrac{-i}{p^0 - i\epsilon} \end{pmatrix} \mathbf{U}^{(0)}$$

where

$$\mathbf{U}^{(0)} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Similar to the quark propagator, but quark-antiquark fields are bosons.

Real-time potential

- Static quark-antiquark potential:

$$\mathbf{V} \quad = \quad \begin{pmatrix} V & 0 \\ -2i\,\mathrm{Im}\,V & -V^* \end{pmatrix} = [\mathbf{U}^{(0)}]^{-1} \begin{pmatrix} V & 0 \\ 0 & -V^* \end{pmatrix} [\mathbf{U}^{(0)}]^{-1}$$

Hence the sum of all insertions of a potential exchange between a free quark and antiquark amounts to the full propagator:

$$\mathbf{U}^{(0)} \begin{pmatrix} \dfrac{i}{p^0 - V + i\epsilon} & 0 \\ 0 & \dfrac{-i}{p^0 - V^* - i\epsilon} \end{pmatrix} \mathbf{U}^{(0)} = \mathbf{S}^{(0)}_{\bar{Q}Q}(p) \sum_{n=0}^{\infty} \left[(-i\mathbf{V}(r))\,\mathbf{S}^{(0)}_{\bar{Q}Q}(p)\right]^n$$

Static potential, energy and decay width at $T \neq 0$

Static quark antiquark at $T \lesssim V$

After having integrated out the scale $1/r$ the EFT is pNRQCD, which is made of
- quark-antiquark states (color singlet S, color octet O),
- low energy gluons and light quarks.

The Lagrangian is organized as an expansion in r:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - V_o \right) \mathrm{O} \right\}$$

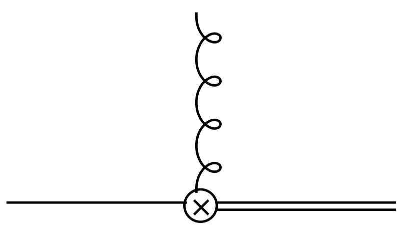
$$+ V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\} + \cdots$$

- At leading order in r, the singlet decouples from the octet and its EOM is: $\left(i\partial_0 - V_s \right) \mathrm{S} = 0$.
- The potentials V_s and V_o are Coulombic: $V_s(r) = -C_F \dfrac{\alpha_{\rm s}}{r}$ and $V_o(r) = \dfrac{\alpha_{\rm s}}{2N_c\, r}$.

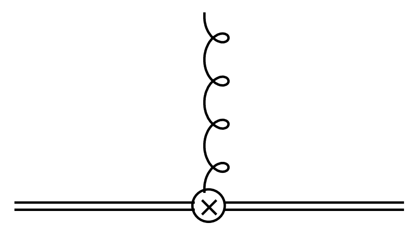
pNRQCD: Feynman rules and loops

$$\theta(T)\, e^{-iTV_s}$$

$$\theta(T)\, e^{-iTV_o} \left(e^{-i \int dt\, A^{\rm adj}} \right)$$

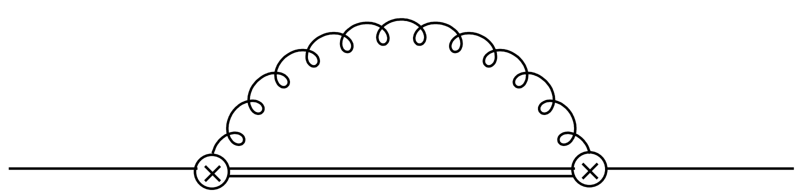


$$\mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$$



$$\mathrm{O}^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$$

- Thermal corrections do not affect the potential, which remains Coulombic, but affect the static energy and the decay width through loop corrections:



Static quark antiquark at $T \lesssim V$: energy and width

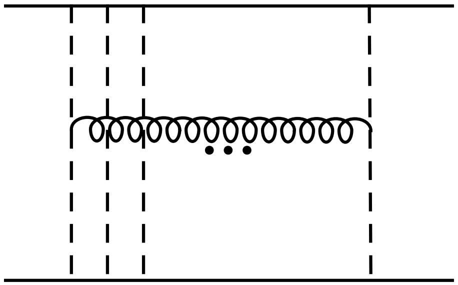
The real part of the diagram gives:

$$\delta E = \frac{2}{3} N_c C_F \frac{\alpha_s^2}{\pi} r T^2 f(N_c \alpha_s/(2rT)), \qquad f(z) \equiv \int_0^\infty dx \frac{x^3}{e^x - 1} \mathrm{P} \frac{1}{x^2 - z^2}$$

The imaginary part of the diagram gives

$$\Gamma = \frac{N_c^3 C_F}{6} \frac{\alpha_s^4}{r} n_B(N_c \alpha_s/(2r))$$

- Corrections coming from the scale m_D are suppressed by powers of m_D/T.
- The width Γ originates from the fact that thermal fluctuations of the medium at short distances may destroy a color-singlet $\bar{Q}Q$ into an octet plus gluons. This process is specific of QCD at finite T; in QCD the relevant diagrams are of the type



Static quark antiquark at $T \ll V$

In this limiting case

$$\delta E = -\frac{8}{45}\,\pi^3\,\frac{C_F}{N_c}\,r^3\,T^4 = -\frac{4}{3}\,\pi\,\frac{C_F}{N_c}\,r^3\,\langle \vec{E}^a(0)\cdot\vec{E}^a(0)\rangle_T$$

and

$$\Gamma = \text{exponentially suppressed}$$

- δE provides the leading gluon condensate correction to the quark-antiquark static energy.

Static quark antiquark at $1/r \gg T \gg V$

Integrating out T from pNRQCD modifies pNRQCD into pNRQCD$_{\rm HTL}$ whose

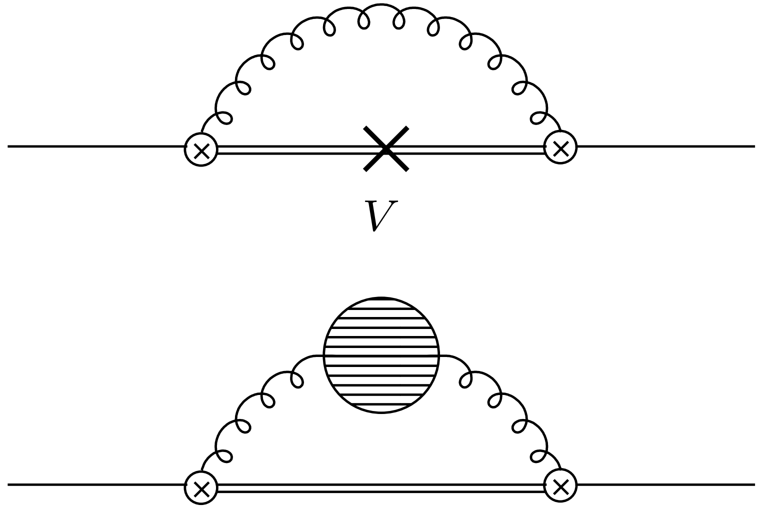
- Yang–Mills Lagrangian gets the additional hard thermal loop (HTL) part; e.g. the longitudinal gluon propagator at $k^0 = 0$ becomes

$$\frac{i}{\vec{k}^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \frac{i}{\vec{k}^2 + m_D^2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \pi \frac{T}{|\vec{k}|} \frac{m_D^2}{\left(\vec{k}^2 + m_D^2\right)^2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- potentials get an additional thermal correction δV to the Coulomb potential.

Static quark antiquark at $1/r \gg T \gg V$: real part

a)



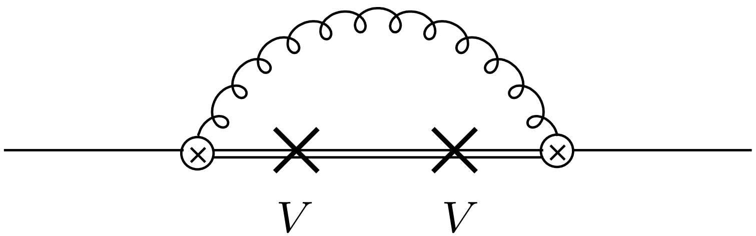
b)

$$\mathrm{Re}\,\delta V_s(r) = \frac{\pi}{9} N_c C_F \alpha_s^2 \, r \, T^2 \qquad a) \quad \sim g^2 r^2 T^3 \times \frac{V}{T}$$

$$-\frac{3}{2}\zeta(3)\, C_F \,\frac{\alpha_s}{\pi}\, r^2\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_s^2\, r^2\, T^3 \qquad b) \quad \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2$$

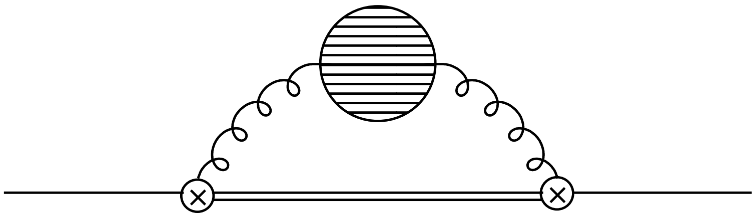
Static quark antiquark at $1/r \gg T \gg V$: imaginary part

a)



Singlet to octet break up contribution

b)



Landau-damping contribution

$$\begin{aligned}\mathrm{Im}\,\delta V_s(r) \;=\; & -\frac{N_c^2 C_F}{6}\,\alpha_{\rm s}^3\,T \\ & +\frac{C_F}{6}\alpha_{\rm s}\,r^2\,T\,m_D^2\,\left(\frac{1}{\epsilon}+\gamma_E+\ln\pi-\ln\frac{T^2}{\mu^2}+\frac{2}{3}-4\ln 2-2\frac{\zeta'(2)}{\zeta(2)}\right) \\ & +\frac{4\pi}{9}\ln 2\;N_c C_F\,\alpha_{\rm s}^2\,r^2\,T^3\end{aligned}$$

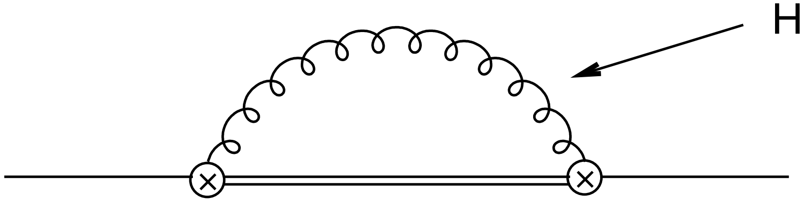
$a)\quad \sim g^2 r^2 T^3 \times \left(\frac{V}{T}\right)^2$

$b)\quad \sim g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2$

Static quark antiquark at $1/r \gg T \gg m_D \gg V$

Divergences appear in the imaginary part of the potential at order $g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^2$.
They cancel in physical observables against loop corrections from lower energy scales.

We consider the case $1/r \gg T \gg m_D \gg V$. Integrating out m_D from pNRQCD$_{\rm HTL}$ leads to an extra contribution δV_s to the potential coming from



$$\mathrm{Re}\,\delta V_s(r) \;\sim\; g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^3$$

$$\mathrm{Im}\,\delta V_s(r) \;=\; -\frac{C_F}{6}\,\alpha_{\rm s}\,r^2\,T\,m_D^2\left(\frac{1}{\epsilon} - \gamma_E + \ln\pi + \ln\frac{\mu^2}{m_D^2} + \frac{5}{3}\right)$$

Static quark antiquark at $1/r \gg T \gg m_D \gg V$: energy and width

$$\delta E = \frac{\pi}{9} N_c C_F \, \alpha_{\rm s}^2 \, r \, T^2 - \frac{3}{2}\zeta(3) \, C_F \, \frac{\alpha_{\rm s}}{\pi} \, r^2 \, T \, m_D^2 + \frac{2}{3}\zeta(3) \, N_c C_F \, \alpha_{\rm s}^2 \, r^2 \, T^3$$

$$\Gamma = \frac{N_c^2 C_F}{3} \, \alpha_{\rm s}^3 \, T - \frac{C_F}{3}\alpha_{\rm s} \, r^2 \, T \, m_D^2 \left(2\gamma_E - \ln \frac{T^2}{m_D^2} - 1 - 4\ln 2 - 2\frac{\zeta'(2)}{\zeta(2)} \right) - \frac{8\pi}{9} \ln 2 \; N_c C_F \, \alpha_{\rm s}^2 \, r^2 \, T^3$$

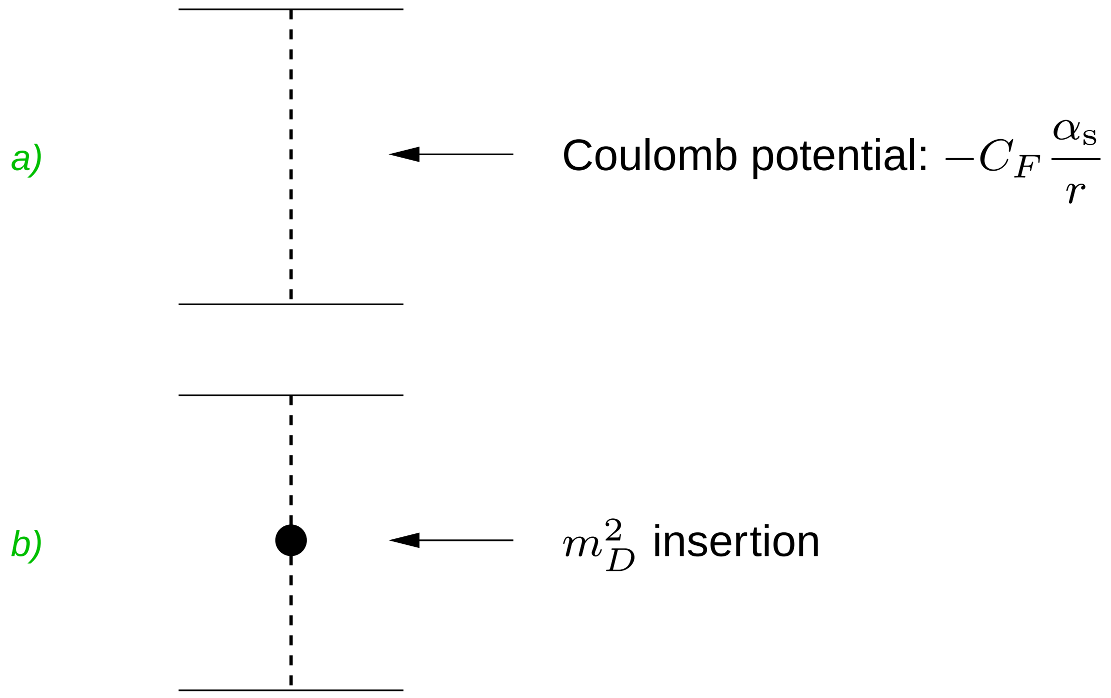
- The non-thermal part of the static energy is the Coulomb potential $-C_F \alpha_{\rm s}/r$.
- The thermal width has two origins. The first term comes from the thermal break up of a quark-antiquark color singlet state into a color octet state. The other terms come from imaginary contributions to the gluon self energy that may be traced back to the Landau damping phenomenon. The first one is specific of QCD, the second one would also show up in QED. Having assumed $m_D \gg V$, the term due to the singlet to octet break up is parametrically suppressed by $(V/m_D)^2$ with respect to the imaginary gluon self-energy contributions.

Static quark antiquark at $T \gg 1/r \gg m_D$

In this situation integrating out T from static QCD leads to static NRQCD$_{\rm HTL}$, which, at one loop, is static NRQCD with the Yang–Mills Lagrangian supplement by the HTL Lagrangian.

Subsequently, integrating out $1/r$ leads to a specific version of pNRQCD$_{\rm HTL}$ where the Coulomb potential gets corrections from HTL insertions.

Static quark antiquark at $T \gg 1/r \gg m_D$: real part

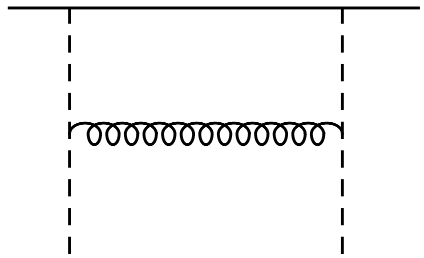


$$\mathrm{Re}\,\delta V_s(r) = -\frac{C_F}{2}\,\alpha_s\,r\,m_D^2$$

$b) \quad \sim \frac{\alpha_s}{r} \times (r m_D)^2$

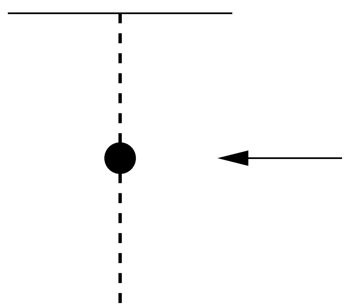
Static quark antiquark at $T \gg 1/r \gg m_D$: imaginary part

a)



Singlet to octet break up contribution

b)



$-i\pi m_D^2 T/|\vec{k}|$ insertion

Landau-damping contribution

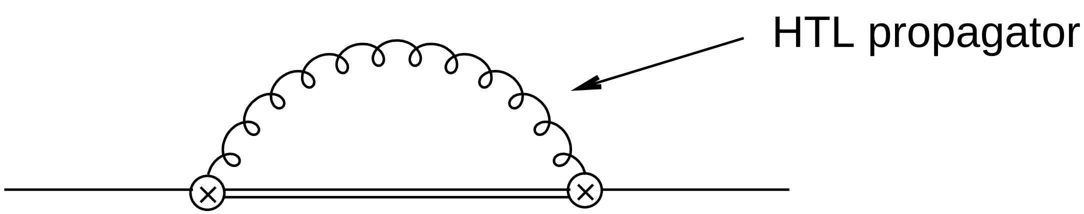
$$\mathrm{Im}\,\delta V_s(r) = -\frac{N_c^2 C_F}{6}\,\alpha_{\mathrm{s}}^3\,T \qquad a) \quad \sim \frac{\alpha_{\mathrm{s}}}{r} \times (rV)^2 \times (Tr)$$

$$+\frac{C_F}{6}\,\alpha_{\mathrm{s}}\,r^2\,T\,m_D^2\left(\frac{1}{\epsilon} + \gamma_E + \ln\pi + \ln(r\,\mu)^2 - 1\right) \qquad b) \quad \sim \frac{\alpha_{\mathrm{s}}}{r} \times (r m_D)^2 \times (Tr)$$

Static quark antiquark at $T \gg 1/r \gg m_D \gg V$

Divergences appear in the imaginary part of the potential at order $\frac{\alpha_s}{r} \times (rm_D)^2 \times (Tr)$.
They cancel in physical observables against loop corrections from lower energy scales.

We consider the case $T \gg 1/r \gg m_D \gg V$. Integrating out m_D from pNRQCD$_{\rm HTL}$ leads to an extra contribution δV_s to the potential coming from



$$\mathrm{Re}\,\delta V_s(r) \;\sim\; g^2 r^2 T^3 \times \left(\frac{m_D}{T}\right)^3$$

$$\mathrm{Im}\,\delta V_s(r) \;=\; -\frac{C_F}{6}\,\alpha_s\, r^2\, T\, m_D^2 \left(\frac{1}{\epsilon} - \gamma_E + \ln\pi + \ln\frac{\mu^2}{m_D^2} + \frac{5}{3}\right)$$

Static quark antiquark at $T \gg 1/r \gg m_D \gg V$:

energy and width

$$\delta E = -\frac{C_F}{2}\,\alpha_{\rm s}\, r\, m_D^2$$

$$\Gamma = \frac{N_c^2 C_F}{3}\,\alpha_{\rm s}^3\, T + \frac{C_F}{3}\,\alpha_{\rm s}\, r^2\, T\, m_D^2 \left(-2\gamma_E - \ln(r m_D)^2 + \frac{8}{3}\right)$$

- The non-thermal part of the static energy is the Coulomb potential $-C_F\alpha_{\rm s}/r$.
- Again the thermal width has two origins. The first term comes from the thermal break up of a quark-antiquark color singlet state into a color octet state. The other terms come from imaginary contributions to the gluon self energy that may be traced back to the Landau damping phenomenon. Having assumed $m_D \gg V$, the term due to the singlet to octet break up is parametrically suppressed by $(V/m_D)^2$ with respect to the imaginary gluon self-energy contributions.

$T \gg 1/r \gg m_D \gg V$: Quarkonium melts in the medium

The quarkonium melts in the medium when

$$E_{\rm binding} \sim \Gamma$$

i.e.

$$\frac{g^2}{r} \sim g^2 T m_D^2 r^2 \ln \frac{1}{m_D r}$$

for $1/r \sim m\, g^2$ and $m_D \sim g\, T$

$$T \sim m\, g^{4/3} \, (\ln 1/g)^{-1/3}$$

∘ `Escobedo Soto arXiv:0804.0691, Laine arXiv:0810.1112`

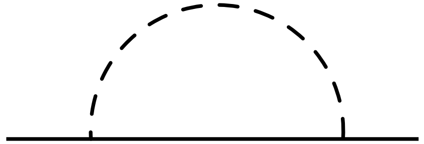
Static quark antiquark at $T \gg 1/r \sim m_D$

In this situation integrating out T from static QCD leads to static NRQCD$_{\rm HTL}$, which, at one loop, is static NRQCD with the Yang–Mills Lagrangian supplement by the HTL Lagrangian.

Subsequently, we have to integrate out both $1/r$ and m_D at the same time, by using HTL resummed gluon propagators.

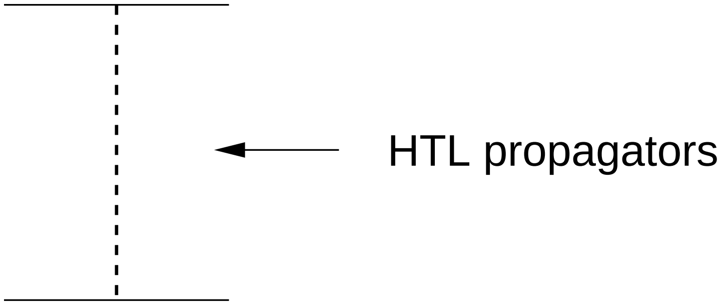
Static quark antiquark at $T \gg 1/r \sim m_D$: real part

a)



mass contribution

b)



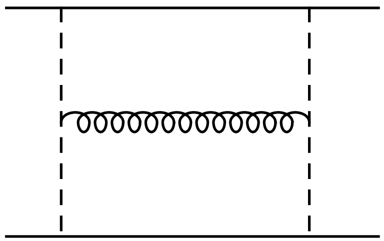
potential contribution

$$\delta E = \mathrm{Re}\,[2\delta m + \delta V_s(r)] \;=\; -C_F\,\alpha_s\,m_D - C_F\,\frac{\alpha_s}{r}\,e^{-m_D r} \qquad a) + b) \qquad \sim \alpha_s m_D$$

- Gava Jengo PLB 105(81)285
- Nadkarni PRD 34(86)3904

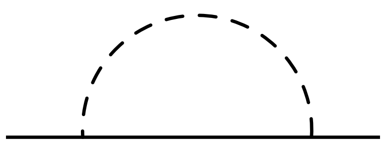
Static quark antiquark at $T \gg 1/r \sim m_D$: imaginary part

a)



Singlet to octet break up contribution

b)



damping rate of a static quark/antiquark

c)

Landau damping contribution

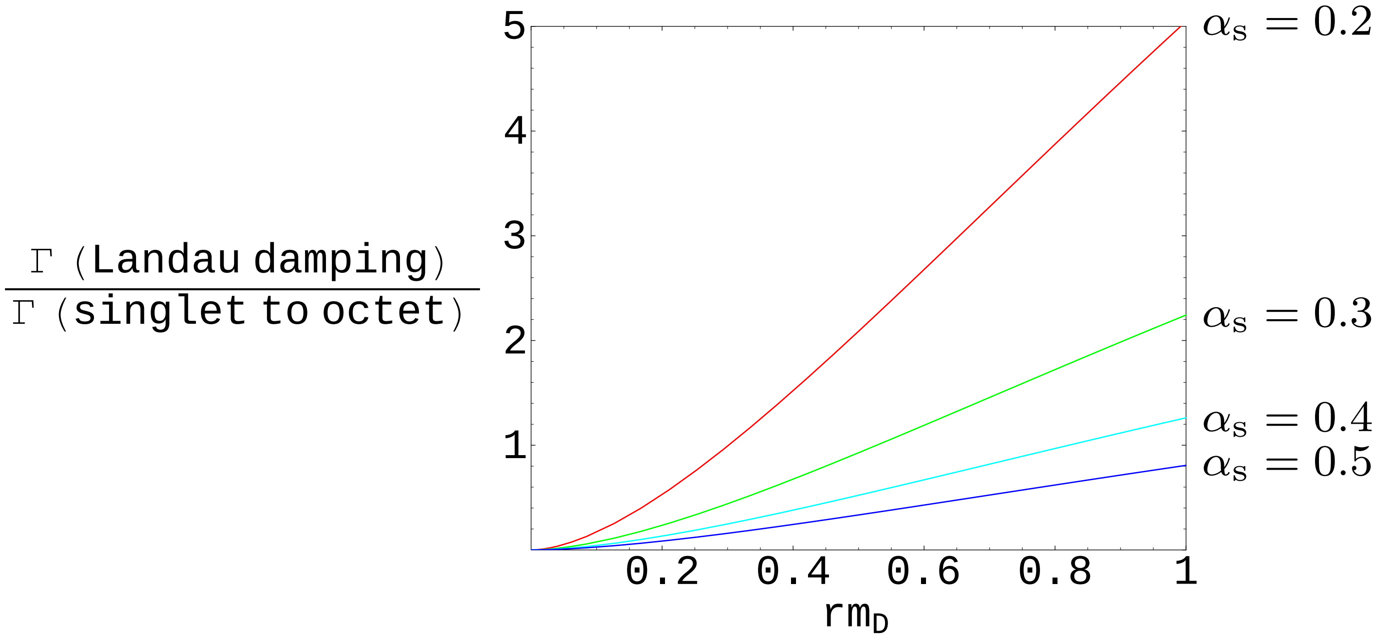
$$\Gamma = -2\mathrm{Im}\,\delta V_s(r) = \frac{N_c^2 C_F}{3}\,\alpha_{\rm s}^3\,T \qquad a) \quad \sim \alpha_{\rm s} m_D \times (Vr)^2 \times \frac{T}{m_D}$$

$$+2\,C_F\,\alpha_{\rm s}\,T\left[1 - \frac{2}{r m_D}\int_0^\infty dx\,\frac{\sin(m_D r\,x)}{(x^2+1)^2}\right] \qquad b) + c) \quad \sim \alpha_{\rm s} m_D \times \frac{T}{m_D} \gg \alpha_{\rm s} m_D$$

- Pisarski PRD 47(93)5589
- Laine Philipsen Romatschke Tassler JHEP 0703(07)054

Static quark antiquark at $T \gg 1/r$

- Under the condition $1/r \sim m_D$ the width is larger by a factor T/m_D than the potential and the bound state dissolves.
- Both Landau damping and singlet to octet break up contribute to the decay width. Parametrically the ratio of the two contributions is proportional to $(m_D/V)^2$, hence Landau damping dominates when $m_D \gg V$ and singlet to octet break up when $V \gg m_D$. Numerically, the singlet to octet contribution may be large also when parametrically suppressed:



Γ(Landau damping)$/\Gamma$(singlet to octet) vs rm_D for different values of $\alpha_s(1/r)$

Conclusions I

- In a framework that makes close contact with modern effective field theories for non-relativistic bound states at zero temperature, we have studied the real-time evolution of a static quark-antiquark pair in a medium of gluons and light quarks at finite temperature.
- For temperatures T ranging from values larger to smaller than the inverse distance of the quark and antiquark, $1/r$, and at short distances, we have derived the potential between the two static sources, their energy and thermal decay width.
- In the medium, the quarkonium melts at a temperature $T \sim m\, g^{4/3}\, (\ln 1/g)^{-1/3}$.

Conclusions II

- The derived potential/energy is neither the quark-antiquark free energy nor the internal energy. It is the real-time potential that describes the real-time evolution of a quarkonium state in a thermal medium. It encodes all contributions coming from modes with energy and momentum larger than the binding energy.
- For $T < V$ the potential is the Coulomb potential. For $T > V$ the potential gets thermal contributions.
- Two mechanisms contribute to the thermal decay width: the imaginary part of the gluon self energy induced by the Landau damping phenomenon, and the quark-antiquark color singlet to color octet thermal break up. Parametrically, the first mechanism dominates for temperatures such that the Debye mass m_D is larger than the binding energy, while the latter dominates for temperatures such that m_D is smaller than the binding energy.

Points for discussion for Panel III

- The potential to be used in the Schrödinger equation is not the lattice free energy nor the internal energy
- The quarkonium disappears due to the imaginary part of the potential
- When the screening sets in $T \gg \frac{1}{r} \sim m_D$ the quarkonium is already gone
- The imaginary part of the potential is produced by Landau damping and by singlet to octet break up.This last effect exist only in QCD