Charmonium correlators and spectral functions at finite temperature

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Outline

Charmonium correlation functions

- Matsubara correlator

$$G_H(\tau, T) = \sum_{\vec{r}} < J_H(\tau, \vec{r}) J_H^\dagger(0, \vec{0}) >, \quad J_H(\tau, \vec{r}) = \bar{q}(\tau, \vec{r}) \Gamma_H q(\tau, \vec{r})$$

- Charmonium states

Γ	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$ n=1	$c\bar{c}$ n=2
γ_5	$^1S_0$	0^{-+}	η_c	η_c'
γ_μ	$^3S_1$	1^{--}	J/ψ	ψ'
1	$^3P_0$	0^{++}	χ_{c0}	
$\gamma_5\gamma_\mu$	$^3P_1$	1^{++}	χ_{c1}	

Charmonium spectral functions

- The Euclidean correlator is related to the spectral function:

$$G(\tau, T, \vec{p}) = \int_0^\infty \mathrm{d}\omega K(\tau, \omega)\sigma(\omega, T, \vec{p});\ K(\tau, \omega) = \frac{cosh(\omega(\tau - \frac{1}{2T}))}{sinh(\frac{\omega}{2T})}$$

- Dissociation patterns of charmonia
- Thermal dilepton rate
- Diffusion constant

Difficulties on the lattice

Obstacles to extract spectral function from correlation functions:

- Temporal extent is restricted within 1/2T
  - can never be improved

$$G(\tau, T, \vec{p}) = \int_0^\infty \mathrm{d}\omega \frac{cosh(\omega(\tau - \frac{1}{2T}))}{sinh(\frac{\omega}{2T})} \sigma(\omega, T, \vec{p})$$

- Difficult to invert the above equation
  - more data points

Maximum Entropy Method

- Based on Bayesian theorem, provide a statistical inference of the most probable spectral function [Asakawa et al., 01]

- Conditional probability of having $\sigma(\omega)$: $\propto exp(\alpha S - L)$
  - L is the standard χ^2
  - S is the Shannon-Jaynes entropy
    $S = \int_0^\infty \left[\sigma(\omega) - m(\omega) - \sigma(\omega)\ log(\frac{\sigma(\omega)}{m(\omega)})\right] \mathrm{d}\omega$
  - $m(\omega)$, default model, provides prior knowledge on $\sigma(\omega)$
  - α, positive parameter, indicates the relative weight of S and L

Maximum Entropy Method (cont.)

- The final output image $\sigma_{out}(\omega)$ is given by a weighted average over α:

$$\sigma_{out}(\omega) = \int \sigma_\alpha \, P[\alpha|Dm] \, \mathrm{d}\alpha$$

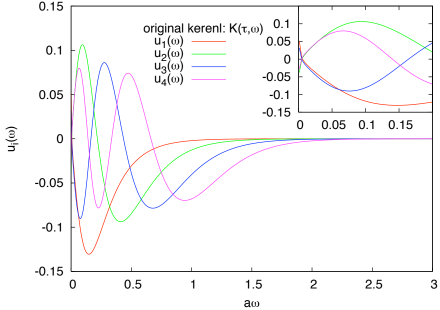
  where σ_α is given by maximizing $Q = \alpha S - L$,
  $P[\alpha|Dm]$ is the weight factor distributions

- During the maximization step of $Q(\sigma, \alpha)$, singular value decomposition of the kernel $K(\tau, \omega)$ is usually used:

$$K(\tau, \omega) = V \, \Sigma \, U^T, \quad K : N_\tau \times N_\omega, \quad V : N_\tau \times N_s, \quad U : N_\omega \times N_s$$

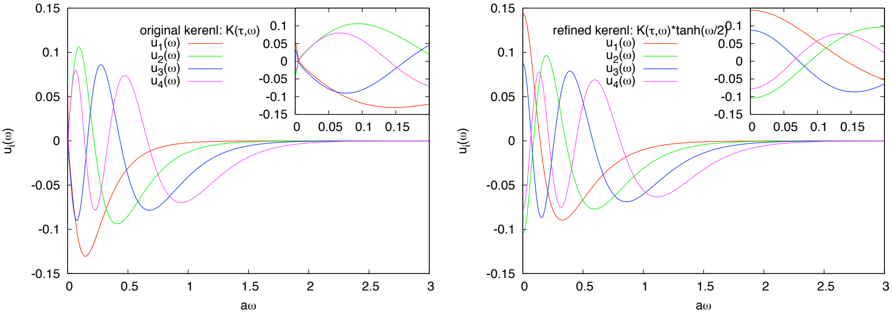
$$\sigma(\omega) = m(\omega) \exp\Big\{ \sum_{i=1}^{N_s} b_i U_i(\omega) \Big\}, \quad (Ns \leq N_\tau)$$

Maximum Entropy Method (cont.)



$$\lim_{\omega \to 0} K(\omega, \tau) = \frac{2T}{\omega} + \frac{\omega}{T}[\frac{1}{6} - \tau T(1 - \tau T)] + \mathcal{O}(\frac{\omega^3}{T^3})$$

[Aarts et al., 07]

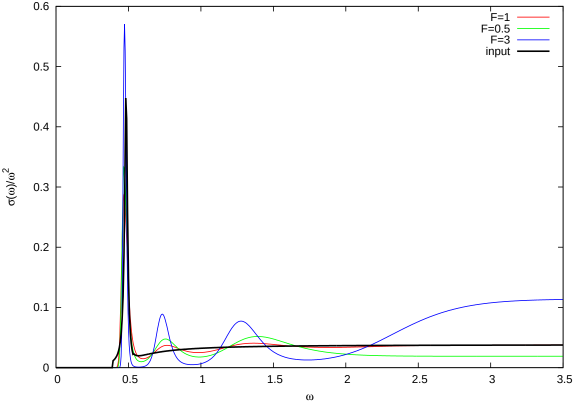


$$K' = K * tanh(\omega/2), \quad \sigma' = \sigma/tanh(\omega/2)$$

1. MEM could probe small ω region more reliably with kernel redefined
2. MEM is not sensitive to the large ω region of spf, which means one has to provide correct large ω information into DM

MEM test on mock data

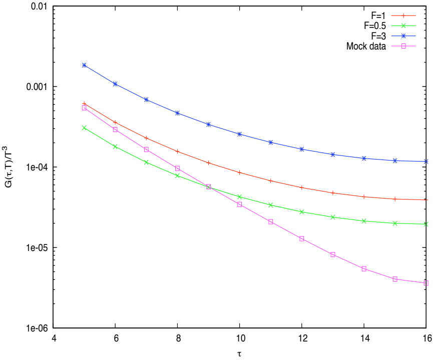
1. spectral function: one relativistic Breit-Weigner resonance plus free continuum spf
2. mock data is obatained through the above spf with random guassan noise

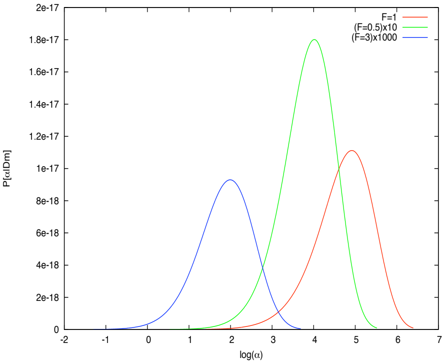


- large ω behavior of input spf: $\frac{3}{4\pi}\omega^2$
- DM : F $\times \frac{3}{4\pi}\omega^2$

MEM test on mock data (cont.)

1. MEM output images always reproduce the correlator data points within the errors, irrepective of DM
2. The correlators calculated from DM and the weight factor distributions will be helpful





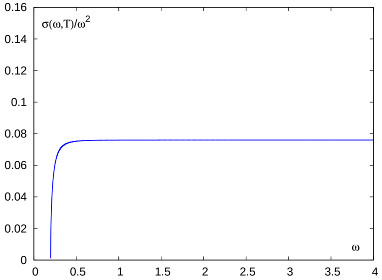
HTD et al., 09

Prior knowledge put into DM

1. large energy behaviour should resemble the non-interacting case

$$\sigma_H = \frac{N_c}{8\pi^2}\Theta(\omega^2 - 4m^2)\ \omega^2\tanh(\frac{\omega}{4T}) \times\sqrt{1-(\frac{2m}{\omega})^2}\Big[a_H + (\frac{2m}{\omega})^2\ b_H\Big] + \ \frac{N_c}{3}\ \frac{T^2}{2}\ f_H\ \omega\delta(\omega)$$

Karsch et al., 03 , Aarts et al., 05



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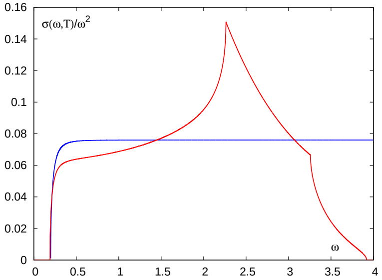

$$\omega_{max} a\ =\ 2 * log(7 + ma)$$

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2. zero mode contribution at $\omega \approx 0$ [Umeda 07]

$$\delta(\omega) \rightarrow \frac{1}{\pi}\frac{\eta}{\omega^2+\eta^2}$$

Petreczky,Teany 06, Aarts, Martinez-Resco 05

MEM analysis on the differences of correlators

$$G(\tau, T, \vec{p}) = \int_0^\infty \mathrm{d}\omega \, \sigma(\omega, T, \vec{p}) \, \frac{cosh(\omega(\tau - \frac{1}{2T}))}{sinh(\frac{\omega}{2T})}$$

$$G(\tau + 1, T, \vec{p}) = \int_0^\infty \mathrm{d}\omega \, \sigma(\omega, T, \vec{p}) \, \frac{cosh(\omega(\tau + 1 - \frac{1}{2T}))}{sinh(\frac{\omega}{2T})}$$

$$\boxed{G(\tau, T, \vec{p}) - G(\tau + 1, T, \vec{p}) = \int_0^\infty \mathrm{d}\omega \sigma(\omega, T, \vec{p}) K^*(\tau, T)}$$

$$K^*(\tau, T) = 2 \sinh(\frac{\omega}{2}) \sinh(\omega(\frac{1}{2T} - \tau - \frac{1}{2}))/\sinh(\frac{\omega}{2T})$$

1. Differences of correlators get rid of τ independent constant $\longrightarrow$ suppress zero mode contribution
2. Output is same as that from the standard way of MEM analysis except zero mode contribution

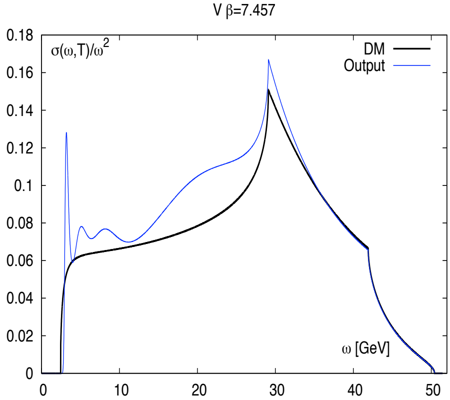
Lattice setup

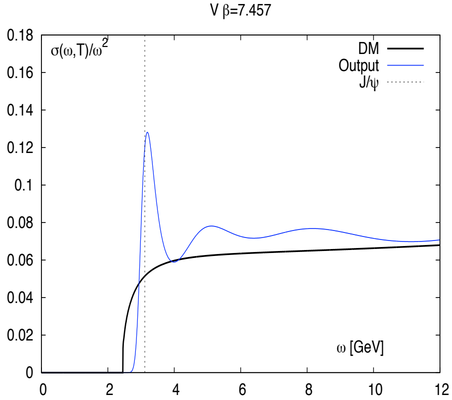
- clover improved wilson fermions
- isotropic quenched lattice

β	κ	a [fm]	$N_\sigma^3 \times N_\tau$	T/T_c	# of conf.	Tag
6.872	0.13035	0.031	$128^3 \times 32$	0.75	126	I A
			$128^3 \times 16$	1.5	198	I B
7.457	0.13179	0.015	$128^3 \times 64$	0.75	179	II A
			$128^3 \times 32$	1.5	108	II B

- fine lattice: $m_c a \approx 0.0987 \ll 1$
- temporal extent: $\tau_{max} \approx 0.48$ fm (0.75 T_c)
- mass tuning: $M_{J/\psi}$ = 3.113(7) GeV, M_{η_c} = 3.046(12) GeV

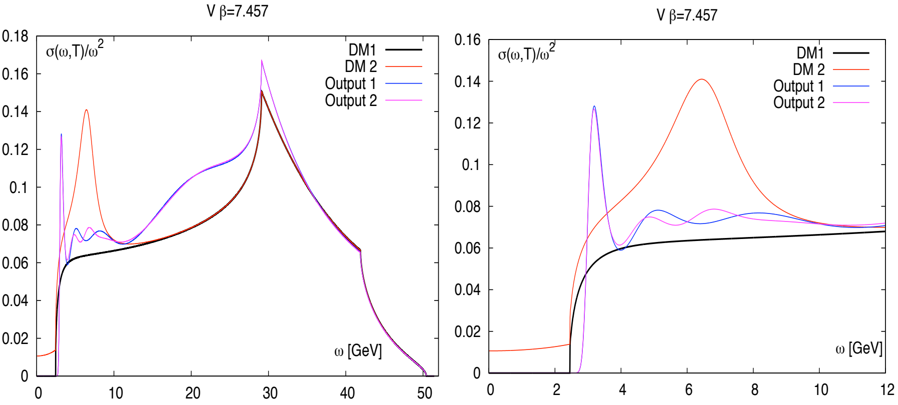
Charmonium spectral functions at 0.75 T_c: Vector channel





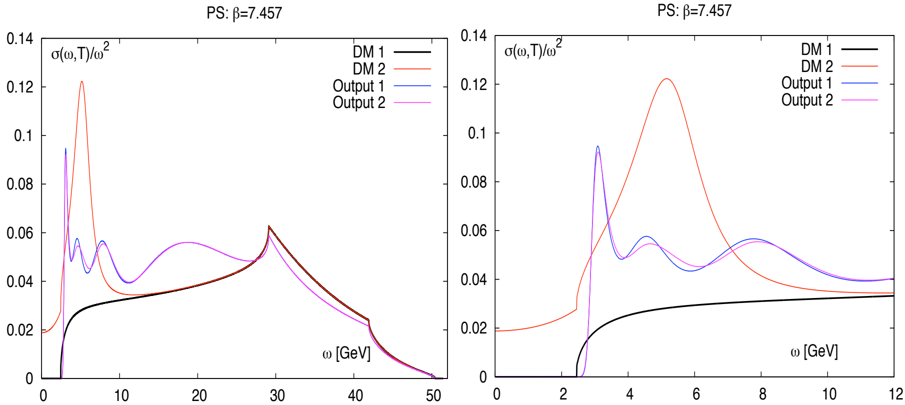
- DM: free lattice spectral function
- N_ω = 8000, $a\,\omega_{min} = 10^{-6}$
- A pronounced ground peak at mass of J/ψ is observed
- Small ω part is well separated from large ω part

Charmonium spectral functions at 0.75 T_c: Vector channel



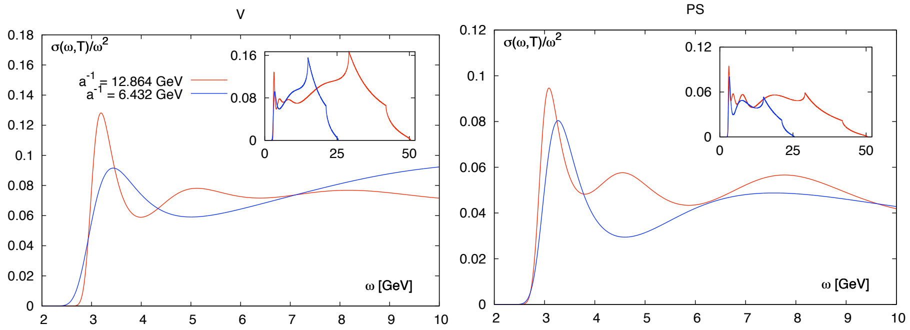
- DM dependence is small, which appears only when $\omega \gtrsim 4$ GeV
- Ground state remains robust
- Below T_c no zero mode contribution is found

Charmonium spectral functions at 0.75 T_c: PS channel



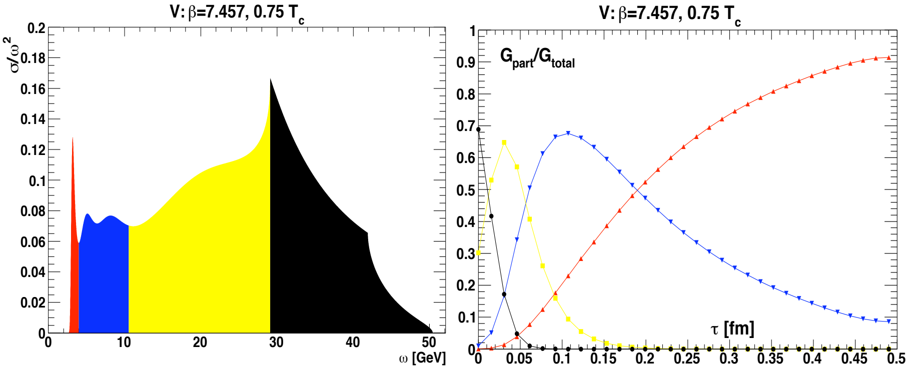
- Similar situation as that of the vector channel

Comparison on different lattice sizes



- The outputs from different lattices are comparable when $\omega \lesssim 4$ GeV
- The one with $\frac{N_\tau}{2} = 32$ is more reliable

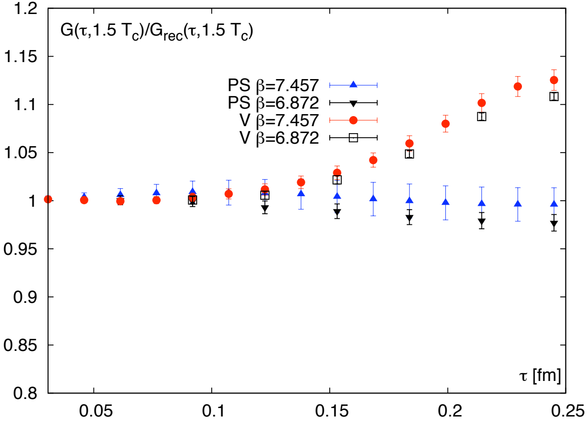
Which part is Which part



- Contribution from small energy region starts to dominate at $\tau \approx 0.2$ fm and overwhelmingly dominates at the largest distance of correlator
- Small distances, which see more lattice cutoff effects, are dominated by the large energy region

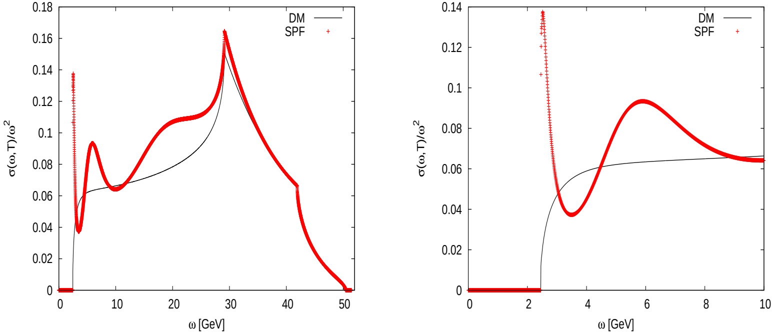
Temperature dependence of charmonia

$$G_{rec}(\tau, T) = \int_0^\infty \mathrm{d}\omega \; \sigma(\omega, T^*) \; \frac{cosh(\omega(\tau - \frac{1}{2T}))}{sinh(\frac{\omega}{2T})}$$



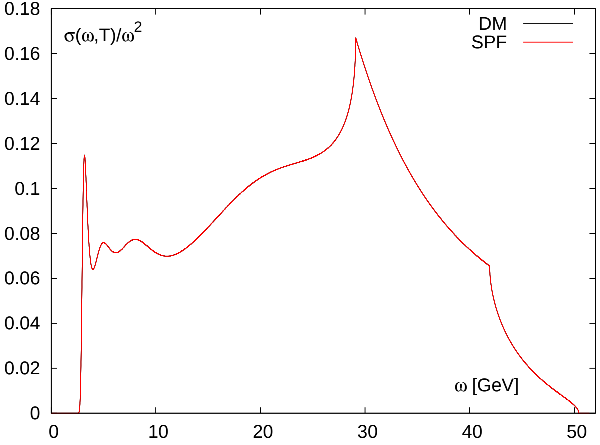
- G/G_{rec} for PS remains unity at all distances within error bars
- G/G_{rec} for Vector deviates from unity up to ≈12% at the largest distance

Charmonium spectral functions at 1.5 T_c: Vector channel



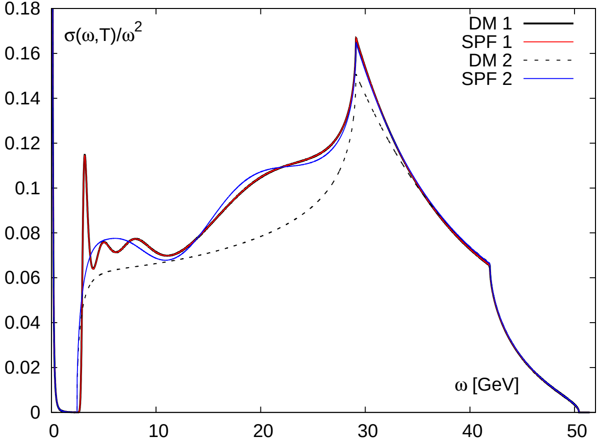
- DM : free lattice spectral function
- some things may appear in the small ω region...

Charmonium spectral functions at 1.5 T_c: Vector channel



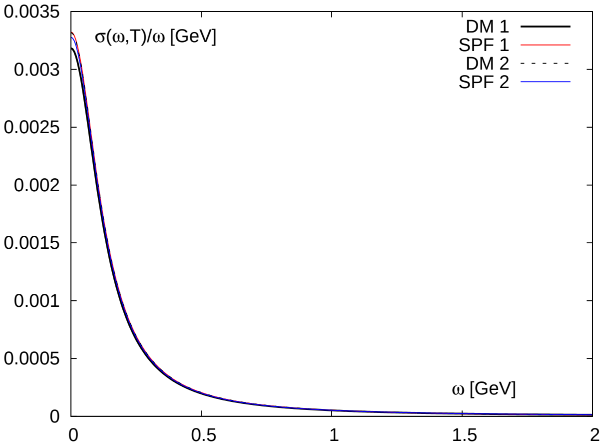
- Analysis on the differences of the neighbouring correlators
- DM : spectral function obtained from MEM at 0.75 T_c.

Charmonium spectral functions at 1.5 T_c: Vector channel



- DM1 : spectral function obtained from MEM at 0.75 T_c plus transport peak around $\omega = 0$
- DM2 : free lattice spf plus transport peak around $\omega = 0$

Charmonium spectral functions at 1.5 T_c: Vector channel



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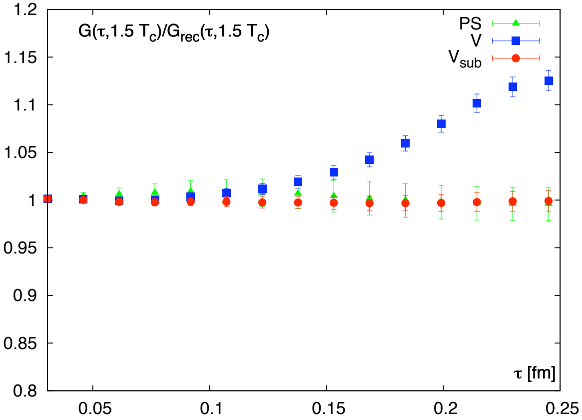

- G/G_{rec} remains unity at all distances after zero mode contribution obatined from MEM is removed, comparable to the results from Mocsy and Petreczky

Summary

Based on the current data we've investigated:

- At 0.75 T_c, the ground state peaks of PS and VC channels obtained from MEM are reliable and robust
- MEM analysis on the differences of neighbouring correlators is a good way to recheck the MEM output with the zero mode contribution suppressed
- Temperature dependece of VC seen from G/G_{rec} plot is mainly from the zero mode contribution
- Reasonable default model should be used in MEM analysis
- More data points are required to do further reliable research

Many thanks to O. Kaczmarek, H. Satz and W. Soeldner !