

Quarkonium in Lattice QCD

What can we learn from lattice correlators?

Olaf Kaczmarek



ECT* Workshop on
Heavy Quarkonium Production in Heavy-Ion Collisions
Trento, May 25-29, 2009

Heavy Quark Free Energies ($m_q=\infty$)

Zero Temperature Potential (as reference)

T-dependent Free energies and Screening

Entropy vs. Internal Energy contributions

Hadronic correlators for light quarks ($m_q=0$)

temporal correlators vs. free correlators

screening masses in the thermodynamic and continuum limit

Charmonium hadronic correlators ($m_q=m_c$)

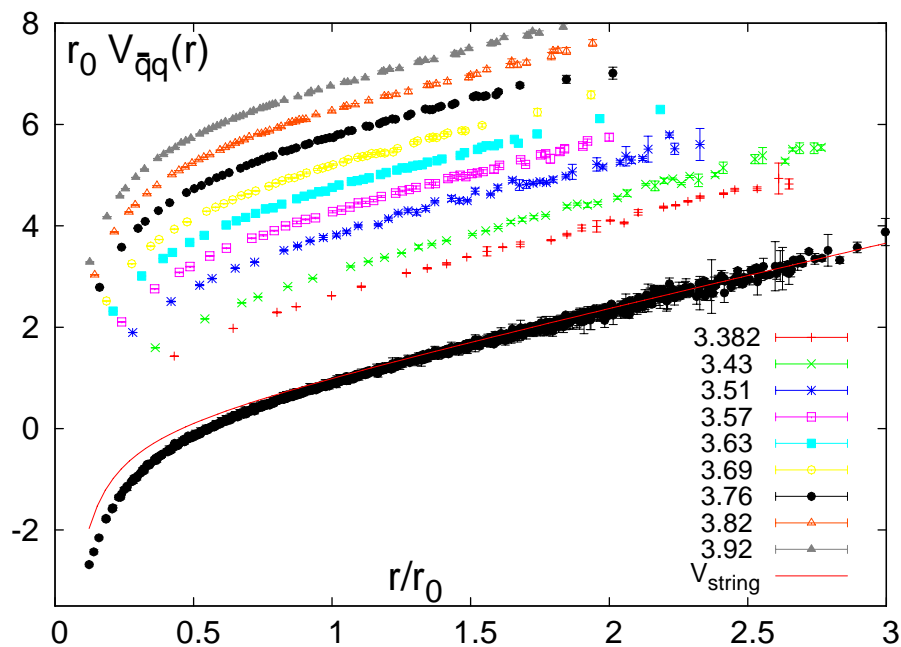
temporal correlators vs. free correlators

temporal correlators vs. reconstructed correlators (from $0.75 T_c$)

charmonium screening masses

zero mode contributions

Zero Temperature Potential and Renormalization



Large distance behaviour:

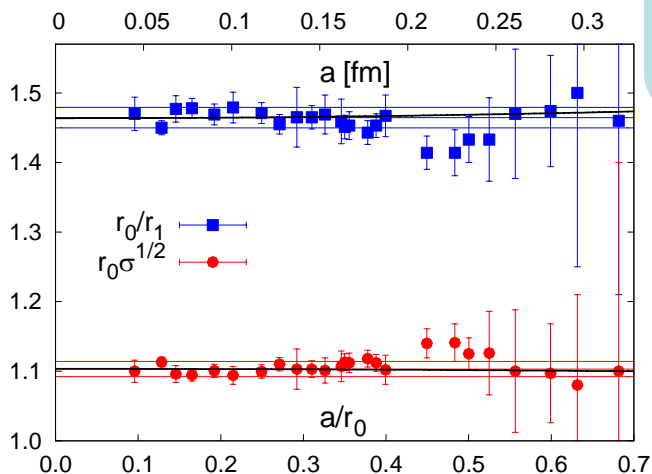
consistent with string model prediction

$$V(r) = -\frac{\pi}{12} \frac{1}{r} + \sigma r$$

used for renormalization

$$V(r) = -\log \left((Z_{\text{ren}}(\beta))^2 \frac{W(r, \tau)}{W(r, \tau + 1)} \right)$$

cut-off effects are small

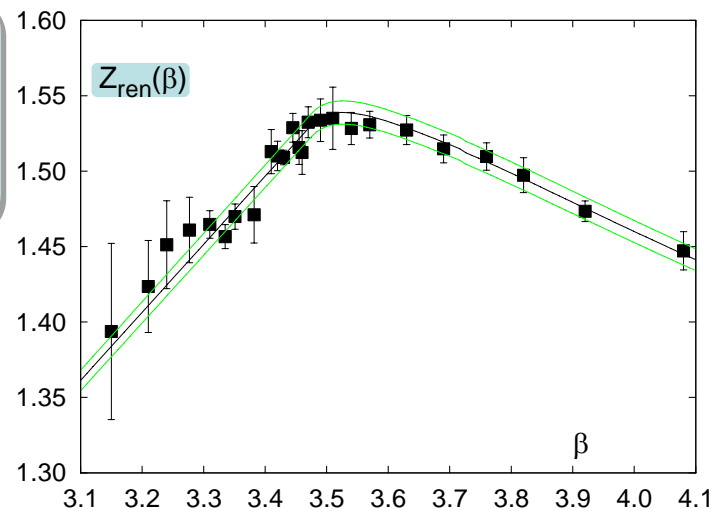


2+1-flavour QCD
highly improved
p4-staggered action
almost realistic quark masses
physical $m_s, m_\pi \approx 220$ MeV

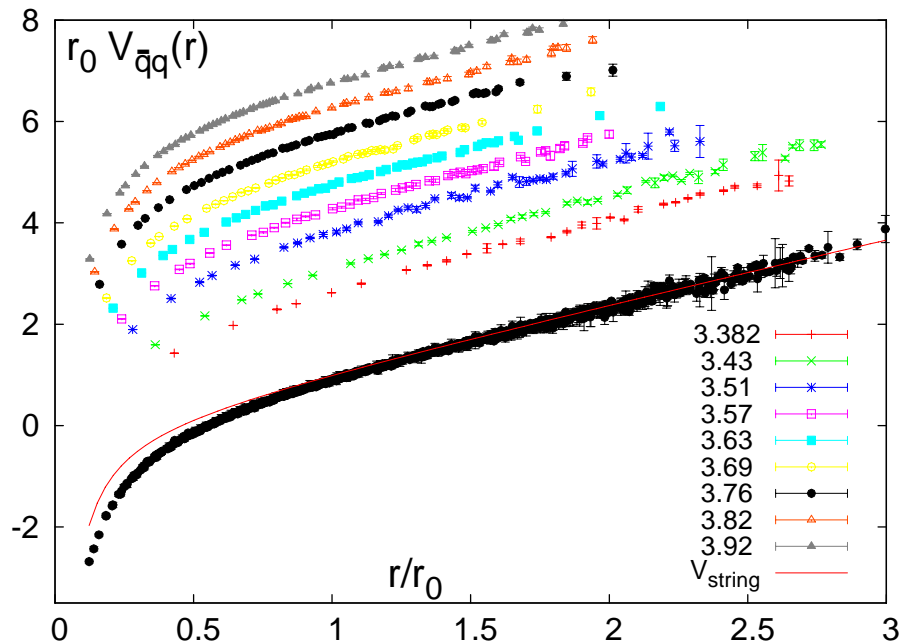
$$r^2 \frac{dV_{\bar{q}q}(r)}{dr} \Big|_{r=r_0} = 1.65$$

$$r^2 \frac{dV_{\bar{q}q}(r)}{dr} \Big|_{r=r_1} = 1.0$$

$$(r_0 = 0.469(7) \text{ fm})$$



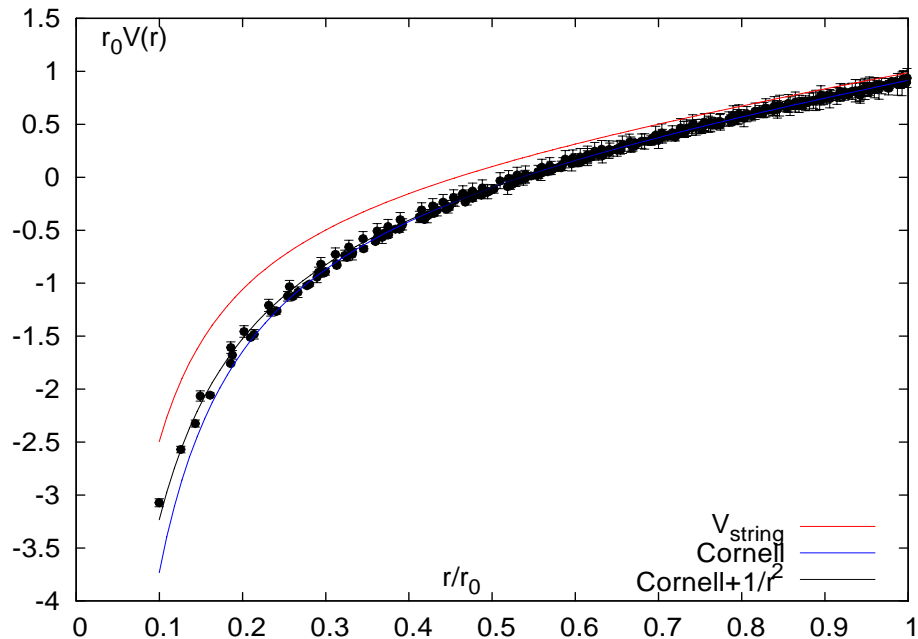
Zero Temperature Potential and Renormalization



Large distance behaviour:

consistent with string model prediction

$$V(r) = -\frac{\pi}{12} \frac{1}{r} + \sigma r$$



Short distance behaviour:

deviations from string model

enhancement of the running coupling

$$V(r) = -\frac{0.392(6)}{r} + \sigma r$$

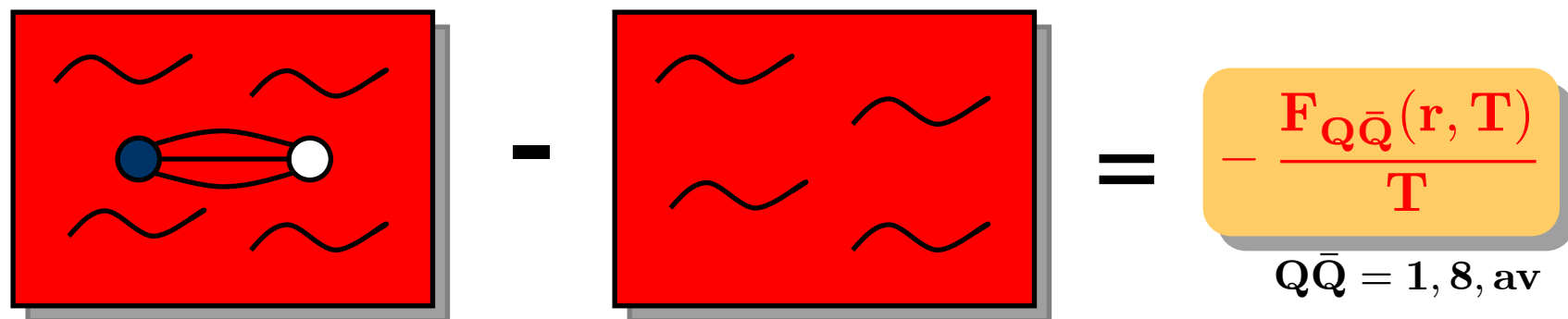
r-dependent running coupling $\alpha(r)$

Heavy Quark Free Energies

Heavy Quark Free Energies defined by Polyakov Loop Correlators

L. McLerran, B. Svetitsky (1981)

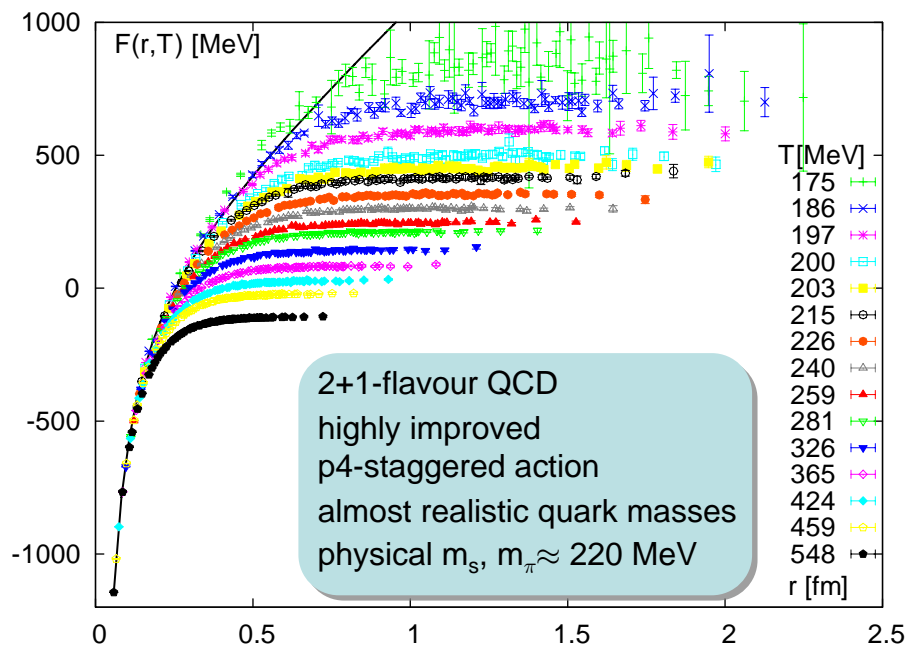
$$\frac{\mathcal{Z}_{Q\bar{Q}}}{\mathcal{Z}(\mathbf{T})} \simeq \frac{1}{\mathcal{Z}(\mathbf{T})} \int \mathcal{D}\mathbf{A} \dots \mathbf{L}(\mathbf{x}) \mathbf{L}^\dagger(\mathbf{y}) \exp \left(- \int_0^{1/\mathbf{T}} dt \int d^3\mathbf{x} \mathcal{L}[\mathbf{A}, \dots] \right)$$



measures the change in free energy induced by the $Q\bar{Q}$ pair

What can we learn about medium modifications of Quarkonium?

Heavy Quark Free Energies – Medium Modifications



Renormalization of $F(r, T)$

$$e^{-F_1(r, T)/T} = (Z_r(g^2))^{2N_\tau} \langle \text{Tr}(L_x L_y^\dagger) \rangle$$

alternative renormalization procedures
all equivalent!

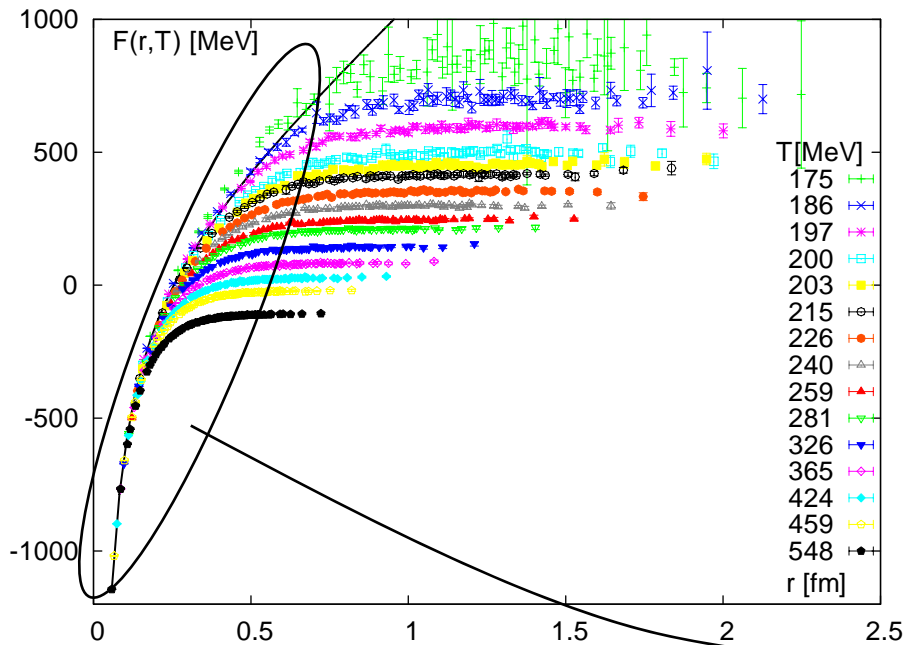
(O. Kaczmarek et al., *PLB*543(2002)41,
S. Gupta et al., *Phys.Rev.D*77(2008)034503)

no temperature effect at small distances

zero temperature (perturbative) at sufficiently small r : $F_1(r) \simeq -\alpha(r)/r$

medium modifications set in at smaller separations with increasing T

Heavy Quark Free Energies – Temperature dependent running coupling



small distance behaviour: $\alpha_{eff}(r,T)$

zero-T (perturbative) behaviour at small r

non-perturbative large values for $r > 0.4$ fm

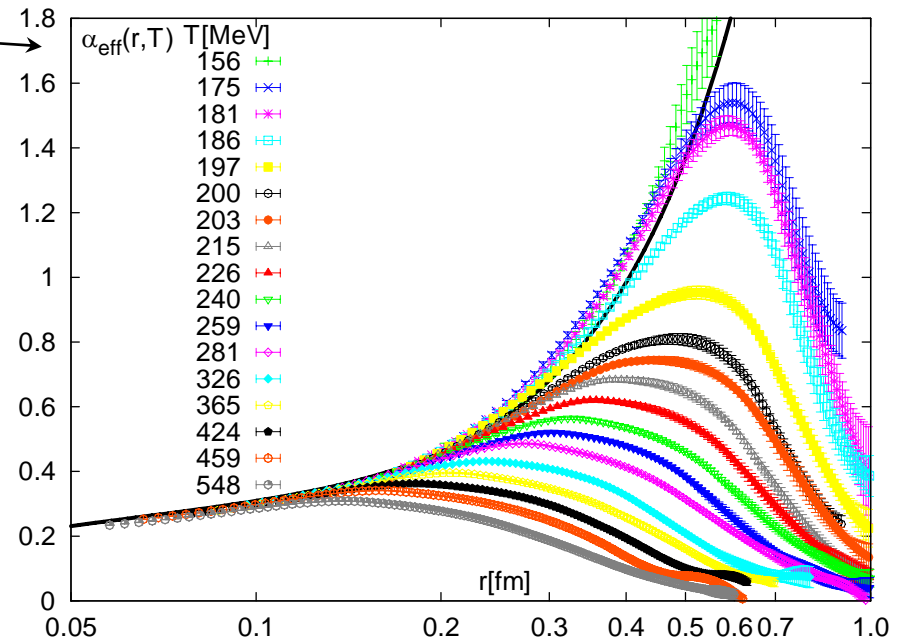
$$\alpha_{eff}(r,T) \simeq \frac{3}{4} r^2 \sigma$$

remnants of confinement forces above T_c

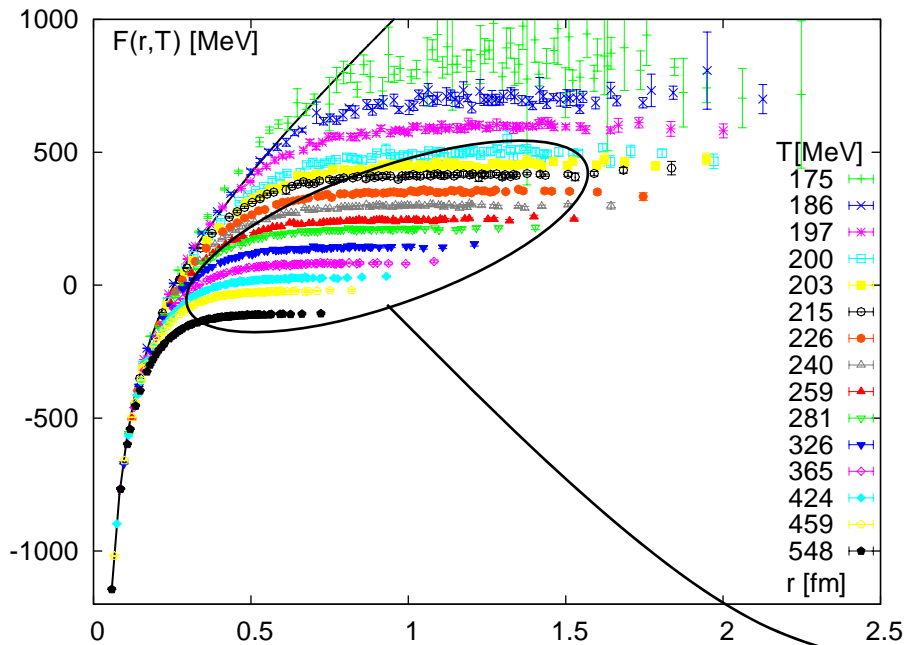
screening sets in at smaller r with incr. T

T and r dependent running coupling:

$$\alpha_{eff}(r,T) = \frac{3}{4} r^2 \frac{dF_1(r,T)}{dr}$$



Heavy Quark Free Energies – Debye Screening



Intermediate distances: screening

screening masses obtained from fits

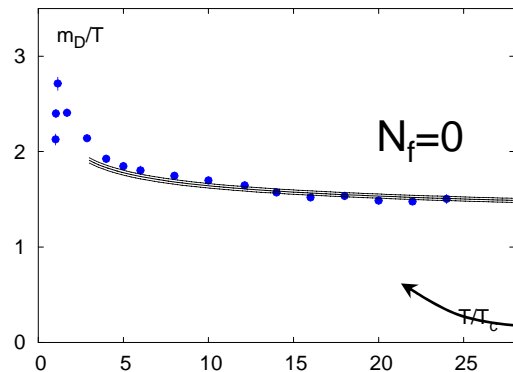
$$F_1(r, T) - F_1(r = \infty, T) = -\frac{4}{3} \frac{\alpha}{r} e^{-m_D(T)r}$$

at large distances $rT \geq 1$

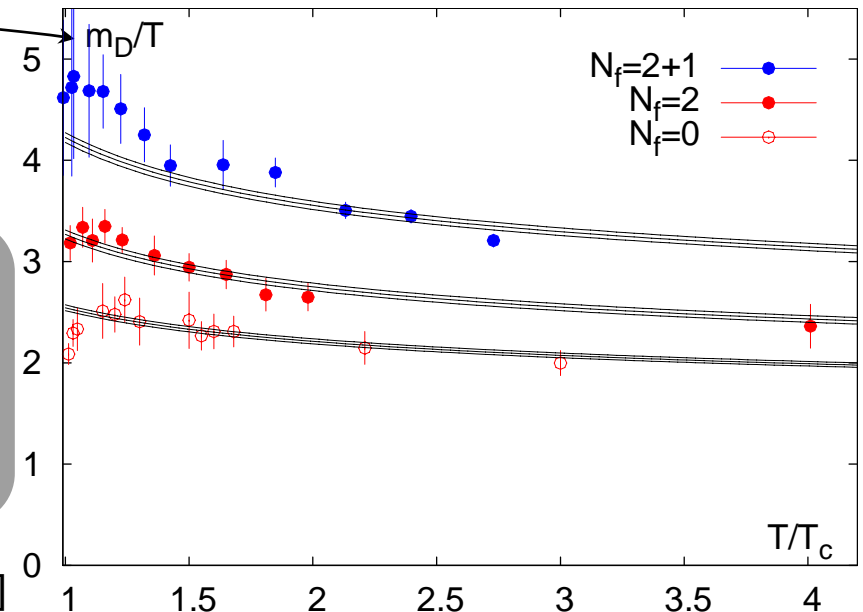
leading order perturbative result:

$$\frac{m_D(T)}{T} = A \left(1 + \frac{N_f}{6}\right)^{1/2} g(T)$$

**perturbative limit reached very slowly
(logarithms at work)**

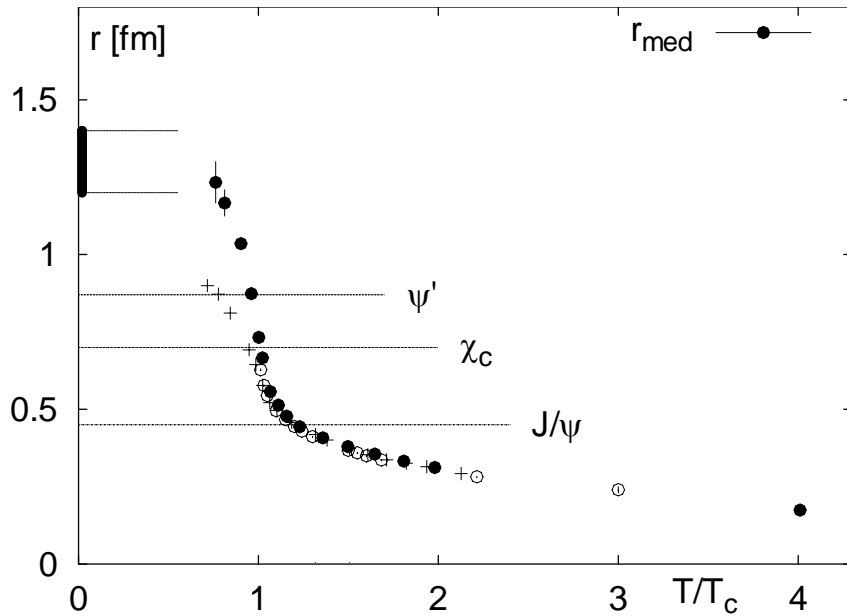


$$\begin{aligned} A_{N_f=2+1} &= 1.66(2) \\ A_{N_f=2} &= 1.42(2) \\ A_{N_f=0} &= 1.52(2) \\ A_{N_f=0} &= 1.39(2) \end{aligned}$$



which $g(T)$, i.e. which scale? $\mu = \mu(m_D)$? [A.Peshier 2006]

Heavy Quark Free Energies – Debye Screening



define screening radius by

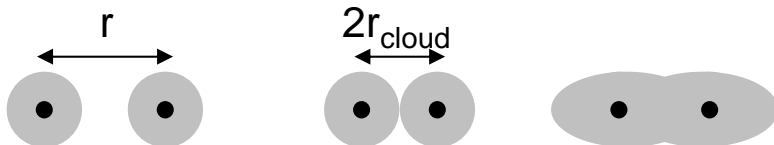
$$V_{T=0}(r_{med}) \equiv F_1(r = \infty, T)$$

equivalently $r_{med} = 2/m_D$

compare to mean charge radii of charmonium states

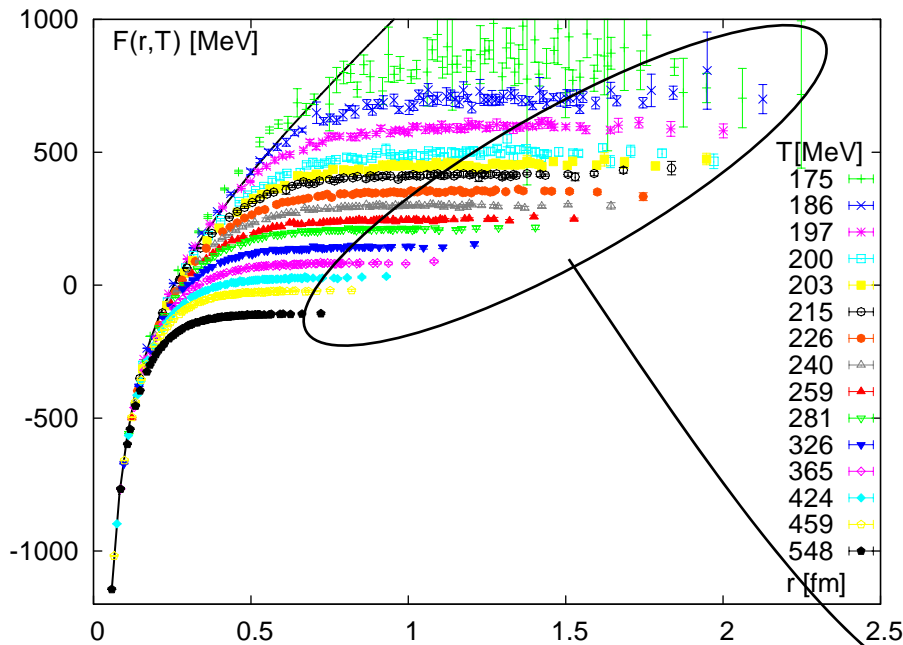
thermal modifications on ψ' and χ_c already close to T_c

J/ψ may survive above deconfinement



The large distance behaviour of the finite temperature energies is rather related to screening than to the temperature dependence of masses of corresponding heavy light mesons!

Heavy Quark Free Energies – Asymptotic behaviour



Asymptotic behaviour:

almost zero-T value close to T_c

strong decrease at critical temperature

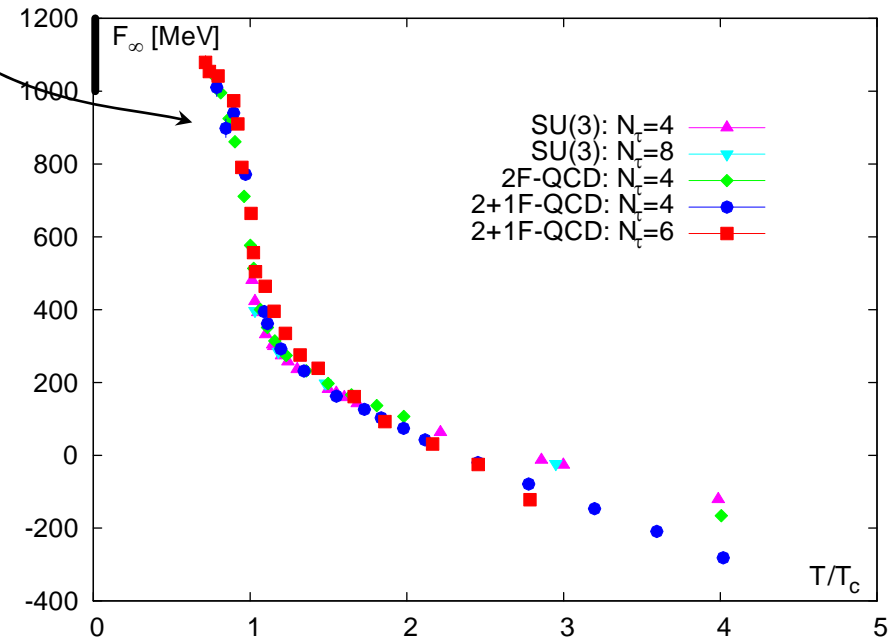
almost linear at high T: $F_1(\infty, T) \sim -T$

$F_1(r, T)$ not only determined by potential energy

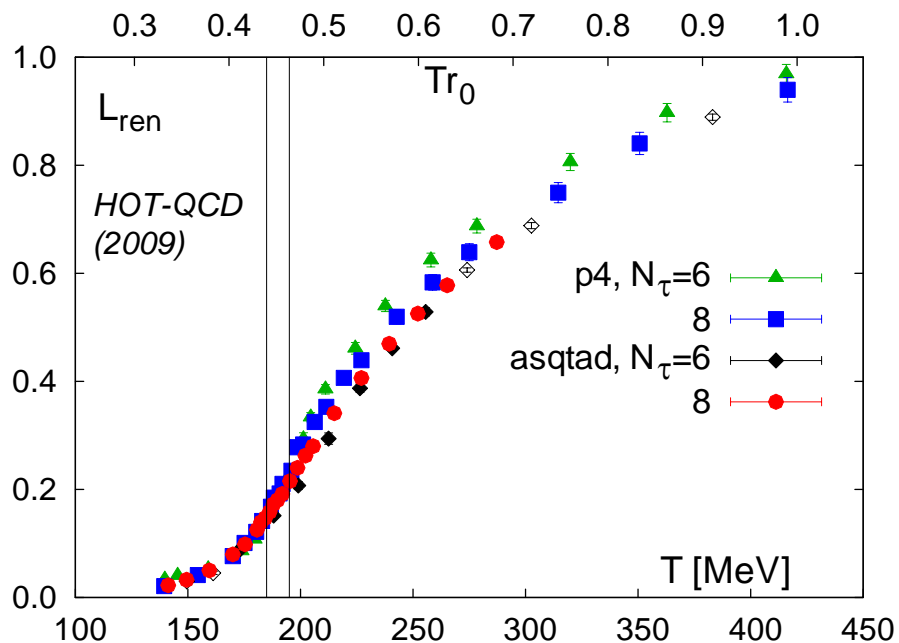
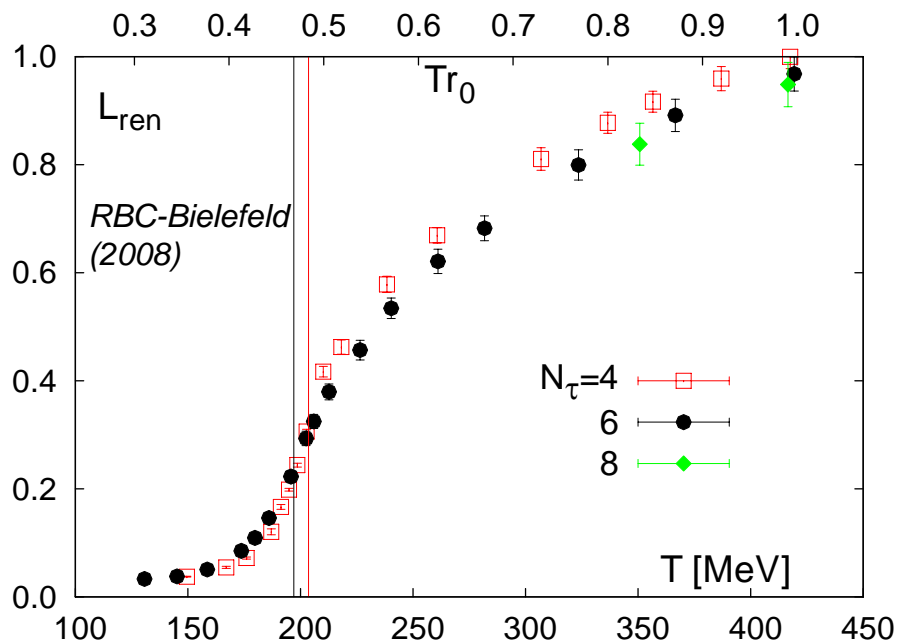
$$F_1(r, T) = U_1(r, T) - TS_1(r, T)$$

Entropy contributions become important

at finite temperature!



Renormalized Polyakov Loop



Renormalization of free energies

$$e^{-\frac{F_1(r,T)}{T}} = \left(Z_R(g^2) \right)^{2N_\tau} \langle \text{Tr}(L_x L_y^\dagger) \rangle$$

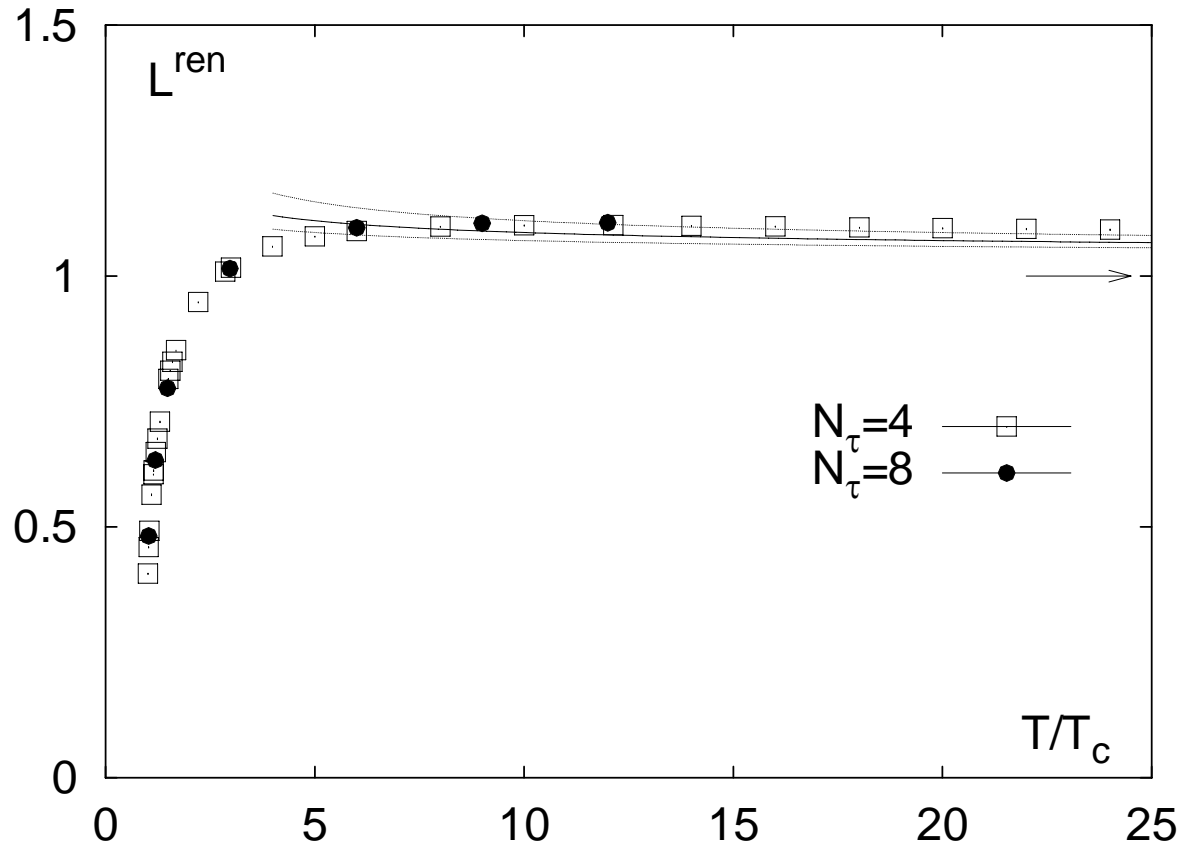
equivalent to renormalization
of Polyakov Loop

$$L_{\text{ren}} = \left(Z_R(g^2) \right)^{N_\tau} L_{\text{lattice}}$$

equivalent to definition by
large distance behaviour of $F_1(r,T)$

$$L_{\text{ren}} = \exp \left(-\frac{F_1(r = \infty, T)}{2T} \right)$$

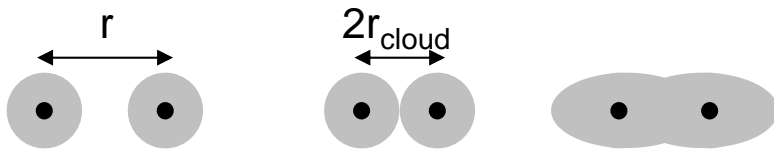
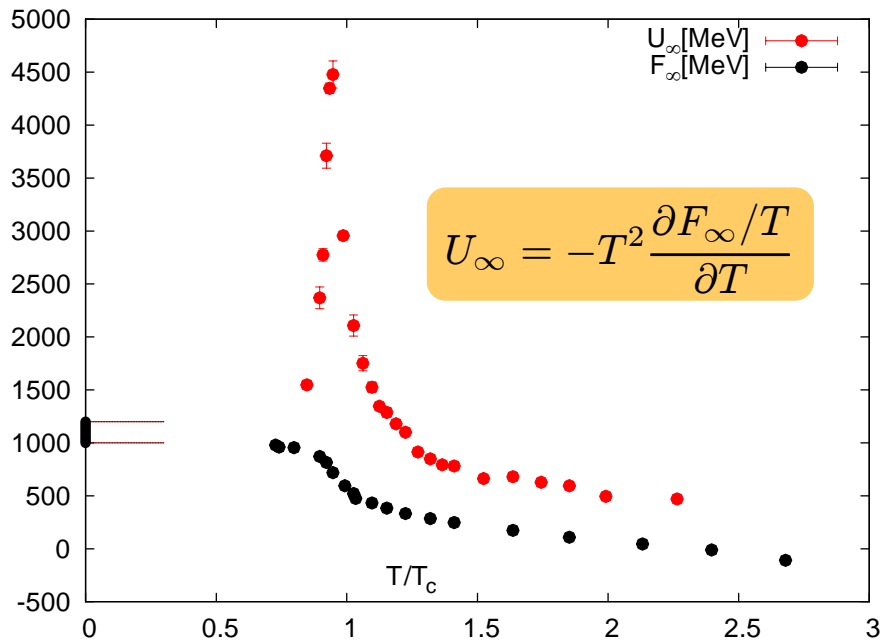
SU(3) pure gauge result



high temperature limit, $L_{\text{ren}}=1$, reached from above as expected in PT

Non-perturbative effects dominate below $5 T_c$

Internal Energy vs. Entropy contributions – asymptotic behaviour



The large distance behaviour of the finite temperature energies is rather related to screening than to the temperature dependence of masses of corresponding heavy light mesons!

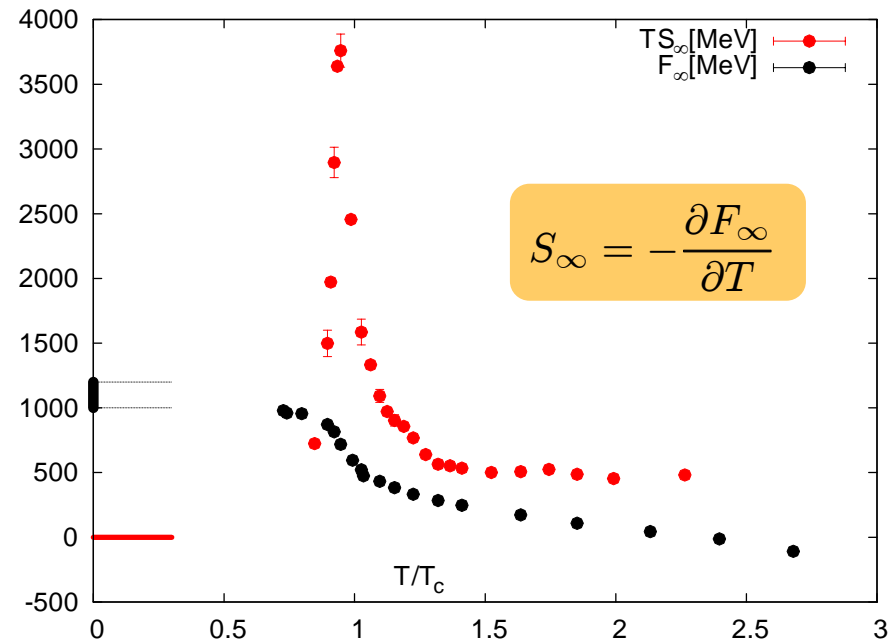
High Temperature behaviour:

$$F_\infty(\mathbf{T}) \simeq -\frac{4}{3} m_D(\mathbf{T}) \alpha(\mathbf{T}) \simeq -\mathcal{O}(g^3 T)$$

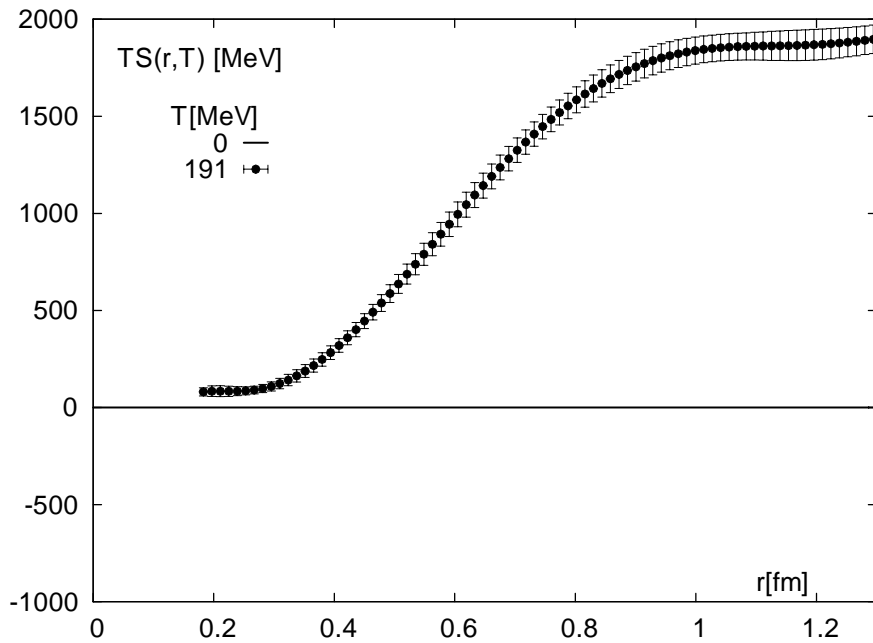
$$TS_\infty(\mathbf{T}) \simeq +\frac{4}{3} m_D(\mathbf{T}) \alpha(\mathbf{T})$$

$$U_\infty(\mathbf{T}) \simeq -4 m_D(\mathbf{T}) \alpha(\mathbf{T}) \frac{\beta(\mathbf{g})}{g}$$

$$\simeq -\mathcal{O}(g^5 T)$$



r-dependent internal energy contributions

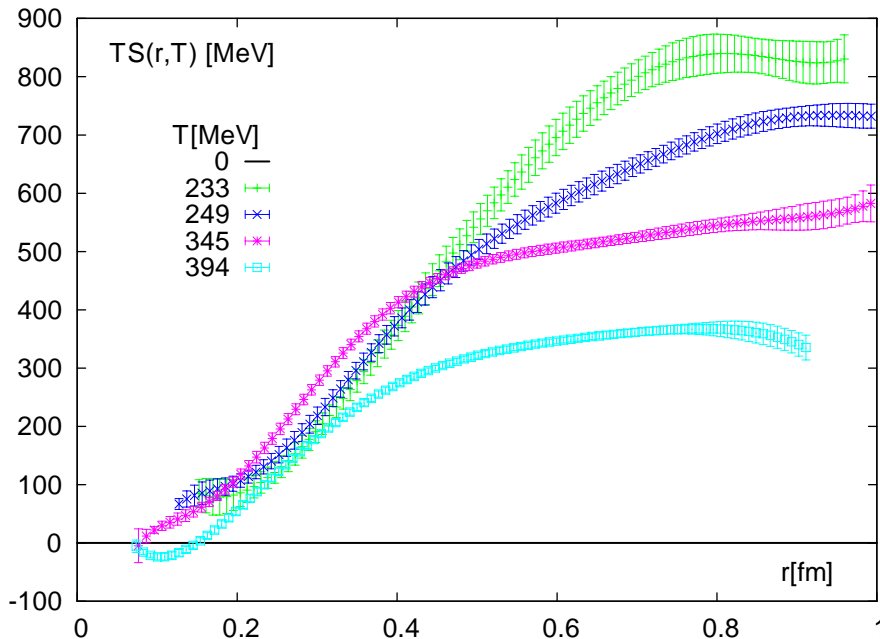


$$F_1(r, T) = U_1(r, T) - TS_1(r, T)$$

$$S_1(r, T) = \frac{\partial F_1(r, T)}{\partial T}$$

$$U_1(r, T) = -T^2 \frac{\partial F_1(r, T)/T}{\partial T}$$

Entropy contributions vanish in the limit $r \rightarrow 0$

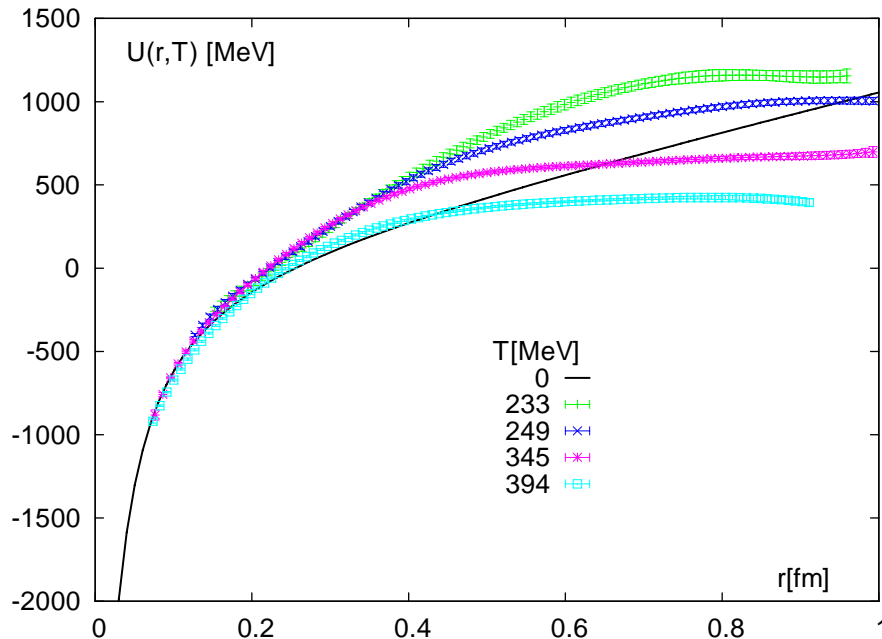
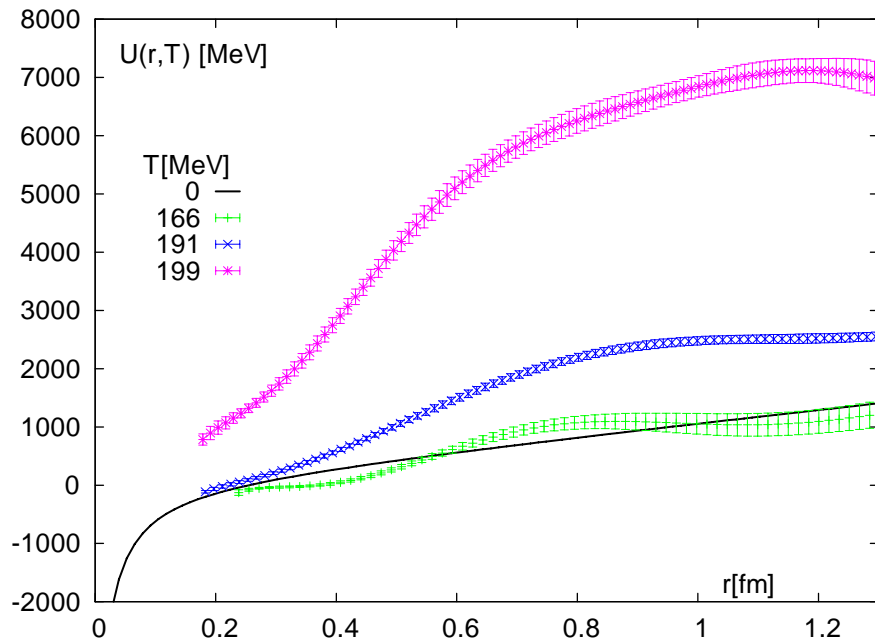


$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

But they play an important role at intermediate r

especially close to T_c

r-dependent Entropy contributions



$$F_1(r, T) = U_1(r, T) - T S_1(r, T)$$

$$S_1(r, T) = \frac{\partial F_1(r, T)}{\partial T}$$

$$U_1(r, T) = -T^2 \frac{\partial F_1(r, T)/T}{\partial T}$$

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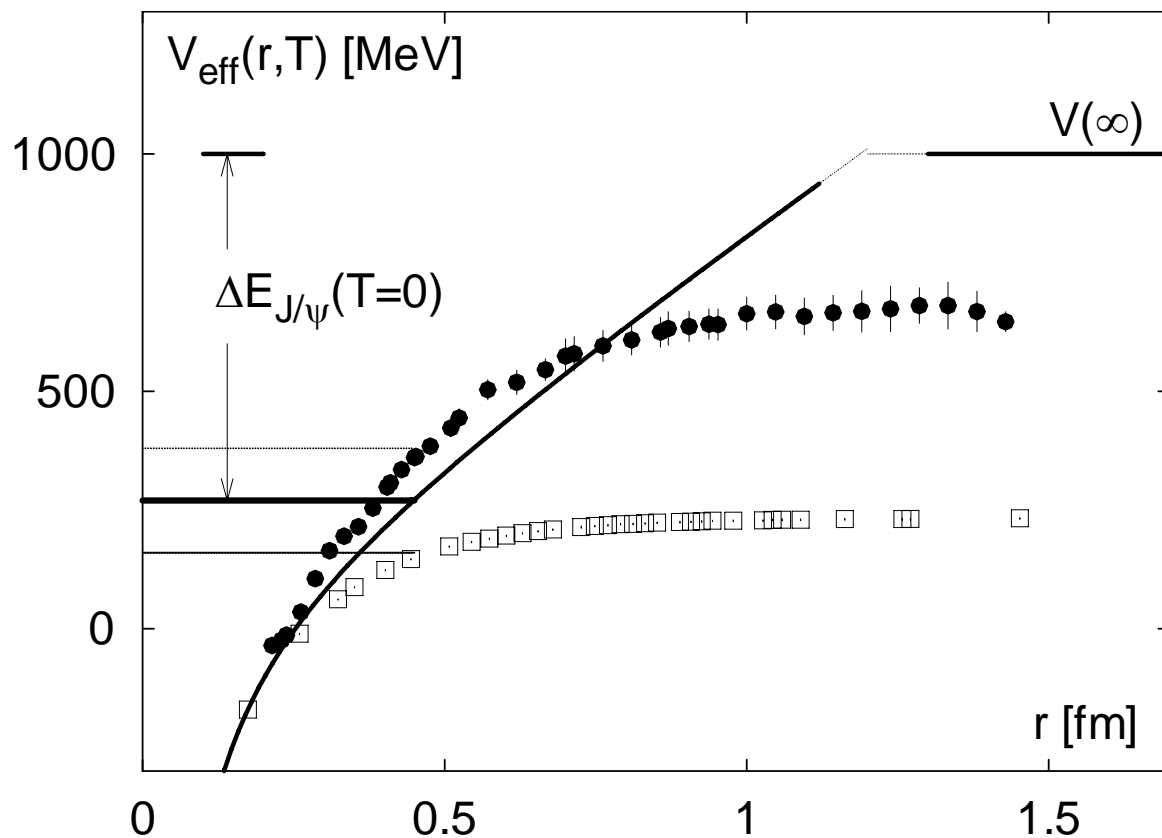
$$F_1(r \ll 1, T) = U_1(r \ll 1, T) \equiv V_1(r)$$

But they play an important role at intermediate r
especially close to T_c

Implications on Heavy Quark bound states ?

What is the correct $V_{\text{eff}}(r, T)$?

Heavy Quark Bound States – Potential Models



steeper slope of $V_{\text{eff}}(r, T) = U_1(r, T)$

→ J/ψ stronger bound using $V_{\text{eff}}(r, T) = U_1(r, T)$

→ dissociation at higher temperatures compared to $V_{\text{eff}}(r, T) = F_1(r, T)$

What is the correct effective potential at finite temperature?

$V_{\text{eff}}(r,T) = F_1(r,T)$ or $U_1(r,T)$ or linear combination?

Gauge or operator dependence of color singlet free energy? [Jahn+Philipsen]

Do potential models at finite T make sense at all?

is a two-particle (Schrödinger) Ansatz justified?

What quark mass should be used?

$m_c(\mu)$ depends on the scale (PDG: $m_c(m_c) = 1.27$ GeV)

what is the charm quark mass in the medium?

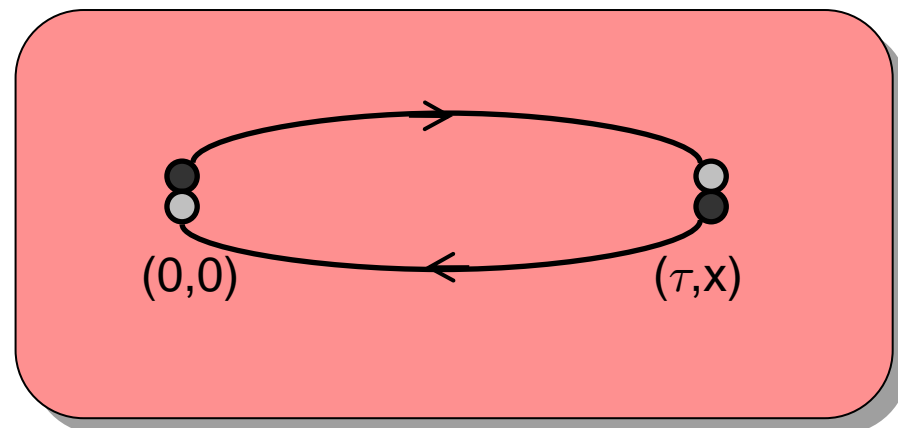
What are the relevant processes and are they included in a potential model?

scattering and Landau damping → real-time potential

Thermal hadronic correlation functions

$$J_H = \bar{q}(\tau, \vec{x}) \Gamma_H q(\tau, \vec{x})$$

$$G_H(\tau, T, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \langle J_H(0, 0) J_H^\dagger(\tau, \vec{x}) \rangle$$



O(a)-improved Clover improved fermionic action

on large quenched lattice configurations up to $128^3 \times 16$ and 32

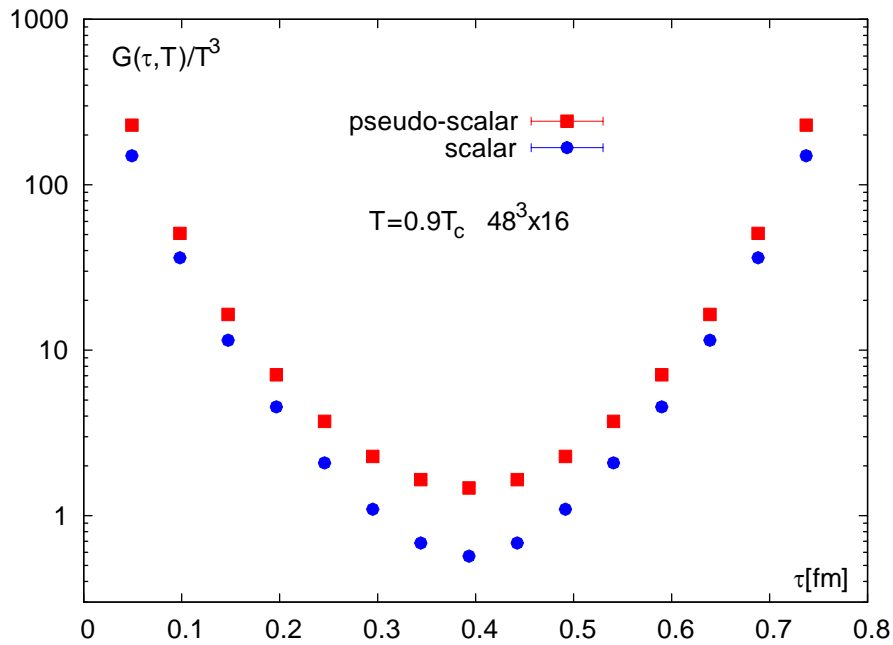
includes all the physics

how to extract it?

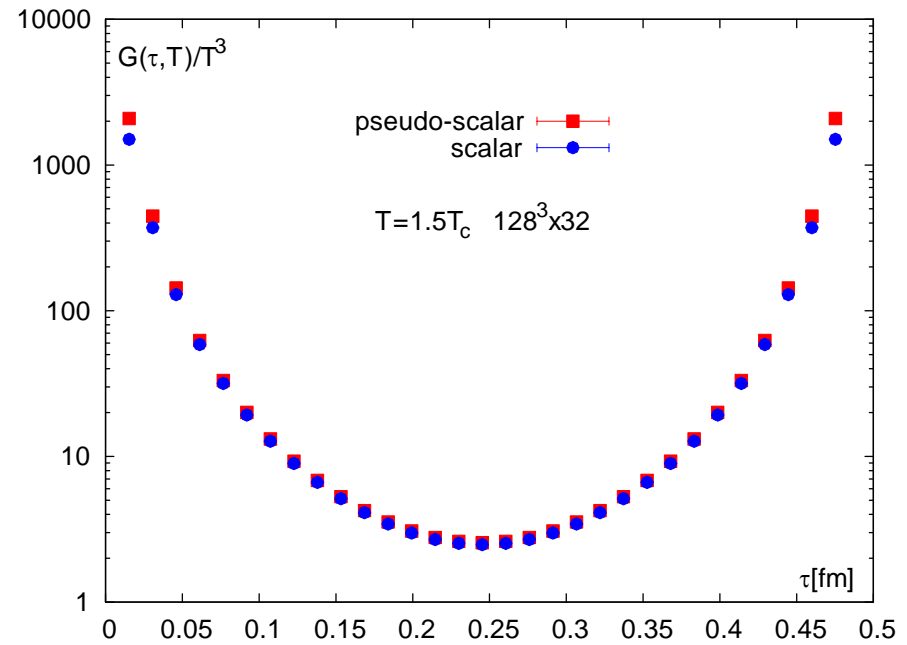
directly from the correlators?

spectral functions using MEM?

Temporal Correlators:

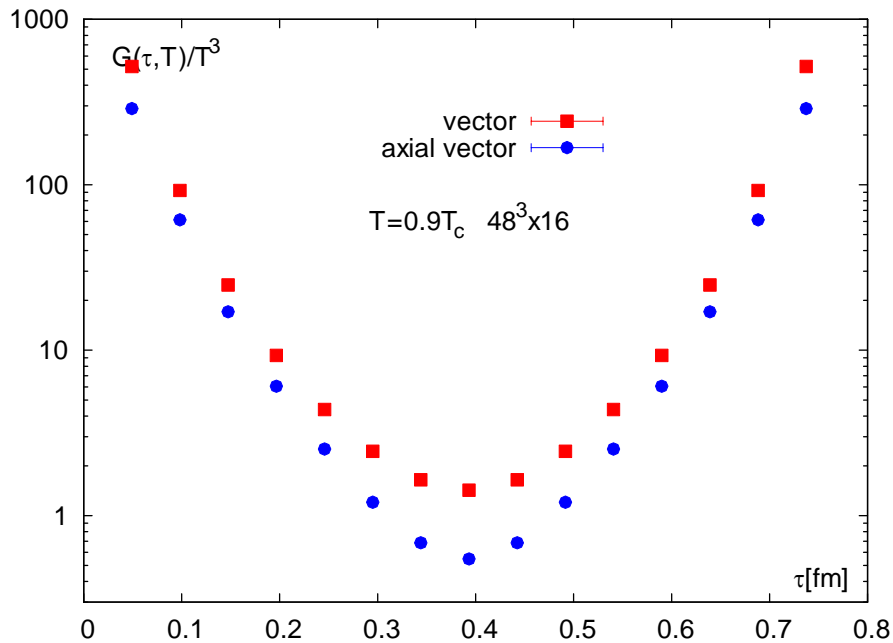


splitting below T_c due to
chiral and axial U(1) symmetry breaking

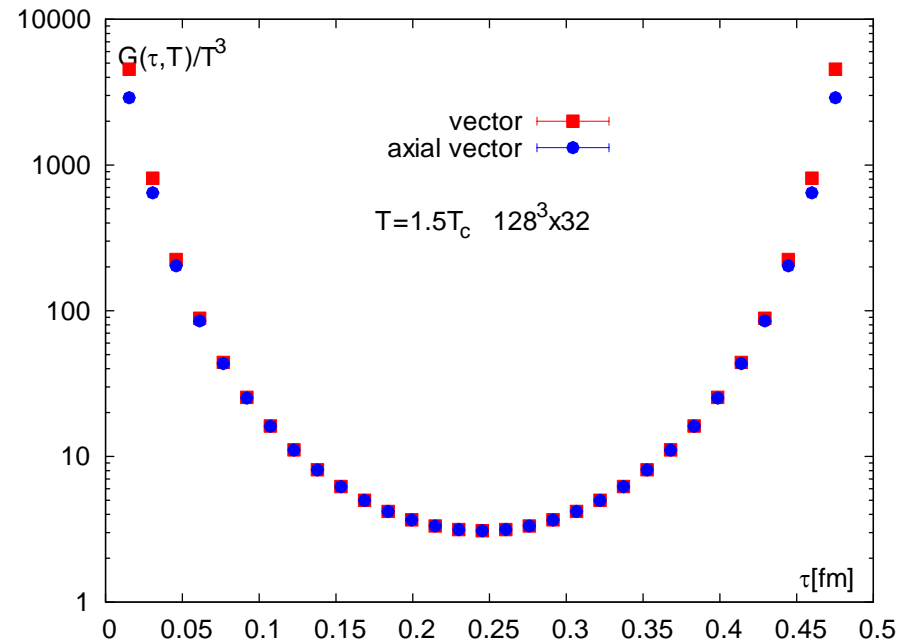


symmetry restoration above T_c

Temporal Correlators:

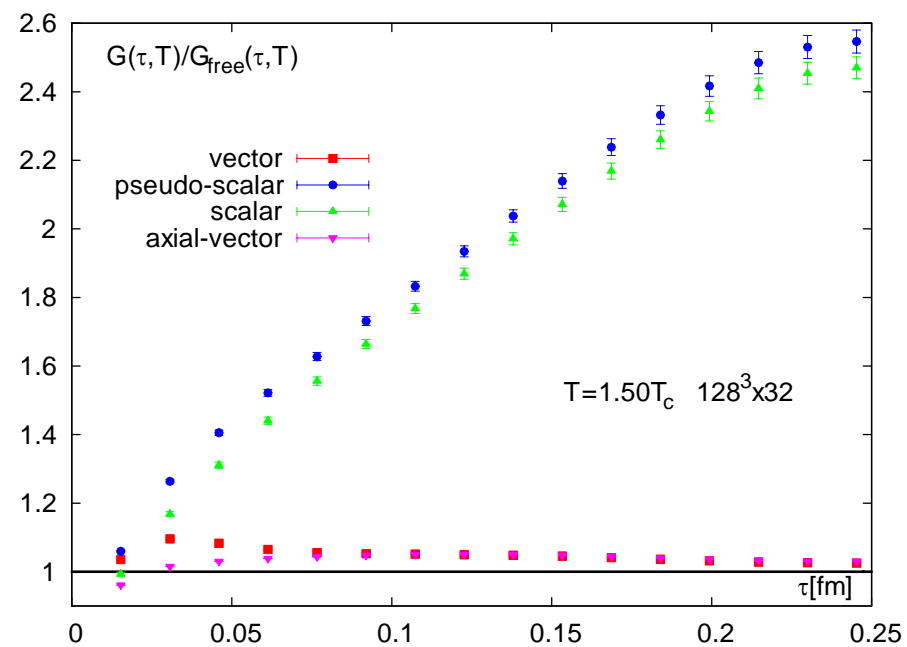
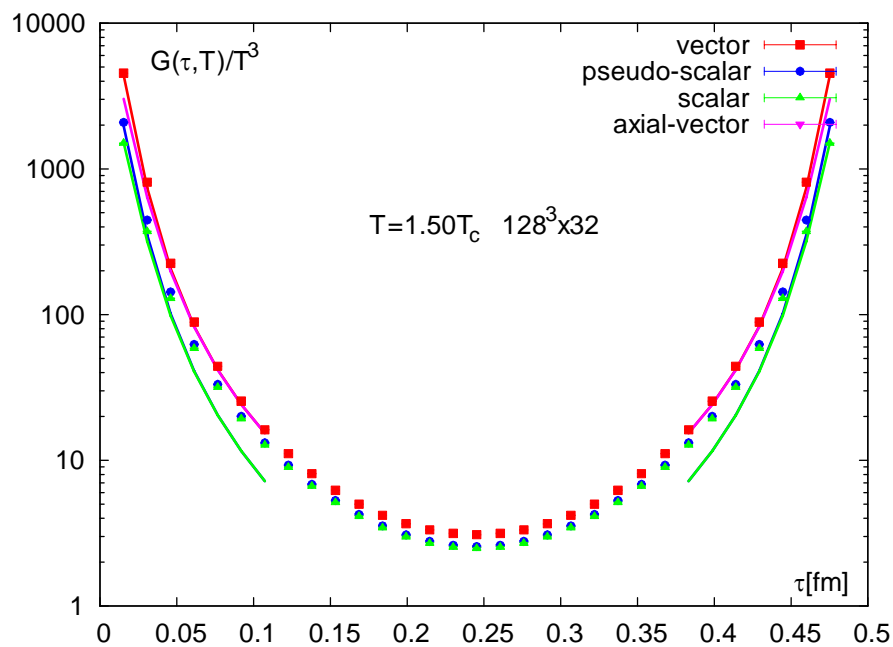


splitting below T_c due to
chiral and axial $U(1)$ symmetry breaking



symmetry restoration above T_c

Light Quark Correlators vs Free Correlators



comparison with free high temperature lattice correlator

vector and axial-vector close to free case

still strong correlations in scalar and pseudo-scalar channel

Light Quark Screening Masses – Definition

use spatial correlators

$$G_H(z, T, \vec{p}_\perp) = \sum_{\tau, \vec{x}_\perp} e^{-i\vec{p}_\perp \cdot \vec{x}_\perp} \langle J_H(0, 0) J_H^\dagger(\tau, \vec{x}) \rangle$$

correlation function depends on the same spectral density,
but the relation is more involved

$$G_H(z) = \int_{-\infty}^{+\infty} \frac{dp_z}{2\pi} e^{ip_z z} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma_H(p_0, \vec{0}_\perp, p_z)}{p_0} \xrightarrow{z \rightarrow \infty} \text{Ampl.} \times \exp(-m_{\text{screen}} z)$$

however, $m_{\text{screen}}(T) \neq m_{\text{pole}}(T)$ in general :

look for zeros of $G^{-1}(p) = p_0^2 + \vec{p}^2 + m_0^2 + \Pi(p_0, \vec{p}, T)$

$$\vec{p} = 0 : \quad -p_0^2 = m_0^2 + \Pi(p_0, \vec{0}, T) = (m_{\text{pole}}(T))^2$$

$$p_0 = 0 : \quad -\vec{p}^2 = m_0^2 + \Pi(0, \vec{p}, T) = (m_{\text{screen}}(T))^2$$

$$\implies m_{\text{screen}}(T) = \frac{m_{\text{pole}}(T)}{A(T)}$$

Light Quark Screening Masses – Thermodynamic Limit, $V \rightarrow \infty$

large collection of lattices ranging from $16^3 \times 8$ to $128^3 \times 16$

allowing for thermodynamic limit $V \rightarrow \infty$ at $N_t=8,12$ and 16

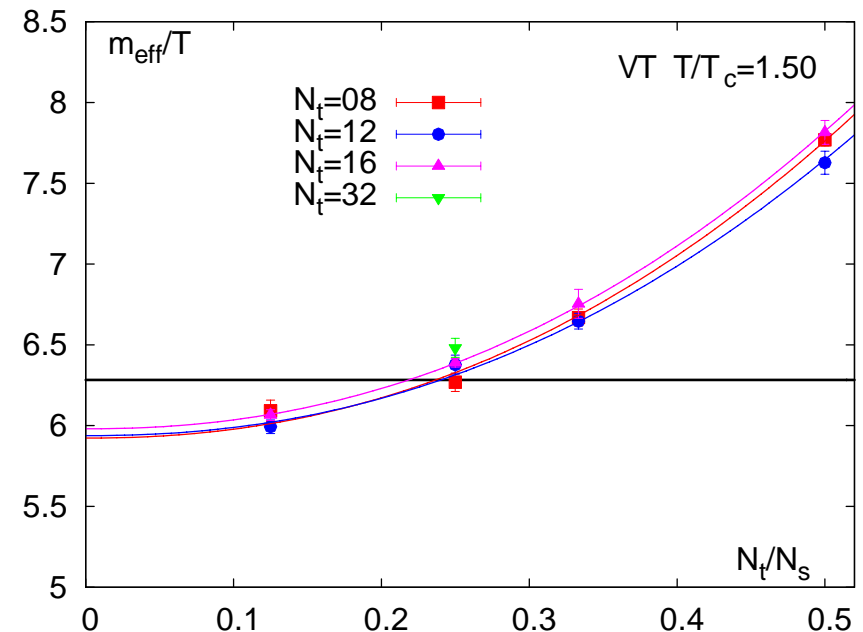
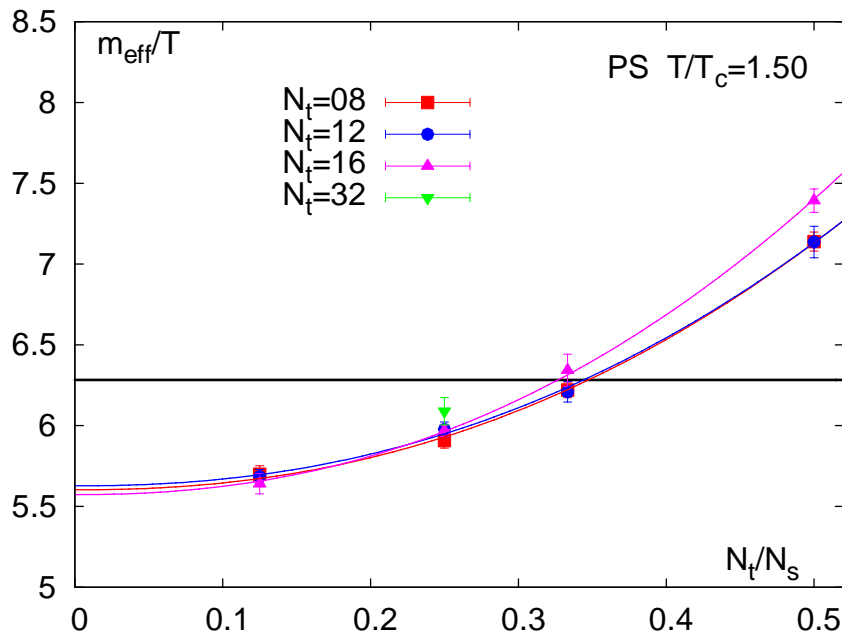
$$T \leq T_c : m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma} \right)^3 \right]$$

$$T > T_c : m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma} \right)^p \right]$$

$$T = \infty : m_{\text{screen}}(L, a) = m_{\text{screen}}(a) \left[1 + \gamma_V \left(\frac{N_\tau}{N_\sigma} \right)^1 \right]$$

combined fit:

p	PS	V
$1.5 T_c$	2.22(10)	2.18(13)
$3.0 T_c$	2.06(7)	2.05(11)



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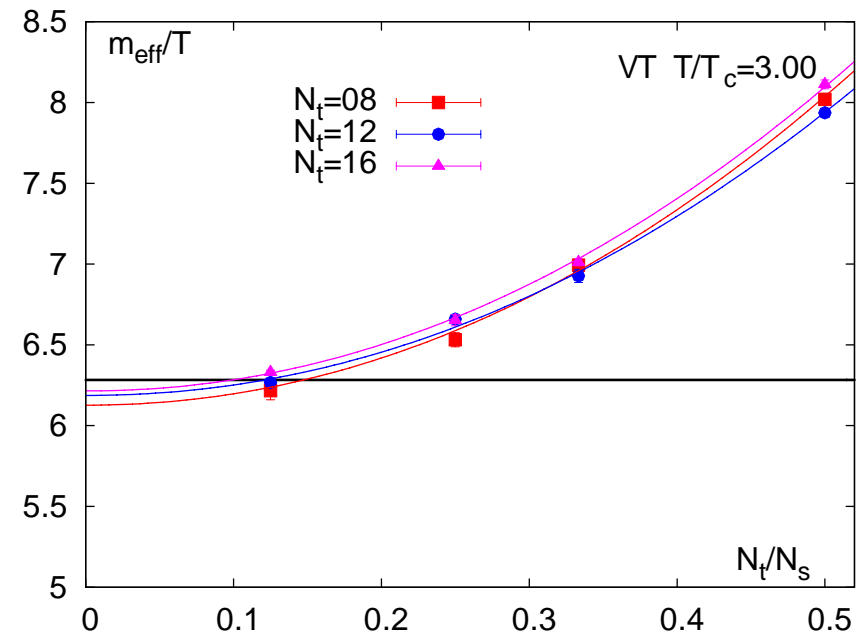
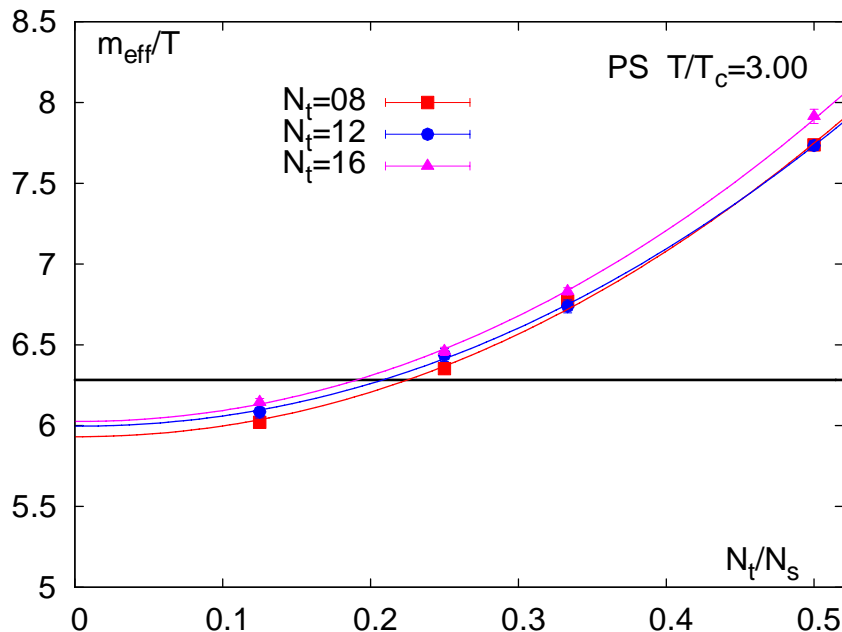
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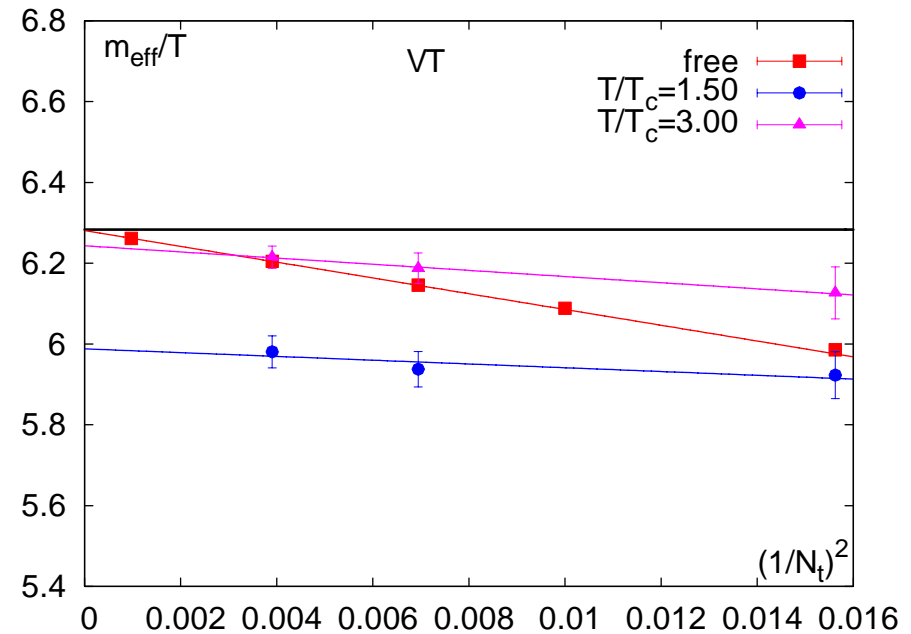
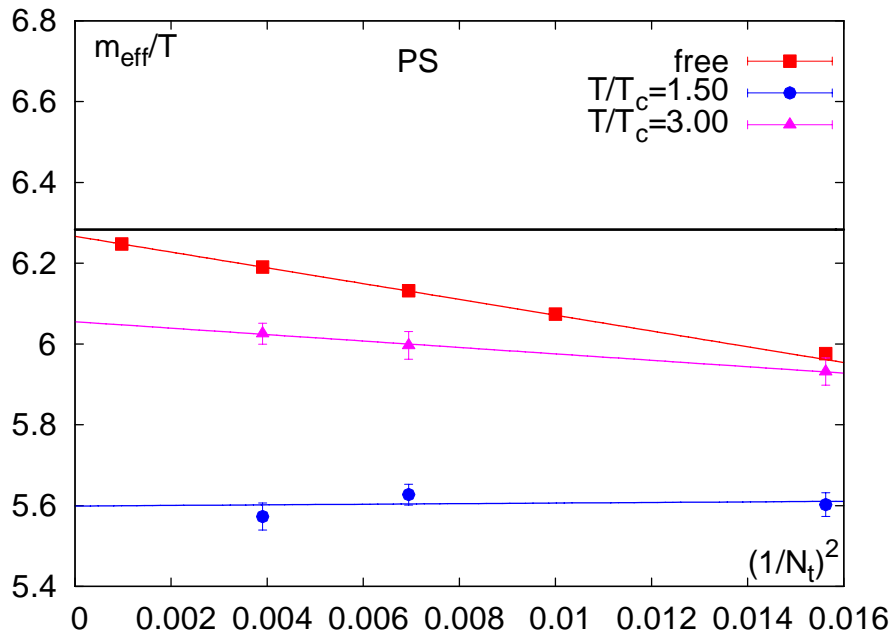
Continuum Limit

lattice spacing $a \rightarrow 0$

Non-perturbatively improved action

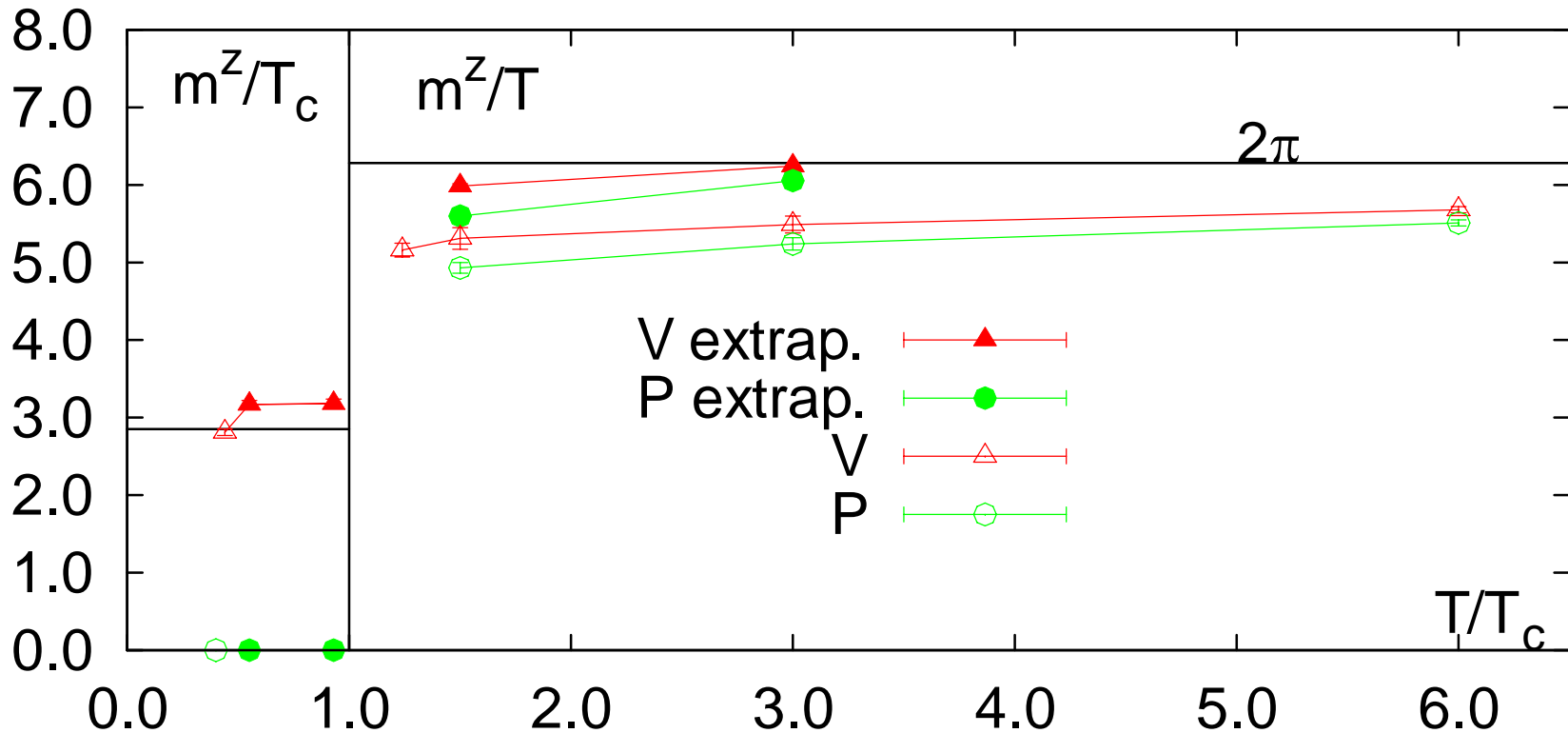
\Rightarrow discretization errors $O(a^2)$

$$T = \infty : \quad \frac{m^z(a)}{T} = \frac{m^z}{T} - \lambda \left(\frac{1}{N_\tau} \right)^2$$



Thermodynamic and Continuum Limit:

$$V \rightarrow \infty \quad \text{and} \quad a \rightarrow 0$$

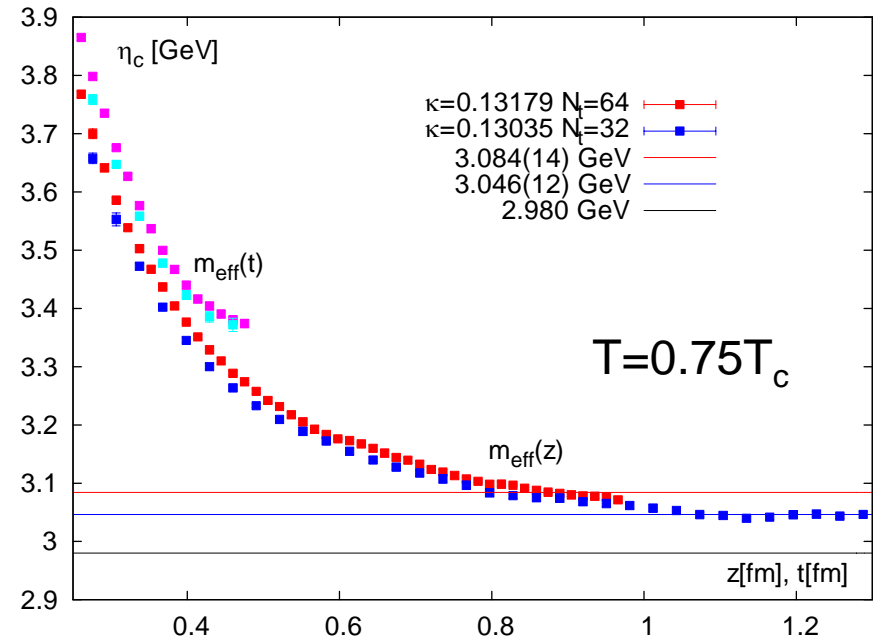
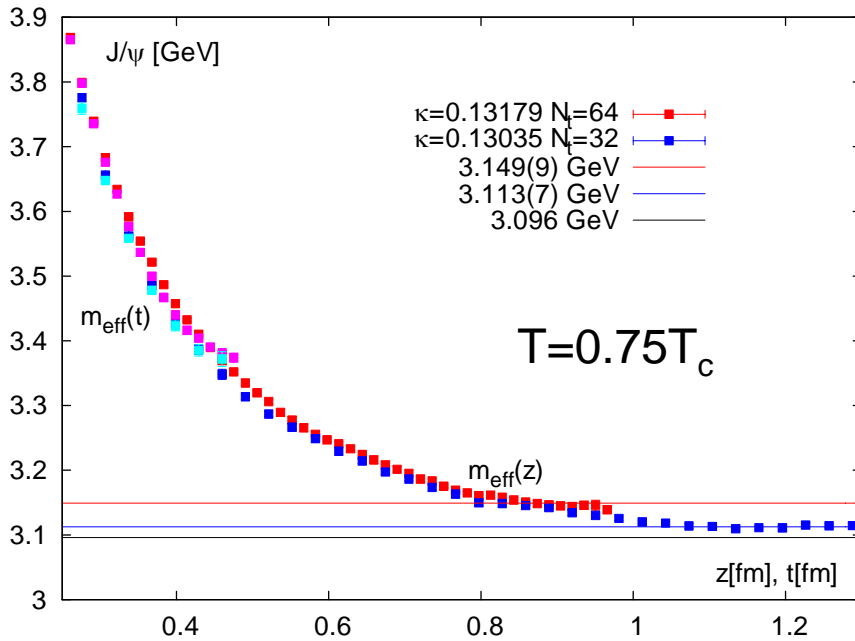


weak temperature dependence below T_c

data still below free limit ($2\pi T$) at $3 T_c$, vector closer to free case

at variance with [Laine, Vepsäläinen] → need higher temperatures

Charmonium Correlators – Quark Mass Tuning



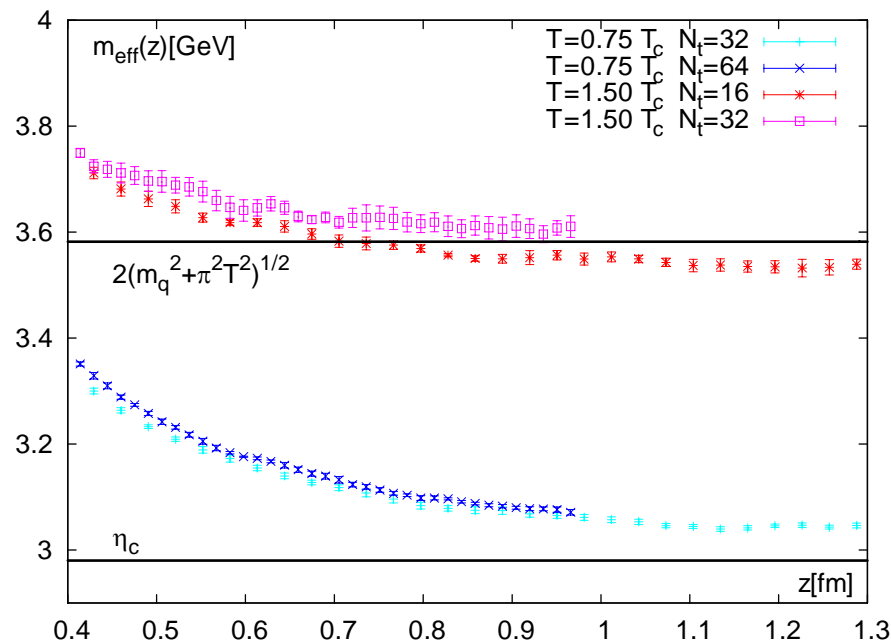
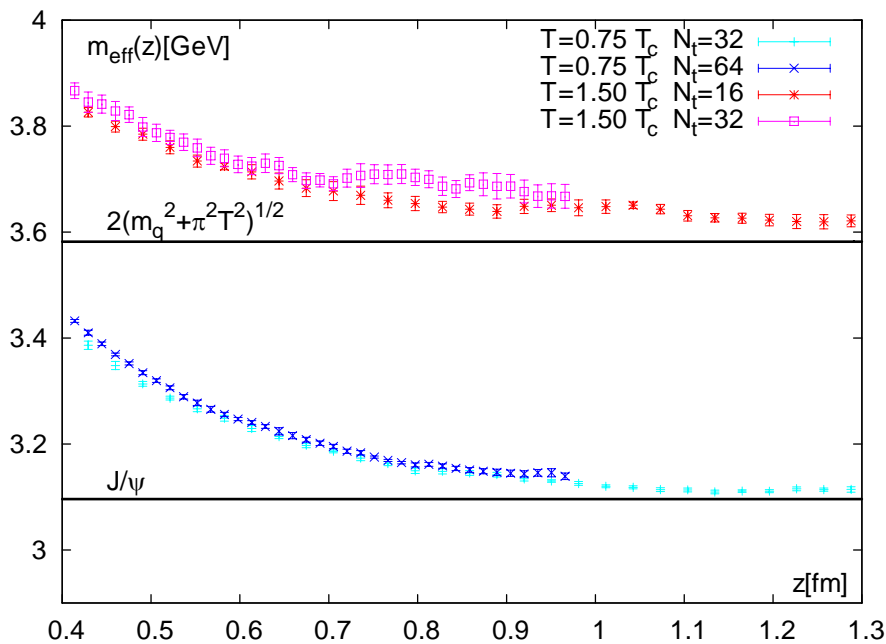
we use the spatial screening mass for quark mass tuning, as $m_{\text{screen}} = m_{\text{pole}}$ at small T

$$\frac{G(z)}{G(z+1)} = \frac{\cosh \left[m_{\text{eff}}(z) \left(\frac{N_\sigma}{2} - z \right) \right]}{\cosh \left[m_{\text{eff}}(z) \left(\frac{N_\sigma}{2} - z - 1 \right) \right]}$$

Lattice parameters and quark masses from the Axial-Ward-Identity and in the $\overline{\text{MS}}$ scheme at the scale of the quark mass:

β	$a[\text{fm}]$	$1/a[\text{GeV}]$	κ	$m_{AWI}(\mu = 1/a)[\text{GeV}]$	$m_{\overline{\text{MS}}}(\mu = m_c)[\text{GeV}]$
6.872	0.031	6.43	0.13035	0.858(2)	1.286(12)
7.457	0.015	12.86	0.13179	0.841(1)	1.282(9)

Charmonium Correlators – Screening Masses



screening masses at $1.50 T_c$ already close to the free case

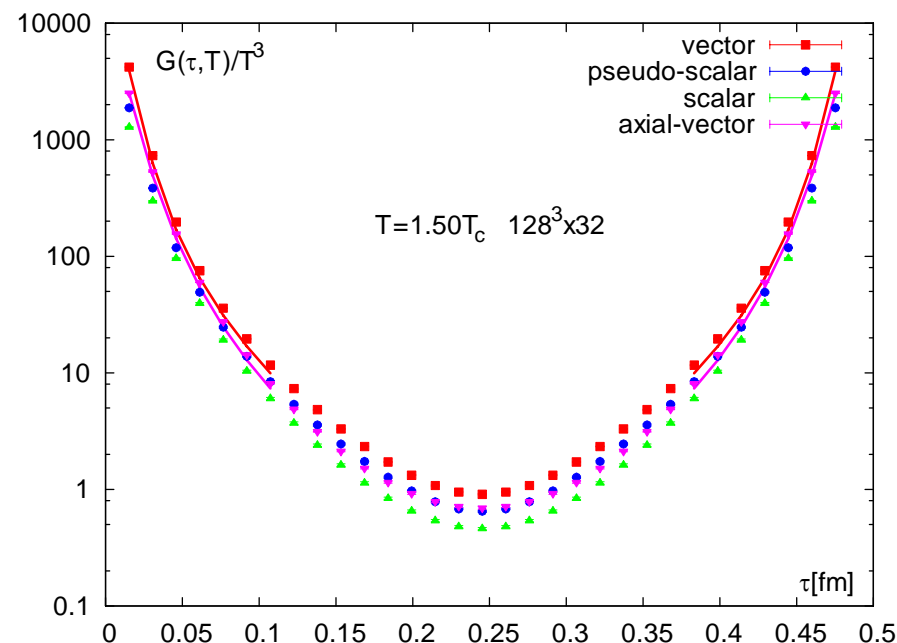
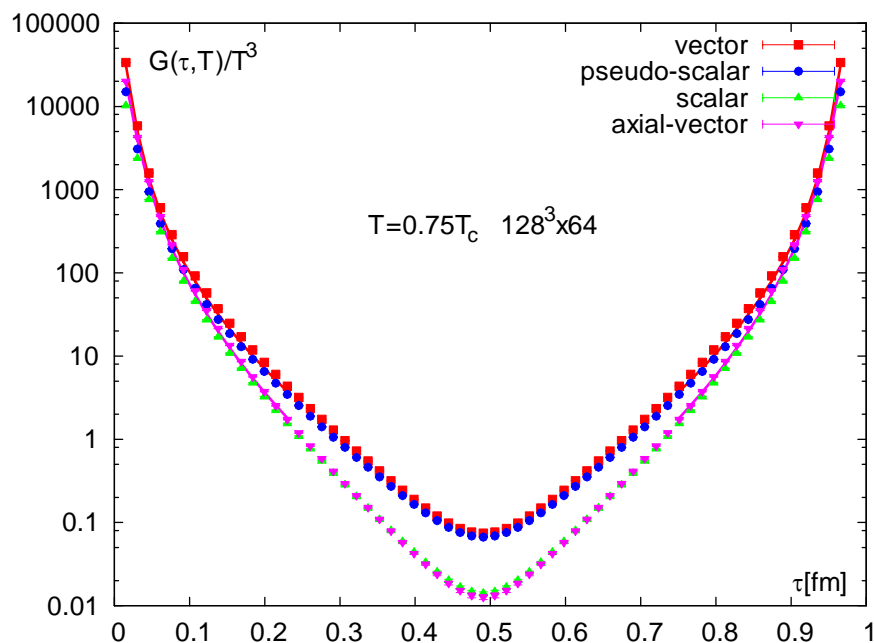
$$m_{free}^{scr}(T) = 2\sqrt{(\pi T)^2 + m_c^2}$$

does this tell us anything about dissociation?

need to understand the momentum dependence of $G_H(\tau, T, p)$ and $\sigma(\omega, T, p)$ in detail

thermodynamic and continuum limit not performed yet

Charmonium Correlators – Temporal Correlators



non-degenerate states still at $1.50 T_c$

(almost) close to free correlators at (very) small separations

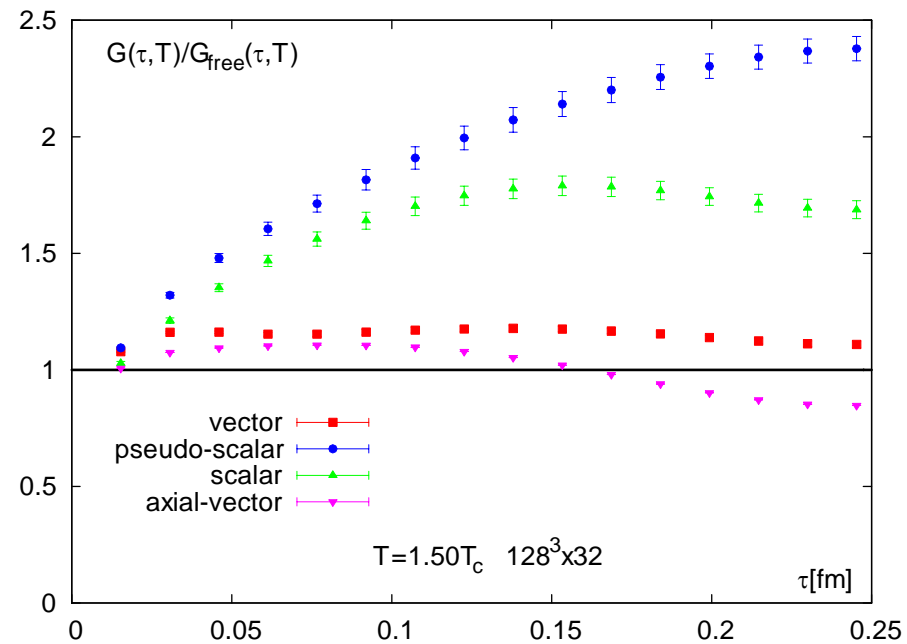
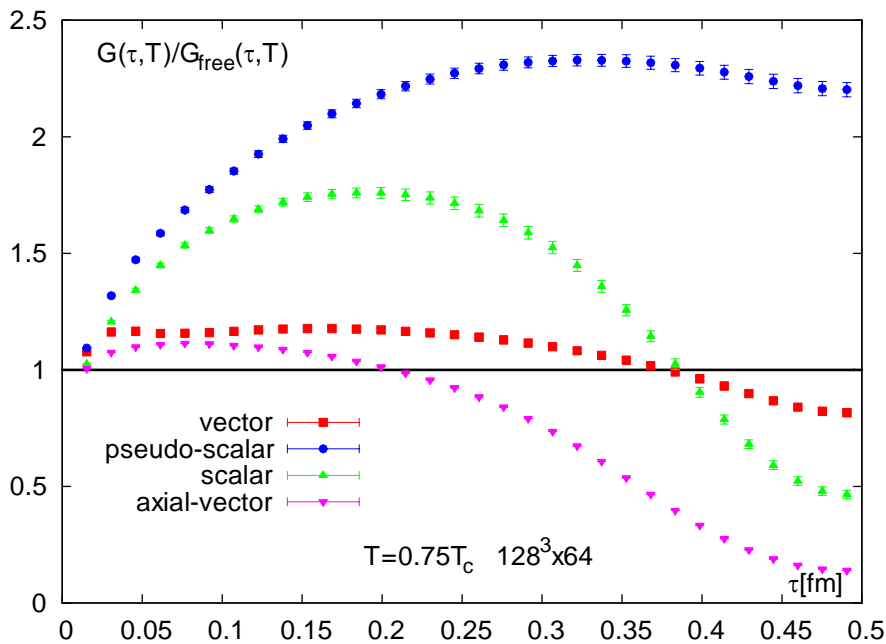
largest distance 0.25 fm due to compact temporal direction

only small distance regime (0.1-0.25 fm) relevant

for thermal effects

for bound state effects

Charmonium Correlators vs Free Correlators



non-degenerate states still at $1.50 T_c$

(almost) close to free correlators at (very) small separations

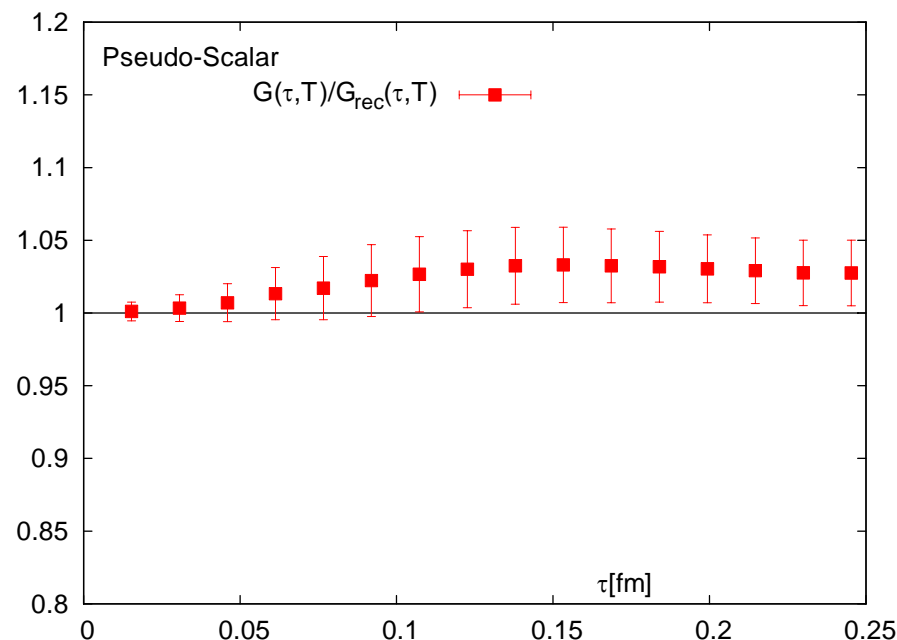
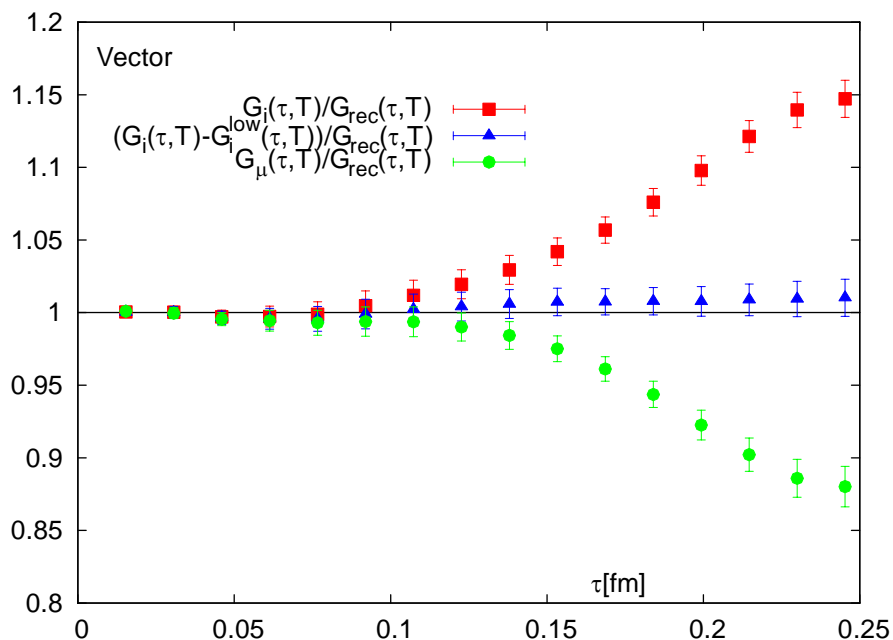
largest distance 0.25 fm due to compact temporal direction

only small distance regime relevant

for thermal effects

for bound state effects

Charmonium Correlators – Zero Mode Contributions



$$G_{rec}(\tau, T) = \int \sigma_0(\omega, 0.75T_c) K(\omega, \tau, T)$$

$$G_{rec}^{low}(\tau, T) = \int_0^{2m_c} \sigma_T(\omega, T) K(\omega, \tau, T)$$

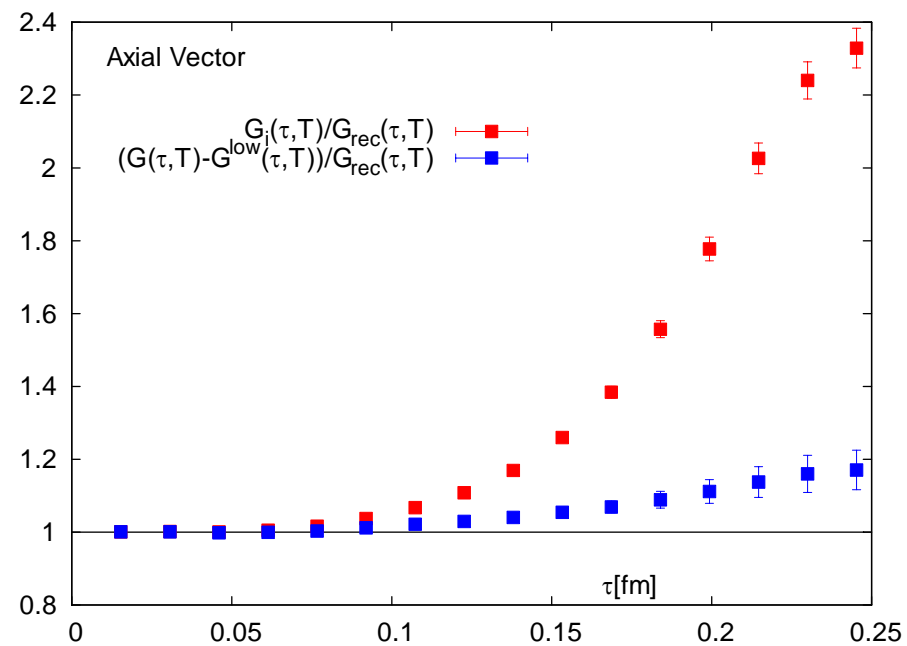
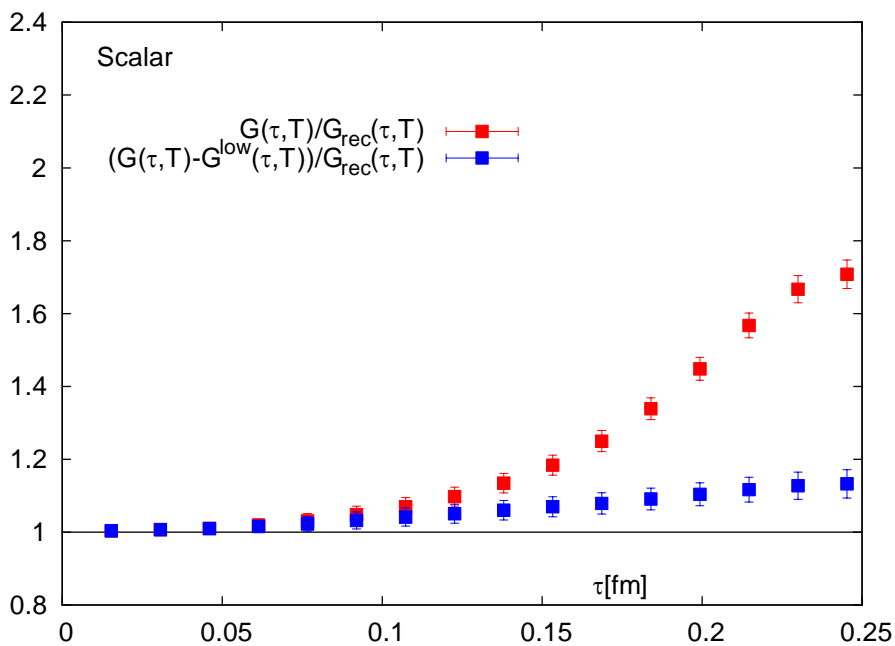
$$G_{\mu\mu}(\tau, T) = G_{ii}(\tau, T) + G_{44}(\tau, T)$$

(see talk of HengTong Ding for SPF)

- main T-effect due to zero-mode contribution
- well described by small ω -part of $\sigma_T(\omega, T)$
- smaller than $G_{44}(\tau, T) = \chi(T)T$
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

Charmonium Correlators – Zero Mode Contributions



$$G_{rec}(\tau, T) = \int \sigma_0(\omega, 0.75T_c) K(\omega, \tau, T)$$
$$G_{rec}^{low}(\tau, T) = \int_0^{2m_c} \sigma_T(\omega, T) K(\omega, \tau, T)$$

- larger zero-mode contribution in S-wave
- larger T-effect in the S-wave states

systematic uncertainties in reconstruction and low- ω part of spectral function

high quality data and small lattice spacing + large momenta (volume) needed

Heavy Quark Free Energies ($m_q=\infty$)

Detailed knowledge of F, U, S and Screening effects!

But still many open questions! Application on Quarkonium?

Hadronic correlators for light quarks ($m_q=0$)

Thermodynamic and Continuum Limit of screening masses!

Spectral functions \rightarrow Dilepton rates, Transport coefficients?

Charmonium hadronic correlators ($m_q=m_c$)

Can we really trust any of the spectral functions obtained with MEM?

What can we learn from Hadronic correlators on Dissociation?

Momentum dependence vs. Spatial correlators/screening masses?

Spectral functions \rightarrow Dilepton rates, Transport coefficients?

Many Thanks to

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