Langevin dynamics and charmonium in sQGP

Clint Young at the ECT*
Workshop on Quarkonia Production in HICs,

28 May 2009

Outline

- Overview of Langevin dynamics
- Effects on charmonium in a HIC
- Results for PHENIX at midrapidity
- Conclusions
- Future work

Overview

- Anomalous J/ψ suppression in a HIC may be explained by treating the heavy charm quarks as deconfined, in a hot medium, for approximately 5 fm/c [1].
- During this time, the dynamics of the charm is best described as a stochastic process (not deterministically, but dependent on a random function $\xi(t)$, over which any observable must then be functionally integrated) [2, 3].

Langevin dynamics

- Each charm quark experiences a drag force, a random kick which rapidly decorrelates in time, and a force due to its interaction with its oppositely charged partner:

$$\frac{dp^i}{dt} = -\eta p^i + \xi^i - \nabla^i U, \left\langle \xi^i(t)\xi^j(0) \right\rangle = \kappa \delta^{ij} \delta(t)$$

- By forcing an ensemble of these quarks to evolve towards a thermal distribution, we obtain the Einstein relation between the drag force and the kicks: $\eta = \frac{\kappa}{2MT}$

Langevin dynamics

When a stochastic process satisfies certain conditions, this differential equation may be discretized (Ito integration):

$$p_{n+1}^{i} = (1 - \eta \Delta t) p_{n}^{i} + \xi_{n}^{i} \Delta t$$

where ξ^{i}_{n} is now selected from a Gaussian distribution with expectation value κ, thanks to the central limit theorem (the sum of a sufficiently large number of random variables will approach a Gaussian distribution) [2].

Fokker-Planck equation

This stochastic process has a dual description: the Fokker-Planck equation is a PDE which describes the phase space evolution of a system experiencing Langevin dynamics. By considering how a distribution evolves over a short time, averaging over the stochastic force, and setting all higher moments of the stochastic force to zero, we obtain

$$\frac{\partial P}{\partial t} = -\frac{\vec{p}}{M} \cdot \vec{\nabla}_x P + \frac{T^2}{D} \vec{\nabla}_p \left(f_0 \vec{\nabla}_p \frac{P}{f_0}\right) + 2D \vec{\nabla}_x \left(f_0 \vec{\nabla}_x \frac{P}{f_0}\right),$$

$$f_0 = \exp\left(-\frac{p^2}{2MT} - \frac{U(\vec{x})}{T}\right), \eta = \frac{T}{MD}$$

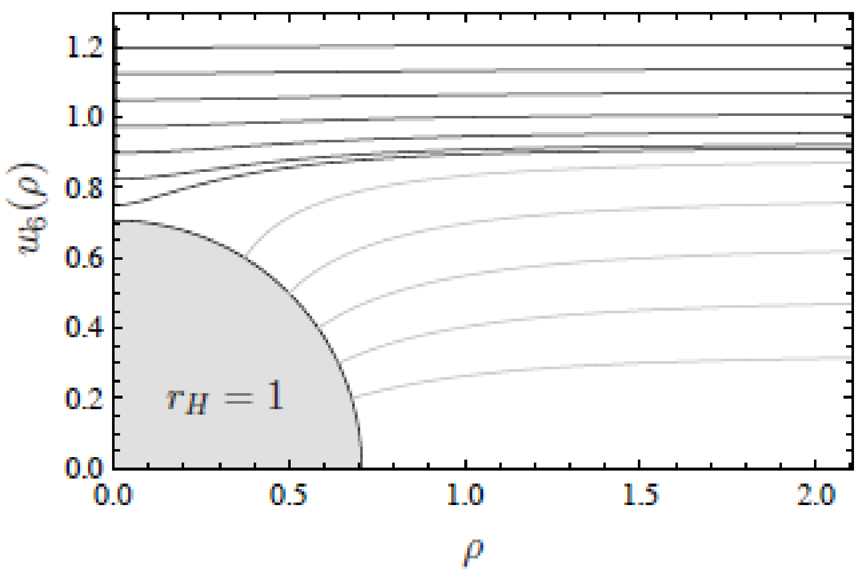
Does the Langevin approach apply to charm quarks in HICs?

- There is no problem with the random force being sufficiently well-behaved so that it can be integrated. The real question is: does the force decorrelate rapidly enough so that it may be treated as random, with
  $$\left\langle \xi^i(t)\xi^j(0)\right\rangle \propto \delta^{ij}\delta(t) \quad ?$$
- The answer seems to be yes: the position of the heavy quark changes on the timescale of M/T^2, the force decorrelates on the timescale of $1/T$. Therefore we need $M \gg T$, which is realized for a charm quark with mass 1.4 GeV in a plasma at a temperature of 200 MeV [2].

However, could there be a non-trivial correlation between the forces on different quarks in the pair, which could effect our results?

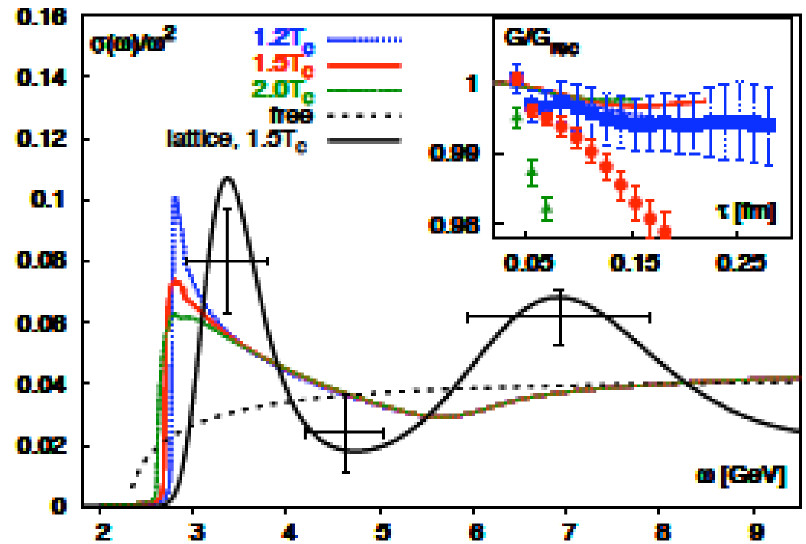
- With Dusling et al., we calculated the the diffusion coefficient for a dipole whose effective action had a dual description as a D7 probe brane in AdS5*S5 , and we found the diffusion coefficient to be significantly suppressed compared to the weak coupling result [4].
- However, while this limit might be appropriate for bottomonium it seems to be inappropriate for charmonium: where the binding energy B~T.
- Charmonium seems to be between the limits of the "photoelectric effect" and "Rayleigh scattering".

$$\mathcal{L}_{\text{eff}} = -\phi_v^\dagger i v \cdot \partial \phi_v + \frac{c_E}{N^2}\phi_v^\dagger \mathcal{O}_E \phi_v + \frac{c_B}{N^2}\phi_v^\dagger \mathcal{O}_B \phi_v$$



Internal vs. free energy

- Shuryak and Zahed argued that based on the timescales of the problem, the interaction energy appropriate for describing the dynamics of bound states is the internal energy, not the free energy [5].
- However, Mocsy and Petreczky have shown that potential models using the free energy as the interaction yield spectral densities for various charmonium states which agree with the spectral densities from the lattice (extracted using the maximal entropy method).
- I will show that for our model, it might be hard to distinguish the difference from HICs.



The value of $2\pi TD_c$

- Phenomenology (Teaney 2005), the gauge-gravity correspondence for strongly-coupled field theories (Casalderrey-Solana and Teaney 2006), and MD simulations converge on the fact that the diffusion coefficient should be much smaller than the perturbative result $1.5/\alpha_s^2$ [7,2].
- We estimate the coefficient to be 1.5

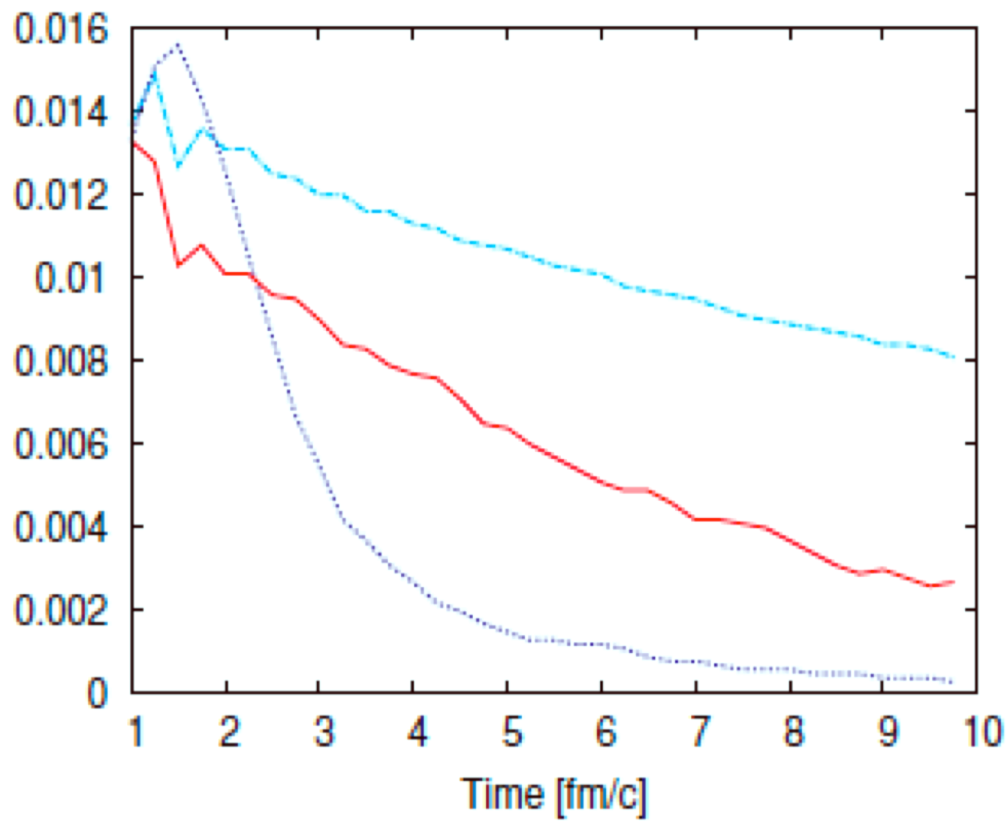
Initialization of the simulation

The quark pairs in fp.exe originate from PYTHIA event generation (LO), and their spatial location is determined by the Glauber model (the distribution in the transverse plane is proportional to the number of collisions in the transverse plane), and the evolution of the pairs begins at t=1 fm/c (the approximate thermalization time).

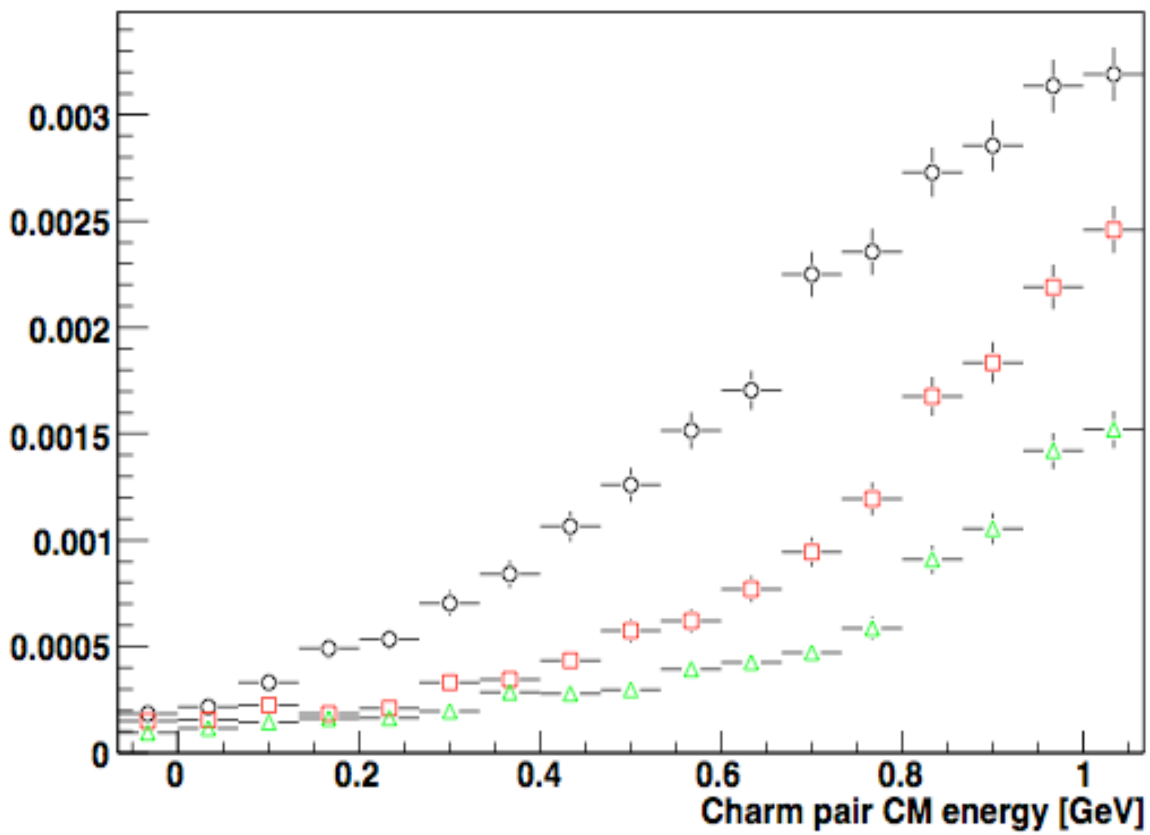
Could this assumption of scaling up from pp collisions be wrong? Could there be significant differences between the initial production in HICs due to enhancement of n-gluon events or a saturated CGC [8]?

Evolution of an ensemble of charm-anticharm pairs

Green: $2\pi TD_c = 1.5$, red: $2\pi TD_c = 3.0$, blue: $2\pi TD_c = 1.5$ with the potential off.



Evolution of an ensemble of charm-anticharm pairs II: quasi-equilibrium

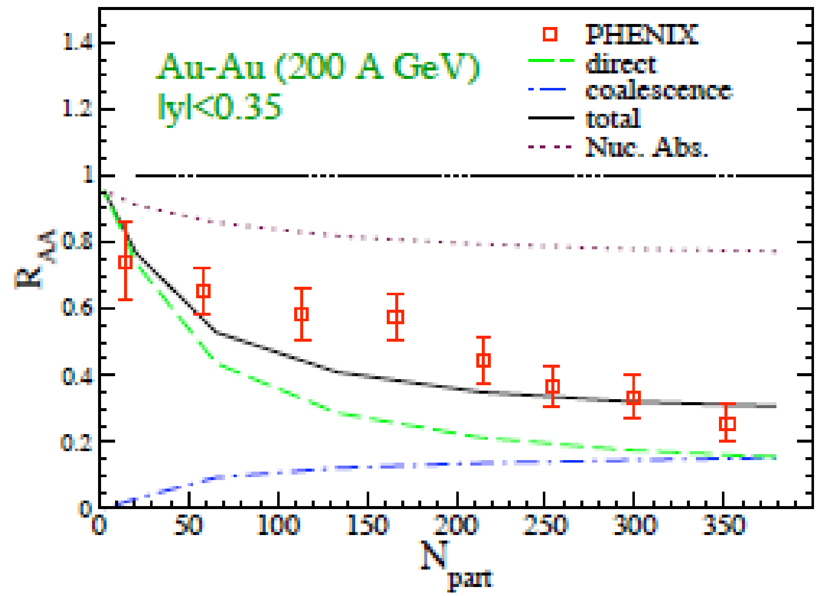


Evolution of an ensemble III

- With a small diffusion coefficient, the ensemble equilibrates in momentum space rapidly, but then slowly diffuses.
- In this example changing the potential would have dramatic effects on the overall J/ψ yield.
- Timescales of the evolution are important: Langevin dynamics for longer periods of time causes more suppression of charm bound states.
- Although the evolution of the ensemble does not end until no bound states remain, “quasi-equilibrium” should develop in relative abundances of various bound states.

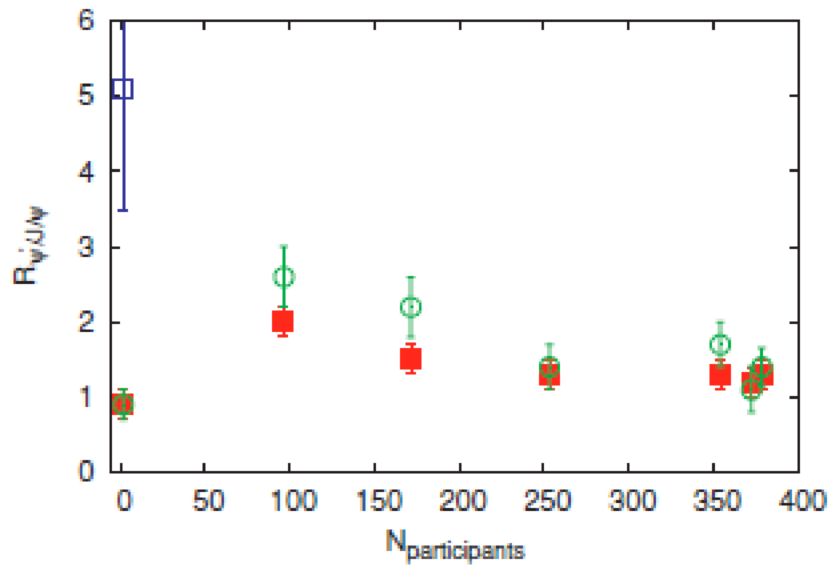
How does this compare with the two-component model (Rapp, Zhao)[9]?

Although our model supposes a different physical process for charm dynamics in QGP, it really is very similar to the two-component model: much of the observed J/ψ states in central HICs originate from “thermalized” charm. Also, their work has the equivalent of slow spatial diffusion (in correlation volumes) and incomplete thermalization in momentum space as well.

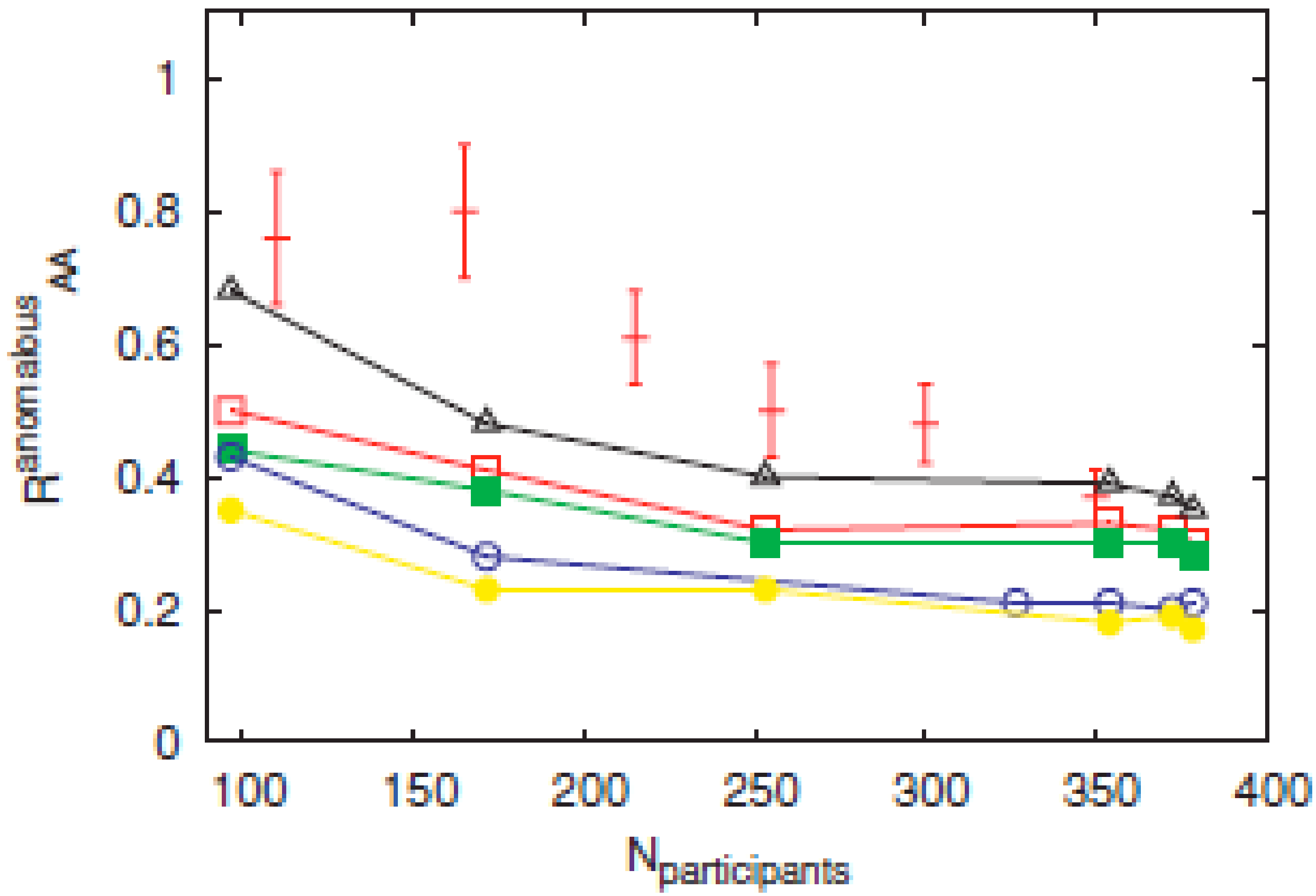


Feeddown?

- Karsch, Kharzeev, and Satz showed that they could get a parameter-free fit to R_{AA} vs. N_{part} by simply saying that all of the higher charmonium states are destroyed in an HIC and the J/ψ state persists, and the observed suppression is an absence of feeddown[10].
- Something similar is observed: the higher states are suppressed to the point where their ratios become nearly thermal.
- Clearly suppression of the higher states is part of the observed suppression.



Results: R_{AA} vs. N_{part} at mid-rapidity for PHENIX Au+Au



Forward rapidity: greater suppression due to the Cronin effect?

- Zhao and Rapp identified the greater suppression of J/ψ at small transverse momentum at forward rapidity as an indicator of a stronger Cronin effect at these rapidities.
- This makes sense in our framework: not only could the Cronin effect cause lower initial production, but by modifying the initial p_t distributions it could cause greater spatial diffusion of quarks experiencing Langevin dynamics.

Conclusions

- Langevin dynamics provides a link between the open and hidden charm in pp collisions with the nearly thermal properties of the charm in HICs.
- Understanding the timescales of a collision is important for understanding J/ψ suppression in sQGP because the spatial diffusion can be relatively slow.
- The interaction between a heavy quark pair explains how the J/ψ states can survive even in a plasma with a long lifetime.

Work in progress: charmonium and dissipation in a quantum system

I have shown a classical simulation of charmonium experiencing Langevin-with-interaction dynamics, where a distribution of J/ψ particles, for example, would evolve to a thermal distribution.

However, quantum mechanically, this is non-trivial (we need dissipative evolution of a density matrix).

Let's try seeing how this might work…

The influence functional for the evolution of a density matrix

When a system interacts with a bath at some temperature whose degrees of freedom we "don't care about" [11]:

$$L = \frac{1}{2} M \dot{x}^2 - V(x) + \Sigma_i m_i \dot{R}_i^2 - \Sigma_{i,j} V_{i,j}(R_i, R_j) - \Sigma_i V_i(x, R_i)$$

we may integrate out these degrees of freedom to obtain a reduced density matrix

$$\rho_{red}(x, x', t, \beta) = \int DxDy \exp(iS[x] - iS[y]) \, F[x, y]$$

The influence functional II

- This functional of *F[x,y]* (the influence functional), if an appropriate way of describing the medium’s effect on charmonium, forbids describing charmonium as a pure state in general, instead it should instead be described as a density matrix.
- Now let’s show how the system could be dissipative…

Path-integral approach to quantum Brownian motion [12]

If the limit is taken where the heavy particle interacts with a bath with a continuous density of degrees of freedom, the influence functional's effect on the path integral for the heavy particle becomes dissipative (non-trivial in a quantum-mechanical system!):

$$\rho_D(\omega)C^2(\omega) = \frac{2m\eta\omega^2}{\pi}, \omega < \Omega,$$

$$\rightarrow \langle F(t)F(0)\rangle = 2\eta T \frac{1}{\pi}\frac{\sin(\Omega t)}{t}$$

In the classical limit, this particle now experiences Langevin motion.

Why quantum Brownian motion?

The J/ψ particle is the lowest bound state of charmonium, so if any charm state, open or hidden, should not be described classically, it is this one.

Modelling charmonium as a quantum Brownian motion system could have many applications and could be generalized in interesting ways.

THANK YOU!

References

[1] E.V. Shuryak, Phys. Lett. **B78**, 150 (1978), Yad. Fiz. **28**, 796 (1978), Phys. Rep. **61**, 71 (1980), D. Kharzeev and H. Satz, Phys. Lett. B334, 155 (1994), T. Matsui and H. Satz, Phys. Lett. B178, 416 (1986).

[2] G. Moore and D. Teaney, Phys. Rev. **C71**, 064904 (2005)

[3] C. Young and E. Shuryak, Phys. Rev. **C79**, 034907 (2009)

[4] K. Dusling, J. Erdmenger, M. Kaminski, F. Rust, D. Teaney, and C. Young, JHEP **0810**, 098 (2008)

References

[5] E. Shuryak and I. Zahed, Phys. Rev. **D70**, 054507

[6] A. Moscy and P. Petreczky, “Quarkonium correlators at finite temperature and potential models”, 2007

[7] J. Casalderrey-Solana and D. Teaney, “Heavy quark diffusion in strongly-coupled N=4 Yang-Mills

[8] D. Kharzeev, E. Levin, M. Nardi, and K. Tuchin, “J/Ψ production in heavy-ion collisions and gluon saturation”, 2008

References

- [9] X. Zhao and R. Rapp, “Forward and midrapidity charmonium production at RHIC”, 2008
- [10] V. Karsch, D. Kharzeev, and H. Satz, “Sequential charmonium disassociation”, 2006
- [11] R. Feynman and A. Hibbs, *Quantum Mechanics and Path Integrals*, 1965
- [12] A. Caldeira and A. Leggett, Physica **121A**, 587-616 (1983)