The GPD E, single spin asymmetries and Ji's sum rule

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Outline:

- **Exclusive processes and GPDs**
- **What do we know about E?**
- **Ji's sum rule**
- **Single spin asmmetries in electroproduction of vector mesons**
- **Summary**

based on work done in collaboration with S. Goloskokov arXiv:0809.4126

The handbag approach in hard exclusive processes

handbag factorization (rigorous proof) Radyushkin, Collins et al, Ji-Osborne

for $Q^2 \to \infty$:

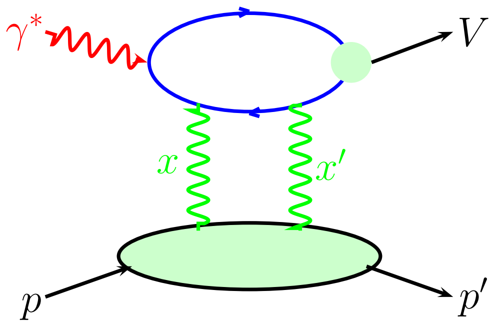
hard subprocesses

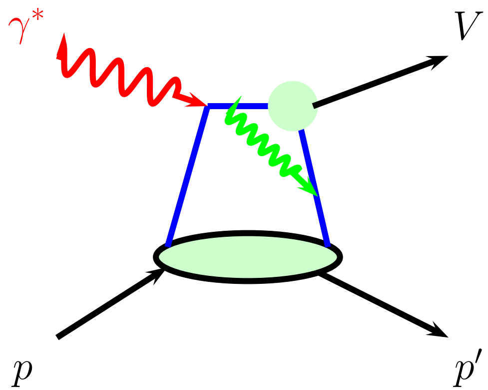
$\gamma^* g \to V g\,,$

$\gamma^* q \to V(P,\gamma) q$

and GPDs and meson w.f.

(encode the soft physics)





dominant transitions $\gamma^*_L \to V_L(P)$, $\gamma^*_T \to \gamma_T$

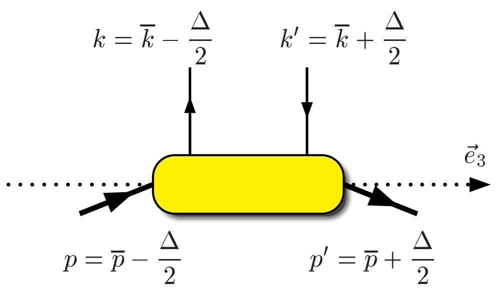
others power suppressed but often non-negligible (e.g. $\gamma^*_T \to V_T$ large)

Generalized Parton Distributions

D. Müller et al (94), Ji(97), Radyushkin (97)

$$\xi = \frac{(p-p')^+}{(p+p')^+} \qquad \bar{x} = \frac{\overline{k}^+}{\overline{p}^+}$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \qquad x' = \frac{\bar{x}-\xi}{1-\xi} \qquad t = \Delta^2$$



$$\overline{p}^+ \int \frac{dz^-}{2\pi}\, e^{i\,\bar{x}\,\overline{p}^+ z^-} \langle p' |\, \overline{\psi}_q(-\overline{z}/2)\, \gamma^+\, \psi_q(\overline{z}/2)\, |p\rangle =$$

$$\overline{u}(p')\gamma^+ u(p) H^q(\bar{x},\xi;t) + \overline{u}(p') i\sigma^{+\alpha}\frac{\Delta_\alpha}{2m} u(p)\ E^q(\bar{x},\xi;t)$$

(gauge $A^+ = 0$; $\overline{z} = [0, z^-, \mathbf{0}_\perp]$) $\qquad \gamma^+\gamma_5 \longrightarrow \widetilde{H}^q, \widetilde{E}^q$

$\xi < \bar{x} \leq 1 \quad (x > 0,\ x' > 0)$ emission and absorption of quarks

$-\xi < \bar{x} < \xi \quad (x > 0,\ x' < 0)$ emission of a $q\overline{q}$ pair

$-1 \leq \bar{x} \leq -\xi \quad (x < 0,\ x' < 0)$ emission and absorption of antiquarks

(quarks with a negative x interpreted as antiquarks with positive x)

gluon GPD defined analogously

Properties of GPDs

- reduction formulas $H^q(\bar{x},0;0) = q(\bar{x})$ $\widetilde{H}^q(\bar{x},0;0) = \Delta q(\bar{x})$

  $H^g(\bar{x},0;0) = \bar{x}g(\bar{x})$ $\widetilde{H}^g(\bar{x},0;0) = \bar{x}\Delta g(\bar{x})$

- sum rules $F_1^q(t) = \int_{-1}^{1} d\bar{x} H^q(\bar{x},\xi,t)$ $F_1 = \sum_q e_q F_1^q$

  $E^q \to F_2^q$ $\widetilde{H}^q \to F_A^q$ $\widetilde{E}^q \to F_P^q$

- polynomiality $\int_{-1}^{1} d\bar{x} \bar{x}^{n-1} H^q = \sum_{j=0}^{[n/2]} h^q_{nj}(t)\xi^{2j}$

  and analogously for other GPDs, h^q_{nj} generalized form factors

  consequence of Lorentz covariance

- universality

- evolution ($\bar{x} < \xi$ ERBL; $\bar{x} > \xi$ DGLAP)

- Ji's sum rule $\langle J_q \rangle = \frac{1}{2} \int_{-1}^{1} d\bar{x}\, \bar{x}\, [H^q(\bar{x},\xi,t=0) + E^q(\bar{x},\xi,t=0)]$

- positivity constraints for $\xi < \bar{x} < 1$

$$\left|\mathcal{F}^q_{++}\right|^2 + \left|\mathcal{F}^q_{-+}\right|^2 \le \frac{q(x)q(x')}{1-\xi^2}$$

$\mathcal{F}^q_{++} = \mathcal{F}^q_{--} = H^q - \frac{\xi^2}{1-\xi^2} E^q$ refers to proton helicity

$\mathcal{F}^q_{-+} = -\mathcal{F}^{q*}_{+-} = \eta \frac{\sqrt{t-t_0}}{2m\sqrt{1-\xi^2}} E^q$ partons have equal helicities

$t_0 = -4\xi^2 m^2/(1-\xi^2)$ minimal value

E implies orbital angular momentum (η phase of $\Delta_\perp$)

similar bounds for H^g, E^g and for
$H \pm \widetilde{H}$, $E \pm \widetilde{E}$ related to $q \pm \Delta q$ (quarks and gluons with definite helicity)

for more bounds see Pobylitsa (02)

Interpretation

Burkhardt (00): $\xi = 0$ case $(x = x' = \bar{x})$

Fourier transform:

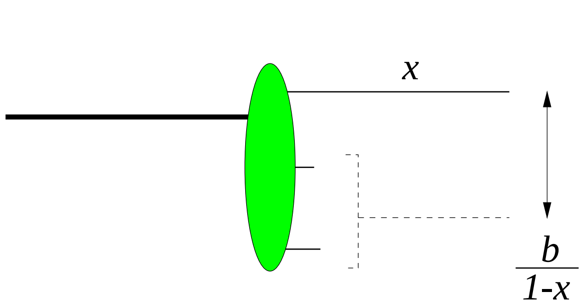
$$q(x, \xi = 0, \mathbf{b}) = \int \frac{d^2\mathbf{\Delta}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{\Delta}} H^q(x, \xi = 0, t = -\Delta^2)$$

and analogue for the other GPDs

density interpretation of $\xi = 0$ GPDs in mixed representation of long. momentum and transverse position space (IMF)

$q(x, \xi = 0, \mathbf{b})$ gives probability to find a quark q with long. momentum fraction x at transverse position $\mathbf{b}$

What is b?



hadron's center of momentum

$\mathbf{b_0} = \sum_i x_i \mathbf{b_i} = 0$ (chosen, $\sum_i x_i = 1$)

$\mathbf{b}$ transverse distance between struck quark and center of momentum

$\mathbf{b}/(1-x)$ relative distance between struck quark and cluster of spectators

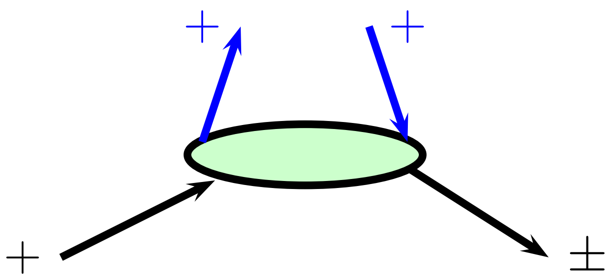
partons with large (small) x_i must (can) have small (large) $\mathbf{b}_i$

$\int dx q(x, \xi = 0, \mathbf{b}) = F_1^q(\mathbf{b})$ two-dimensional density in transverse plane
refers to a rapidly moving hadron (IMF) (fully relativistic)

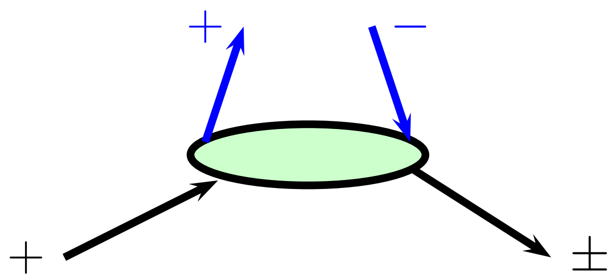
Non-rel. case: FT $\mathbf{p} \to \mathbf{r}$ three-dimensional density
refers to hadron at rest

Helicty-flip GPDs

Hoodbhoy-Ji (98), Diehl (01)



non-flip GPDs $H, \cdots$



$\gamma^+ \Longrightarrow \sigma^{+j} \qquad j = 1, 2$

$H_T\,,\ \widetilde{H}_T\,,\ E_T\,,\ \widetilde{E}_T$ (lead. twist)

parton helicity-flip (or transversity) GPDs

$H_T^q(\bar{x}, 0, 0) = \delta^q(\bar{x})$ (transversity PDFs)

known from SIDIS exp. Anselmino *et al*

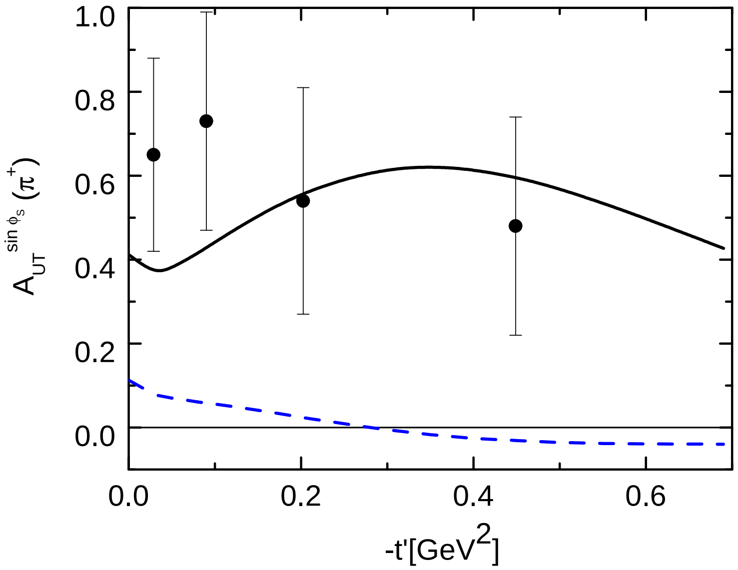
not much known

subprocesses often suppress parton helicity-flip to lead. twist order (e.g. DVCS, meson production)

higher-twist: small contributions, difficult to observe in general

A gold-plated observable: $A_{UT}^{\sin\phi_S}$

HERMES(09)



$Q^2 \simeq 2.5\,\mathrm{GeV}^2$, $W = 3.99\,\mathrm{GeV}$

ϕ_S orientation of target-spin vector

$\sin\phi_s$ moment very large

does not seem to vanish for $t' \to 0$

dominant term:

$A_{UT}^{\sin\phi_S} \propto \mathrm{Im}\Big[M^*_{0-,++} M_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

H_T accompanied by twist-3 π wave fct

Goloskokov-K(09)

a few more applications of transversity GPDs:

Pire *et al*	$\gamma_T^* \to \rho_T$ to twist-3
Ahmad *et al*	π^0
Huang *et al*	wide-angle photoproduction

... but we are not able to calculate GPDs as yet

controlled by non-pert. QCD

lattice QCD: a few moments of GPDs Hägler *et al*

extraction from experiment:

analysis of nucleon FF with help of sum rules

determination of val. quark GPDs $H, E, \widetilde{H}$

Guidal *et al*, Diehl *et al*

analysis of DVCS with parameterizations of GPDs

Müller *et al*, Moutarde, Guidal, Polyakov-Vanderhaeghen

also for π^+ production Bechler-Müller

analysis of meson electropro. with double distribution ansatz

Goloskokov-K

models: Overlap of light-cone wave functions Diehl *et al*, Pasquini *et al*

$\gamma_L^* \gamma \to \pi^+ \pi^-$ with NJL Noguera *et al*

H, E in chiral quark soliton model Goeke *et al*

Double distributions

integral representation (i= valence, sea quarks, gluons)

$$H_i(\bar{x},\xi,t') = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-\bar{x})\, f_i(\beta,\alpha,t') + D_i\, \Theta(\xi^2-\bar{x}^2)$$

f_i double distributions — Mueller *et al* (94), Radyushkin (99)

advantage - polynomiality automatically satisfied

$D_i(\bar{x},t)$ ($i =$gluon, flavor-singlet) additional free function, support $-\xi < \bar{x} < \xi$

useful ansatz with relation to PDFs (reduction formula respected)

$$f_i(\beta,\alpha,t') \;=\; h_i(\beta)\,\exp[(b_i+\alpha_i'\ln(1/|\beta|))t]\,\frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)}\,\frac{[(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\begin{aligned} h_g(t=0) &= |\beta| g(|\beta|)\,, & n_g &= 2 & \alpha_g' &= 0.15\,\mathrm{GeV}^{-2} \\ h^q_{\rm sea}(t=0) &= q(|\beta|)\,\mathrm{sign}(\beta)\,, & n_{\rm sea} &= 2 & \alpha_{\rm sea} &= \alpha_g \\ h^q_{\rm val}(t=0) &= q_{\rm val}(\beta)\,\Theta(\beta)\,, & n_{\rm val} &= 1 & \alpha_v' &= 0.9\,\mathrm{GeV}^{-2} \end{aligned}$$

sea quarks mix with gluons under evolution — E analogously

The $\gamma^* p \to VB$ amplitudes

consider large Q^2, W and small t;
kinematics fixes skewness: $\xi \simeq \frac{x_{\rm Bj}}{2-x_{\rm Bj}}[1+m_V^2/Q^2] \simeq x_{\rm Bj}/2 +$ m.m.c.

$$\mathcal{M}_{\mu+,\mu+}(V) = \frac{e_0}{2}\left\{\sum_a e_a \mathcal{C}_V^{aa}\langle H^g_{\rm eff}\rangle_{V\mu} + \sum_{ab}\mathcal{C}_V^{ab}\langle H^{ab}_{\rm eff}\rangle_{V\mu}\right\},$$

$$\mathcal{M}_{\mu-,\mu+}(V) = -\frac{e_0}{2}\frac{\sqrt{-t}}{M+m}\left\{\sum_a e_a \mathcal{C}_V^{aa}\langle E^g\rangle_{V\mu} + \sum_{ab}\mathcal{C}_V^{ab}\langle E^{ab}\rangle_{V\mu}\right\},$$

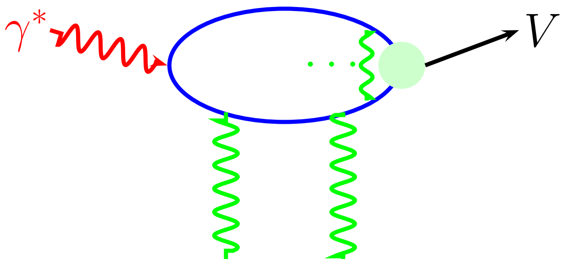
$\mathcal{C}_V^{ab}$ flavor factors, $M(m)$ mass of $B(p)$, $H_{\rm eff} = H - \xi^2/(1-\xi^2)E$
contributions from $\widetilde{H}$ to T-T amplitude not shown
electroproduction with unpolarized protons at small ξ:
E not much larger than H (see below) $\Longrightarrow H_{\rm eff} \to H$ for small ξ
$|M_{\mu-,\mu+}|^2 \propto t/m^2$ neglected $\Longrightarrow$ probes H (exception ρ^+)
trans. polarized target: probes $Im[\langle E\rangle^*\langle H\rangle]$ interference
polarized beam and target: probes $Re[\langle H\rangle^*\langle\widetilde{H}\rangle]$ interference

Subprocess amplitudes

$F = H, E \qquad \lambda$ parton helicities   Goloskokov-K

$$\langle F\rangle^{ab(g)}_{V\mu} = \sum_\lambda \int d\bar{x}\mathcal{H}^{Vab(g)}_{\mu\lambda,\mu\lambda}(\bar{x},\xi,Q^2,t=0)\, F^{ab(g)}(\bar{x},\xi,t)$$

$F^{aa} = F^a\,, \qquad F^{ab} = F^a - F^b \quad (a \neq b)$ (with flavor symmetry)



$$\mathcal{H}^{Vab}_{\mu\lambda,\mu\lambda} = \int d\tau d^2b\, \hat{\Psi}_{V\mu}(\tau,-\vec{b}) \exp[-S(\tau,\vec{b},Q^2)] \times \hat{\mathcal{F}}^{ab}_{\mu\lambda,\mu\lambda}(\bar{x},\xi,\tau,Q^2,\vec{b})$$

LO pQCD
+ quark trans. mom.
+ Sudakov supp.
$\Rightarrow$ lead. twist for $Q^2 \to \infty$

Sudakov factor (Sterman et al)

$S \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b\Lambda_{\rm QCD})}$ + NLL

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. FT of $\propto e_a/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)]$

regularizes also TT amplitude

in collinear appr:

$$\mathrm{TT}: \int_0^1 d\tau \frac{\Phi_V(\tau)}{\tau}\frac{1}{(1-\tau)t+\tau Q^2}$$

IR singular for large Q^2
regular for large $-t$ (wide-angles)

Goloskokov-K. 06, 07, 08

analysis of cross sections and spin density matrix elements
for ρ^0 and ϕ electroproduction
data taken from HERMES, COMPASS, E665, H1, ZEUS

H constructed from CTEQ6 PDFs through the double distr. ansatz
($D = 0$, sum rules and positivity bounds checked numerically)

assume H is fairly well-known at small ξ and $x \lesssim 0.6$

What do we know about E_v?

analysis of Pauli FF for proton and neutron at $\xi = 0$ Diehl et al (04):

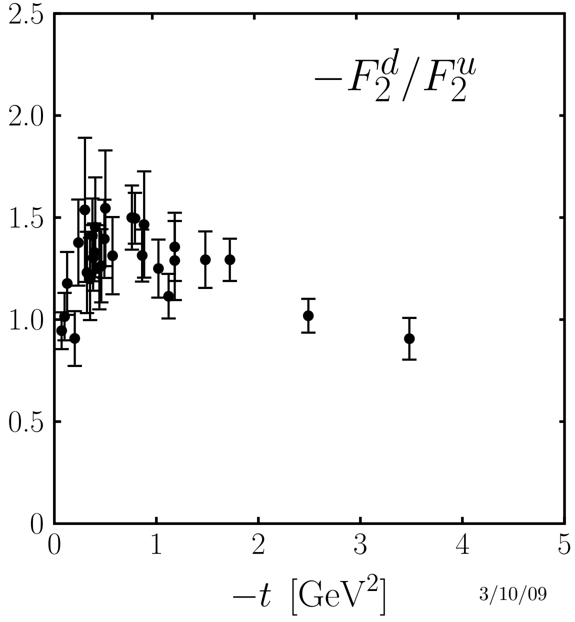
$$F_2^{p(n)} = \int_0^1 dx \Big[e_{u(d)} E_v^u(x, \xi = 0, t) + e_{d(u)} E_v^d(x, \xi = 0, t) \Big]$$

ansatz for small $-t$: $E_v^a = e_v^a(x) \exp\Big\{ t\big(\alpha_v' \ln(1/x) + b_a^e\big) \Big\}$

forward limit: $e_v^a = N_a x^{-\alpha_v(0)} (1-x)^{\beta_v^a}$ (analogously to PDFs)

N_a fixed from $\kappa_a = \int_0^1 dx E_v^a(x, \xi = 0, t = 0)$

(strong $x - t$ corr.; for large x, $-t$ add $C_q x (1-x)^2$ and factor $(1-x)^3$ for others)



fitting FF data provides: $\beta_v^u = 4$, $\beta_v^d = 5.6$

(other powers not excluded in 04 analysis)

new JLab data on $G_{E,M}^n$

up to $3.5(5.0)\,\mathrm{GeV}^2$ favor $\beta_v^u < \beta_v^d$

Input to double distribution model

E for gluons and sea quarks

sum rule (Ji's s.r. and momentum s.r. of DIS) at $t = \xi = 0$

$$\int_0^1 dx x e_g(x) = e_{20}^g = -\sum e_{20}^{a_v} - 2\sum e_{20}^{\bar{a}}$$

valence term very small, in particular if $\beta_v^u \leq \beta_v^d$

$\Rightarrow$ gluon and sea quark moments cancel each other almost completely

parameterization (flavor symm. sea for E assumed)

$$e_i = N_i x^{-\alpha_g(0)}(1-x)^{\beta_i}$$

and exponential t dependence:

$$\propto \exp\left\{t\big(\alpha_i' \ln(1/x) + b_i^e\big)\right\}$$

input to double distribution model

Positivity bounds for E

Fourier transform of zero-skewness GPDs ($f_i^e = \alpha_i' \ln(1/x) + b_i^e$)

$$e_i(x,\mathbf{b}) \;=\; \frac{1}{4\pi}\,\frac{e_i(x)}{f_i^e(x)}\;\exp[-\frac{b^2}{4f_i^e(x)}]$$

$H_i \to q_i(x,\mathbf{b})$ $\qquad\qquad$ $\widetilde{H}_i \to \Delta q_i(x,\mathbf{b})$

bound (Pobylitsa(02), Burkardt(04))

$$\frac{b^2}{m^2}\left(\frac{\partial}{\partial b^2} e_i(x,\mathbf{b})\right)^2 \leq \Big[q_i^2(x,\mathbf{b}) - \Delta q_i^2(x,\mathbf{b})\Big]$$

multiplication with b^2 and integration over $\mathbf{b}$ leads for exponential t dependence with assumption $\tilde{f}_i = f_i$ to: (Diehl et al(04))

$$\left[\frac{e_i(x)}{q_i^2(x) - \Delta q_i^2(x)}\right] \leq 8em^2\left[\frac{f_i^e(x)}{f_i(x)}\right]^3\,\Big[f_i(x) - f_i^e(x)\Big]$$

positivity bound forbids large sea $\Rightarrow$ gluon small too

Solutions for E and Ji's sum rule

$$\langle J^a \rangle = \frac{1}{2}\left[q^a_{20} + e^a_{20}\right] \qquad \langle J^g \rangle = \frac{1}{2}\left[g_{20} + e^g_{20}\right]$$

$(\xi = 0)$ $\langle J \rangle$ means average value of three component of J

var.	β^u_{val}	β^d_{val}	β_g	β_s	N_g	N_s	J^u	J^d	J^s	J^g
1	4	5.6	-	-	0.000	0.000	0.250	0.020	0.015	0.214
2	4	5.6	6	7	-0.873	0.155	0.276	0.046	0.041	0.132
3	4	5.6	6	7	0.776	-0.155	0.225	-0.005	-0.011	0.286
4	10	5	7	-	0.523	0.000	0.209	0.013	0.015	0.257
5	6	6	7	-	0.167	0.000	0.230	0.024	0.015	0.228
6	6	6	-	7	0.000	0.025	0.234	0.028	0.019	0.214

J^i quoted at scale $4\,\text{GeV}^2$ (spread indicates uncertainties of present knowledge)

$\sum J^i = 1/2$, the spin of the proton

characteristic, stable pattern: for all variants J^u and J^g are large, others small

L and S

$$\langle J^{u_v}\rangle = 0.211(17) \qquad \langle J^{d_v}\rangle = 0.000(19) \qquad \text{at scale } 4\,\mathrm{GeV}^2$$

Lattice (Hägler et al (07)): $\langle J^u\rangle = 0.214(27)\,,\ \langle J^d\rangle = -0.001(27)$
at m_π(phys) sea quark contributions seem to be small

orbital angular momenta: subtract contribution from spin

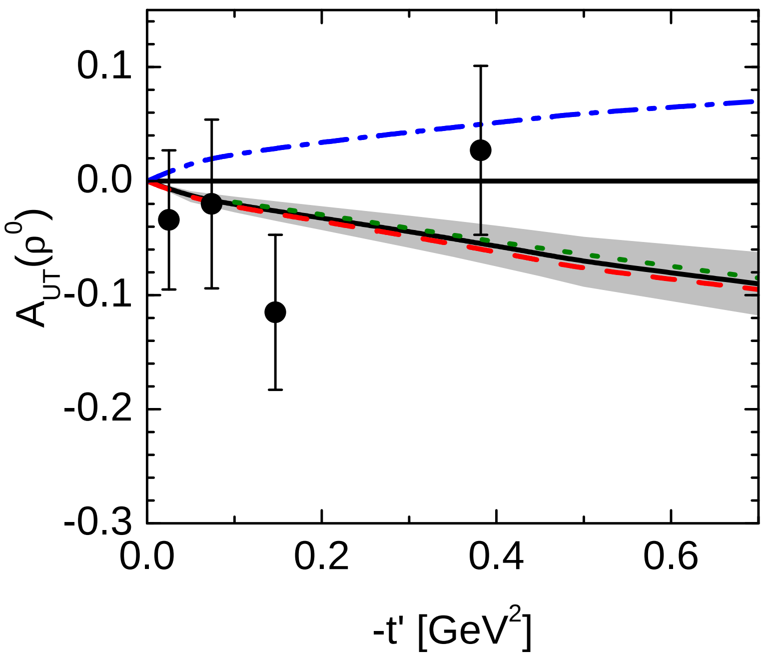
$$\langle L^i\rangle := \langle J^i\rangle - \Delta q^i$$

valence quarks (for variants 1, 2, 3): $\langle L^{u_v}\rangle \simeq -0.241\,,\ \langle L^{d_v}\rangle \simeq 0.155\,,$

note: for gluons no gauge inv. separation into L and S
Ji (96), Burkardt-BC (08))

Results for $A_{UT}(V)$

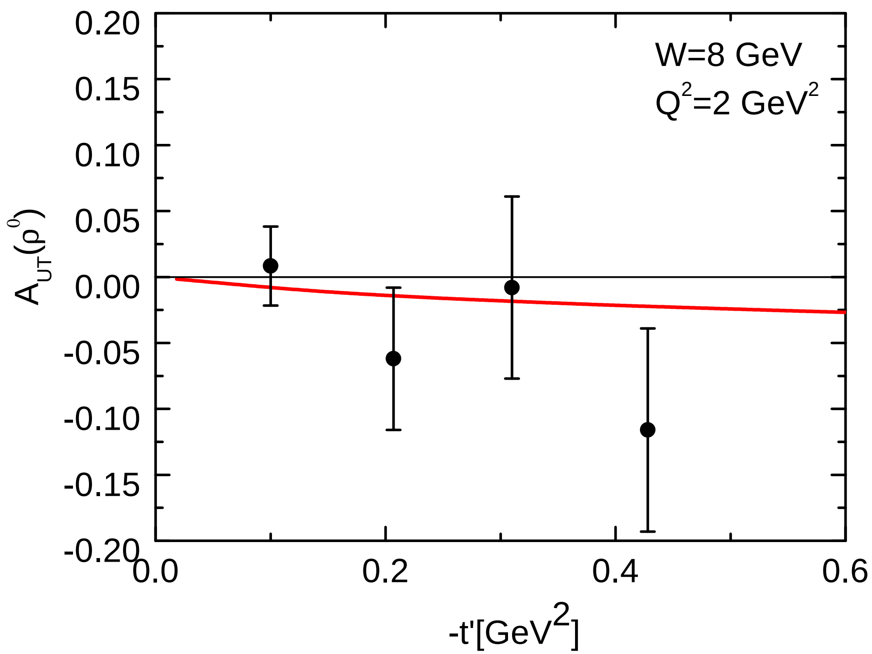
data: HERMES (08)



$W = 5\,\mathrm{GeV} \quad Q^2 = 3\,\mathrm{GeV}^2$

variant 1, 2, 3, 4

COMPASS prel.



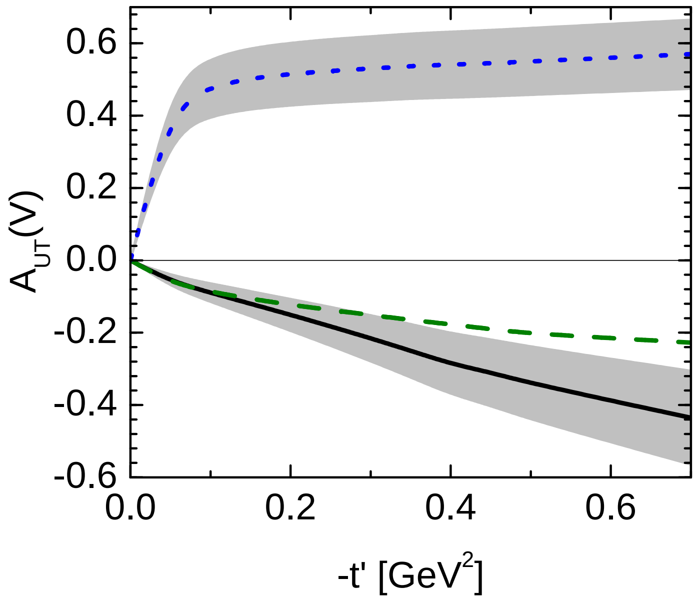
$W = 8\,\mathrm{GeV} \quad Q^2 = 2\,\mathrm{GeV}^2$

variant 1

negative value of A_{UT} favored, variant 4 disfavored

$A_{UT}(\phi) \simeq 0$ in agreement with prel. HERMES data

Results continued



variant 1 for ω, ρ^+, K^{*0}

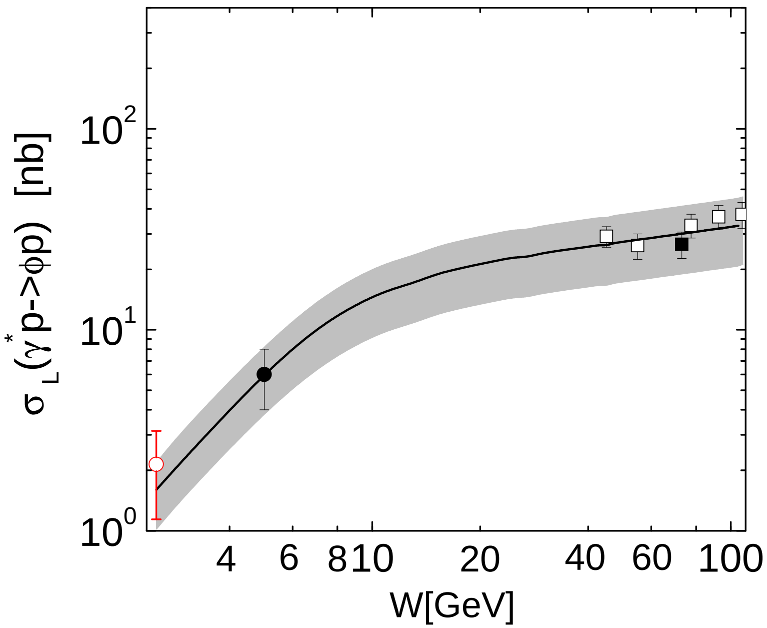
t dependence controlled by trivial factor $\sqrt{-t'}$

except for ρ^+: since $H_v^u - H_v^d$ small and $E_v^u - E_v^d$ large

E non-negligible in cross section, contribution from helicity flip ampl. $\propto t'$

more data on ρ^0, ω, ϕ from HERMES and COMPASS will come

The ϕ cross section

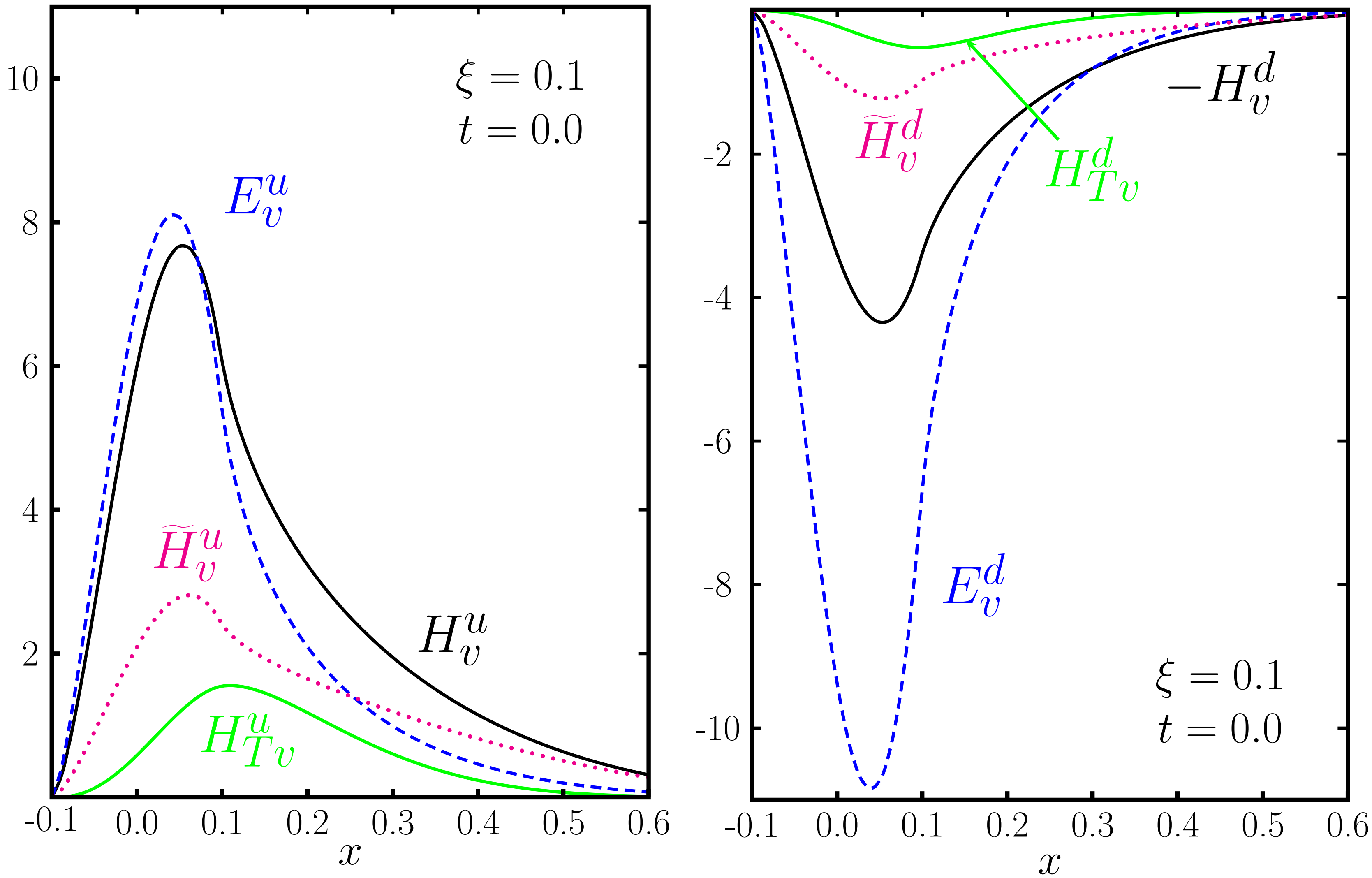


at $Q^2 = 3.8\,\mathrm{GeV}^2$ HERMES (●), ZEUS (□), H1 (■), CLAS (○)

Goloskokov-K (08) calculation with E neglected: hints at small E^g

JLAB12 may measure ϕ cross section at $W = 3, 4\,\mathrm{GeV}$

Valence quark GPDs

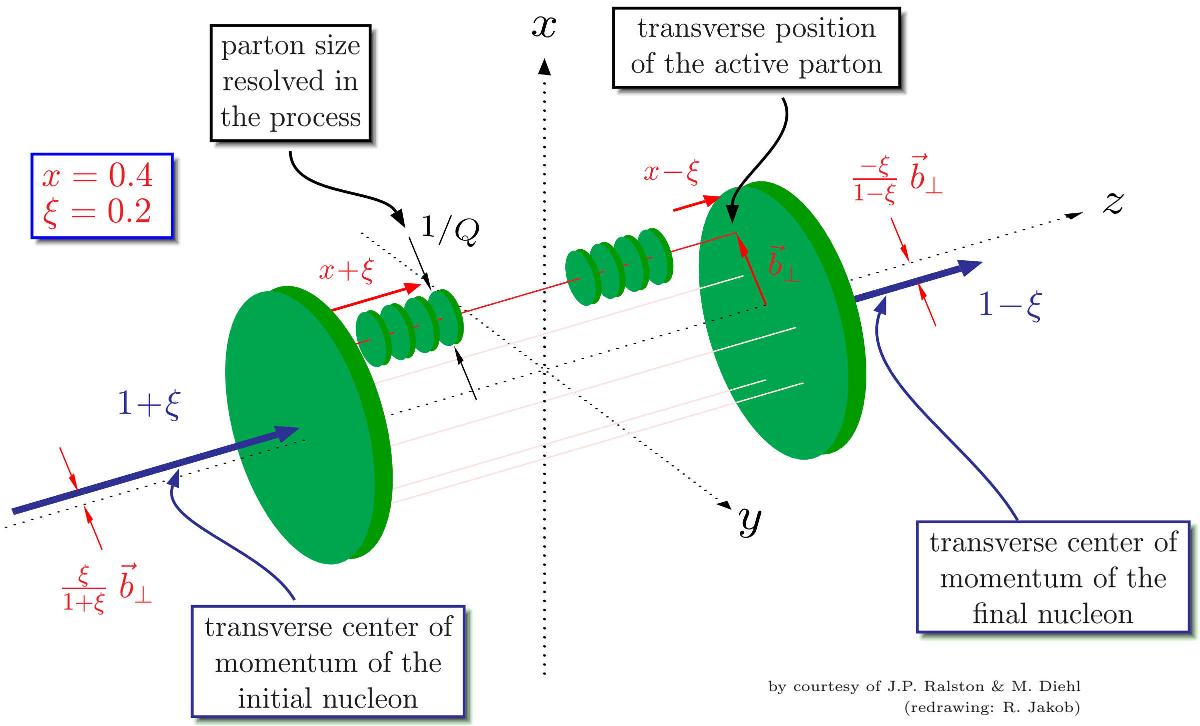


Summary

- an improved form factor analysis which takes into account the new JLAB data on G_E^n, G_M^n and G_E^p/G_M^p, will improve knowledge of E_{val}
- E^g and $E_{\rm sea}$ estimated from a sum rule and positivity bounds
- HERMES and COMPASS measurements of A_{UT} for ρ^0, disfavor extreme solutions for $E_{\rm val}$
- A_{UT} data from ω, ρ^+ and K^{*0} will probe E_{val} further and may tell us whether E_g and $E_{\rm sea}$ are small
- measurement of ϕ cross section at JLAB12 will also probe E_g and E_{sea}
- knowledge of E allows for an evaluation of Ji's sum rule; J^u and J^g large, J^d, J^s small

Impact parameter representation for $\xi \neq 0$

Diehl(02) and Phys. Rep. **388**, 41 (2003)



Fourier trans.

with respect to

$\mathbf{D}_\perp = \frac{\mathbf{p}'_\perp}{1-\xi} - \frac{\mathbf{p}_\perp}{1+\xi}$

GPDs probe partons at transverse distance $\vec{b}_\perp$