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# What is Orbital Angular Momentum?

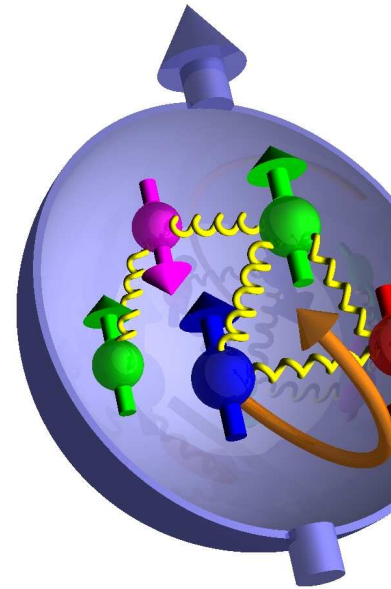
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# Motivation

- polarized DIS: **only  $\sim 30\%$  of the proton spin due to quark spins**
- ↪ ‘spin crisis’  $\rightarrow$  ‘spin puzzle’, because  $\Delta\Sigma$  much smaller than the quark model result  $\Delta\Sigma = 1$
- ↪ quest for the remaining 70%
  - quark orbital angular momentum (OAM)
  - gluon spin
  - gluon OAM
- ↪ How are the above quantities defined?
- ↪ How can the above quantities be measured



# example: angular momentum in QED

- consider, for simplicity, first QED without electrons:

$$\vec{J} = \int d^3r \vec{x} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{x} \times \left[ \vec{E} \times (\vec{\nabla} \times \vec{A}) \right]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[ E^j (\vec{x} \times \vec{\nabla}) A^j + (\vec{x} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]$$

- drop  $2^{nd}$  term (eq. of motion  $\vec{\nabla} \cdot \vec{E} = 0$ ), yielding  $\vec{J} = \vec{L} + \vec{S}$  with

$$\vec{L} = \int d^3r E^j (\vec{x} \times \vec{\nabla}) A^j \quad \vec{S} = \int d^3r \vec{E} \times \vec{A}$$

- note:  $\vec{L}$  and  $\vec{S}$  not separately gauge invariant

# example: angular momentum in QED with electrons

$$\begin{aligned}\vec{J}_\gamma &= \int d^3r \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{x} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\ &= \int d^3r \left[ E^j (\vec{r} \times \vec{\nabla}) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A} \right] \\ &= \int d^3r \left[ E^j (\vec{r} \times \vec{\nabla}) A^j + (\vec{r} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]\end{aligned}$$

- replace  $2^{nd}$  term (eq. of motion  $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$ ), yielding

$$\vec{J}_\gamma = \int d^3r \left[ \psi^\dagger \vec{r} \times e\vec{A}\psi + E^j (\vec{x} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A} \right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$  cancels similar term in electron OAM  $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

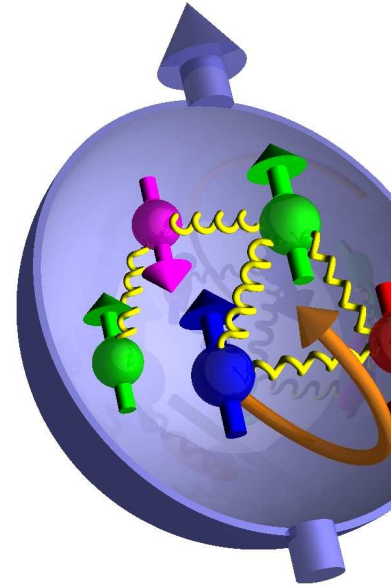
↪ decomposing  $\vec{J}_\gamma$  into spin and orbital also shuffles angular momentum from photons to electrons!

## example (cont.)

- total angular momentum of isolated system uniquely defined
- ambiguities arise when decomposing  $\vec{J}$  into contributions from different constituents
- gauge theories: changing gauge may also shift angular momentum between various degrees of freedom
- ↪ decomposition of angular momentum in general depends on 'scheme' (gauge & quantization scheme)
- does not mean that angular momentum decomposition is meaningless, but
- one needs to be aware of this 'scheme'-dependence in the physical interpretation of exp/lattice/model results in terms of spin vs. OAM
- and, for example, not mix 'schemes', e.t.c.

# Outline

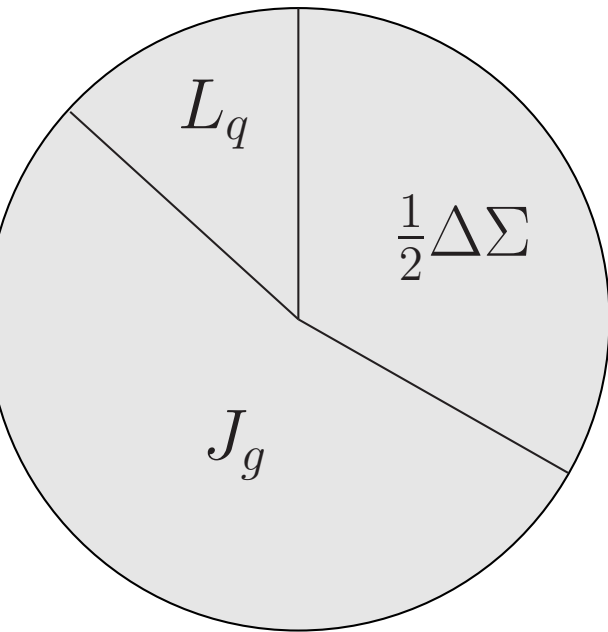
- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe
- Chen-Goldman decomposition



# The nucleon spin pizza(s)

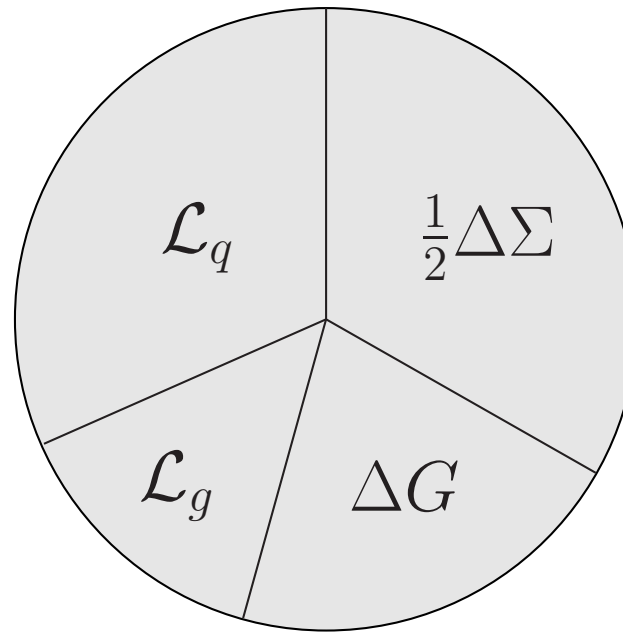


Ji



‘pizza tre stagioni’

Jaffe & Manohar



‘pizza quattro stagioni’

- only  $\frac{1}{2} \Delta \Sigma \equiv \frac{1}{2} \sum_q \Delta q$  common to both decompositions!

# Angular Momentum Operator

• angular momentum tensor  $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$

•  $\partial_\rho M^{\mu\nu\rho} = 0$

↪  $\tilde{J}^i = \frac{1}{2} \varepsilon^{ijk} \int d^3r M^{jk0}$  conserved

$$\frac{d}{dt} \tilde{J}^i = \frac{1}{2} \varepsilon^{ijk} \int d^3x \partial_0 M^{jk0} = \frac{1}{2} \varepsilon^{ijk} \int d^3x \partial_l M^{jkl} = 0$$

•  $M^{\mu\nu\rho}$  contains time derivatives (since  $T^{\mu\nu}$  does)

• use eq. of motion to get rid of time derivatives

• integrate total derivatives appearing in  $T^{0i}$  by parts

• yields terms where derivative acts on  $x^i$  which then 'disappears'

↪  $J^i$  usually contains both

• 'Extrinsic' terms, which have the structure ' $\vec{x} \times$  Operator', and can be identified with 'OAM'

• 'Intrinsic' terms, where the factor  $\vec{x} \times$  does not appear, and can be identified with 'spin'

# Angular Momentum in QCD (Ji)

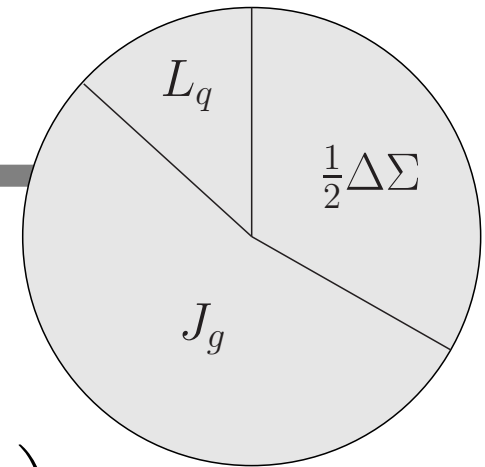
- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[ \psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{x} \times \left( i\vec{\partial} - g\vec{A} \right) \psi + \vec{x} \times \left( \vec{E} \times \vec{B} \right) \right]$$

with  $\Sigma^i = \frac{i}{2} \varepsilon^{ijk} \gamma^j \gamma^k$

- Ji does not integrate gluon term by parts, nor identify gluon spin/OAM separately
  - Ji-decomposition valid for all three components of  $\vec{J}$ , but usually only applied to  $\hat{z}$  component, where the quark spin term has a partonic interpretation
- (+) all three terms manifestly gauge invariant
- (+) DVCS can be used to probe  $\vec{J}_q = \vec{S}_q + \vec{L}_q$
- (-) quark OAM contains interactions
- (-) only quark spin has partonic interpretation as a single particle density

# Ji-decomposition



● Ji (1997)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left( \frac{1}{2}\Delta q + L_q \right) + J_g$$

with  $(P^\mu = (M, 0, 0, 1), S^\mu = (0, 0, 0, 1))$

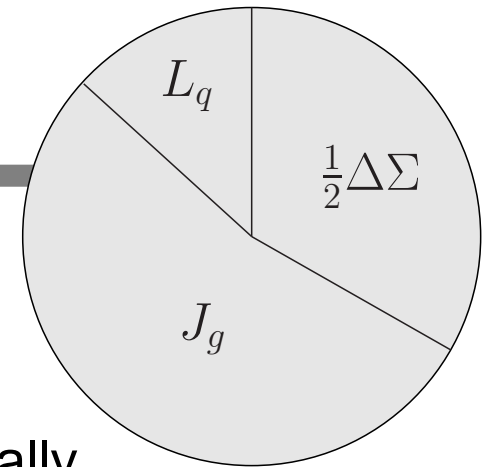
$$\frac{1}{2}\Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle \quad \Sigma^3 = i\gamma^1\gamma^2$$

$$L_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i\vec{D} \right)^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | \left[ \vec{x} \times \left( \vec{E} \times \vec{B} \right) \right]^3 | P, S \rangle$$

●  $i\vec{D} = i\vec{\partial} - g\vec{A}$

# Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left( \vec{r} \times i\vec{D} \right) q + \vec{r} \times \left( \vec{E} \times \vec{B} \right)$   
applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to  $\hat{z}$  component where at least quark spin has parton interpretation as difference between number densities
- $\Delta q$  from polarized DIS
- $J_q = \frac{1}{2} \Delta q + L_q$  from exp/lattice (GPDs)
- $L_q$  in principle independently defined as matrix elements of  $q^\dagger \left( \vec{r} \times i\vec{D} \right) q$ , but in practice easier by subtraction  $L_q = J_q - \frac{1}{2} \Delta q$
- $J_g$  in principle accessible through gluon GPDs, but in practice easier by subtraction  $J_g = \frac{1}{2} - J_q$
- Ji makes no further decomposition of  $J_g$  into intrinsic (spin) and extrinsic (OAM) piece

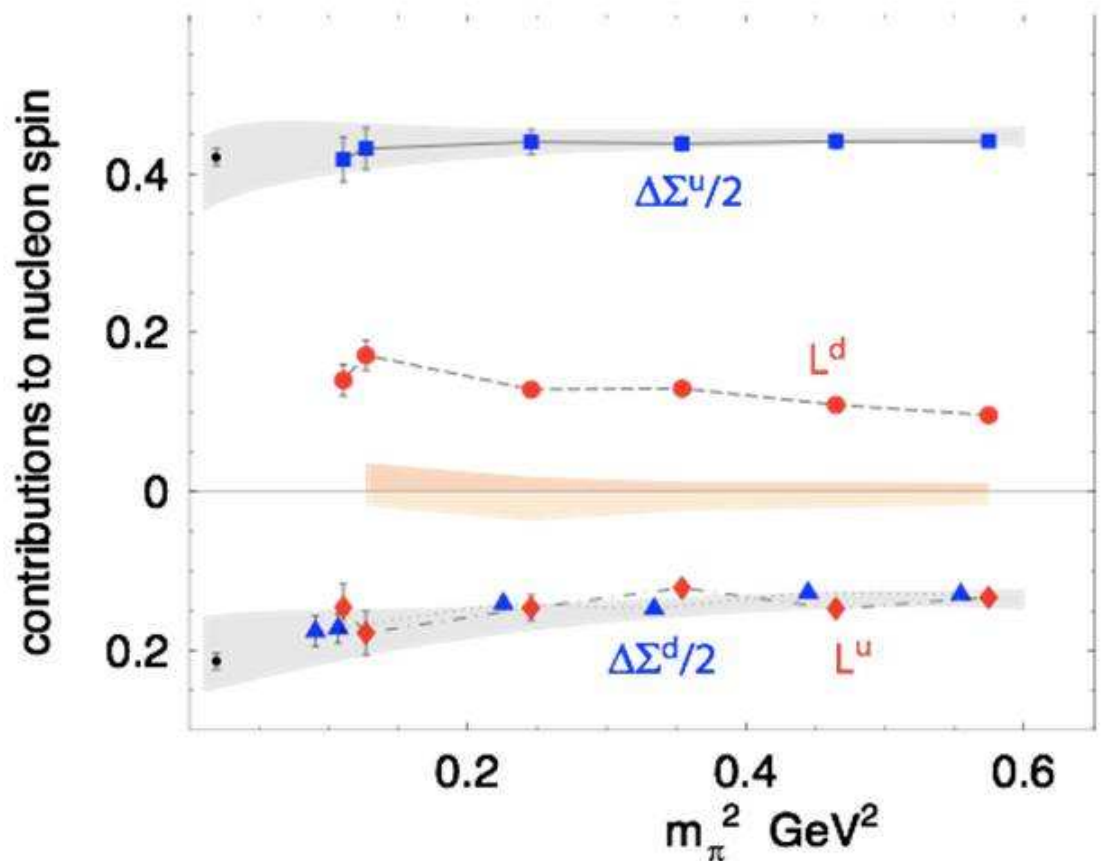
# $L_q$ for proton from Ji-relation (lattice)

- lattice QCD  $\Rightarrow$  moments of GPDs (LHPC; QCDSF)
- $\hookrightarrow$  insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx [H_q(x, 0) + E_q(x, 0)] x.$$

$$\hookrightarrow L_q^z = J_q^z - \frac{1}{2} \Delta q$$

- $L_u, L_d$  both large!
- present calcs. show  $L_u + L_d \approx 0$ , but
  - disconnected diagrams ..?
  - $m_\pi^2$  extrapolation
  - parton interpret. of  $L_q$ ...



# Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypersurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2 x_{\perp} \int dx^{-} M^{12+}$$

where  $x^{-} = \frac{1}{\sqrt{2}} (x^0 - x^1)$  and  $M^{12+} = \frac{1}{\sqrt{2}} (M^{120} + M^{123})$

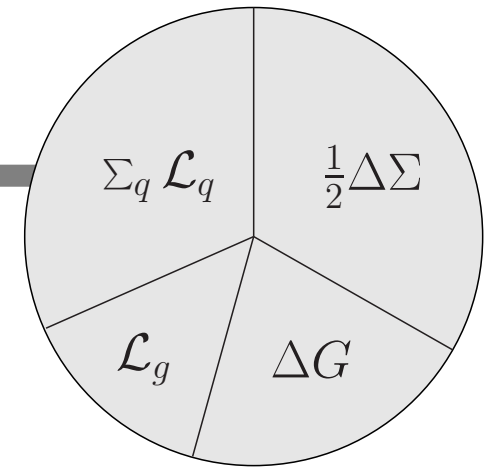
- Since  $\partial_{\mu} M^{12\mu} = 0$

$$\int d^2 \mathbf{x}_{\perp} \int dx^{-} M^{12+} = \int d^2 \mathbf{x}_{\perp} \int dx^3 M^{120}$$

(compare electrodynamics:  $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow$  flux in = flux out)

- use eqs. of motion to get rid of 'time' ( $\partial_{+}$  derivatives) & integrate by parts whenever a total derivative appears in the  $T^{i+}$  part of  $M^{12+}$

# Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge  
 $A^+ = 0$  one finds for  $J^z = \int dx^- d^2 \mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where ( $\gamma^+ = \gamma^0 + \gamma^z$ )

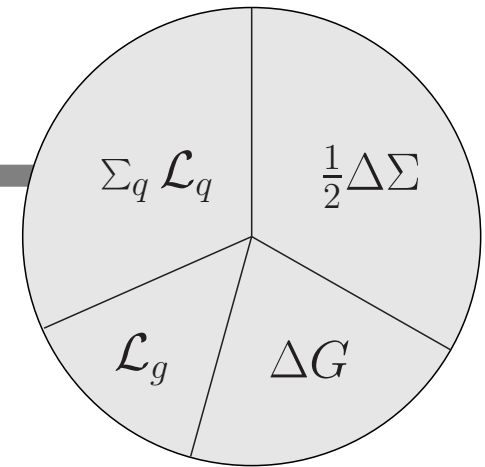
$$\mathcal{L}_q = \int d^3 r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3 r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

$$\mathcal{L}_g = 2 \int d^3 r \langle P, S | \text{Tr} F^{+j} (\vec{x} \times i\vec{\partial})^z A^j | P, S \rangle$$

# Jaffe/Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



- $\Delta\Sigma = \sum_q \Delta q$  from polarized DIS (or lattice)
- $\Delta G$  from  $\overrightarrow{p} \overleftarrow{p}$  or polarized DIS (evolution)
- ↪  $\Delta G$  gauge invariant, but local operator only in light-cone gauge
- $\int dx x^n \Delta G(x)$  for  $n \geq 1$  can be described by manifestly gauge inv. local op. (→ lattice)
- $\mathcal{L}_q, \mathcal{L}_g$  independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when  $A^+ = 0$
- parton net OAM  $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$  by subtr.  $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general,  $\mathcal{L}_q \neq L_q$        $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to ‘mix’ Ji and JM decompositions, e.g.  $J_g - \Delta G$  has no fundamental connection to OAM

$$L_q \neq \mathcal{L}_q$$

- $L_q$  matrix element of

$$q^\dagger \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right]^z q = \bar{q} \gamma^0 \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right]^z q$$

- $\mathcal{L}_q^z$  matrix element of  $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q} \gamma^+ \left[ \vec{r} \times i\vec{\partial} \right]^z q \Big|_{A^+=0}$$

- (for  $\vec{p} = 0$ ) matrix element of  $\bar{q} \gamma^z \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right]^z q$  vanishes (parity!)

- ↪  $L_q$  identical to matrix element of  $\bar{q} \gamma^+ \left[ \vec{r} \times \left( i\vec{\partial} - g\vec{A} \right) \right]^z q$  (nucleon at rest)

- ↪ even in light-cone gauge,  $L_q^z$  and  $\mathcal{L}_q^z$  still differ by matrix element of  $q^\dagger \left( \vec{r} \times g\vec{A} \right)^z q \Big|_{A^+=0} = q^\dagger (xgA^y - ygA^x) q \Big|_{A^+=0}$

# Summary part 1:

- Ji:  $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe:  $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- $\Delta G$  can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or  $\overrightarrow{p} \overleftarrow{p}$
- ↪ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with ‘spin’ only in that gauge
- in general  $L_q \neq \mathcal{L}_q$  or  $J_g \neq \Delta G + \mathcal{L}_g$ , but
- how significant is the difference between  $L_q$  and  $\mathcal{L}_q$ , etc. ?

# OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD 79, 071501 (2009)]

- toy model for nucleon where nucleon (mass  $M$ ) splits into quark (mass  $m$ ) and scalar 'diquark' (mass  $\lambda$ )
- ↪ light-cone wave function for quark-diquark Fock component

$$\psi_{+\frac{1}{2}}^{\uparrow}(x, \mathbf{k}_{\perp}) = \left(M + \frac{m}{x}\right) \phi \quad \psi_{-\frac{1}{2}}^{\uparrow} = -\frac{k^1 + ik^2}{x} \phi$$

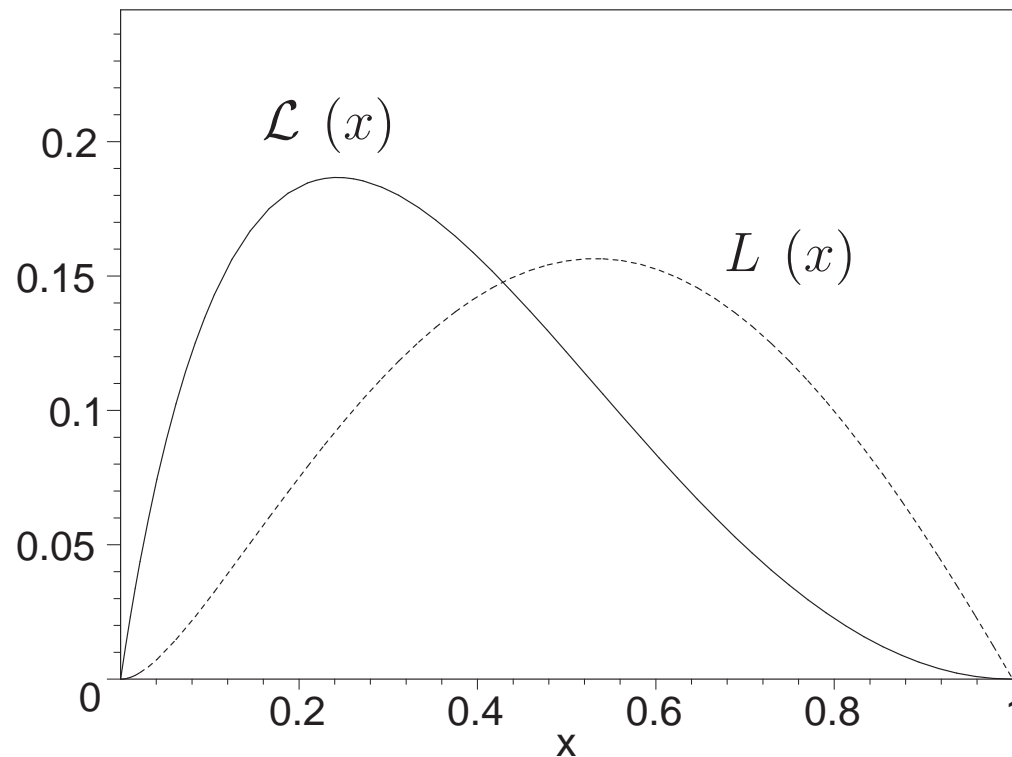
with  $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_{\perp}^2 + m^2}{x} - \frac{\mathbf{k}_{\perp}^2 + \lambda^2}{1-x}}$ .

- quark OAM according to JM:  $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_{\perp}}{16\pi^3} (1-x) \left| \psi_{-\frac{1}{2}}^{\uparrow} \right|^2$
- quark OAM according to Ji:  $L_q = \frac{1}{2} \int_0^1 dx x [q(x) + E(x, 0, 0)] - \frac{1}{2} \Delta q$
- ↪ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e.  $L_q = \mathcal{L}_q$
- not surprising since scalar diquark model is not a gauge theory

# OAM in scalar diquark model

- But, even though  $L_q = \mathcal{L}_q$  in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} (1-x) \left| \psi_{-\frac{1}{2}}^\uparrow \right|^2 \neq \frac{1}{2} \{x [q(x) + E(x, 0, 0)] - \Delta q(x)\} \equiv L_q(x)$$



↪ ‘unintegrated Ji-relation’ does not yield x-distribution of OAM

# OAM in QED

- light-cone wave function in  $e\gamma$  Fock component

$$\begin{aligned}\Psi_{+\frac{1}{2}+1}^\uparrow(x, \mathbf{k}_\perp) &= \sqrt{2} \frac{k^1 - ik^2}{x(1-x)} \phi & \Psi_{+\frac{1}{2}-1}^\uparrow(x, \mathbf{k}_\perp) &= -\sqrt{2} \frac{k^1 + ik^2}{1-x} \\ \Psi_{-\frac{1}{2}+1}^\uparrow(x, \mathbf{k}_\perp) &= \sqrt{2} \left( \frac{m}{x} - m \right) \phi & \Psi_{-\frac{1}{2}+1}^\uparrow(x, \mathbf{k}_\perp) &= 0\end{aligned}$$

- OAM of  $e^-$  according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2 \mathbf{k}_\perp \left[ (1-x) \left| \Psi_{+\frac{1}{2}-1}^\uparrow(x, \mathbf{k}_\perp) \right|^2 - \left| \Psi_{+\frac{1}{2}+1}^\uparrow(x, \mathbf{k}_\perp) \right|^2 \right]$$

- $e^-$  OAM according to Ji  $L_e = \frac{1}{2} \int_0^1 dx x [q(x) + E(x, 0, 0)] - \frac{1}{2} \Delta q$

$$\rightsquigarrow \mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$$

- Likewise, computing  $J_\gamma$  from photon GPD, and  $\Delta_\gamma$  and  $\mathcal{L}_\gamma$  from light-cone wave functions and defining  $\hat{L}_\gamma \equiv J_\gamma - \Delta_\gamma$  yields

$$\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$$

- $\frac{\alpha}{4\pi}$  appears to be small, but here  $\mathcal{L}_e, L_e$  are all of  $\mathcal{O}(\frac{\alpha}{\pi})$

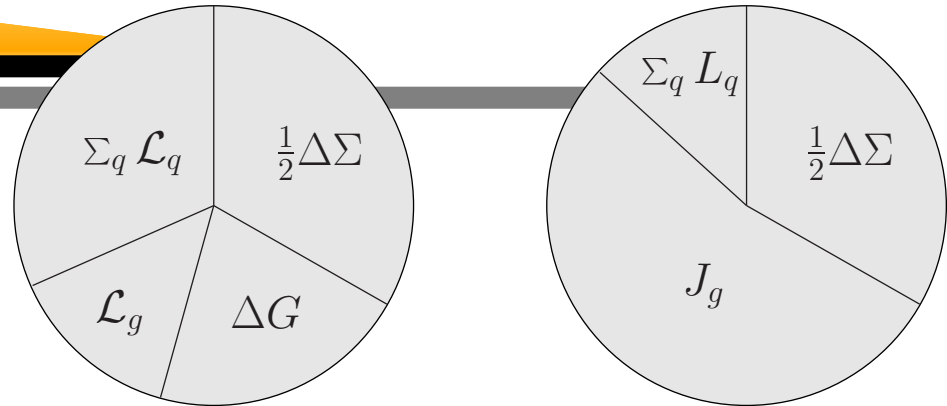
# OAM in QCD

- ↪ 1-loop QCD:  $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD):  $L_u \approx -.15$ ;  $L_d \approx +.15$
- QCD evolution yields negative correction to  $L_u$  and positive correction to  $L_d$
- ↪ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low  $Q^2$ ) and lattice results ( $Q^2 \sim 4\text{GeV}^2$ )
- above result suggests that  $\mathcal{L}_u > L_u$  and  $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- ↪ possible that lattice result consistent with  $\mathcal{L}_u > \mathcal{L}_d$

# Summary part 2

Jaffe & Manohar

Ji



- inclusive  $\overrightarrow{e} \overleftarrow{p} / \overrightarrow{p} \overleftarrow{p}$  provide access to
  - quark spin  $\frac{1}{2}\Delta q$
  - gluon spin  $\Delta G$
  - parton grand total OAM  $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin  $\frac{1}{2}\Delta q$
  - $J_q$  &  $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$  does not yield gluon OAM  $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$  for  $\mathcal{O}(\alpha_s)$  dressed quark

# pizza tre e mezzo stagioni



- Chen, Goldman et al.: integrate by parts in  $J_g$  only for term involving  $\mathbf{A}_{phys}$ , where

$$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys} \quad \text{with} \quad \nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$$

- $\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left( \frac{1}{2} \Delta q + L'_q \right) + S'_g + L'_g$  with  $\Delta q$  as in JM/Ji

$$L'_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) \left( \vec{x} \times i\vec{D}_{pure} \right)^3 q(\vec{x}) | P, S \rangle$$

$$S'_g = \int d^3x \langle P, S | \left( \vec{E} \times \vec{A}_{phys} \right)^3 | P, S \rangle$$

$$L'_g = \int d^3x \langle P, S | E^i \left( \vec{x} \times \vec{\nabla} \right)^3 A_{phys}^i | P, S \rangle$$

- $i\vec{D}_{pure} = i\vec{\partial} - g\vec{A}_{pure}$
- only  $\frac{1}{2}\Delta q$  accessible experimentally



# example: angular momentum in QED

- consider now, QED with electrons:

$$\vec{J}_\gamma = \int d^3r \vec{x} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{x} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})]$$

- integrate by parts

$$\vec{J} = \int d^3r \left[ E^j (\vec{x} \times \vec{\nabla}) A^j + (\vec{x} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]$$

- replace  $2^{nd}$  term (eq. of motion  $\vec{\nabla} \cdot \vec{E} = e j^0 = e \psi^\dagger \psi$ ), yielding

$$\vec{J}_\gamma = \int d^3r \left[ \psi^\dagger \vec{r} \times e \vec{A} \psi + E^j (\vec{x} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A} \right]$$

- $\psi^\dagger \vec{r} \times e \vec{A} \psi$  cancels similar term in electron OAM  $\psi^\dagger \vec{r} \times (\vec{p} - e \vec{A}) \psi$

- ↪ decomposing  $\vec{J}_\gamma$  into spin and orbital also shuffles angular momentum from photons to electrons!

# pizza tre e mezzo stagioni



- Chen, Goldman et al.: integrate by parts in  $J_g$  only for term involving  $\mathbf{A}_{phys}$ , where

$$\mathbf{A} = \mathbf{A}_{pure} + \mathbf{A}_{phys} \quad \text{with} \quad \nabla \cdot \mathbf{A}_{phys} = 0 \quad \nabla \times \mathbf{A}_{pure} = 0$$

# Announcement:

- workshop on **Orbital Angular Momentum of Partons in Hadrons**
- ECT\* 9-13 November 2009
- organizers: M.B. & Gunar Schnell
- confirmed participants: M.Anselmino, H.Avakian, A.Bacchetta, L.Bland, D.Boer, S.J.Brodsky, M.Diehl, D.Fields, L.Gamberg, G.Goldstein, M.Grosse-Perdekamp, P.Hägler, X.Ji, R.Kaiser, E.Leader, S.Liutti, N.Makins, A.Miller, D.Müller, P.Mulders, A.Schäfer, G.Schierholz, O.Teryaev, W.Vogelsang, F.Yuan

# The Ji-relation (poor man's derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that  $L_q$  can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^1 dx x [H_q(x, \xi, 0) + E_q(x, \xi, 0)]$$

(nucleon at rest;  $\vec{S}$  is nucleon spin)

$$\hookrightarrow L_q^z = J_q^z - \frac{1}{2} \Delta q$$

- derivation (MB-version):

- consider nucleon state that is an eigenstate under rotation about the  $\hat{x}$ -axis (e.g. nucleon polarized in  $\hat{x}$  direction with  $\vec{p} = 0$  (wave packet if necessary))

- for such a state,  $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$  and  $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

$$\hookrightarrow \langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$$

- relate  $2^{nd}$  moment of  $\perp$  flavor dipole moment to  $J_q^x$

# The Ji-relation (poor man's derivation)

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the  $\hat{x}$ -axis (e.g. nucleon polarized in  $\hat{x}$  direction with  $\vec{p} = 0$  (wave packet if necessary))
  - for such a state,  $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$  and  $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$ 
    - ↪  $\langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$
    - ↪ relate 2<sup>nd</sup> moment of  $\perp$  flavor dipole moment to  $J_q^x$
  - effect sum of two effects:
    - $\langle T^{++} y \rangle$  for a point-like transversely polarized nucleon
    - $\langle T_q^{++} y \rangle$  for a quark relative to the center of momentum of a transversely polarized nucleon
  - 2<sup>nd</sup> moment of  $\perp$  flavor dipole moment for point-like nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} f(r) \end{pmatrix} \chi \quad \text{with} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

# The Ji-relation (poor man's derivation)

• derivation (MB-version):

- $T_q^{0z} = i\bar{q} (\gamma^0 \partial^z + \gamma^z \partial^0) q$

- since  $\psi^\dagger \partial_z \psi$  is even under  $y \rightarrow -y$ ,  $i\bar{q}\gamma^0 \partial^z q$  does not contribute to  $\langle T^{0z} y \rangle$

↪ using  $i\partial_0 \psi = E\psi$ , one finds

$$\begin{aligned} \langle T^{0z} b_y \rangle &= E \int d^3 r \psi^\dagger \gamma^0 \gamma^z \psi y = E \int d^3 r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y \\ &= \frac{2E}{E+M} \int d^3 r \chi^\dagger \sigma^z \sigma^y \chi f(r) (-i) \partial^y f(r) y = \frac{E}{E+M} \int d^3 r f^2(r) \end{aligned}$$

- consider nucleon state with  $\vec{p} = 0$ , i.e.  $E = M$  &  $\int d^3 r f^2(r) = 1$

↪  $2^{nd}$  moment of  $\perp$  flavor dipole moment  $\langle T_q^{++} y \rangle = \langle T^{0z} b_y \rangle = \frac{1}{2}$

↪ 'overall shift' of nucleon COM yields contribution

$$\frac{1}{2} \int dx x H_q(x, 0, 0) \text{ to } \langle T_q^{++} y \rangle$$

# The Ji-relation (poor man's derivation)

- spherically symmetric wave packet for Dirac particle with  $J_x = \frac{1}{2}$  centered around the origin has  $\perp$  center of momentum  $\frac{1}{M} \langle T_q^{++} b_y \rangle$  not at origin, but at  $\frac{1}{2M}$ !
- consistent with

$$\frac{1}{2} = \langle J_x \rangle = \langle (T^{0z} b^y - T^{0y} b^z) \rangle = 2 \langle T^{0z} b^y \rangle = \langle T^{++} b^y \rangle$$

- 'overall shift of  $\perp$  COM yields  $\langle T_q^{++} b_y \rangle = \frac{1}{2} \int dx x H_q(x, 0, 0)$
- intrinsic distortion adds  $\frac{1}{2} \int dx x E_q(x, 0, 0)$  to that

# Transversely Deformed Distributions and $E(x, 0, -\Delta_{\perp}^2)$

M.B., Int.J.Mod.Phys.A18, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ( $\xi = 0$ ):

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \uparrow \rangle = H(x, 0, -\Delta_{\perp}^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+ x^-} \langle P+\Delta, \uparrow | \bar{q}(0) \gamma^+ q(x^-) | P, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, 0, -\Delta_{\perp}^2).$$

- Consider nucleon polarized in  $x$  direction (in IMF)

$$|X\rangle \equiv |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \uparrow\rangle + |p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, \downarrow\rangle.$$

- ↪ unpolarized quark distribution for this state:

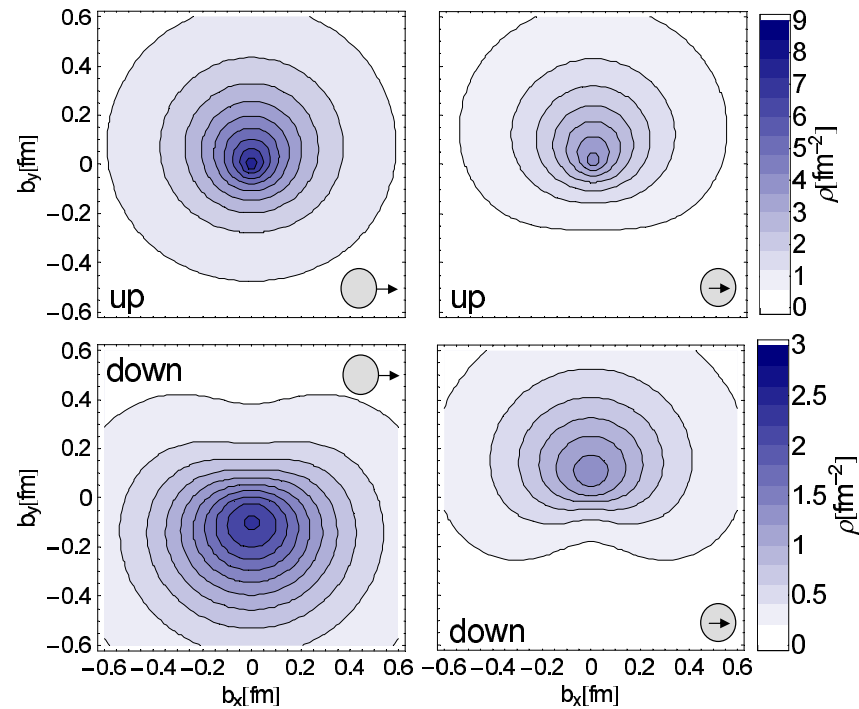
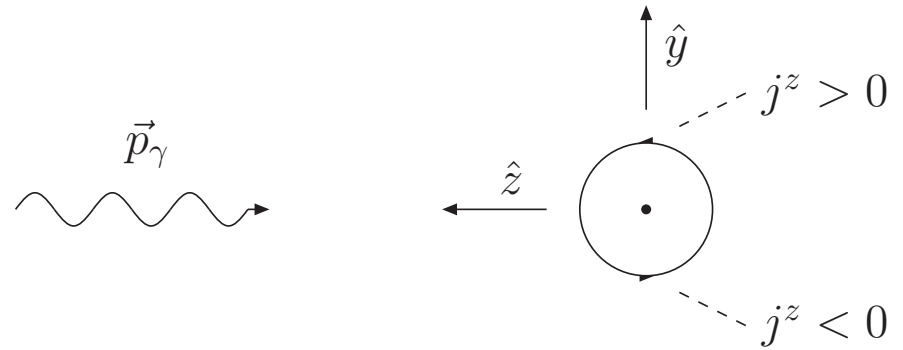
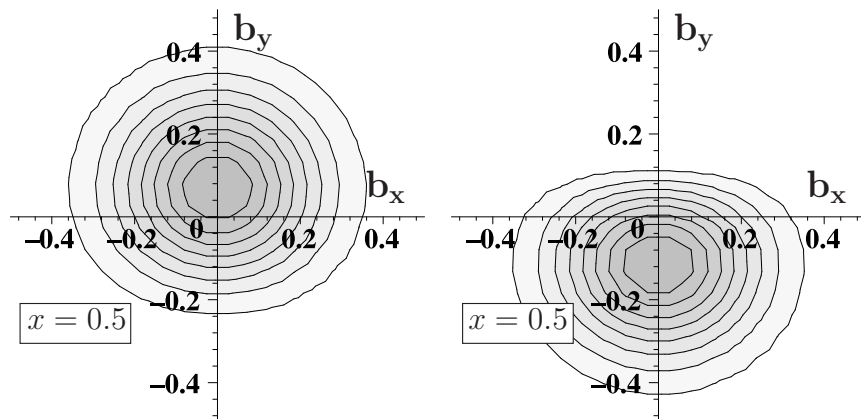
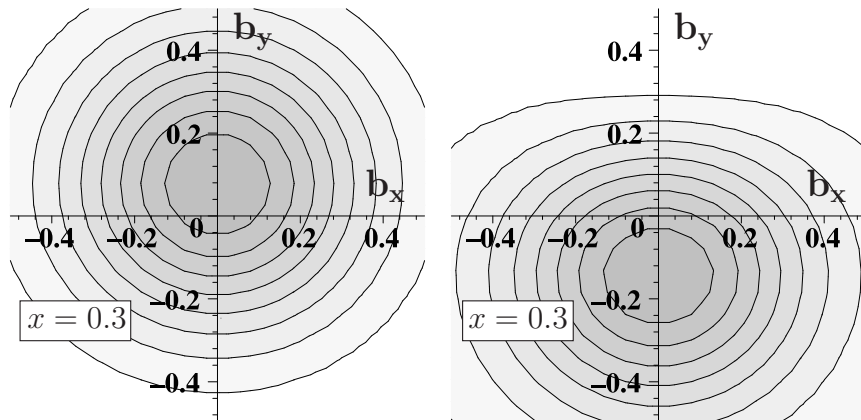
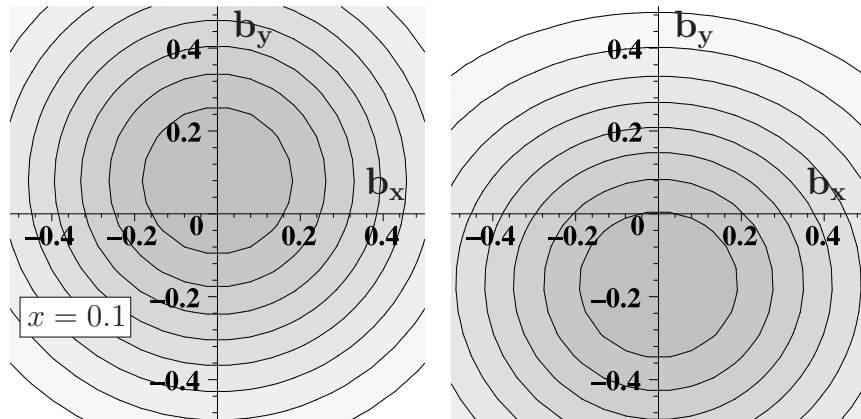
$$q(x, \mathbf{b}_{\perp}) = \mathcal{H}(x, \mathbf{b}_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} E(x, 0, -\Delta_{\perp}^2) e^{-i\mathbf{b}_{\perp} \cdot \Delta_{\perp}}$$

- Physics:  $j^+ = j^0 + j^3$ , and left-right asymmetry from  $j^3$  !  
[X.Ji, PRL 91, 062001 (2003)]

# p polarized in $+\hat{x}$ direction

$u(x, \mathbf{b}_\perp)$

$d(x, \mathbf{b}_\perp)$



lattice results (Hägl er et al.)

# The Ji-relation (poor man's derivation)

- 'overall shift of  $\perp$  COM yields  $\langle T_q^{++} b_y \rangle = \frac{1}{2} \int dx x H_q(x, 0, 0)$
  - intrinsic distortion adds  $\frac{1}{2} \int dx x E_q(x, 0, 0)$  to that
- ↪ Ji relation

$$J_q^x = \frac{1}{2} \int dx x [H_q(x, 0, 0) + E_q(x, 0, 0)]$$

- rotational invariance: should apply to each vector component, but parton interpretation (transverse shift) only for  $\perp$  pol. nucleon