INT. WORKSHOP ON RELATIVISTIC DESCRIPTION OF TWO- AND THREE-BODY SYSTEMS IN NUCLEAR PHYSICS,
ECT OCTOBER 19th - 23th, 2009*

Projecting the Bethe-Salpeter equation onto the light-front

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Outline:

I. A snapshot on LF dynamics

II. LF valence dynamics and BS equation

III. Quasi-Potential & Light-Front B.-S. Equation

IV. Two-boson systems: E.M. current operator & WTI

V. Two-Fermion systems: Yukawa model

VI. LF three-boson dynamics & ladder 4d B.-S. equation

VII. Zero-range model: LO and NLO

VIII. Conclusions and Perspectives

I. A Snapshot on Light-Front dynamics: $x^+ = t+z=0$

S.J. Brodsky et al. / Physics Reports 301 (1998) 299–486

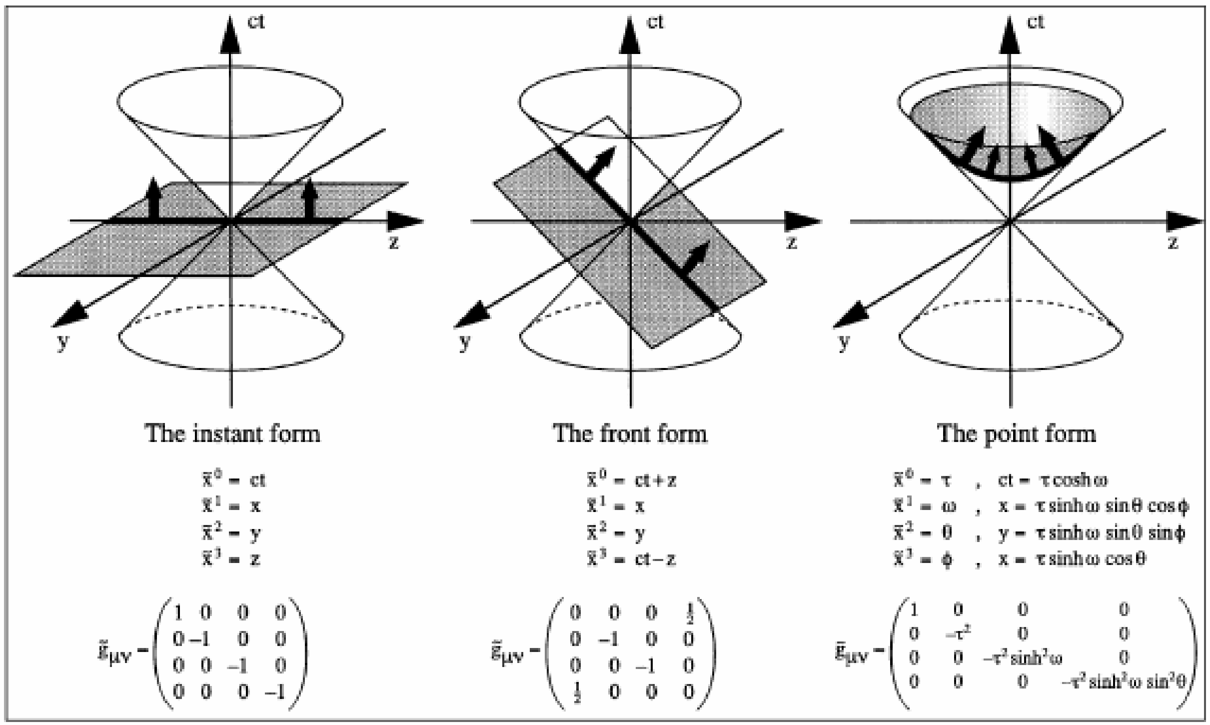


Fig. 1. Dirac's three forms of Hamiltonian dynamics.

Properties of LF quantization

1. **Trivial vacuum - perturbative (except for zero modes);**
2. **Maximal number of 7 kinematical transf. (3 boosts + 1 rot. + 3 transl.)**
3. **Truncation in the Fock-space not stable under rotations around transverse directions (non-kinematical boosts);**

II. LF valence dynamics and BS equation

OUR AIM:
Derive the dynamics of few-constituents on the LF from a given model for the Bethe-Salpeter equation.

- 4d Bethe-Salpeter amplitude $\leftarrow\rightarrow$ ***Valence state on the LF***

(Kinematical momenta: $\boldsymbol{k}^+ = \boldsymbol{k}^0 + \boldsymbol{k}^3$ and $\boldsymbol{k}_\perp$)

Integration on the "Energy:" $k^- = k^0 - k^3$

"Iterated Resolvents-dynamics of the valence state"- Brodsky, Pauli, Pinsky, Phys. Rept. 301(98)299; Frederico et al NPA737(04)260c

In practice the truncation of LF Fock-space is considered...

- Perturbative vacuum (no self-energies/vertex corrections);

- Stable under the maximal number of 7 kinematical boosts;

- Rotational invariance is not fulfilled (non-kinematical boosts).

Not complete list of previous works...

LF two-boson/ two-fermion systems: (C-R Ji,Perry,Miller,Karmanov,Carbonell,Brinet Mathiot, Bakker, Amghar,Desplanques...)

Quasi Potential Approach to LF: Sales et al PRC61(00)044003; 63(01)064003

LF conserved current operators: Kvinikhidze & Blankleider PRD68(03)02581

WTI -QP two-boson/two-fermion - Marinho et al PRD76(07)096001;PRD77(08)116010

→Electroweak observables, form-factors & GPD (Nonvalence contributions)

GPD/two-body syst.: Tiburzi & Miller PRD67(03)054014; 054015;

Pion ff q+ > 0 (SL & TL): de Melo et al PRD73(06) 074013 ;

Pion/GPD: Ji, Mischenko, Radyuskin PRD73(06)114013;

Bakker, Ji, Choi, Pasquini, Salme,Pace...

LF Dynamics of three-body systems: Bakker, Kondratyuk, Terentev, NPB158(79)497

Zero-Range model & BS eq. - Frederico PLB282(92)409; Carbonell & Karmanov PRC67(03)037001; Marinho & Frederico PoS(LC2008)036; Karmanov & Maris PoS LC2008, 037 (2008), FBS 46, 95 (2009).

LFD of 3-constituents: valence state ←→ 4d Bethe-Salpeter eq. 3-legs

qqq - Mitra, Ann.Phys. 318(08)845

Non-perturbative renormalization with truncated Fock-space:

Karmanov, Mathiot, Smirnov PRD77(08) 085028

II. Quasi-Potential & LF B.-S. Equation: 2-bosons

Starting with a 4-dimensional BS equation for 2→2 scattering amplitude (no self energies/vertex corrections):

$$T = V + VG_0T$$

V is the sum of two-body irreducible diagrams

$$T(K) = W(K) + W(K)\widetilde{G}_0(K)T(K)$$

Woloshyn & Jackson NPB64(1973)269

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \qquad \Delta_0(K) = G_0(K) - \widetilde{G}_0(K)$$

LF time projection: integration in *k*⁻

Sales, F., Sauer. PRC61(2000)044003

$$\widetilde{G}_0(K) := G_0(K)|g_0^{-1}(K)|G_0(K)$$

$$\left\langle k_1'^- \right| G_0(K) \left| k_1^- \right\rangle = -\frac{1}{2\pi}\frac{\delta\left(k_1'^- - k_1^-\right)}{\widehat{k}_1^+(K^+ - \widehat{k}_1^+)\left(k_1^- - \frac{\widehat{\vec{k}}_{1\perp}^2 + m_1^2 - io}{\widehat{k}_1^+}\right)\left(K^- - k_1^- - \frac{\widehat{\vec{k}}_{2\perp}^2 + m_2^2 - io}{K^+ - \widehat{k}_1^+}\right)}$$

$$|G_0(K)| := \int dk_1'^- dk_1^- \left\langle k_1'^- \right| G_0(K) \left| k_1^- \right\rangle = \frac{i\theta(K^+ - \widehat{k}_1^+)\theta(\widehat{k}_1^+)}{\widehat{k}_1^+(K^+ - \widehat{k}_1^+)\left(K^- - \widehat{k}_{1on}^- - \widehat{k}_{2on}^- + io\right)}$$

$$k^-{}_{\rm on} = \frac{\vec{k}^2{}_\perp + m^2}{k^+}$$

Valence propagator in global LF time

$$|G(K)| = |G_0(K)| + |G_0(K)T(K)G_0(K)|$$

Valence→ Valence scattering amplitude

$$t(K) := g_0(K)^{-1}|G_0(K)T(K)G_0(K)|g_0(K)^{-1}$$

$$t(K) = w(K) + w(K)g_0(K)t(K)$$

Effective interaction

$$w(K) := g_0(K)^{-1}|G_0(K)W(K)G_0(K)|g_0(K)^{-1}$$

$$W(K) = V(K)\sum_{i=0}^{\infty}[\Delta_0(K)V(K)]^i$$

Bethe-Salpeter amplitude for scattering/bound states

For scattering states: $$|\Psi^+\rangle = |\Psi_0\rangle + G_0(K)V(K)|\Psi^+\rangle$$

and the corresponding homogeneous equation for the bound state

Valence wave function for bound/scattering states: $$|\phi\rangle = \| \Psi\rangle$$

Homogeneous equation for the LF valence wave function of a bound state (projecting the 4-dim BS equation or from the bound-state pole of the 3-dim t-matrix)

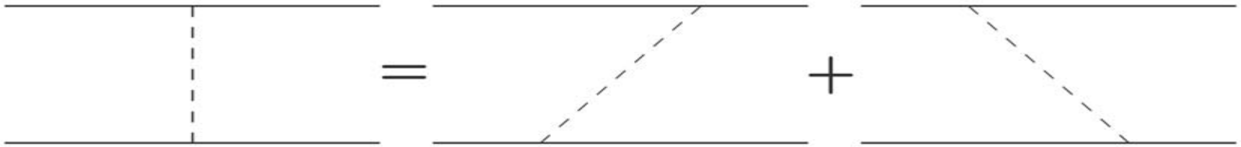
$$|\phi_B\rangle = g_0(K_B)w(K_B)|\phi_B\rangle$$

$$\int dk_1^- \langle k_1^- | \Psi_B\rangle = |\phi_B\rangle$$

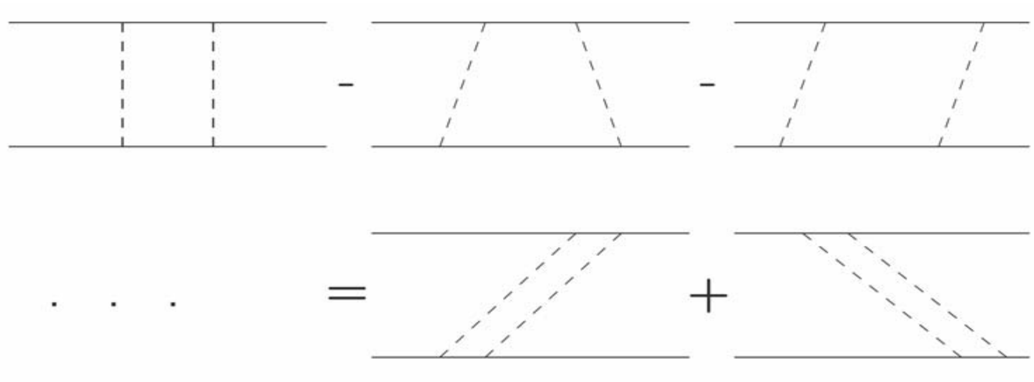
Example: Bosonic Yukawa model

$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

$$w^{(1)} =$$

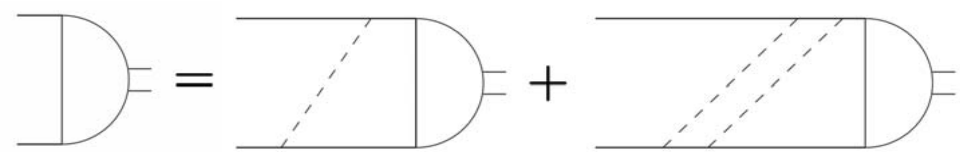


$$w^{(2)} =$$



Mass^2 eigenvalue eq. & valence wf:

$$\boxed{g(K_\lambda)^{-1} \, |\phi_\lambda\rangle = 0}$$



$$\left[g_0^{-1} - w \right]$$

LF Bound state equation

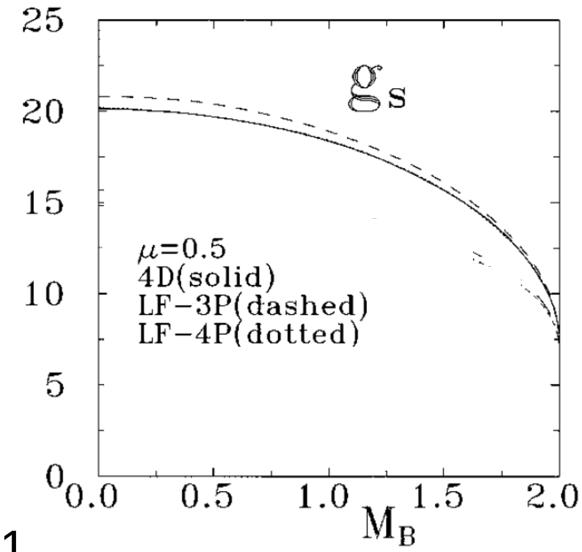
$$\phi_B(y,\vec{k}'_\perp;K) = \frac{\gamma(y,\vec{k}'_\perp;K)}{K^2 - M_0^2}$$

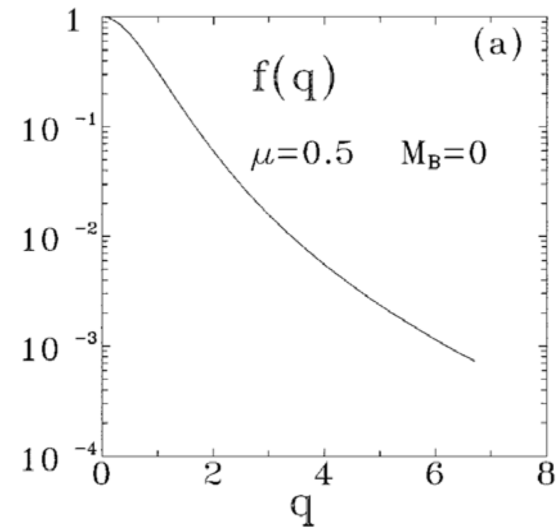
$$\gamma(y,\vec{k}'_\perp;K) = \frac{1}{(2\pi)^3}\int \frac{d^2k_\perp dx}{2x(1-x)}\gamma(x,\vec{k}_\perp;K)\frac{\mathcal{K}^{(2)}(y,\vec{k}'_\perp;x,\vec{k}_\perp) + \mathcal{K}^{(4)}(y,\vec{k}'_\perp;x,\vec{k}_{1\perp})}{K^2 - M_0^2}$$

$$\mathcal{K}^{(2)}(y,\vec{k}'_\perp;x,\vec{k}_\perp) = g_S^2 \frac{\theta(x-y)}{(x-y)\left(K^2 - \left(M_0^{(3)}\right)^2 + i\epsilon\right)} + \left[x \leftrightarrow y, \vec{k}'_\perp \leftrightarrow \vec{k}_\perp\right]$$

$$\left(M_0^{(3)}\right)^2 = \frac{\vec{k}'^2_\perp + m^2}{y} + \frac{\vec{k}^2_\perp + m^2}{1-x} - \frac{(\vec{k}'_\perp + \vec{k}_\perp)^2 + \mu^2}{x-y}$$

Comparision between LF (3d) and 4d results for bound states





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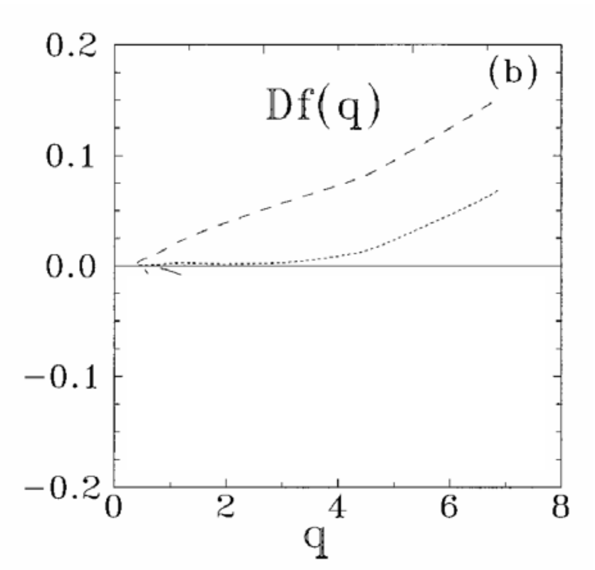


$$f_{\text{exact}}(\sqrt{\vec{k}^2_{1\perp}}) := \int dk_1^- dk_1^+ \langle k_1 | \Psi_B \rangle$$

$$f^{(n)}_{\text{app}}(\sqrt{\vec{k}^2_{1\perp}}) = \int dk_1^+ \langle k_1^+ \vec{k}_{1\perp} | \phi_B \rangle^{(n)}_{\text{app}}$$

$$Df(q) = 1 - f^{(n)}_{\text{app}}(q) / f_{\text{exact}}(q)$$

$\Rightarrow$ Hierarchy Equations in LF Fock-space

J. H.O. Sales PhD thesis ITA (unpublished), Frederico et al NPA737(04)260c

- Interaction m.e.'s:

$$\langle q k_\sigma | v | k \rangle = -2(2\pi)^3 \delta(q + k_\sigma - k) \frac{g_S}{\sqrt{q^+ k_\sigma^+ k^+}} \theta(k_\sigma^+)$$

$$\langle q | v | k_\sigma k \rangle = -2(2\pi)^3 \delta(k + k_\sigma - q) \frac{g_S}{\sqrt{q^+ k_\sigma^+ k^+}} \theta(k_\sigma^+)$$

Interaction Fock-space

- Hierarchy Eqs.:

$$g^{(2)}(K) = g_0^{(2)}(K) + g_0^{(2)}(K) v g^{(3)}(K) v g^{(2)}(K),$$

$$g^{(3)}(K) = g_0^{(3)}(K) + g_0^{(3)}(K) v g^{(4)}(K) v g^{(3)}(K),$$

$$g^{(4)}(K) = g_0^{(4)}(K) + g_0^{(4)}(K) v g^{(5)}(K) v g^{(4)}(K),$$

$\ldots$

$$g^{(N)}(K) = g_0^{(N)}(K) + g_0^{(N)}(K) v g^{(N+1)}(K) v g^{(N)}(K),$$

$\ldots$

Subtraction of divergences?

- Truncation: $|\Phi_1, \Phi_2, (N-2)\sigma\rangle$

Iterated resolvents: Brodsky, Pauli, Pinsky, Phys. Rep. **301** (98) 299

Reconstructing 4-d B.S. amplitude from the LF valence wf:

$$|\Psi\rangle = G(K)|g(K)^{-1}|\phi\rangle$$

(*i e* → 0)

$$g(K_\lambda)^{-1}\,|\phi_\lambda\rangle = 0$$

(projecting back to the LF retrieves the valence wf.)

<BS Ampl.| 4d operator |BS Ampl> → <val.|3d operator |val.>

Reverse LF time projection operation: expansion W

$$G(K)|g(K)^{-1} = [1 + \Delta_0(K)W(K)]\,G_0(K)|g_0(K)^{-1}$$

III. Two-boson systems: E.M. current operator & WTI

$$\langle\Psi_f|\,\mathcal{J}^\mu(Q)\,|\Psi_i\rangle \qquad\qquad Q = K_f - K_i$$

$$\mathcal{J}^\mu(Q) = \mathcal{J}_0^\mu(Q) + \mathcal{J}_I^\mu(Q)$$

Ward-Takahashi in operator form: (Gross & Riska PRC36(1987)1928)

$$\boxed{Q_\mu \mathcal{J}^\mu(Q) = \left[G^{-1}, \widehat{e}_1\right] + (1 \leftrightarrow 2)}$$

$$\langle k_i|\widehat{e}_i|p_i\rangle = e_i \delta^4\left(k_i - p_i - Q\right)$$

$$G^{-1}(K_\lambda)\,|\Psi_\lambda\rangle = 0 \quad \Longrightarrow \quad \boxed{Q_\mu\,\langle\Psi_f|\,\mathcal{J}^\mu(Q)\,|\Psi_i\rangle = 0}$$

Light-front e.m. current operator: valence states

$$\langle \Psi_f | \mathcal{J}^\mu(Q) | \Psi_i \rangle = \langle \phi_f | j^\mu(K_f, K_i) | \phi_i \rangle$$

$$|\Psi\rangle = G(K)|g(K)^{-1}|\phi\rangle$$

$$\boxed{j^\mu(K_f, K_i) := g(K_f)^{-1}|G(K_f)\mathcal{J}^\mu(Q)G(K_i)|g(K_i)^{-1}}$$

Gauging method for bound states: Kvinikhidze and Blankleider (PRD68 (2003) 025021)

LF Projection WTI:

$$Q_\mu |G(K_f)\mathcal{J}^\mu(Q)G(K_i)| = |\,[\widehat{e}_1, G]\,| + (1 \leftrightarrow 2)$$

$$\boxed{Q^\mu j_\mu(K_f, K_i) = \left[g^{-1}, \widehat{e}_{LF}\right]}$$

$$\langle k_i^+, \vec{k}_{i\perp} | \widehat{e}_{i,LF} | p_i^+, \vec{p}_{i\perp} \rangle = e_i \delta\left(k_i^+ - p_i^+ - Q^+\right) \delta^2\left(\vec{k}_{i\perp} - \vec{p}_{i\perp} - \vec{Q}_\perp\right)$$

LF charge operator

LF e.m. current is conserved

$$j^\mu(K_f,K_i) = g_0(K_f)^{-1}|G_0(K_f)\,[1+W(K_f)\Delta_0(K_f)]\times$$
$$\mathcal{J}^\mu(Q)\,[1+\Delta_0(K_i)W(K_i)]\,G_0(K_i)|g_0(K_i)^{-1}$$

$$\left.\begin{aligned} W^{(n)}(K) &= \sum_{i=1}^{n} W_i(K) \\ W_n &= V(K)[\Delta_0(K)V(K)]^{n-1} \end{aligned}\right\} \quad w^{(n)}(K) = \sum_{i=1}^{n} g_0(K)^{-1}|G_0(K)W_i(K)G_0(K)|g_0(K)^{-1}$$

Truncation in W keeps c.c.? No!

$$Q^\mu\langle\phi_f^{(n)}|j_\mu^{(n)}(K_f,K_i)|\phi_i^{(n)}\rangle = \langle\Psi_f^{(n)}|Q^\mu\mathcal{J}_\mu(Q)|\Psi_i^{(n)}\rangle$$
$$= \langle\Psi_f^{(n)}|\left[\widehat{e},G^{-1}\right]|\Psi_i^{(n)}\rangle \neq 0$$

Conserved & truncated LF e.m. current: WTI

Marinho, F., Sauer, PRD 76, 096001(07)

$$Q^\mu j_\mu^{c(n)}(K_f, K_i) = \left[g_n^{-1}, \widehat{e}_{LF}\right]$$

$$j^{c\mu(n)} = g_0^{-1}|G_0 \left[\mathcal{J}^\mu(Q) + \sum_{i=1}^{n-1}(W_i\Delta_0\mathcal{J}^\mu(Q) + \mathcal{J}^\mu(Q)\Delta_0 W_i) + \sum_{i=2}^{n-1}\sum_{j=1}^{i-1} W_j\Delta_0\mathcal{J}^\mu(Q)\Delta_0 W_{i-j}\right.$$

$$\left. + W_n\Delta_0\mathcal{J}_0^\mu(Q) + \mathcal{J}_0^\mu(Q)\Delta_0 W_n + \sum_{i=1}^{n-1} W_i\Delta_0\mathcal{J}_0^\mu(Q)\Delta_0 W_{n-i}\right] G_0|g_0^{-1}$$

Message:

Keep in the current all LF two-body irreducible terms consistent with the truncation of the interaction

⇨ ***Conserved e.m. current!***

Current in the Yukawa model for 2-boson systems

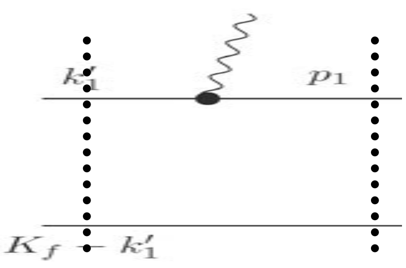
4d Ladder B.-S.

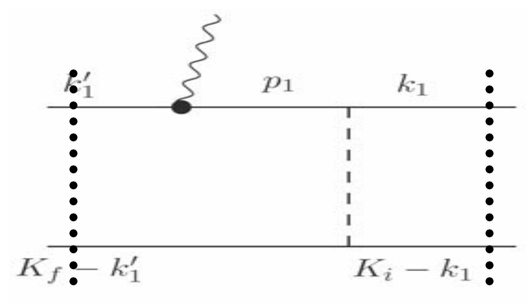
$$\mathcal{L}_I = g_S \phi_1^\dagger \phi_1 \sigma + g_S \phi_2^\dagger \phi_2 \sigma$$

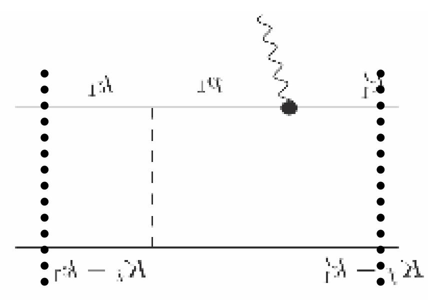
$$\langle k_1 | \mathcal{J}_0^\mu(Q) | p_1 \rangle = -2\pi [e_1 (k_1 + p_1)^\mu \delta^4 (k_1 - p_1 - Q)((K_f - k_1)^2 - m_2^2)]$$

$$j^{c\mu(1)} = j^{c\mu(0)} + g_0^{-1} | G_0 \left[W_1 \Delta_0 \mathcal{J}_0^\mu(Q) + \mathcal{J}_0^\mu(Q) \Delta_0 W_1 \right] G_0 | g_0^{-1}$$

$$j^{c\mu(0)} = g_0^{-1} | G_0 \mathcal{J}_0^\mu(Q) G_0 | g_0^{-1}$$

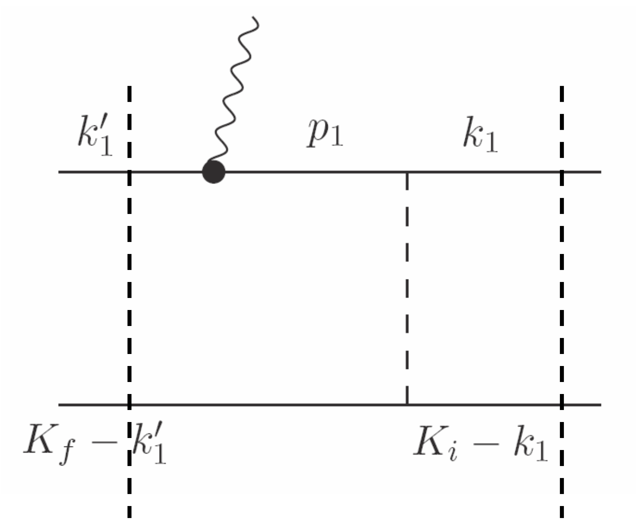


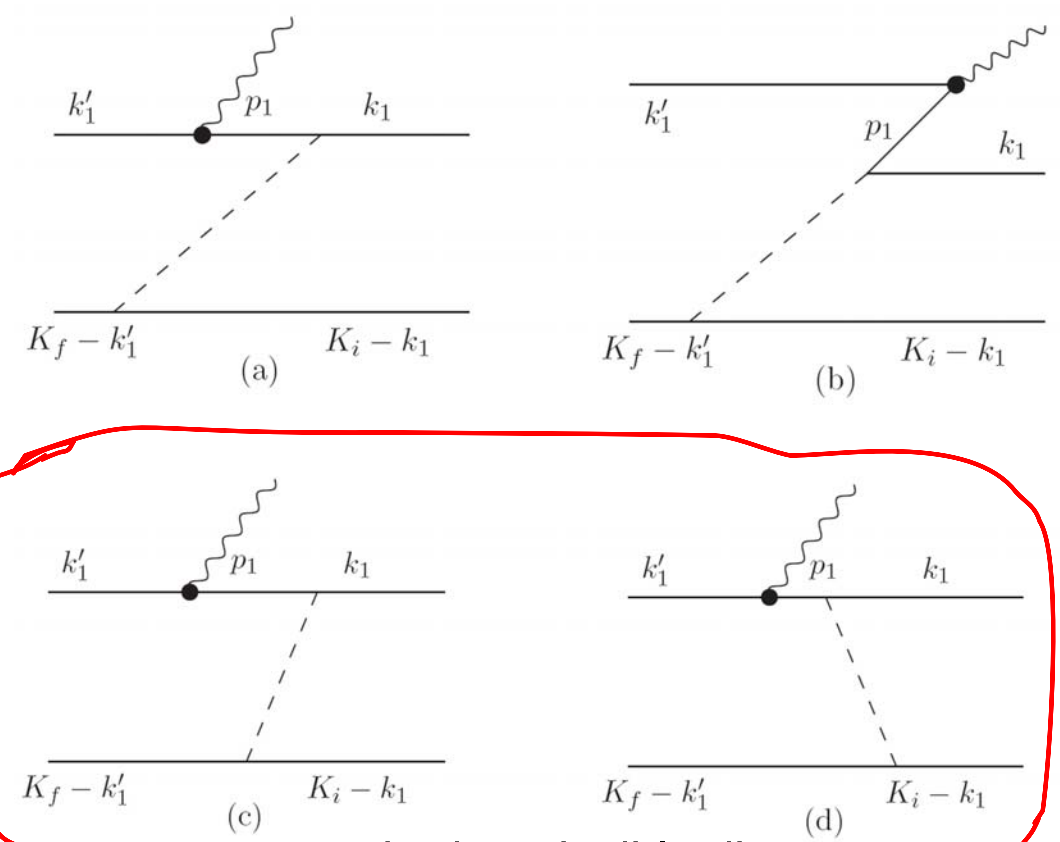




- 2-body LF reducible terms

LF current in 1st order



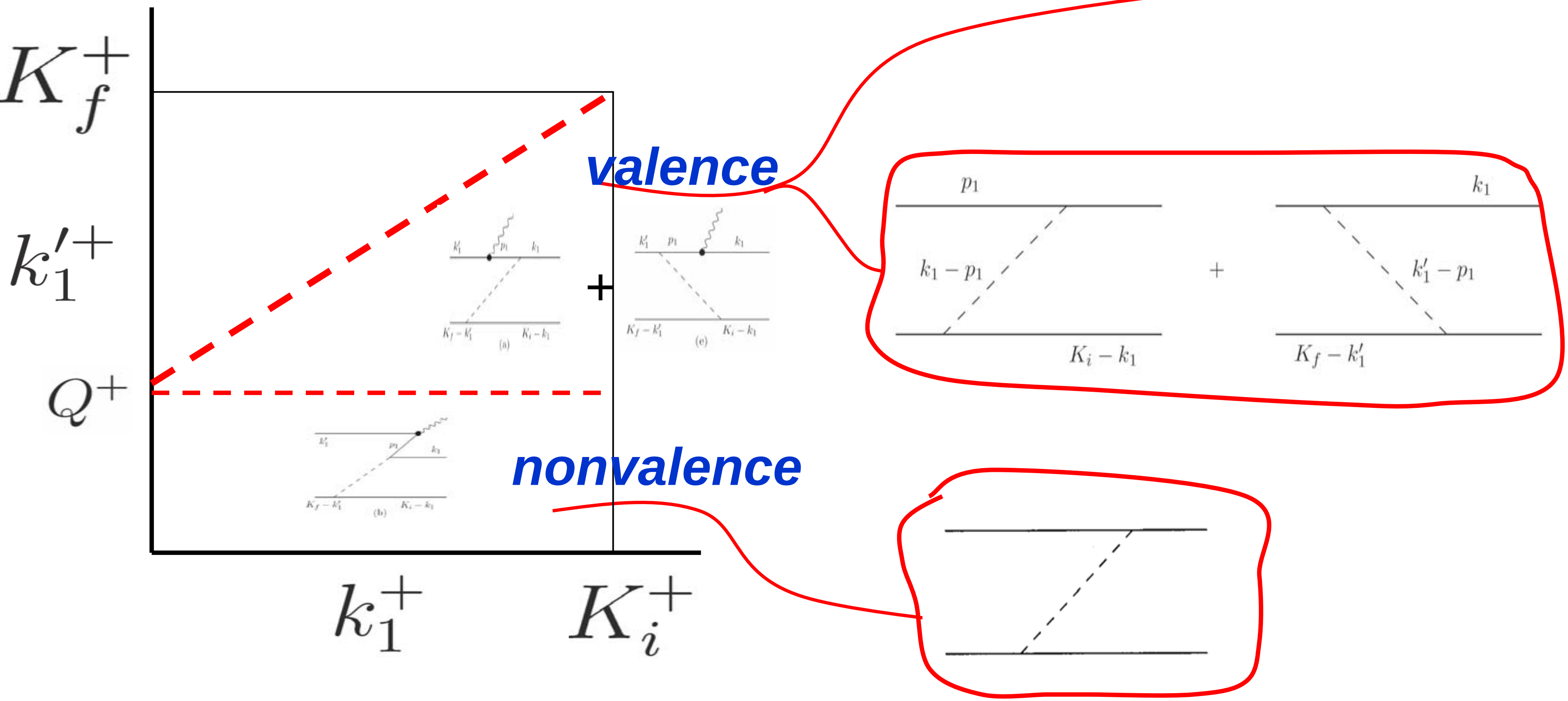


LF 2-body reducible diagrams

$$K_i^+ > 0 \text{ and } Q^+ \geq 0$$

Kinematical regions:

$$\langle k_1'^+ \vec{k}_{1\perp}' | Q \cdot j^{c(1)} | k_1^+ \vec{k}_{1\perp} \rangle_{(a)+(e)} = \langle k_1'^+ \vec{k}_{1\perp}' | \left[\widehat{e}_{LF}, w^{(1)} \right] | k_1^+ \vec{k}_{1\perp} \rangle \theta\left(k_1'^+ - Q^+ \right)$$



Current conservation is OK!

IV. Two-Fermion systems: Yukawa model

Starting with a 4-dimensional BS equation for 2→2 scattering amplitude (no self energies/vertex corr.):

$$T = V + V G_0 T$$

V is the sum of two-body irreducible diagrams

$$T(K) = W(K) + W(K)\widetilde{G}_0(K)T(K)$$

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \qquad \Delta_0(K) = G_0(K) - \widetilde{G}_0(K)$$

Separation of the instantaneous term in the fermion propagator:

$$\frac{\not{k}_i + m_i}{k_i^2 - m_i^2 + i\varepsilon} = \frac{\not{k}_{ion} + m_i}{k_i^2 - m_i^2 + i\varepsilon} + \frac{\gamma^+}{k_i^+}$$

(Sales et al PRC63(2001)064003)

$$\overline{G}_0 = (\hat{\not{k}}_{1on} + m_1)(\hat{\not{k}}_{2on} + m_2) G_0$$

$$G_0^F = \overline{G}_0 + \Delta G_0^F$$

$$G_0 = \frac{i}{\hat{k}_1^2 - m_1^2 + io}\,\frac{i}{\hat{k}_2^2 - m_2^2 + io}$$

$$|\overline{G}_0(K)| := g_0(K)$$

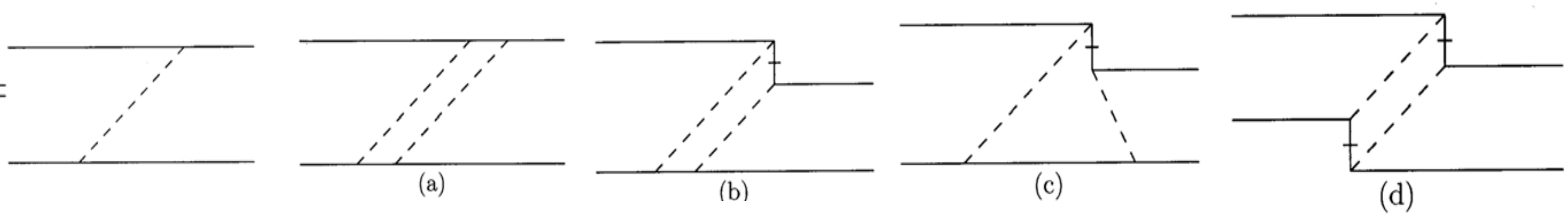
$$\tilde{G}_0 := \overline{G}_0 | g_0^{-1} | \overline{G}_0$$

Mass^2 eigenvalue eq. & valence wf:

$$\left[g_0^{-1} - w\right] |\phi_\lambda\rangle = 0$$

Yukawa model: $\mathcal{L}_I = g_S \bar{\Psi} \Psi \sigma$

$$w^{(2)} =$$



Covariant Box-diagram decomposed in LF time-ordered diagrams

B. L. G. Bakker, J. K. Boomsma, C.-R. Ji, Phys. Rev. **D 75**, 065010 (2007)

Fermions: conserved & truncated LF e.m. current & WTI

Marinho, F., Pace,Salme,Sauer, PRD77,116010(2008)

$$j^\mu = g_0^{-1}|\overline{G}_0[1 + W\Delta_0]J^\mu[1 + \Delta_0 W]\overline{G}_0|g_0^{-1}$$

(Sales et al PRC63(2001)064003)

$$\Delta_0 := G_0 - \tilde{G}_0$$

$$\tilde{G}_0 := \overline{G}_0|g_0^{-1}|\overline{G}_0$$

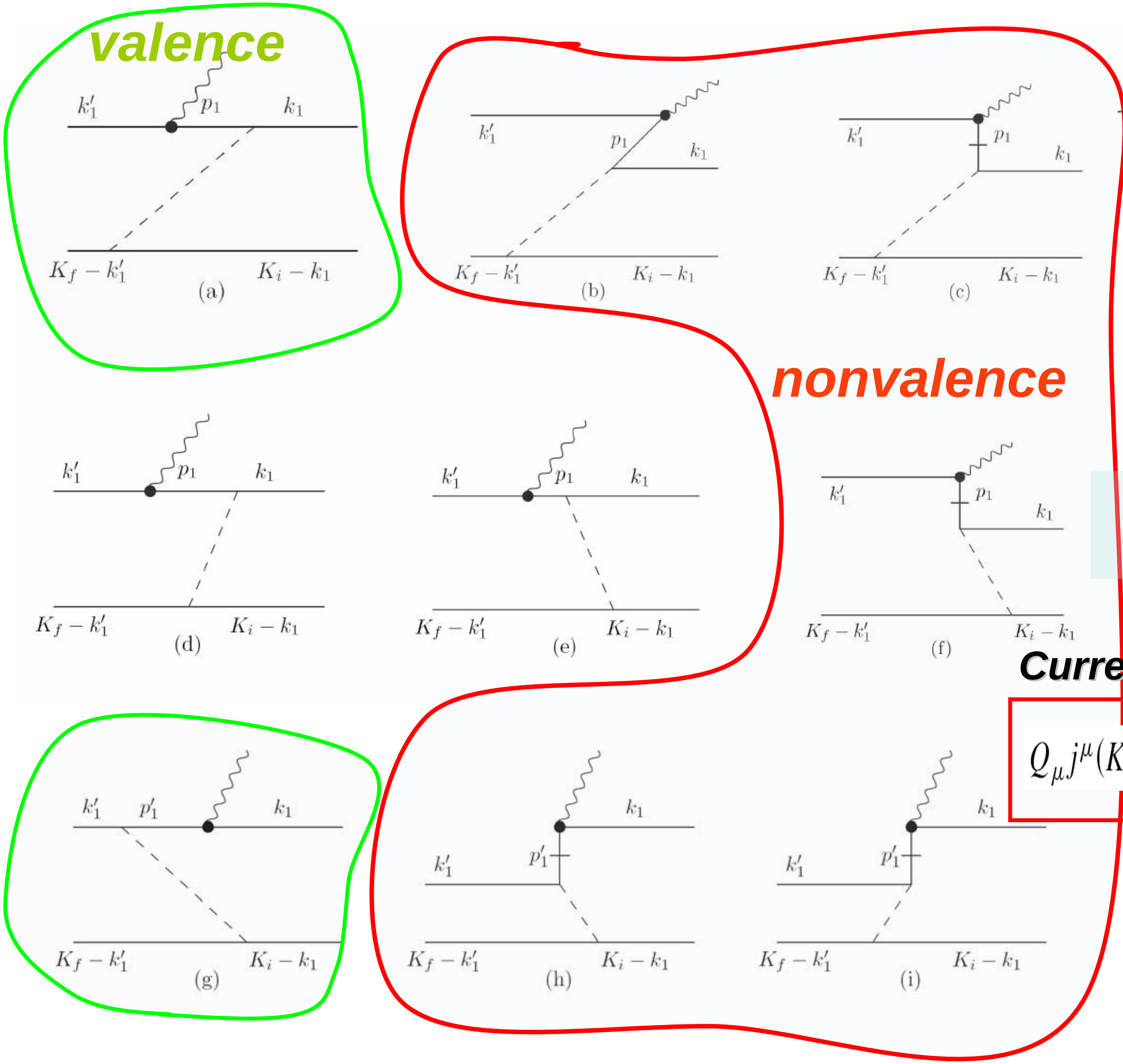
WTI for the conserved current:

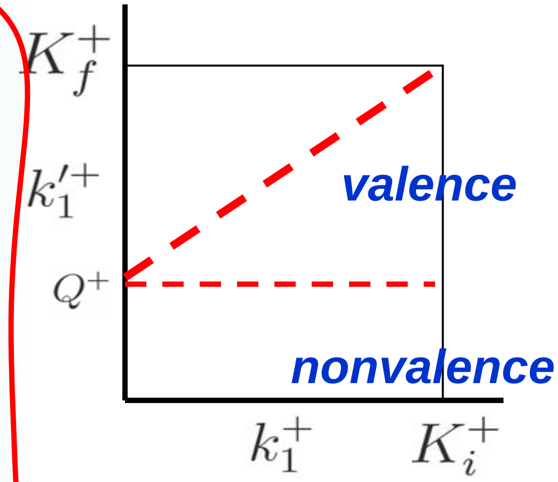
$$Q_\mu j^\mu(K_f, K_i) = g^{-1}(K_f)\hat{\mathcal{Q}}^L_{\rm LF} - \hat{\mathcal{Q}}^R_{\rm LF} g^{-1}(K_i)$$

Expansion...

$$e_1\delta\left(k_1'^+ - k_1^+ - Q^+\right)\delta^2\left(\vec{k}'_{1\perp} - \vec{k}_{1\perp} - \vec{Q}_\perp\right)\Lambda_+(k'_{1on})\frac{m_1}{k_1'^+}\gamma_1^+\Lambda_+(k_{1on})\Lambda_+(k_{2on})$$

LF current in 1st order



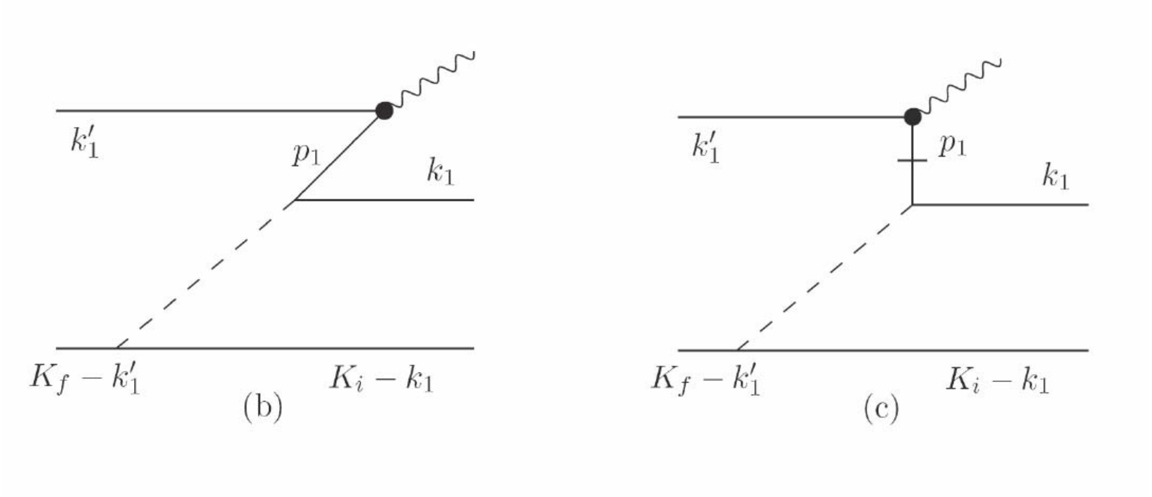


$K_i^+ > 0$ and $Q^+ \geq 0$

Current conservation holds!

$$Q_\mu j^\mu(K_f, K_i) = g^{-1}(K_f)\hat{Q}^L_{\mathrm{LF}} - \hat{Q}^R_{\mathrm{LF}} g^{-1}(K_i)$$

End-point singularity in j^- for $Q^+ \rightarrow 0$



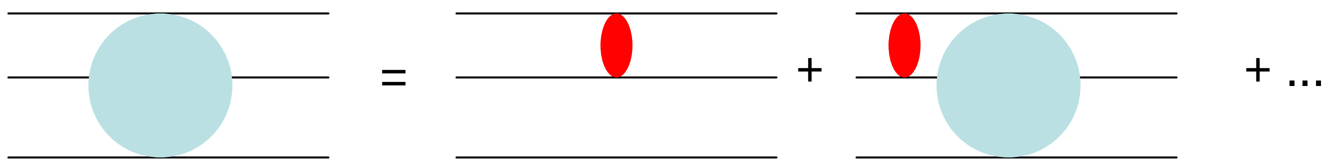
$\sim 1 / Q^+ \rightarrow \infty$

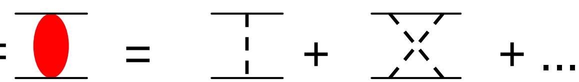
Finite contribution to the WTI !!!

V. Three-boson systems and ladder 4d BS equation

Marinho, PhD thesis ITA/2007

$$T = V + V G_0 T \qquad V = \sum_{i=1}^{3} V_i \quad ; \quad V_i = V_{(2)jk} S_i^{-1}$$



$V_{(2)jk}$ = 

$$\langle k_1^-, k_2^- | G_0 | k_1'^-, k_2'^- \rangle = \frac{-i}{(2\pi)^2} \frac{\delta(k_1^- - k_1'^-)}{\hat{k}_1^+ \hat{k}_2^+ (K^+ - \hat{k}_1^+ - \hat{k}_2^+)(k_1^- - \hat{k}_{1on}^-)} \frac{\delta(k_2^- - k_2'^-)}{(k_2^- - \hat{k}_{2on}^-)(K^- - k_1^- - k_2^- - (K - \hat{k}_1 - \hat{k}_2)_{on}^-)}$$

Integration over k^- for 1 and 2 → free 3-boson resolvent

$$g_0(\underline{k}_1, \underline{k}_2) = \frac{i\theta(K^+ - k_1^+ - k_2^+)\theta(k_1^+)\theta(k_2^+)}{k_1^+ k_2^+ (K^+ - k_1{}^+ - k_2^+)(K^- - k_{1on}^- - k_{2on}^- - (K - k_1 - k_2)_{on}^-)}$$

$$\underline{k} \equiv (k^+, \vec{k}_\perp)$$

Faddeev decomposition:

$$W(K) = V(K) + V(K)\Delta_0(K)W(K) \quad \begin{cases} \Delta_0(K) = G_0(K) - \widetilde{G}_0(K) \\ \widetilde{G}_0(K) := G_0(K)|g_0^{-1}(K)|G_0(K) \end{cases}$$

$$V = \sum_{i=1}^{3} V_i \rightarrow W_i = V_i + V_i \Delta_0 W \qquad W = \sum_{i=1}^{3} W_i$$

$$(1 - V_i\Delta_0)W_i = V_i + V_i\Delta_0(W_j + W_k)$$

$$W_{(2)i} = V_i + V_i\Delta_0 W_{(2)i}$$

$$W_i = W_{(2)i} + W_{(2)i}\Delta_0(W_j + W_k)$$

$$t = \sum_{i=1}^{3} t_i \ ; \qquad w = \sum_{i=1}^{3} w_i$$

$$t_i = w_i + w_i g_0 t$$

$$w_i = g_0^{-1} \ \left| G_0 W_i G_0 \right| \ g_0^{-1}$$

In practice W_i is obtained from a power expansion in V:

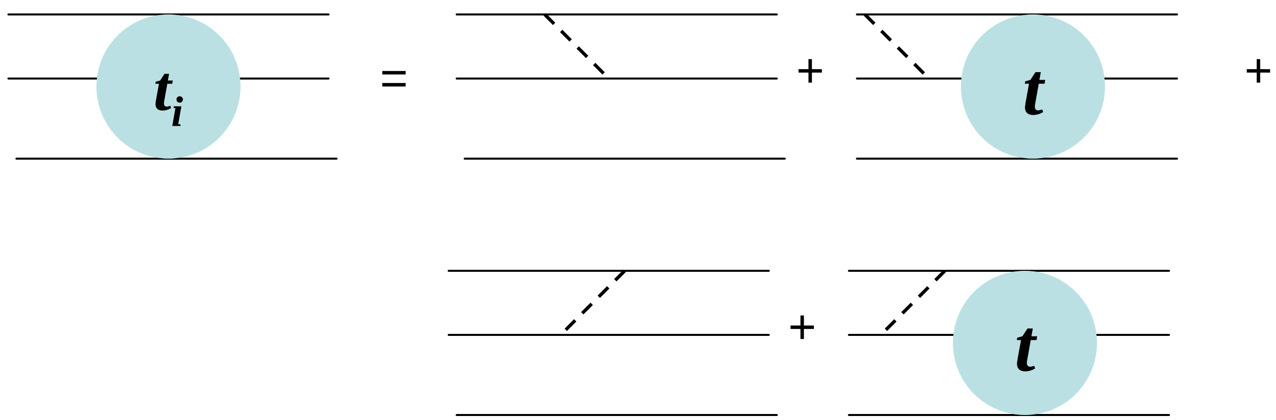
$$W_i = \boxed{V_i} + \boxed{V_i \Delta_0 (V_i + V_j + V_k)} + V_i \Delta_0 (V_i + V_j + V_k) \Delta_0 (V_i + V_j + V_k) + \ldots$$

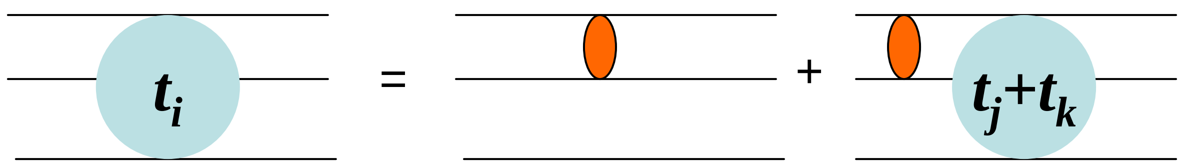
LO

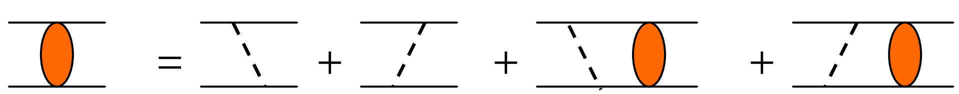
NLO

Bosonic Yukawa model: LO

$$w_i^{LO} = g_0^{-1}\ |G_0 V_i G_0|\ g_0^{-1}$$





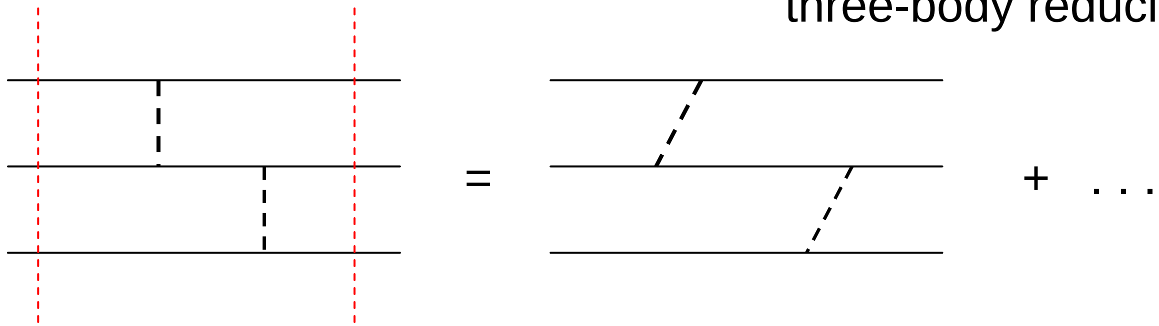


Cluster separability satisfied!

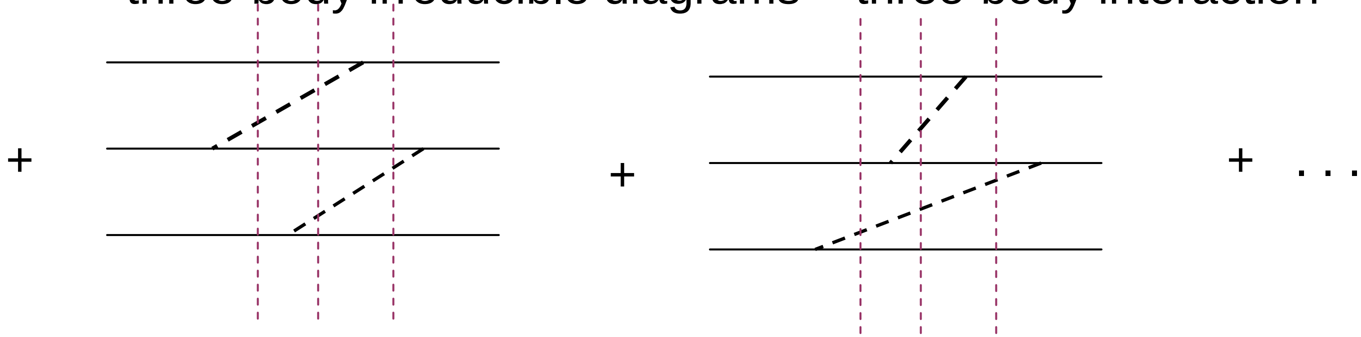
Bosonic Yukawa model: NLO

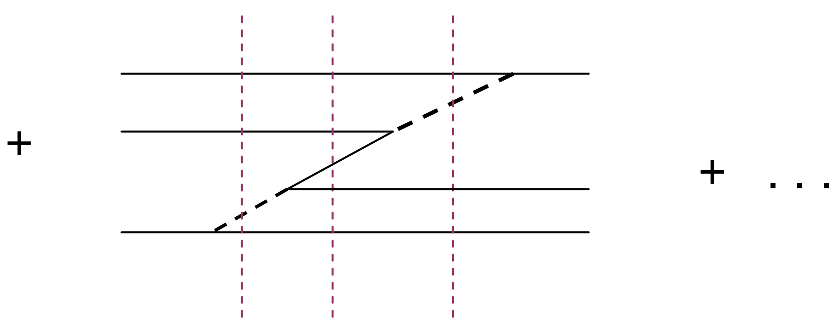
$$w_i^{NLO} = w_i^{LO} + g_0^{-1} \; |G_0 V_i \Delta_0 (V_i + V_j + V_k) G_0| \; g_0^{-1}$$

three-body reducible diagrams

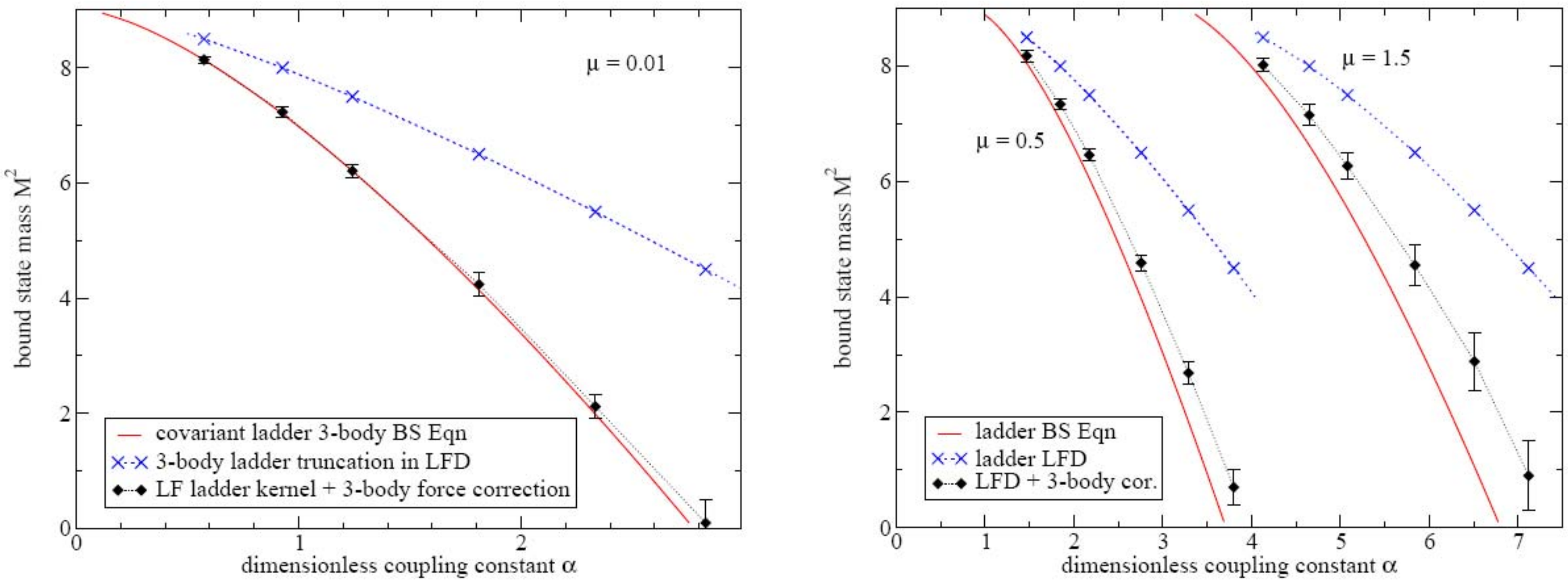


three-body irreducible diagrams = three-body interaction





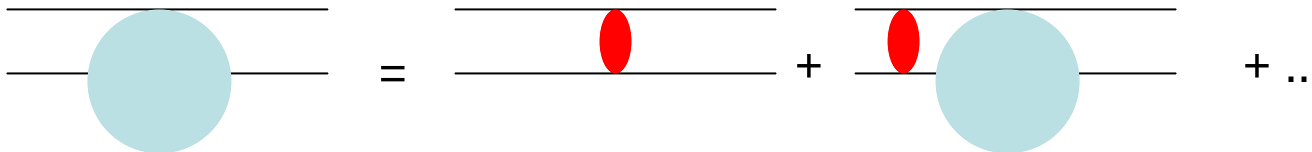
Perturbative contribution of the 3-body interaction to the 3-boson mass Karmanov & Maris PoS LC2008, 037 (2008), Few Body Syst.46, 95 (2009).

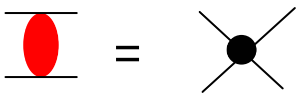


VI. Zero-range model: LO and NLO

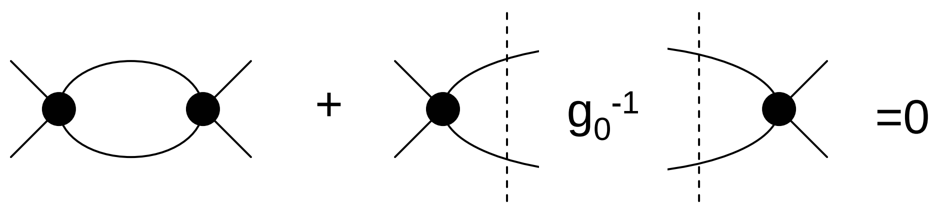
$$\mathcal{L}_I = \frac{1}{4!}\lambda\phi^4$$

4d ladder



$V_{(2)jk}$ = 

$$\langle k_i, k_j | V_k | k_i', k_j' \rangle = \lambda (2\pi)^2 \delta^4(k_k - k_k')(k_k^2 - m^2)$$

$V_i \Delta_0 V_i =$ 

Ladder+contact→
$$\begin{cases} W_{(2)i} = V_i + V_i \Delta_0 W_{(2)i} = V_i \\ W_i = V_i + V_i \Delta_0 (W_j + W_k) \end{cases}$$

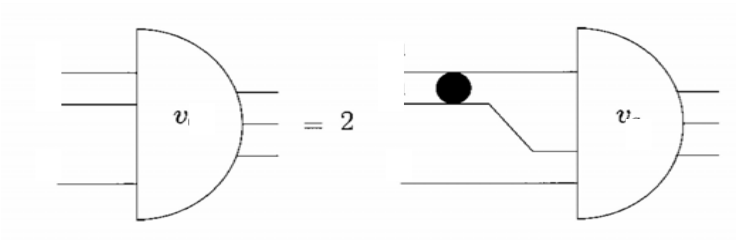
Leading order bound-state 3-boson equation

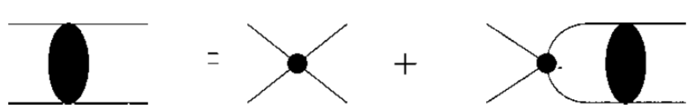
Bound state eq.: $\left\{ \begin{array}{l} v_i = w_i g_0 (v_i + v_j + v_k) \\ V_i \textit{ Faddeev components} \end{array} \right.$

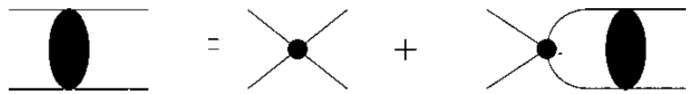
$$w_i^{LO} = g_0^{-1} | G_0 V_i G_0 | g_0^{-1} \qquad W_i^{LO} = V_i$$

$$w_i^{LO}(\underline{k}_j, \underline{k}_k; \underline{k}'_j, \underline{k}'_k) = -2\pi i \lambda \; k_i^+ \delta^3(\underline{k}_i - \underline{k}'_i)$$

$$\boxed{v_i^{LO} = (1 - w_i^{LO} g_0)^{-1} w_i^{LO} g_0 (v_j^{LO} + v_k^{LO})}$$

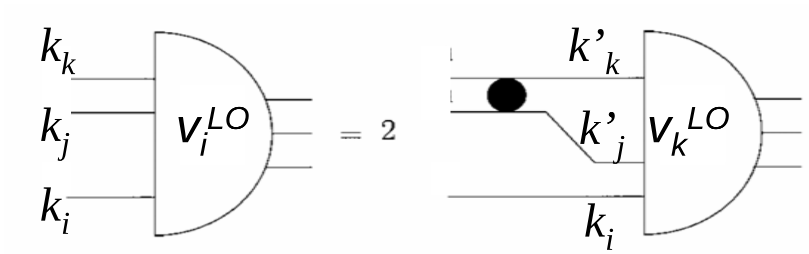






$$\tau(M_{jk}^2) = 2\pi \left[\lambda^{-1} + \frac{1}{8\pi^2} \int_0^1 dx \int d^2 k_\perp \frac{1}{x(1-x)\left(M_{jk}^2 - \frac{k_\perp^2 + m^2}{x(1-x)}\right)} \right]^{-1}$$

The divergence is eliminated by fixing ***lambda*** through the 2-boson bound state mass!



$$v_i^{LO}(\underline{k}_j, \underline{k}_k) = -2\tau(M_{jk}^2) \int d^3 \underline{k}'_j \frac{\theta(k'^+_j)\theta(K^+ - k^+_i - k'^+_j) \qquad v_k^{LO}(\underline{k}_i, \underline{k}'_j)}{k'^+_j (K^+ - k^+_i - k'^+_j)(K^- - k^-_{ion} - k'^-_{jon} - (K - k_i - k'_j)^-_{on})}$$

Frederico 92: regularization M_{jk} real → no collapse
Carbonell & Karmanov 03: no-regularizaton collapse for M_{2B}< critical value

Next-to-Leading order bound-state 3-boson equation

$$W_i^{NLO} = W_i^{LO} + V_i \Delta_0 (V_j + V_k)$$

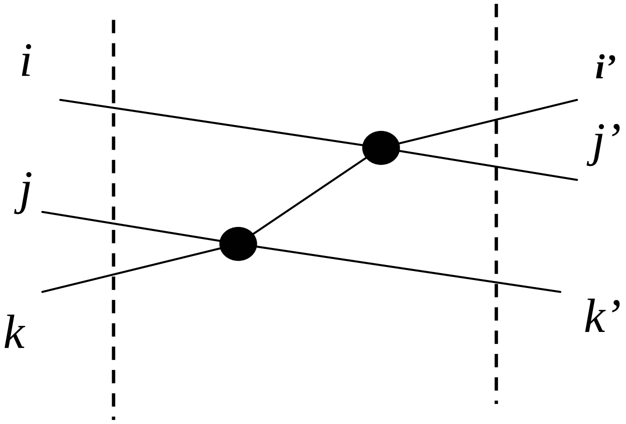
$$w_i^{NLO} = w_i^{LO} + \Delta w_i^{NLO}$$

$$\boxed{v_i^{NLO} = (1 - w_i^{LO} g_0)^{-1} w_i^{LO} g_0 \sum_{n \neq i} v_n^{NLO} + (1 - w_i^{LO} g_0)^{-1} \Delta w_i^{NLO} g_0 \sum_n v_n^{NLO}}$$

LO kernel

NLO kernel

$$\Delta w_i^{NLO} = g_0^{-1} |G_0 V_i \Delta_0 V_j G_0| g_0^{-1} + g_0^{-1} |G_0 V_i \Delta_0 V_k G_0| g_0^{-1}$$



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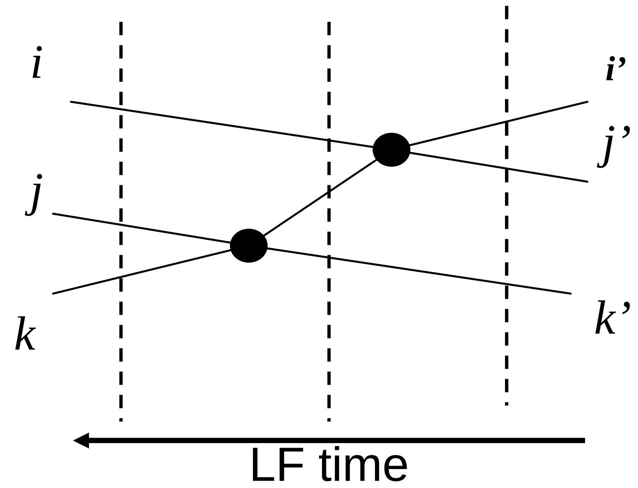
Three-body reducible diagram



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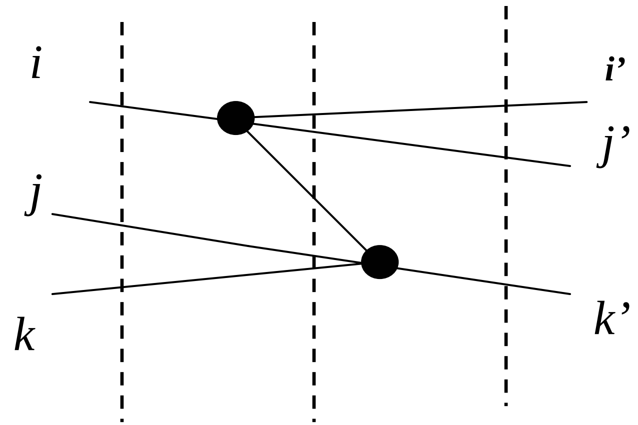
Three-body irreducible diagram



$$|G_0 V_i \Delta_0 V_k G_0|$$

NLO 3-boson bound-state equation

(Marinho & F. PoS(LC2008)036)

LO kernel

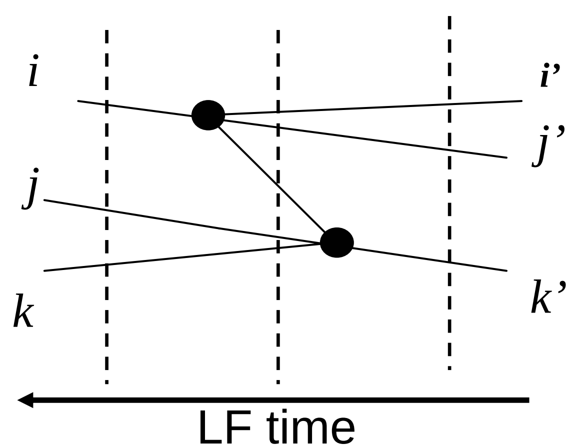
$$v_i^{NLO}(\underline{k}_j,\underline{k}_k) = -\tau(M^2_{jk})\left(2\int d^3\underline{k}'_j \frac{\theta(k_i^+)\theta(k_j'^+)\theta(K^+-k_i^+-k_j'^+)}{k_j'^+k_k'^+(K^- - k^-_{ion} - k'^-_{jon} - (K-k_i-k'_j)^-_{on})} v_k^{NLO}(\underline{k}_i,\underline{k}'_j) + \right.$$

NLO kernel

$$2\pi\lambda\int d^3\underline{k}'_j d^3\underline{k}'_k \frac{\theta(k_i^+ - K^+ + k_k'^+)\theta(K^+ - k_j'^+ - k_k'^+)\theta(k_j'^+)\theta(k_k'^+)}{k_j'^+k_k'^+(k_i^+ - K^+ + k_k'^+)(K^+ - k_j'^+ - k_k'^+)} \times$$

$$\left. \frac{v_i^{NLO}(\underline{k}'_j,\underline{k}'_k) + v_j^{NLO}(\underline{k}'_k,\underline{k}'_i) + v_k^{NLO}(\underline{k}'_i,\underline{k}'_j)}{(K^- - k^-_{ion} - k^-_{kon} - k'^-_{jon} - (k_i - K + k'_k)^-_{on} - (K - k'_j - k'_k)^-_{on})} + (k'_k \leftrightarrow k'_j)\right)$$

Effective 3-body interaction



VII. Conclusions

- "LF Few-body dynamics": Valence w.f. dynamics
- Quasi-potential Approach to LF → LF dynamics
- 4-d Bethe-Salpeter amplitude ←→ valence w.-f.
- 4-d operators ←→ 3-d operators acting valence w.f.
- Conserved current operator & WTI:

  Conserved LF e.m. current operator expanded systematically and consistent with the mass squared operator;

  Valence and nonvalence contributions to the current required by current conservation;

  Current conservation in LF is a weaker requirement than covariance (covariance under non-kinematical boosts).

Perspectives:

- 2-particle scattering with valence wf ←→ 4d calculations in Minkowski space with Nakanishi integral representation (Karmanov and Carbonell, EJPA27, 1(06); scattering for 3-particles... (bosonic and fermionic systems)
- GPD's, conserved current operator for 3-body systems
- 3→3 scattering amplitude B→$pi^-pi^+k^-$ (CP violation)
- Applications to deuteron, trinucleon, n-d scattering...
- Excitons in graphene!