ECT*, Trento

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The Bethe-Salpeter Equation and the Nakanishi representation

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• Plan

- Bethe-Salpeter (BS) equation.
- Wick rotation and Euclidean solution.
- Our method in Minkowski space.
- Solution for spinless system
- New development and solution for two-fermion system.
- Conclusions

• BS amplitude

Bethe-Salpeter amplitude (Phys. Rev. **84**, 1232, (1951)):

$$\Phi(x_1, x_2, p) = \langle 0 | T\Big(\varphi(x_1)\varphi(x_2)\Big) | p \rangle$$

Separating c.m. motion:

$$\Phi(x_1, x_2, p) = \Phi(x, p)\ \exp(p(x_1 + x_2)),$$

$$x_{1,2} = (t_{1,2}, \vec{r}_{1,2}), \quad p = (p_0, \vec{p}), \quad x = \frac{1}{2}(x_1 - x_2)$$

• Momentum space

Fourier transformation:

$$\Phi(k,p) = \int \Phi(x,p)\exp(-ikp)d^4k$$

$\Phi(k,p)$ depends on relative four-momentum k and on total four-momentum $p = k_1 + k_2$.

$\Phi(k,p)$ depends on two scalar variables $k^2, k{\cdot}p$.

• Projecting on light-front

LF light-front wave function:

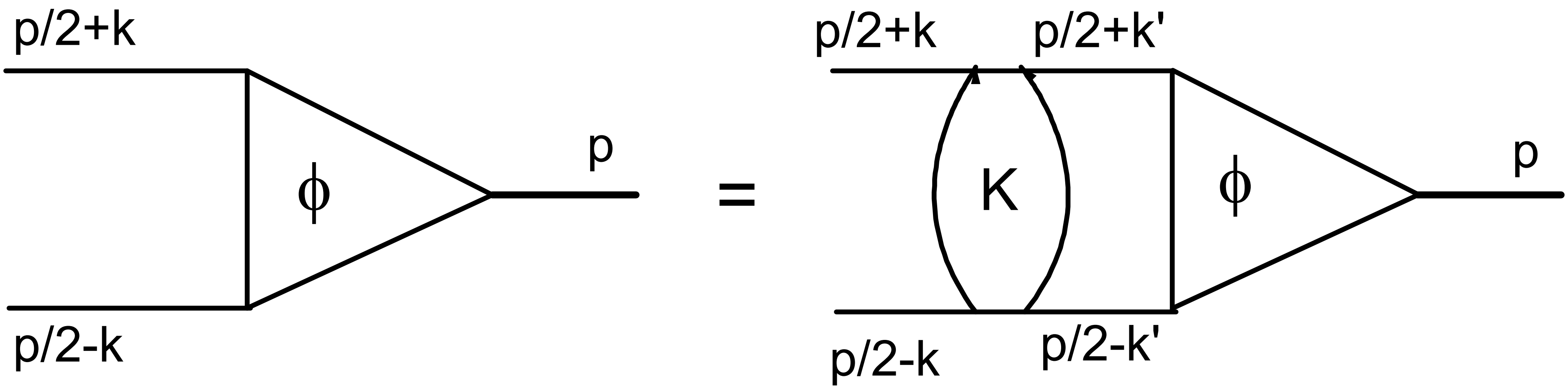
$$\psi(\vec{k}_\perp, x) = \int \Phi(k,p) dk_-$$

$$k_\pm = k_0 \pm k_z, \quad x = k_+/p_+, \quad \vec{k}_\perp = (k_x, k_y)$$

Remark: Light-front wave function $\psi(\vec{k}_\perp, x)$ is non-singular.

Therefore, the integral over k_- kills the singularities in physical domain.

• BS equation



$$\left((\frac{p}{2}+k)^2-m^2\right)\left((\frac{p}{2}-k)^2-m^2\right)\Phi(k,p)$$

$$=-i\int\frac{d^4k'}{(2\pi)^4}K(k,k',p)\Phi(k',p)$$

• Singularity

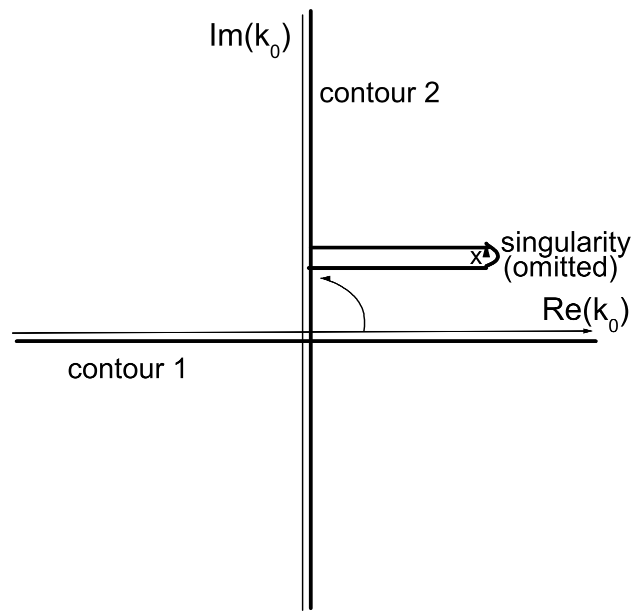
$$\Phi(k,p) = \frac{\Gamma(p,k)}{\left((\frac{p}{2}+k)^2 - m^2 + i\epsilon\right)\left((\frac{p}{2}-k)^2 - m^2 + i\epsilon\right)}$$

$$\Gamma(k,p) = -i\int \frac{d^4k'}{(2\pi)^4}\frac{K(k,k',p)\Gamma(p,k')}{\left((\frac{p}{2}+k')^2 - m^2 + i\epsilon\right)\left((\frac{p}{2}-k')^2 - m^2 + i\epsilon\right)}$$

It is not a problem in principle (it is normal).

But it is a problem for numerical solution.

• Wick rotation



$$\int \ldots d^4k = \int d^3k \int_{-\infty}^{\infty} \ldots dk_0 = \int_{-i\infty}^{i\infty} \ldots dk_0 = \int_{-\infty}^{\infty} \ldots i dk_4$$

• Euclidean space

Euclidean BS amplitude:

$$\Phi(\vec{k}, k_0) \to \Phi_E(\vec{k}, k_4) = \Phi(\vec{k}, ik_0)$$

Euclidean BS equation (non-singular):

$$\left[\left(m^2 - \frac{M^2}{4} + \vec{k}^2 + k_4^2\right)^2 + M^2 k_4^2\right] \Phi_E(\vec{k}, k_4)$$
$$= \int \frac{d^3k' dk_4}{(2\pi)^4} K_E(k, k') \Phi_E(\vec{k'}, k_4)$$

• Advantage of Euclidean space solution

Finding solution in Euclidean space, we can easily find binding energies.

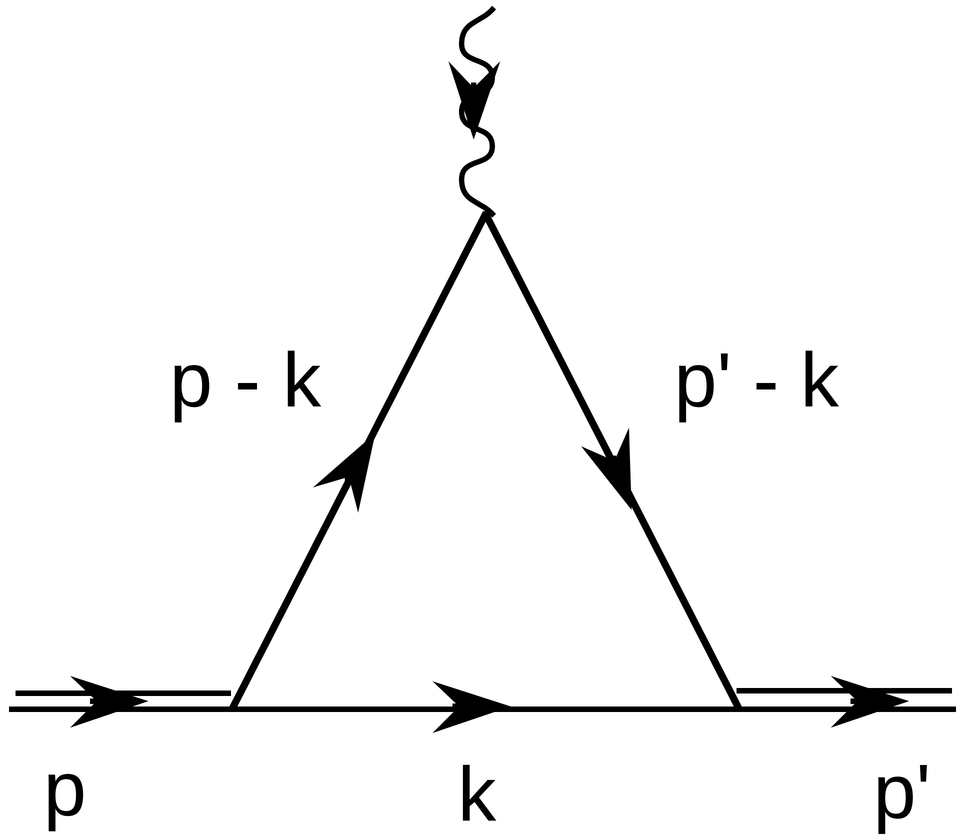
• Disadvantage of Euclidean space solution

We cannot extrapolate numerically the Euclidean $\Phi_E(\vec{k}, k_4)$ back in Minkowski space numerically (extrapolation is extremely unstable).

But we need Minkowski space solution to calculate observables (form factors, etc.)

• Why do we need Minkowski BS solution?

To calculate observables, e.g., EM form factor.



It is expressed through Euclidean BS amplitudes approximately only (in so called static approximation).

Thursday October 22, 2009 9.30-10.00

Jaume Carbonell (LPSC, Grenoble, France)

Bethe-Salpeter solutions in Minkowski space: numerical methods and electromagnetic form factors

• Aim

Our aim is to find not only the binding energies,
but the BS amplitude in Minkowski space.

• Minkowski space solution

(first solution, for the ladder kernel only)

K. Kusaka, A.G. Williams, Phys.Rev. **D51** *(1995) 7026;*

K. Kusaka, K. Simpson, A.G. Williams, Phys.Rev. **D56** *(1997) 5071.*

It is based on Nakanishi integral representation for BS amplitude (*N. Nakanishi, Phys. Rev.* **130**, *1230 (1963)*):

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{-ig(\gamma',z')}{\left[\gamma' + m^2 - \frac{1}{4}M^2 - k^2 - p{\cdot}k\ z' - i\epsilon\right]^3}.$$

For ladder kernel an equation for $g(\gamma,z)$ is found and solved. Then the BS amplitude $\Phi(k,p)$ is easily found in Minkowski space.

However, the method by K. Kusaka et al. worked for ladder kernel only.

• Example

We set $-ig(\gamma, z) = 1$, calculate the integral and find

$$\Phi_M(k;p) = \frac{i^2}{\left[(\frac{p}{2}+k)^2 - m^2 + i\epsilon\right]\left[(\frac{p}{2}-k)^2 - m^2 + i\epsilon\right]},$$

i.e. just the product of two free propagators.
BS amplitude is, of course, still singular.

All the non-trivial dynamics is in the function $g(\gamma, z)$.

• Our (exact) method

V. A. Karmanov and J. Carbonell, Eur. Phys. J. **A27** (2006) 1.

- Take BS amplitude in the Nakanishi form:

$$\Phi(k,p)=\int_{-1}^{1}dz'\int_0^{\infty}d\gamma'\frac{-ig(\gamma',z')}{\left[\gamma'+m^2-\frac{1}{4}M^2-k^2-p\cdot k\;z'-i\epsilon\right]^3}$$

- Substitute it in the BS equation.

$$\Phi(k,p)=\frac{-i}{\left((\frac{p}{2}+k)^2-m^2\right)\left((\frac{p}{2}-k)^2-m^2\right)}\int\frac{d^4k'}{(2\pi)^4}K(k,k',p)\Phi(k',p)$$

- Apply to both sides of the BS equation the LF projection:

$$\int dk_-\Phi(k,p)=\int dk_-\frac{-i}{\left((\frac{p}{2}+k)^2-m^2\right)\left((\frac{p}{2}-k)^2-m^2\right)}\int\frac{d^4k'}{(2\pi)^4}K(k,k',p)\Phi(k',p)$$

- Obtain equation for $g(\gamma,z)$.

• Equation for $g(\gamma, z)$

(Obtained analytically, without any approximation.)

$$\int_0^\infty \frac{g(\gamma', z)d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + (1 - z^2)\kappa^2\right]^2}$$
$$= \int_0^\infty d\gamma' \int_{-1}^1 dz'\, V(\gamma, z; \gamma', z') g(\gamma', z')$$

where $\kappa^2 = m^2 - \frac{1}{4}M^2$.

• This equation is equivalent to the initial BS equation.

Matrix form:

$$\lambda B x = A x$$

It is just standard form for well known fortran subroutine.

• Kernel

Calculate:

$$I(k,p) = \int \frac{d^4k'}{(2\pi)^4} \frac{iK(k,k',p)}{\left[k'^2 + p\cdot k' z' - \gamma' - \kappa^2 + i\epsilon\right]^3}.$$

Substitute in:

$$\begin{aligned} V(\gamma, z; \gamma', z') &= \frac{p_+}{\pi} \int_{-\infty}^{\infty} \frac{-iI(k,p)dk_-}{\left[(\frac{p}{2}+k)^2 - m^2 + i\epsilon\right]} \\ &\times \frac{1}{\left[(\frac{p}{2}-k)^2 - m^2 + i\epsilon\right]}, \end{aligned}$$

For given BS kernel K we can calculate the kernel V of equation for $g(\gamma, z)$.

• LF wave function

(As a by-product)

$$
\begin{aligned}
\psi(\vec{k}_\perp, x) &= \int_{-\infty}^{\infty} \Phi(k,p) dk_- \\
&= \int_0^{\infty} \frac{g(\gamma', 1-2x) d\gamma'}{\left[\gamma' + \vec{k}_\perp^2 + m^2 - x(1-x)M^2\right]^2}
\end{aligned}
$$

• Wick-Cutkosky model (OBE, $\mu = 0$)

For $\mu = 0$, we are looking the solution in the form:

$$g(\gamma, z) = \delta(\gamma) g(z)$$

The kernel is very simplified. We get:

$$g(z) = \frac{\alpha}{2\pi} \int_{-1}^{1} dz' \; K(z, z') g(z')$$

with

$$K(z', z') = \frac{m^2}{m^2 - \frac{1}{4}(1 - z'^2) M^2} \left\{ \begin{array}{ll} \frac{(1-z)}{(1-z')}, & \text{if } -1 \leq z' \leq z \\ \frac{(1+z)}{(1+z')}, & \text{if } \quad z \leq z' \leq 1 \end{array} \right.$$

This is the Wick-Cutkosky equation (1954).

• OBE (ladder) kernel ($\mu \neq 0$)

One-boson exchange (ladder) kernel: $K(k,k',p) = \frac{-g^2}{(k-k')^2-\mu^2+i\epsilon}$
$g^2 = 16\pi m^2\alpha$. Equation:

$$\int_0^\infty \frac{g(\gamma',z)d\gamma'}{\Big[\gamma'+\gamma+z^2m^2+(1-z^2)\kappa^2\Big]^2} = \int_0^\infty d\gamma' \int_{-1}^1 dz'\, V(\gamma,z;\gamma',z')g(\gamma',z')$$

Kernel $V(\gamma,z;\gamma',z')$:

$$V(\gamma,z;\gamma',z') = \frac{\alpha m^2(1-z)^2}{2\pi\Big[\gamma+z^2m^2+(1-z^2)\kappa^2\Big]} \int_0^1 \frac{v^2dv}{B_1^2}$$

$B_1 = B_1(\gamma,z;\gamma',z';v)$ is a polynomial. Integral $\int_0^1 \frac{v^2dv}{B_1^2}$ is calculated analytically. Equation is solved numerically.

• Kernel $V(\gamma, z; \gamma', z')$ v.s. z'

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(* Spinless case  *)
z = 0.8
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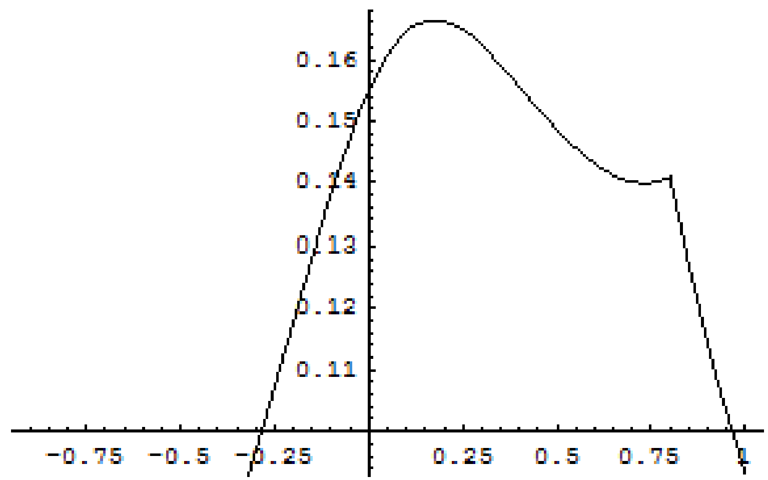


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- Graphics -
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Kernel $V(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.8$

• Numerical results (ladder, $\mu \neq 0$)

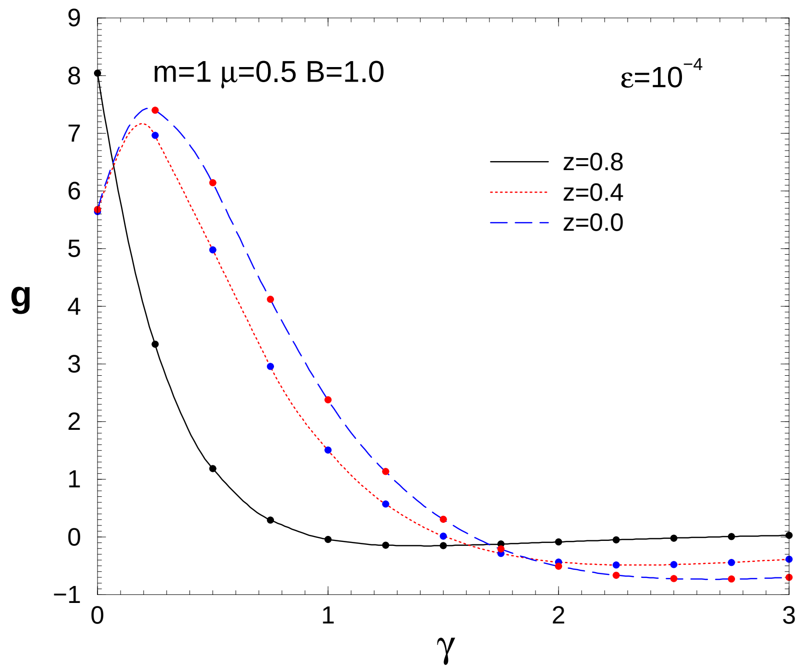
Coupling constant $\alpha = \frac{g^2}{16\pi m^2}$ as a function of the binding energy for $\mu = 0.15$ and $\mu = 0.5$

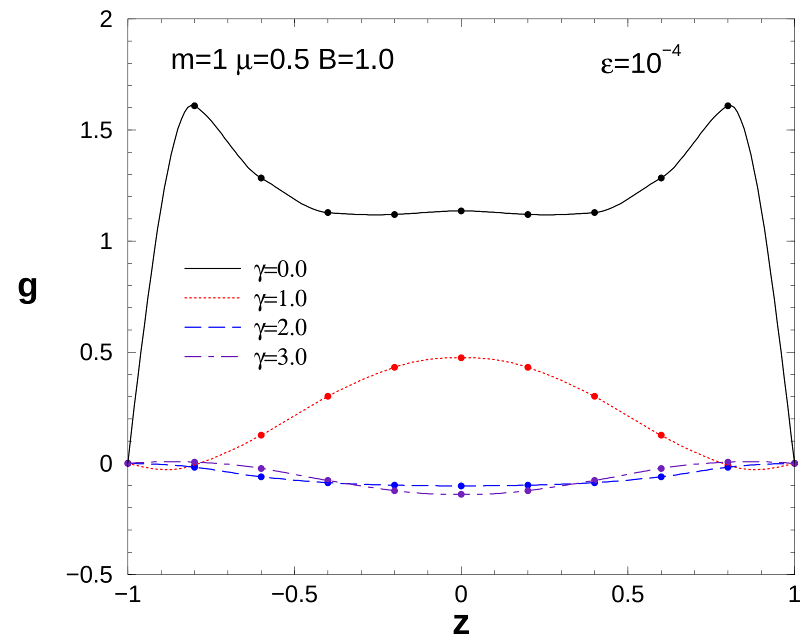
B	$\alpha(\mu = 0.15)$	$\alpha(\mu = 0.50)$
0.01	0.5716	1.440
0.10	1.437	2.498
0.20	2.100	3.251
0.50	3.611	4.901
1.00	5.315	6.712

These results, **with all shown digits**, coincide with ones obtained in Euclidean space (by Wick rotation).

• This is a test of the method.

• Function $g(\gamma, z)$

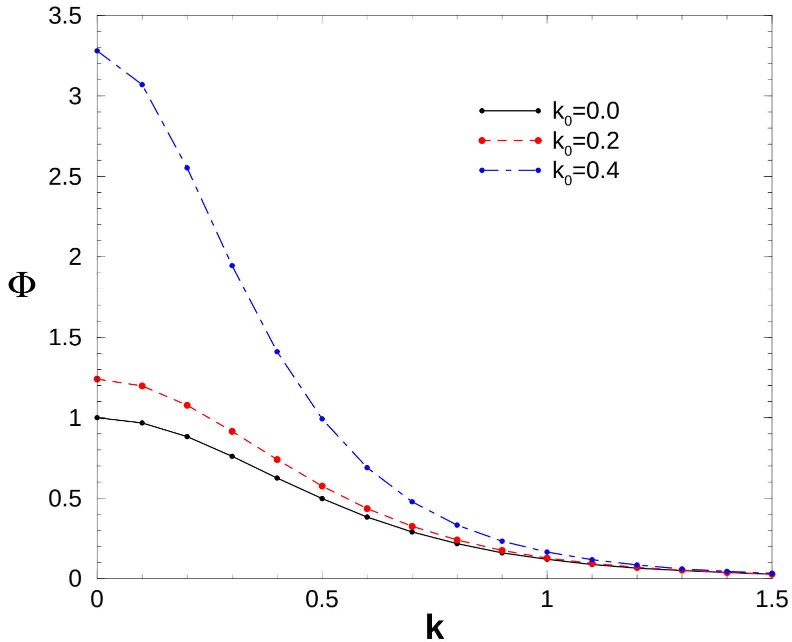


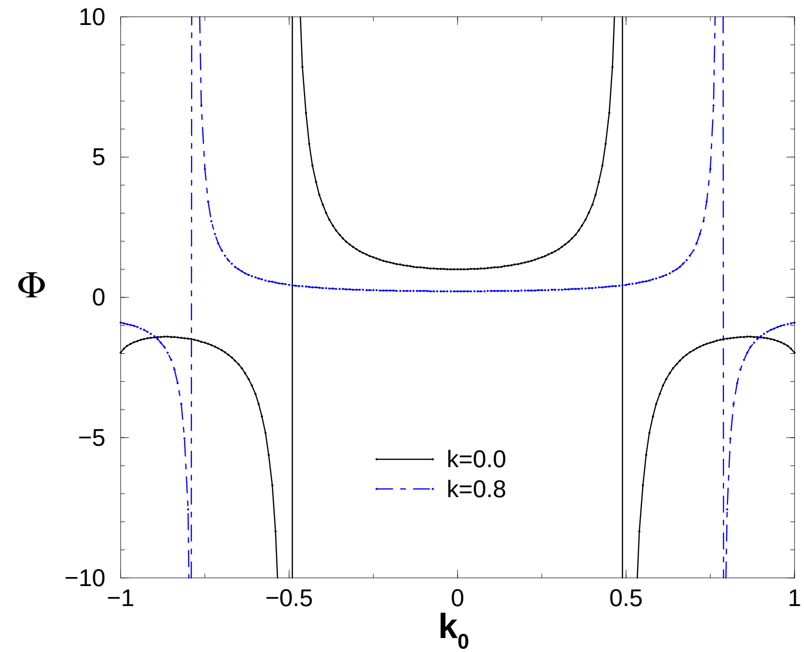


Function $g(\gamma, z)$ for $\mu = 0.5$ and $B = 1.0$. On left – versus γ for fixed values of z and on right – versus z for a fixed values of γ.

• BS amplitude $\Phi(k_0, k)$, $\vec{p} = 0$

in Minkowski space

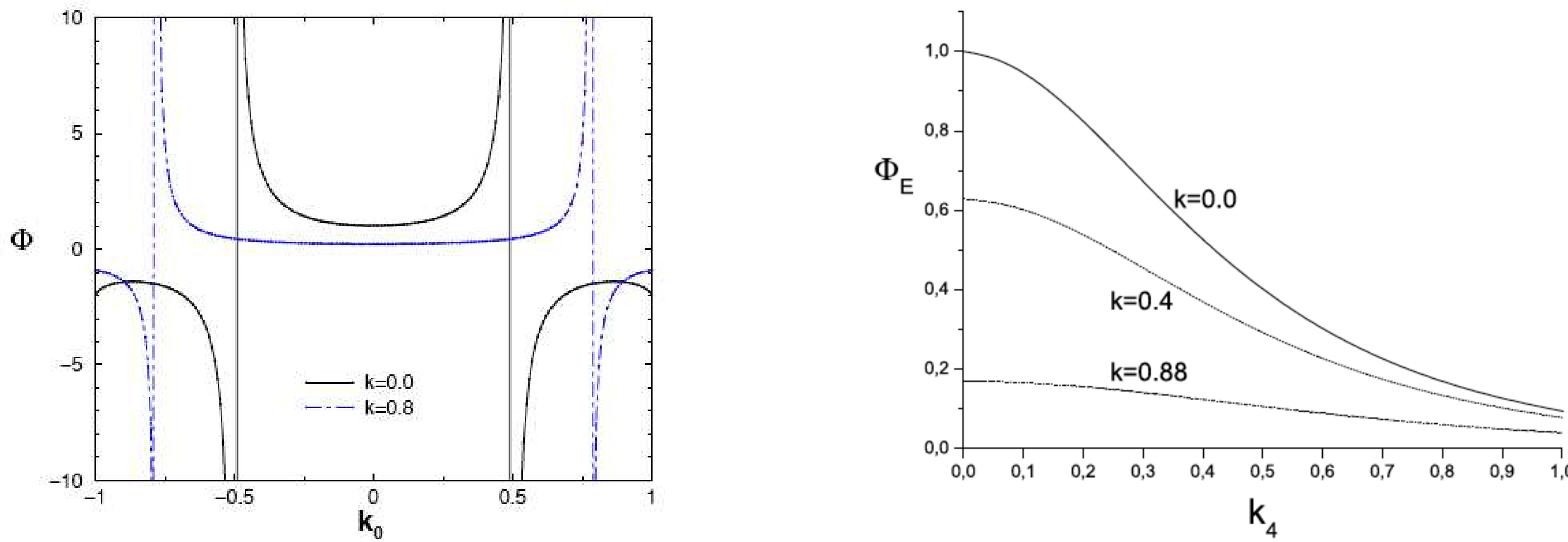




Left: BS amplitude $\Phi(k_0, k)$ vs. k for a fixed values of k_0.
Right: BS amplitude $\Phi(k_0, k)$ vs. k_0 for a fixed values of k.

• BS amplitude

Comparison of Minkowski and Euclidean spaces

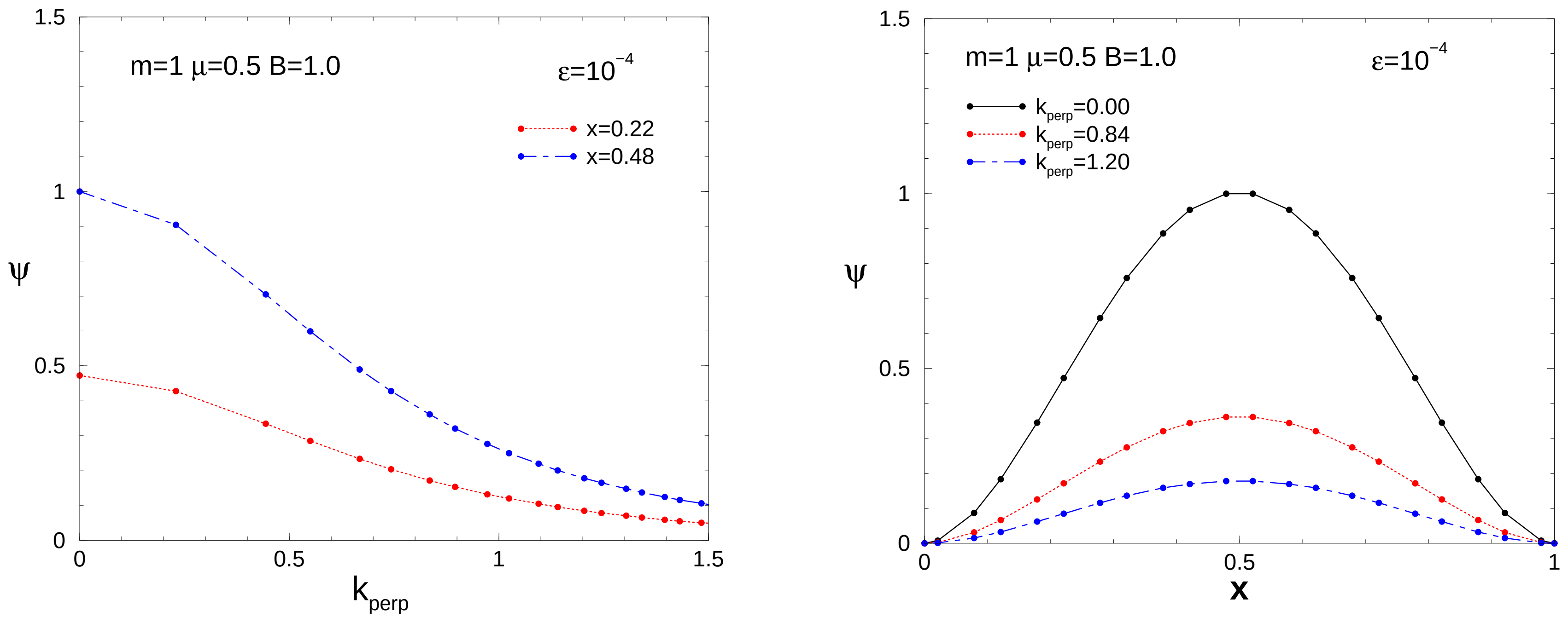


Left: BS amplitude $\Phi(k_0, k)$ in Minkowski space.
Right: BS amplitude $\Phi_E(k_4, k)$ in Euclidean space.

Continuation of Minkowski $\Rightarrow$ Euclidean space exactly coincides with direct solution in Euclidean space.

• LF wave function $\psi(k_\perp, x)$

$$\psi(\vec{k}_\perp, x) = \int_0^\infty \frac{g(\gamma', 1-2x)d\gamma'}{\left[\gamma' + \vec{k}_\perp^2 + m^2 - x(1-x)M^2\right]^2}$$



Left: LFWF $\psi(k_\perp, x)$ versus $k_\perp$ for fixed values of x.

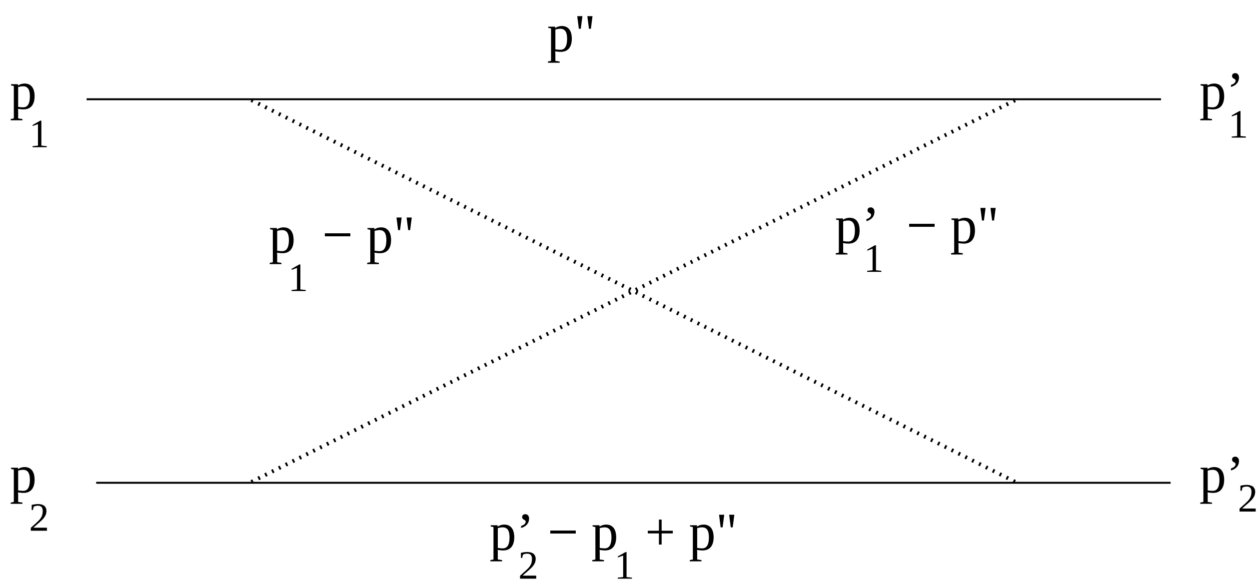
Right: $\psi(k_\perp, x)$ versus x for a few fixed values of $k_\perp$.

• Cross-ladder kernel

Euclidean space: *M.J. Levine and J. Wright, Phys. Rev.* **D2**, *2509 (1970); J.R. Cooke and G.A. Miller, Phys. Rev.* **C62**, *054008 (2000). A. Amghar, B. Desplanques and L. Theusl, Nucl. Phys.* **A 694** *(2001) 439.*

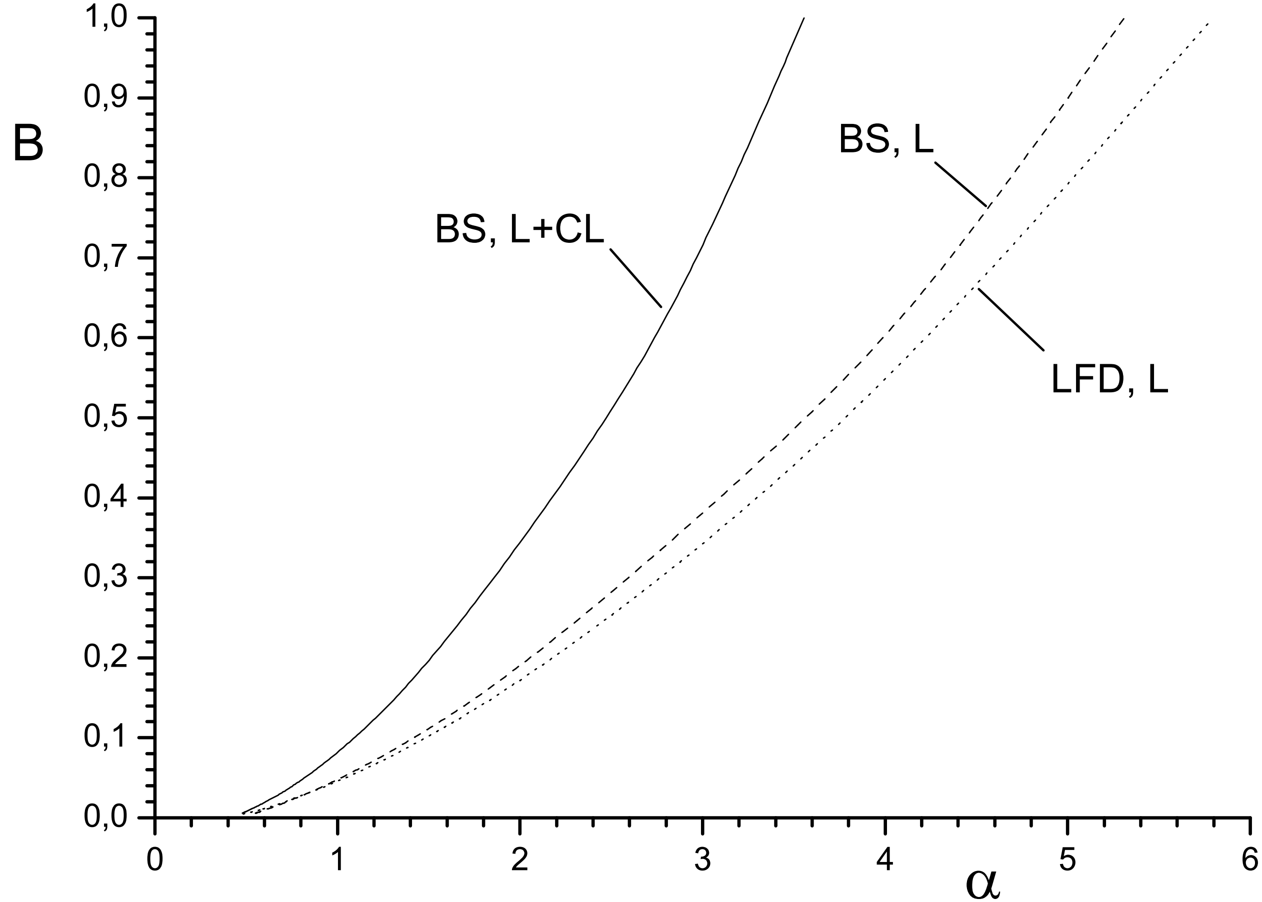
Minkowski space solution:

J. Carbonell and V. A. Karmanov, Eur. Phys. J. **A27** *(2006) 11*



Feynman cross ladder

• Numerical results (L +CL), $\mu = 0.15$



Binding energy B vs. coupling constant α for BS and LFD equations with the ladder (L) kernels only and with the ladder +cross-ladder (L+CL) one for exchange mass $\mu = 0.15$.

• Ladder +cross-ladder kernel

Comparison of Minkowski and Euclidean solutions. $\mu = 0.5$

B	α(Eucl.)	α(Mink.)
0.01	1.205	1.206
0.05	1.608	1.607
0.1	1.930	1.930
0.2	2.417	2.416
0.5	3.448	3.446
1.0	4.551	4.549

This coincidence confirms

- Our method for cross-ladder kernel;
- Possibility of Wick rotation for cross-ladder kernel.

• Two fermions

This is much more realistic case.

BS amplitude depends of two spin indices of fermions. It is 2×2 matrix. Decompose it in terms of a basis:

$$\Phi(k,p) = (S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4)$$

where

$$S_1 = \gamma_5, \quad S_2 = \frac{1}{M}\hat{p}\,\gamma_5, \quad S_3 = \frac{k\cdot p}{M^3}\hat{p}\,\gamma_5 - \frac{1}{M}\hat{k}\,\gamma_5,$$
$$S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p_\mu k_\nu\,\gamma_5$$

with

$$\hat{p} = p_\mu\gamma^\mu, \quad \sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

Four scalar functions $\phi_{1-4}(k,p)$.

Properties of symmetry (Pauli principle):

$$\phi_{1,2,4}(k,p) = \phi_{1,2,4}(-k,p), \quad \phi_3(k,p) = -\phi_3(-k,p).$$

Nakanishi representation for all components ϕ_i.

$$\begin{aligned}\phi_i(k,p) &= \frac{-i}{\sqrt{4\pi}} \int_{-1}^{1} dz' \\ &\times \int_0^\infty d\gamma' \frac{g_i(\gamma', z')}{\left[\gamma' + m^2 - \frac{1}{4}M^2 - k^2 - p\cdot k\ z' - i\epsilon\right]^3}.\end{aligned}$$

• System of equations

$$\int_0^\infty \frac{g_i(\gamma', z) d\gamma'}{\left[\gamma' + \gamma + z^2 m^2 + (1 - z^2)\kappa^2\right]^2} =$$

$$\sum_{j=1,2,3,4} \int_0^\infty d\gamma' \int_{-1}^{1} dz' \; V_{ij}(\gamma, z; \gamma', z') g_j(\gamma', z')$$

The 4×4 kernel matrix is calculated similarly to the spinless case.

• Meson exchange Lagrangians

Scalar meson exchange Lagrangian:

$$\mathcal{L}^{int} = g_s \, \bar{\psi}\psi\phi^{(s)}$$

Pseudoscalar meson exchange Lagrangian:

$$\mathcal{L}^{int} = i \, g_{ps} \, \bar{\psi}\gamma_5\psi \, \phi^{(ps)}$$

Vertex form factor:

$$F(q) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 - q^2}$$

• Numerical results

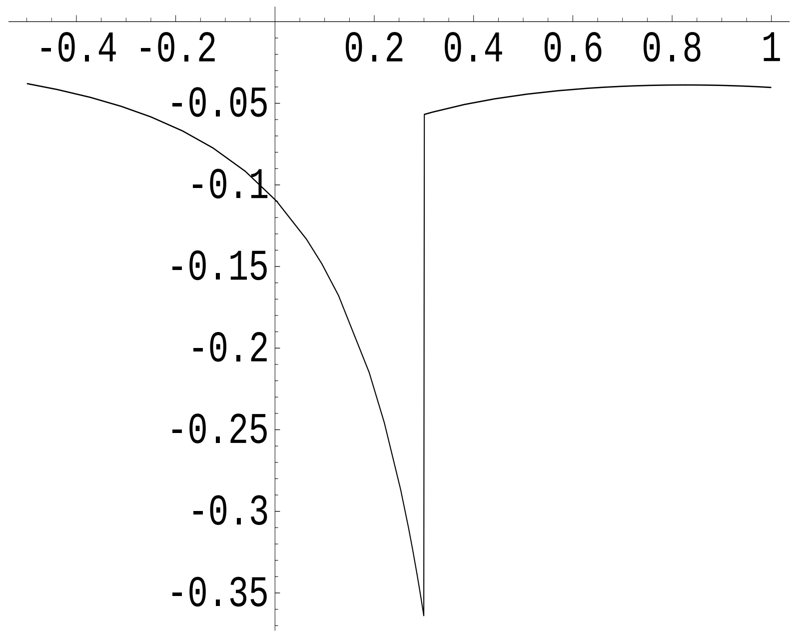
Euclidean solution: S.M. Dorkin, M. Beyer, S.S. Semykh and L.P. Kaptari, Few-Body Systems, **42**, 1, (2008).

Scalar exchange (Yukawa model)
$\mu = 0.15,\ \Lambda = 2$

B	g^2 (Dorkin et al.)	g^2 (We, Eucl.)	g^2 (We, Mink.)
0.08104	20.23	20.23	20.7
0.14773	30.34	30.34	31.7
0.27765	50.57	50.57	52.15

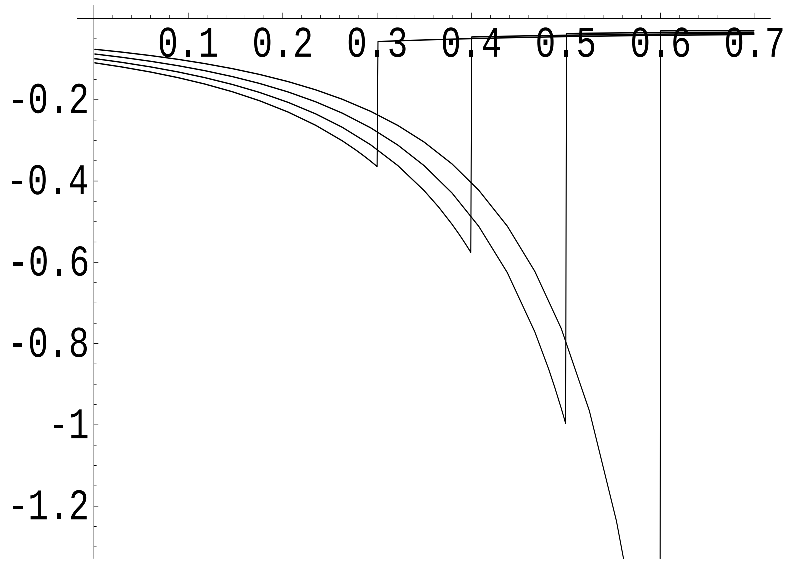
Binding energies, found via Mink. and Euclid, coincide within 2%. Not enough precision, after 4 digits coincidence in the spinless case!

• Discontinuity of $V_{ij}(\gamma, z; \gamma', z')$



One of the matrix elements V_{ij} at $z = 0.3$ v.s. z'. One can see the discontinuity at $z' = z$.

• Discontinuity of $V_{ij}(\gamma, z; \gamma', z')$



Family of matrix elements at $z = 0.3, 0.4, 0.5, 0.6$ v.s. z'.

No catastrophe, but we should take care, choosing a method of the z' integration.

• Improving the method

- Take the BS equation and multiply both sides by $\eta(k,p)$:

$$\eta(k,p)\ \Phi(k,p) = \frac{-i\eta(k,p)}{\left((\frac{p}{2}+k)^2-m^2+i\epsilon\right)\left((\frac{p}{2}-k)^2-m^2+i\epsilon\right)}\int\frac{d^4k'}{(2\pi)^4}K(k,k',p)\Phi(k',p)$$

where

$$\begin{aligned}\eta(k,p) &= \frac{(m^2-L^2)}{\left(k_1^2-L^2+i\epsilon\right)}\frac{(m^2-L^2)}{\left(k_2^2-L^2+i\epsilon\right)}\\ &= \frac{(m^2-L^2)}{\left((\frac{p}{2}+k)^2-L^2+i\epsilon\right)}\frac{(m^2-L^2)}{\left((\frac{p}{2}-k)^2-L^2+i\epsilon\right)}\end{aligned}$$

Equation remains the same!

- Use Nakanishi representation and apply to both sides the LF projection $\int\ldots dk_-$.
- Obtain new equation for $g(\gamma,z)$. L appears in the equation, but the result does not depend on it!

• New equation for $g(\gamma, z)$

$$\int_0^\infty d\gamma' \int_{-1}^1 dz' F(\gamma, z; \gamma', z')\, g_i(\gamma', z') = \int_0^\infty d\gamma' \int_{-1}^1 dz' \sum_{ij} V_{ij}(\gamma, z; \gamma', z') g_j(\gamma', z')$$

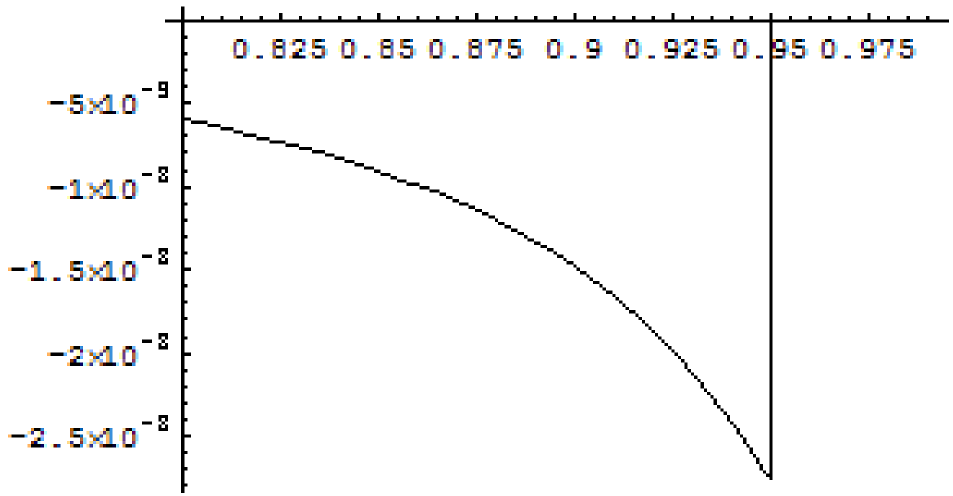
where

$$F(\gamma, z; \gamma', z') = \begin{cases} f(\gamma, z; \gamma', z'), & \text{if } -1 \le z' \le z \le 1 \\ f(\gamma, -z; \gamma', -z'), & \text{if } -1 \le z \le z' \le 1 \end{cases}$$

$$f(\gamma, z; \gamma', z') = \frac{(L^2 - m^2)}{\left[\gamma \frac{(1-z')}{(1-z)} + \gamma' + (1-z')(1+z)\kappa^2 + \Big(z' - z(1-z')\Big) m^2 + \frac{(z-z')}{(1-z)} L^2\right]^3}$$

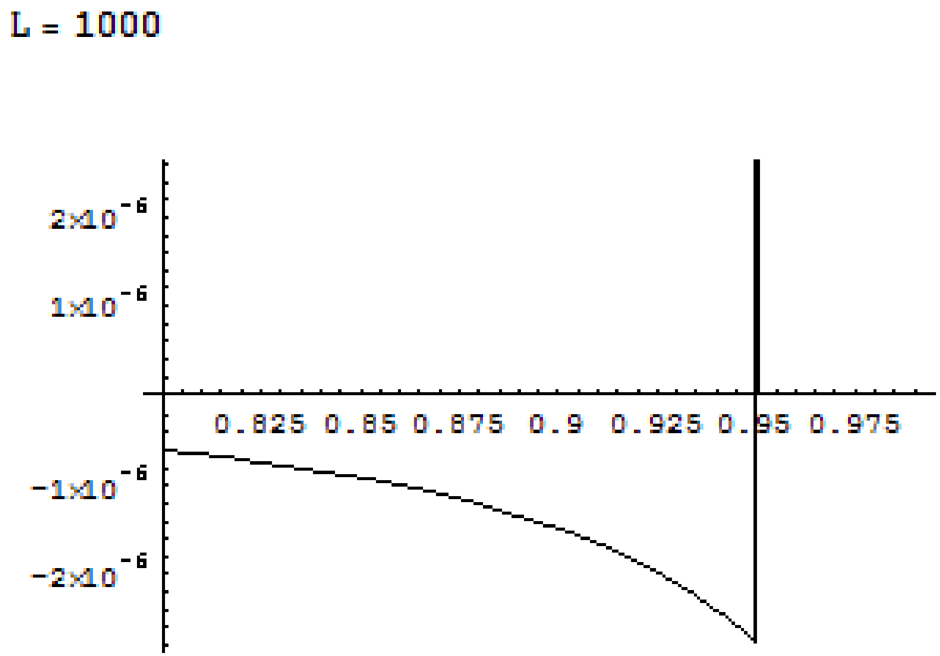
Double integral in l.h.s.

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 10000$



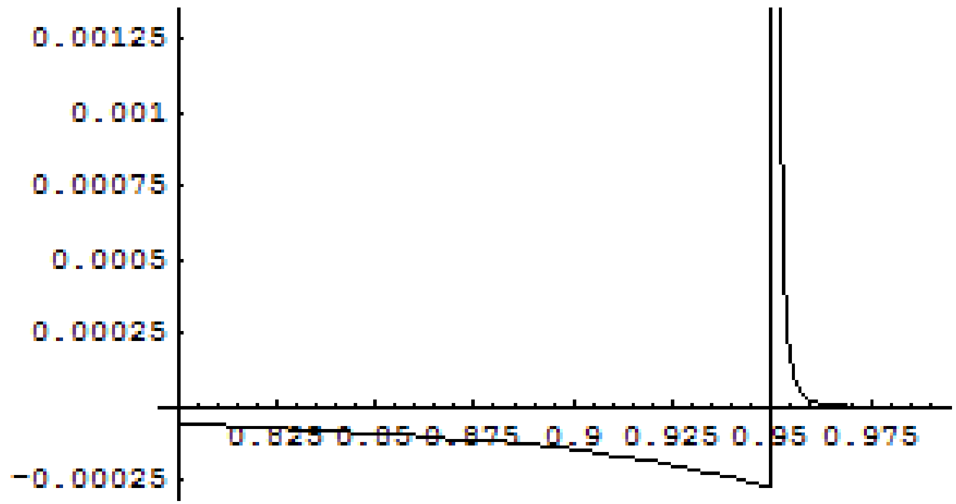
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 10000$

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 1000$



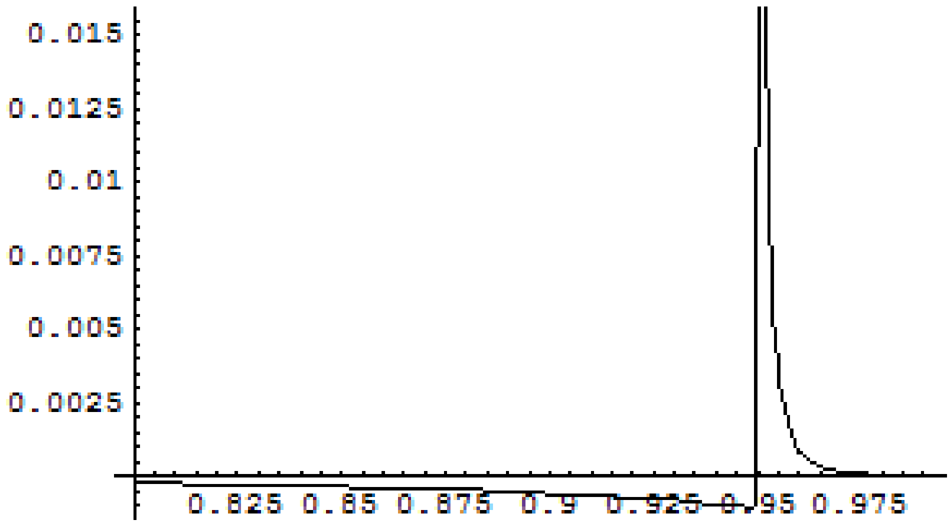
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 1000$

• Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z', $L = 100$



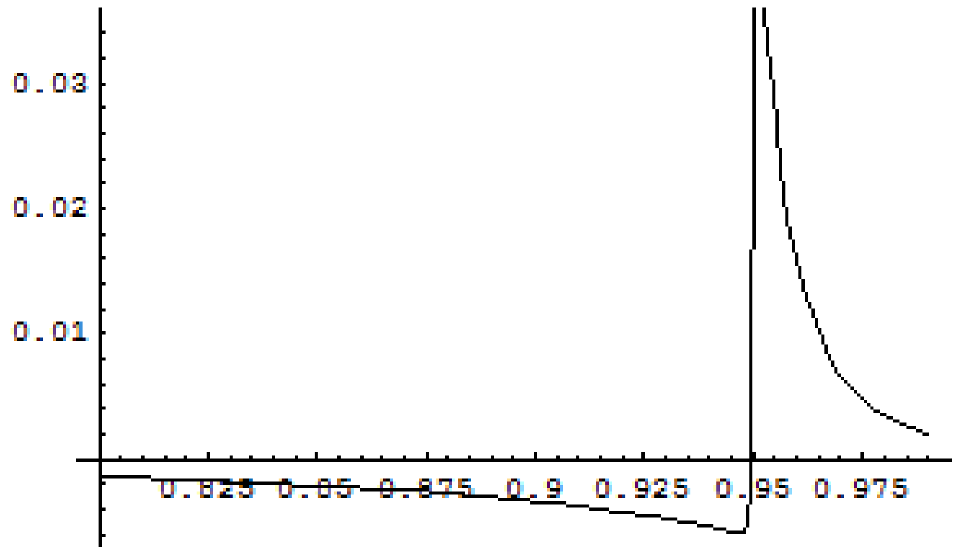
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 100$

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 50$



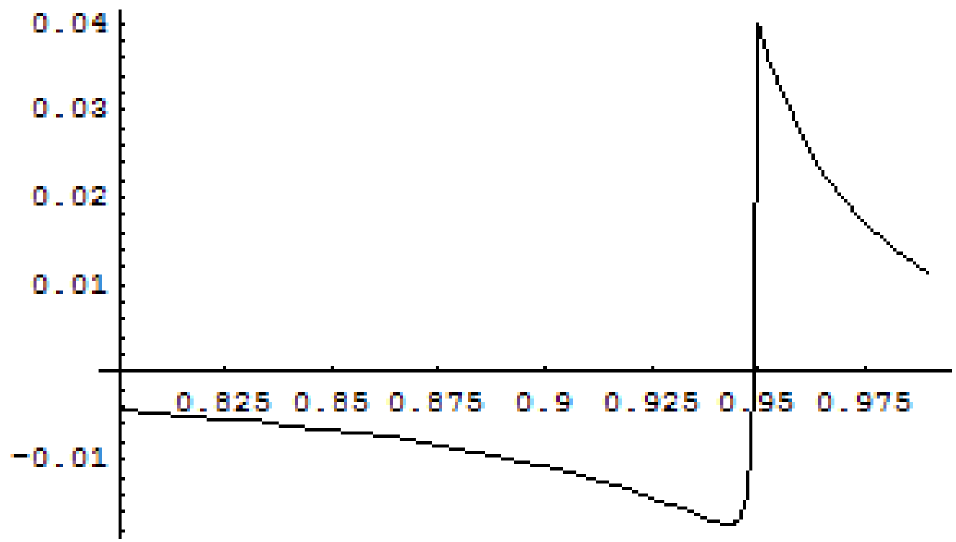
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 50$

• Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z', $L = 20$



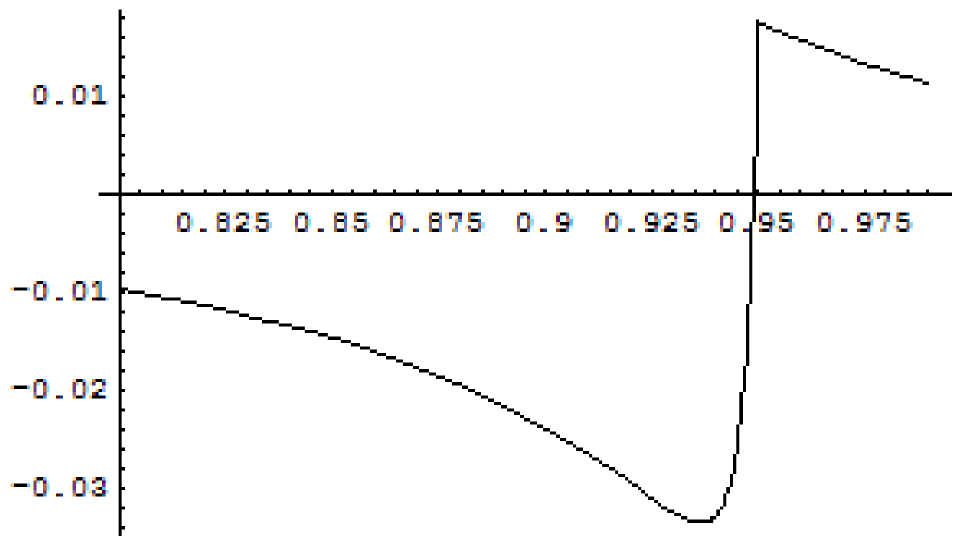
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 20$

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 10$



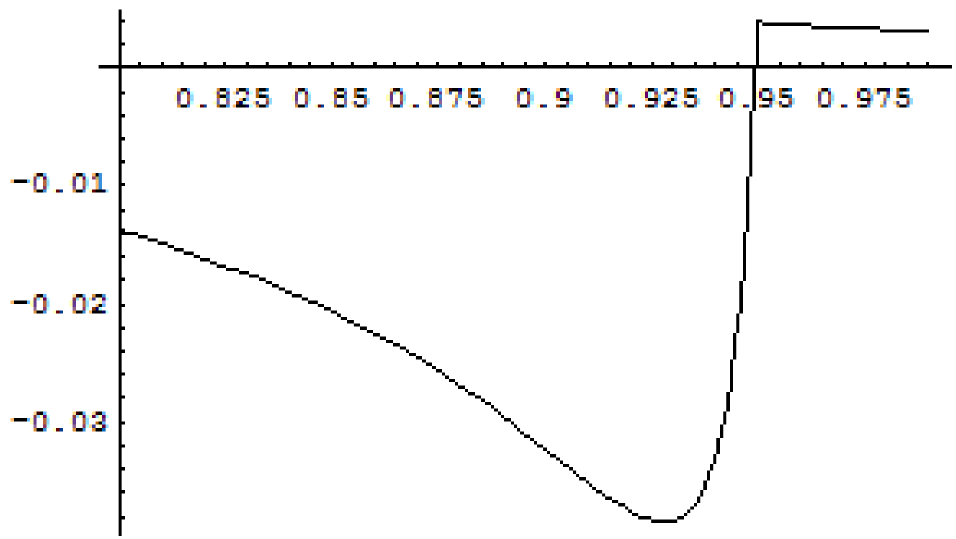
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 10$

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 5$



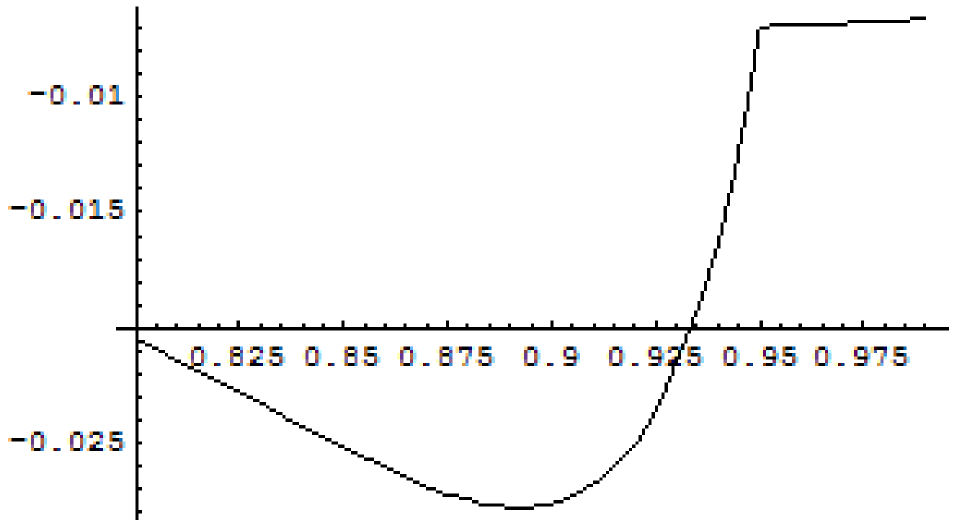
Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 5$

• **Kernel** $V_{14}(\gamma, z; \gamma', z')$ **v.s.** z'**,** $L = 3$



Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 3$

• Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z', $L = 1.1$



Kernel $V_{14}(\gamma, z; \gamma', z')$ v.s. z' for fixed $z = 0.95$, $L = 1.1$

• Numerical results

Scalar exchange (Yukawa model)

$\mu = 0.15$, $\Lambda = 2$, $L = 1.1$

B	g^2 (Dorkin et al.)	g^2 (We, Eucl.)	g^2 (We, Mink.)
0.08104	20.23	20.23	20.23
0.14773	30.34	30.34	30.34
0.27765	50.57	50.57	50.57

Binding energies, found via Mink. and Euclid, coincide now within 4 digits. Good precision!

• Numerical results

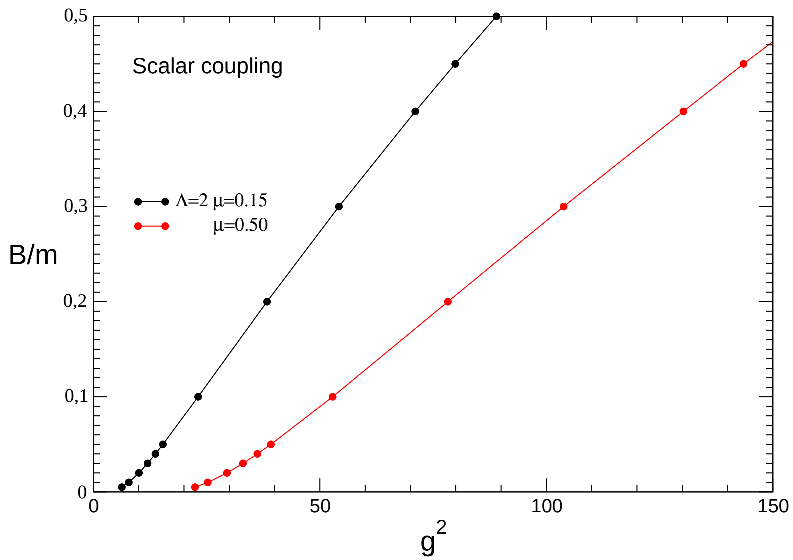
Pseudo scalar exchange

$\mu = 0.15,$, $\Lambda = 2$, $L = 1.1$

B	g^2 (Dorkin et al.)	g^2 (We, Eucl.)	g^2 (We, Mink.)
0.1	260.8	262.1	262.1

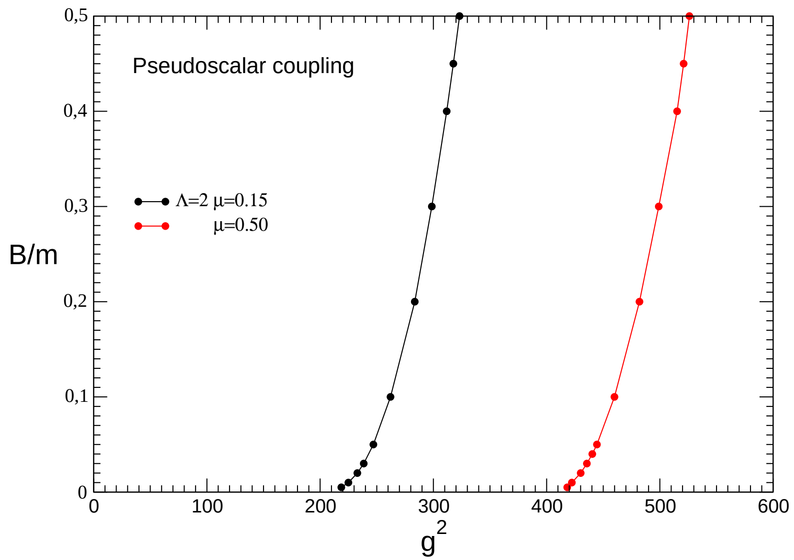
Our binding energies, found via Mink. and Euclid, coincide within 4 digits. Difference with Dorkin et al. is 0.5%. Good precision!.

Binding energy for scalar exchange v.s. g



Binding energy for scalar exchange v.s. g^2 for $\Lambda = 2$, $L = 1.1$, $\mu = 0.15$ and $\mu = 0.5$

nding energy for pseudo scalar exchange v.



Binding energy for pseudo scalar exchange v.s. g^2 for $\Lambda = 2$, $L = 1.1$, $\mu = 0.15$ and $\mu = 0.5$

• Further improvements

After introducing $\eta(L)$

$$\eta(L) = \frac{(m^2 - L^2)}{\left((\frac{p}{2} + k)^2 - L^2 + i\epsilon\right)} \frac{(m^2 - L^2)}{\left((\frac{p}{2} - k)^2 - L^2 + i\epsilon\right)}$$

one of 16 matrix elements (V_{23}) remains discontinuous. If we replace

$$\eta(L) \to \eta(L_1)\eta(L_2)$$

then all the matrix elements become continuous. One can take

$$L_1 = L_2 \quad \Rightarrow \quad \eta(L) \to \eta^2(L)$$

• Scattering states

Nakanishi representation for the scattering amplitude

$$T(s,t;k_1^2,k_2^2)$$

exists but seems too complicated technically (requires four Nakanishi parameters).

However, one can solve inhomogeneous BS equation in continuous spectrum (scattering states).

It should give the phase shifts.

• Conclusions

- A method to find Bethe-Salpeter amplitude in Minkowski space, both for spinless particles and fermions, is developed.
- The method is applied to the ladder (OBE) kernel and to the ladder +cross-ladder one.
- It can be applied to any kernel given by a Feynman graphs.
- It gives, as a by-product, LF wave function.
- Generalization for the scattering states seems possible.