Baryon structure in chiral effective field theory on the light front

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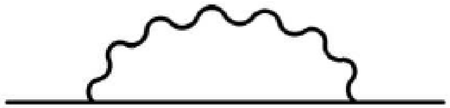
Outline

- Introduction
- General formalism: covariant light-front dynamics
- Chiral effective field theory
- Vertex functions
- Observables
- Perspectives

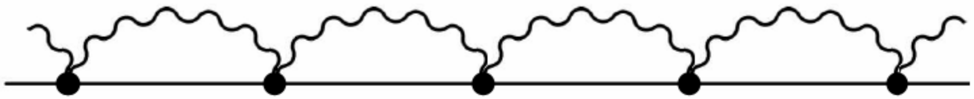
Introduction

Need of a non-perturbative framework to calculate bound state properties

easy with πNN coupling



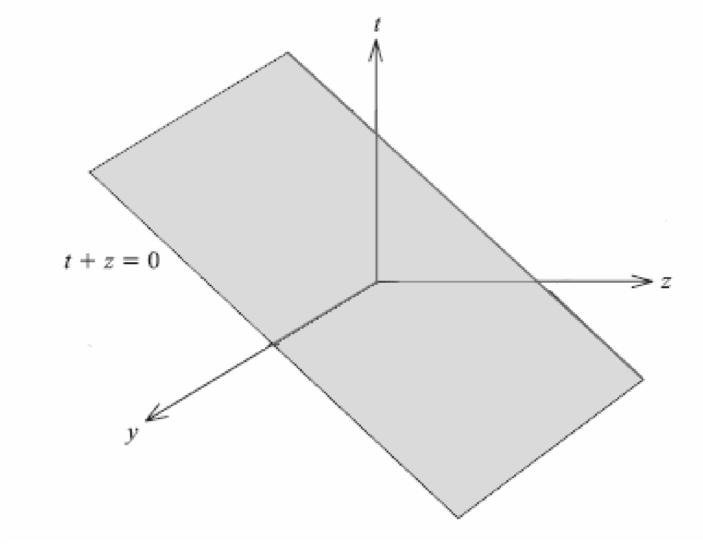
to be generalized for $\pi\pi NN$ case



Light-front dynamics

- Relativistic description of bound states
- The state vector is expanded in Fock components
- The state vector is defined on a surface in 4-dimensional space-time

Standard version of LFD: plane $t + z = const$

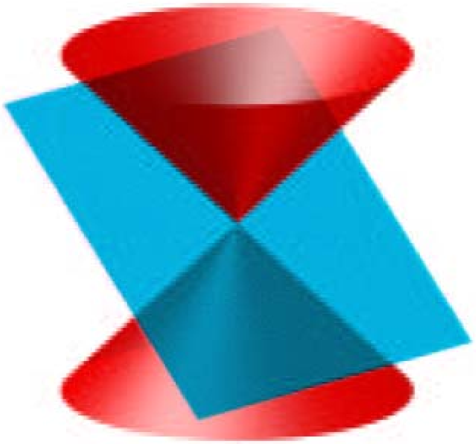


Rotational invariance is broken!

Covariant light-front dynamics

Plane: $\omega \cdot x = 0$

$\omega^2 = 0$



(Any plane tangential to the red cone)

$\omega = (1, 0, 0, -1)$ corresponds to standard LFD

Advantages:

- no vacuum fluctuation contributions
- transparent connection with non-relativistic approaches
- clear separability of unphysical components in approximate calculations

Fock representation of the state vector

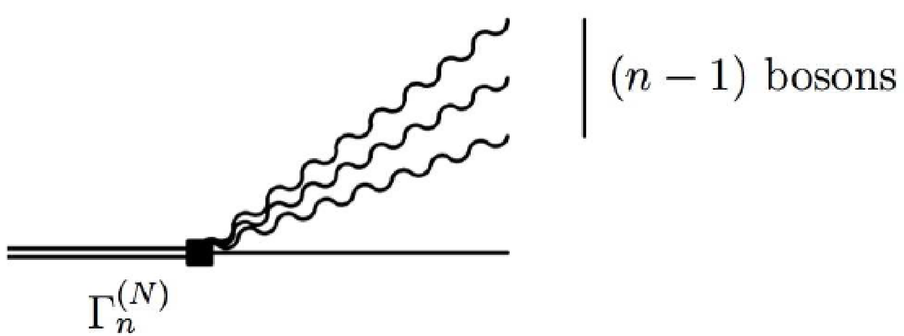
The state vector

$$\varphi(p) = |1\rangle + |2\rangle + \ldots + |N\rangle + \ldots$$

N – the maximal number of Fock sectors under consideration

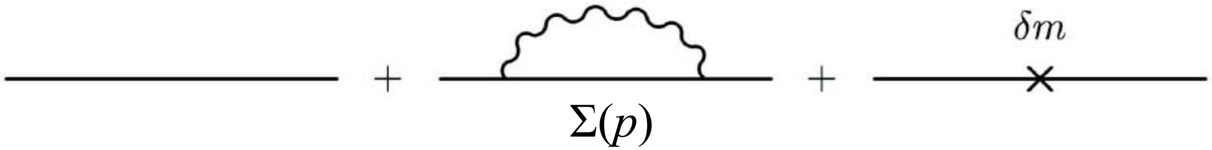
n – number of constituents in a given Fock sector, $n \leq N$

The many-body vertex fuction

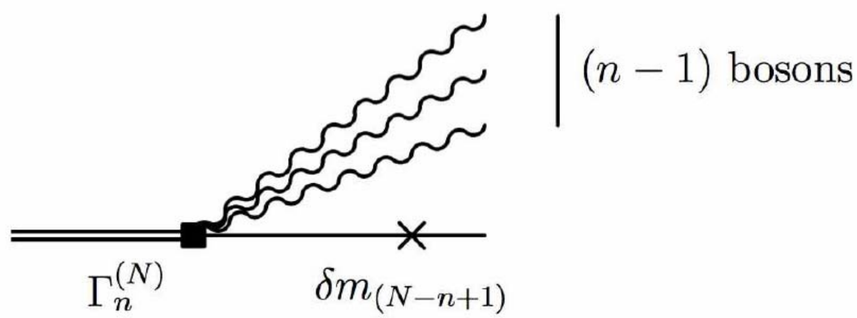


Renormalization scheme

Contribution to the physical fermion propagator



The general case: dependence on the Fock sector

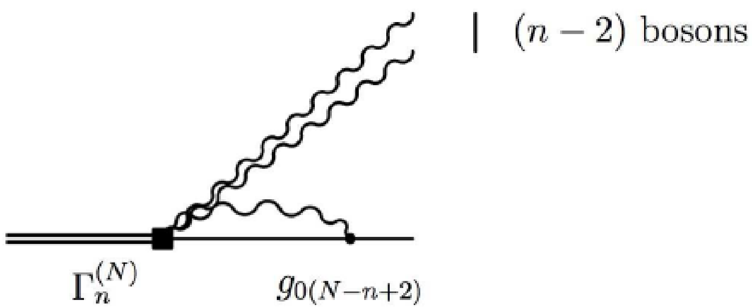


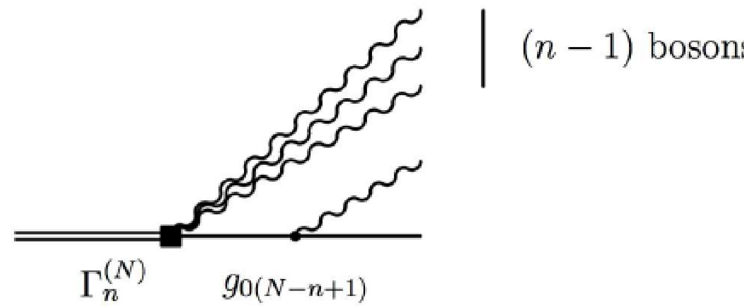
(maximal number of particles in which
the fermion line can fluctuate)

Renormalization scheme

Bare coupling constant: the same strategy

- Interaction with internal bosons

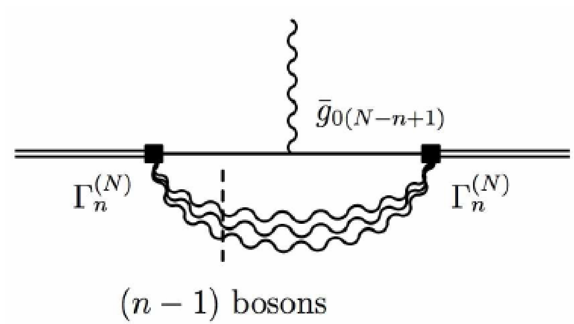




the number of particles in which the fermion line can fluctuate after absorbtion

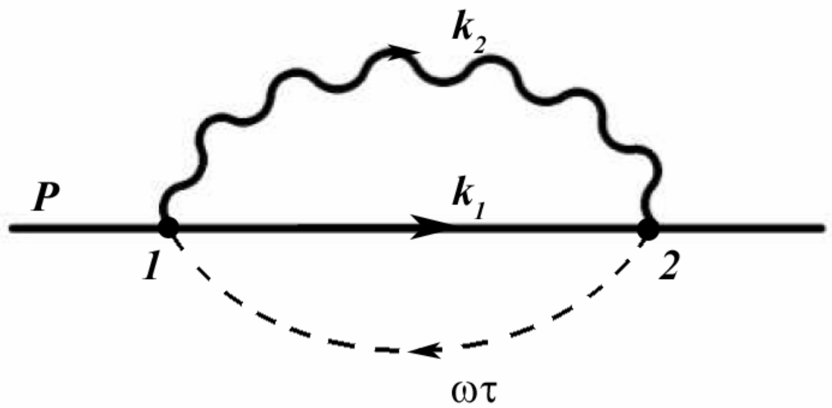
… before emission

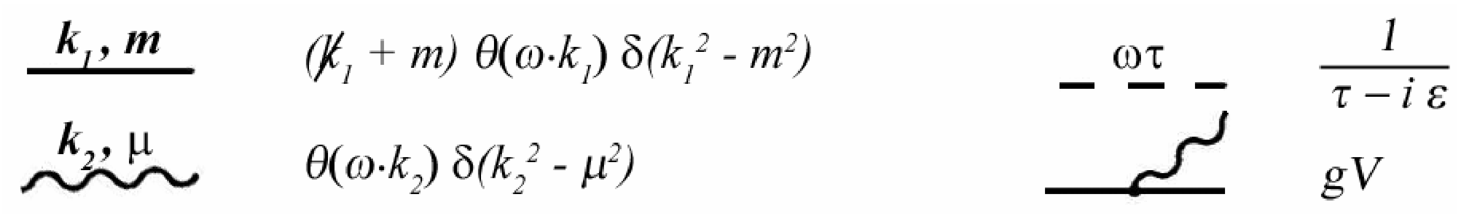
- Interaction with external bosons



Covariant light-front graph technique

- Particles are on their mass shell
- New fictitious particles – spurions
- Conservation laws include all physical particles and spurions





Conservation law: $P + \omega\tau = k_1 + k_2$

Integration over τ and unfixed 4-momentum

ChPT Lagrangian

- Lagrangian is formulated in terms of u fields

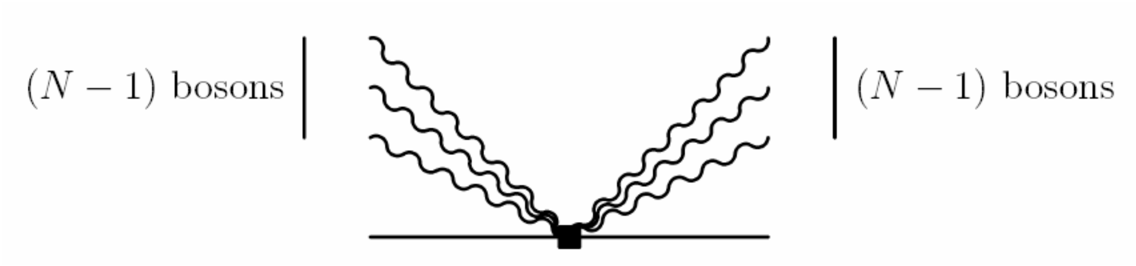
$$u = e^{i\frac{\vec{\tau}\cdot\vec{\pi}}{2F_0}}$$

F_0 is a pion decay constant

- Expansion in a finite number of pion fields

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots + \mathcal{L}^{(N)} + \ldots$$

- N-body Fock space truncation: $2(N-1)$ pion fields



ChPT Lagrangian

2-body Fock space truncation: $\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)}$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i \not{D} - \bar{m} + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu)\Psi,$$

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \mathrm{Tr}(\chi_+)\bar{\Psi}\Psi - \frac{c_2}{4M^2}\mathrm{Tr}(u_\mu u_\nu)(\bar{\Psi}D^\mu D^\nu \Psi + H.c.) + \frac{c_3}{2}\mathrm{Tr}(u^\mu u_\mu)\bar{\Psi}\Psi$$
$$- \frac{c_4}{4}\bar{\Psi}\gamma^\mu\gamma^\nu[u_\mu, u_\nu]\Psi + c_5\bar{\Psi}\left(\chi_+ - \frac{1}{2}\mathrm{Tr}(\chi_+)\right)\Psi + \bar{\Psi}(\frac{c_6}{2}f_{\mu\nu}^+ + \frac{c_7}{2}v_{\mu\nu}^{(s)})\sigma^{\mu\nu}\Psi$$

In the absence of external fields:

$$D_\mu \Psi = (\partial_\mu + \Gamma_\mu)\Psi$$
$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u\partial_\mu u^\dagger)$$
$$u_\mu = i(u^\dagger \partial_\mu u - u\partial_\mu u^\dagger)$$
$$\chi_+ = u^\dagger \chi u^\dagger + u\chi^\dagger u$$
$$\chi = \mu^2 \hat{I}$$
$$f_{\mu\nu}^+ = v_{\mu\nu}^{(s)} = 0$$

ChPT Lagrangian

In terms of pion fields

$$\mathcal{L}^{(1)} = -\frac{g_A}{2F_0}\bar{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi - \frac{1}{4F_0^2}\bar{\Psi}\,\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\gamma^\mu\Psi$$

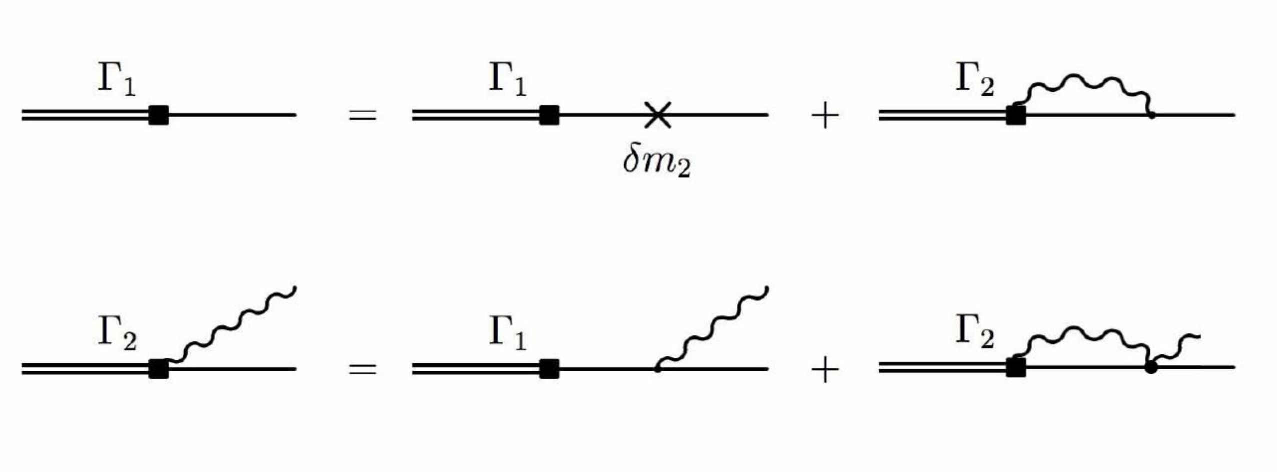
$$\mathcal{L}^{(2)} = 2c_1\mu^2(2-\frac{\vec{\pi}^2}{F_0^2})\bar{\Psi}\Psi - \frac{c_2}{2M^2F_0^2}\partial_\mu\vec{\pi}\cdot\partial_\nu\vec{\pi}(\bar{\Psi}\partial^\mu\partial^\nu\Psi + H.c.)$$

$$+\frac{c_3}{F_0^2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}\bar{\Psi}\Psi - \frac{ic_4}{2F_0^2}\bar{\Psi}\gamma^\mu\gamma^\nu\vec{\tau}\cdot\partial_\mu\vec{\pi}\times\partial_\nu\vec{\pi}\Psi$$

Vertex functions

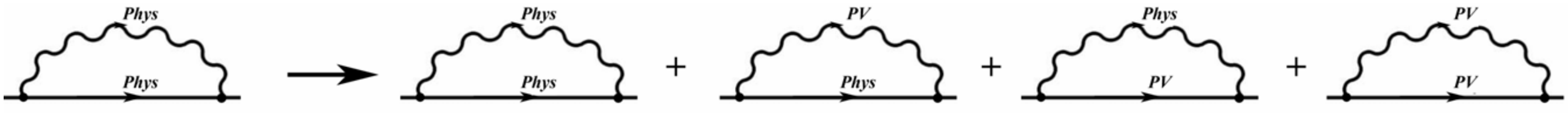
$$\mathcal{L} = -\frac{g_A}{2F_0}\bar{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi - \frac{1}{4F_0^2}\bar{\Psi}\,\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\gamma^\mu\Psi$$

Equations for the πN vertex functions in the case of the two-body Fock space truncation:



Regularization scheme

Pauli-Villars scheme: extension of Fock space



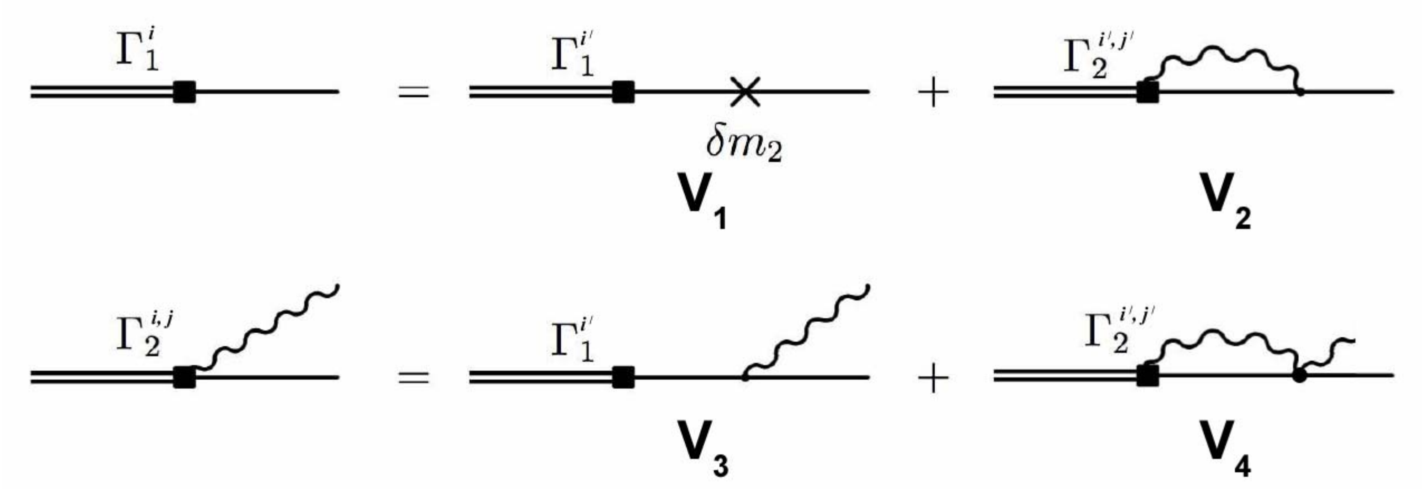
We use

i = *0* (physical) and *1* (Pauli-Villars) fermions

j = *0* (physical) and *1, 2* (Pauli-Villars) bosons

Vertex functions

System of equations:



$$\begin{aligned}
\bar{u}(p_{1i})\Gamma_1^i u(p) &= \bar{u}(p_{1i})\left(V_1 + V_2\right)u(p), \\
\bar{u}(k_{1i})\Gamma_2^{ij} u(p) &= \bar{u}(k_{1i})(V_3 + V_4)u(p).
\end{aligned}$$

Vertex functions representation

$$\bar{u}(k_{1i})\Gamma_1^i u(p) = (m_i - m^2) a_1^i \bar{u}(k_{1i}) u(p) \ ,$$

$$\bar{u}(k_{1i})\Gamma_2^{ij} u(p) = i\bar{u}(k_{1i}) \left((\not{k}_{2j} - \not{\omega}\tau)\, b_1^{ij}(R_\perp, x) + \frac{m\ \not{\omega}}{\omega \cdot p}\, b_2^{ij}(R_\perp, x) \right) \gamma_5 u(p) \ .$$

$$\tau = \frac{s - m^2}{2\,\omega \cdot p} \qquad x = \frac{\omega \cdot k_{2j}}{\omega \cdot p}$$

$$s = (k_{1i} + k_{2j})^2 \qquad R = k_{2j} - x\,p\,, \quad R = (R^0, R_\perp, R_\|)$$

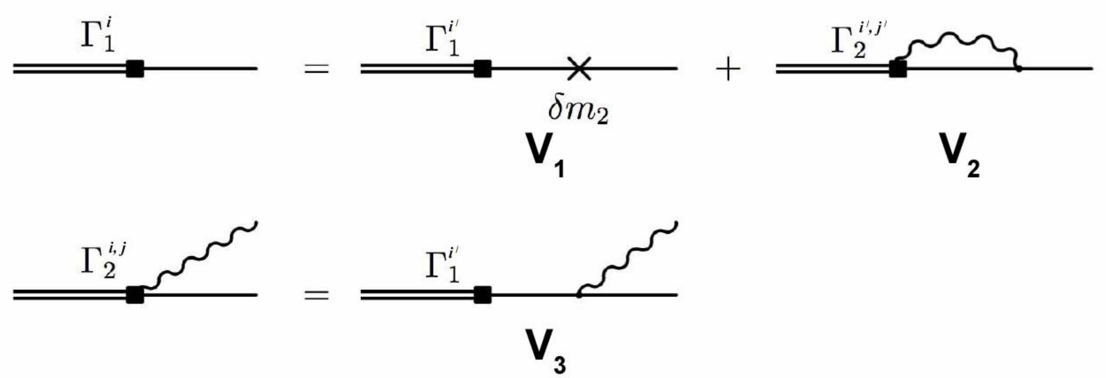
In our case a_1^i, b_1^{ij}, b_2^{ij} are constants depending on i only

Necessary condition: the physical on-mass shell vertex function should not depend on ω

$$b_2^{i=0,\, j=0}\,(s = m^2) = 0$$

Solution without contact term

Equations:



Solution:

$$b_1^{ij} = \frac{g_A}{F_0} m a_1^0 - \frac{g_A}{2F_0} \frac{(m_1 + m)(m_1 + m_i)}{(m + m_i)} a_1^1 \,,$$

$$b_2^{ij} = \frac{g_A}{2F_0} \frac{(m_1^2 - m^2)(m - m_i)}{2m} a_1^1 \,.$$

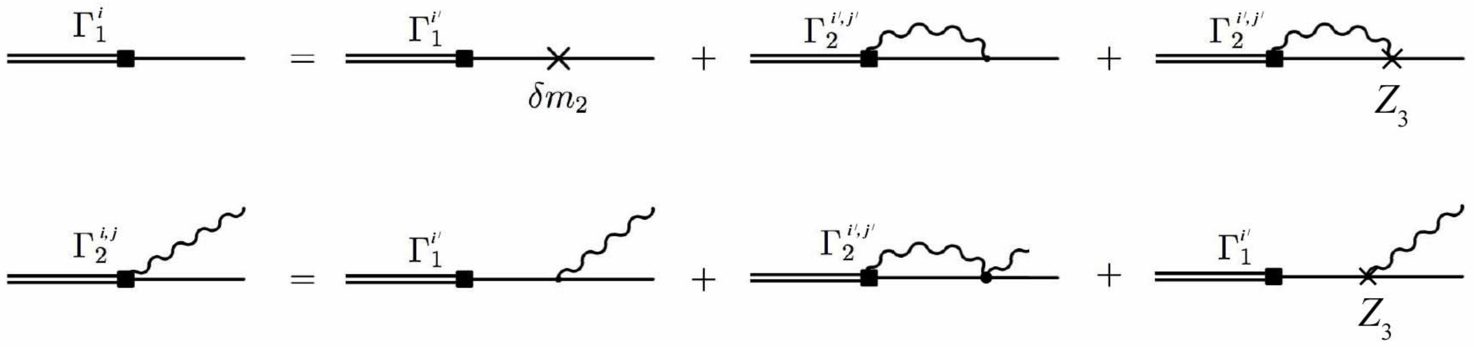
The condition $b_2{}^{i=0,\, j=0}\,(s = m^2) = 0$ is satisfied automatically

Solution with contact term

$$b_2^{\,i=0,\,j=0}\,(s = m^2) = const \neq 0$$

We need a new ω-dependent counterterm to kill it

$$L_{int} = -\frac{1}{2}\frac{g_A}{F_0}\bar{\Psi}\,\gamma^\mu\,\gamma_5\,\tau^b\,\partial_\mu\phi^b\,\Psi - \frac{1}{4F_0^2}\bar{\Psi}\,\gamma^\mu\,\vec{\tau}\cdot\vec{\phi}\times\partial_\mu\vec{\phi}\,\Psi + i\,Z_3\,\bar{\Psi}\,\not{\omega}\,\gamma_5\,\tau^b\,\phi^b\,\Psi + \ldots$$



With this counterterm $b_2^{\,i=0,\,j=0}\,(s = m^2) = 0$

Nucleon mass as a function of pion mass

$$\Sigma = A + \frac{\not{p}}{m}B + \frac{\not{\omega}m}{\omega p}C$$

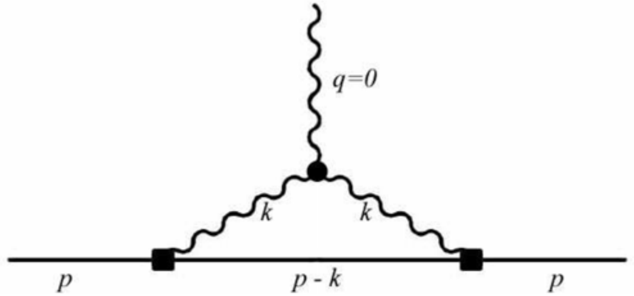
$C = 0$ (unphysical contribution)

$$m_N = \bar{m} + (f(m_N, \mu) - f(\bar{m}, 0))$$

$$f = A + B$$

$$m_N = \bar{m} + \frac{3g_A^2\bar{m}}{32F_0^2\pi^2}\mu^2 - \frac{3g_A^2}{32F_0^2\pi}\mu^3$$

Scalar form factor



Relation to the pion-nucleon sigma-term:

$$\sigma = \mu^2 \frac{\partial \Sigma}{\partial \mu^2}$$

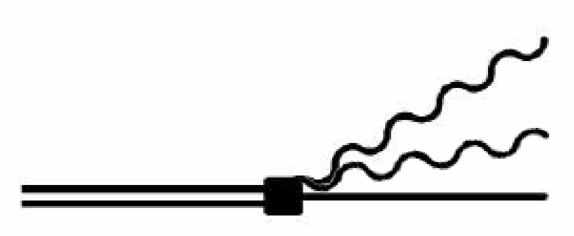
Analysis in progress…

Perspectives

- Finish test for 1 nucleon and 1 pion

    scalar and electromagnetic form factors calculation

- Calculations for 1 nucleon and 2 pions



} Δ and Roper resonance contributions

Calculations are already done for the Yukawa model (scalar meson)