Pion FF in QCD Sum Rules with NLCs

A. Bakulev, **A. Pimikov & N. Stefanis**

Bogoliubov Lab. Theor. Phys., JINR (Dubna, Russia)

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).
- AAV correlator and corresponding OPE diagrams.

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).
- AAV correlator and corresponding OPE diagrams.
- Non-local condensates in QCD calculations.

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).
- AAV correlator and corresponding OPE diagrams.
- Non-local condensates in QCD calculations.
- QCD SR with non-local condensates .

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).
- AAV correlator and corresponding OPE diagrams.
- Non-local condensates in QCD calculations.
- QCD SR with non-local condensates .
- QCD SR vs experimental data.

Content:

- Historical introduction: $\boldsymbol{A.V.E.}$ and others.
- Definition of pion form factor (FF).
- AAV correlator and corresponding OPE diagrams.
- Non-local condensates in QCD calculations.
- QCD SR with non-local condensates .
- QCD SR vs experimental data.
- Local Duality approach and NLC SRs.

Historical

introduction

How has all that started to evolve...

A. V.

How has all that started to evolve...

A. V.

A. Radyushkin

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

G. Korchemsky

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

A. B.

G. Korchemsky

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

A. B.

G. Korchemsky

A. Belitsky

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

A. B.

G. Korchemsky

A. Belitsky

O. T. & A. V.& S. M.

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

A. B.

G. Korchemsky

A. Belitsky

A. Pimikov

BMS & A. V.

O. T. & A. V.& S. M.

How has all that started to evolve...

A. V.

A. Radyushkin

O. Teryaev

S. Mikhailov

A. B.

G. Korchemsky

A. Belitsky

A. Vladimirov

BMS & A. V.

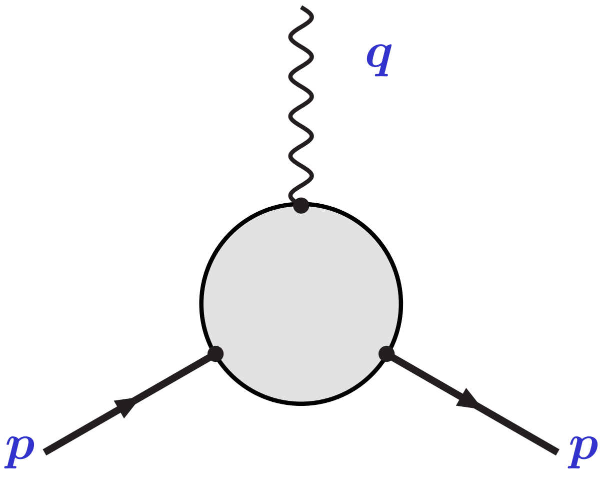
O. T. & A. V.& S. M.

Pion FF
in
QCD Sum Rule Approach

Definition of pion FF

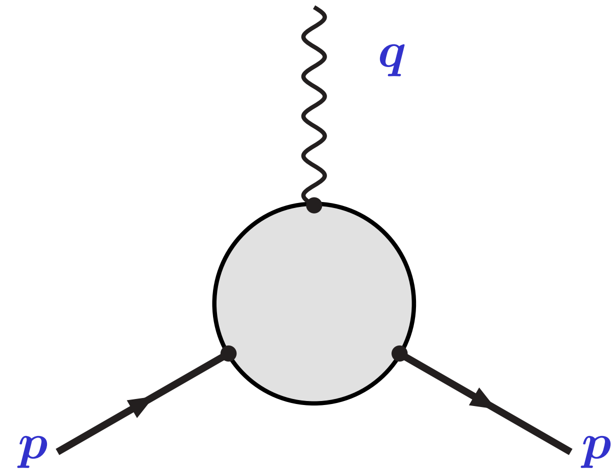
Pion FF F_π is defined by the matrix element

$$\langle\pi^+(p')|J_\mu(0)|\pi^+(p)\rangle = (p+p')_\mu F_\pi(Q^2)\,,$$



where J_μ is the electromagnetic current, $(p'-p)^2 = q^2 \equiv -Q^2$ is the photon virtuality, and pion FF is normalized to $F_\pi(0)=1$.

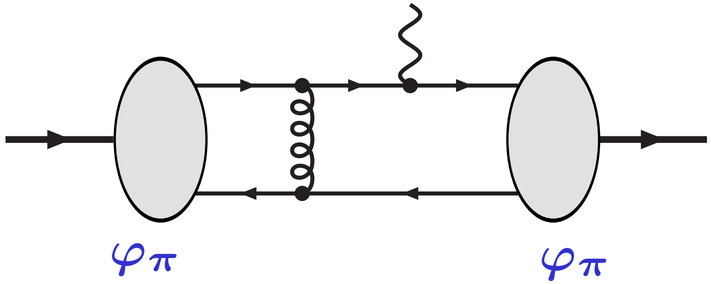
Definition of pion FF



Pion FF F_π is defined by the matrix element

$$\langle\pi^+(p')|J_\mu(0)|\pi^+(p)\rangle = (p+p')_\mu F_\pi(Q^2)\,,$$

where J_μ is the electromagnetic current, $(p'-p)^2 = q^2 \equiv -Q^2$ is the photon virtuality, and pion FF is normalized to $F_\pi(0)=1$.
At asymptotically large Q^2 pQCD factorization gives pion FF



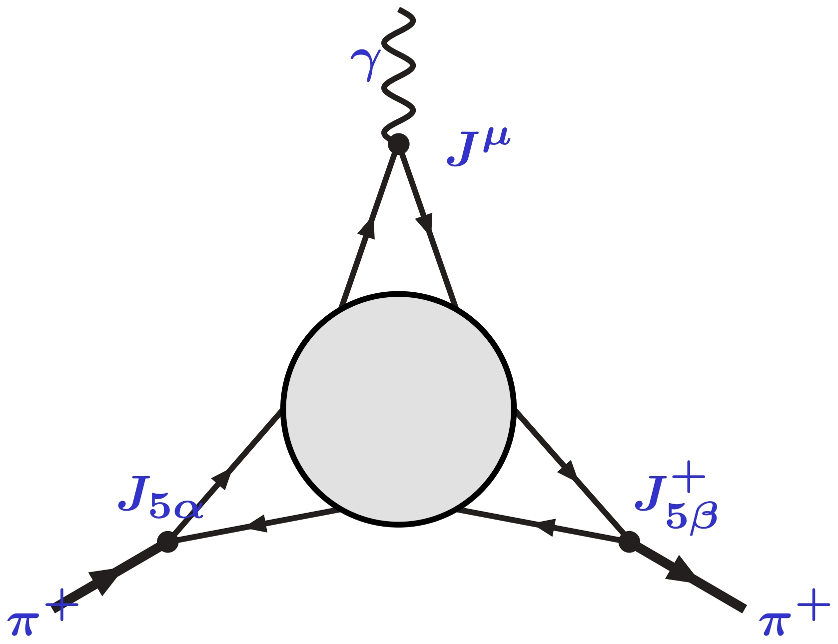
$$F_\pi(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{9Q^2}\left|\int_0^1 \frac{\varphi_\pi(x,Q^2)}{x}\,dx\right|^2$$

in terms of twist-2 pion DA $\varphi_\pi(x,Q^2)$.

AAV correlator

Axial-Axial-Vector correlator can be used for studying pion FF by QCD SR technique:

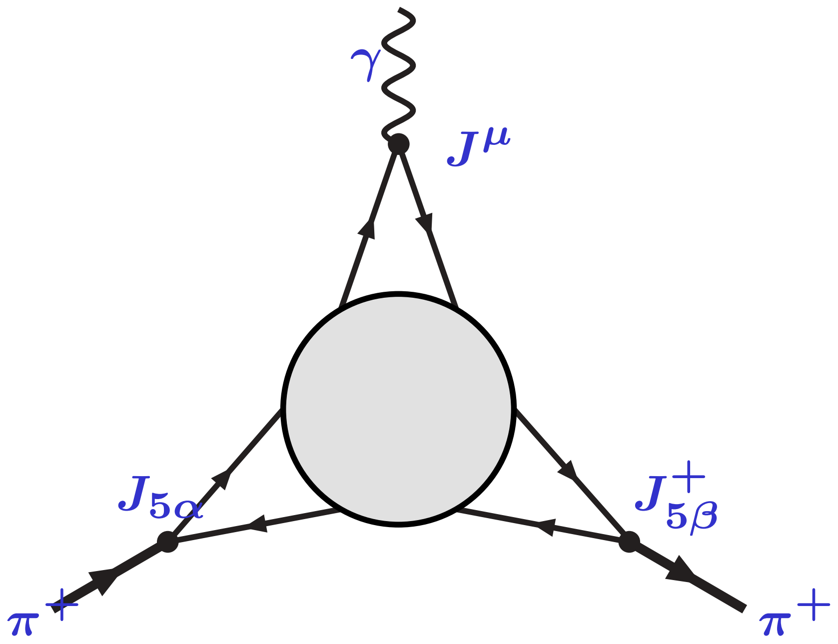
$$\int\!\!\int d^4x\, d^4y\, e^{i(qx-p_2y)} \langle 0|T\left[J^{+}_{5\beta}(y) J^{\mu}(x) J_{5\alpha}(0)\right]|0\rangle$$



AAV correlator

Axial-Axial-Vector correlator can be used for studying pion FF by QCD SR technique:

$$\int\!\!\int d^4x\, d^4y\, e^{i(qx-p_2y)} \langle 0|T\left[J^{+}_{5\beta}(y) J^{\mu}(x) J_{5\alpha}(0)\right]|0\rangle$$

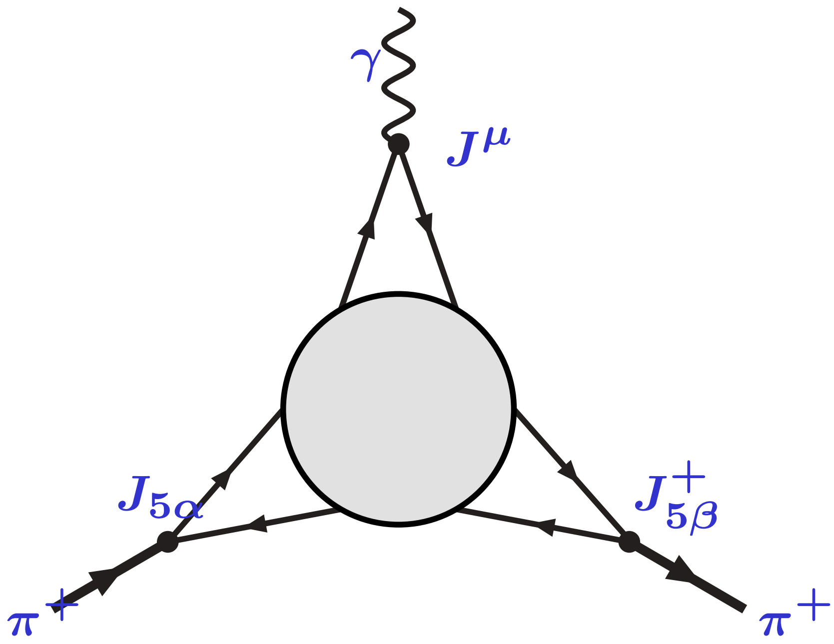


where EM current $J^{\mu}(x) = e_u\,\overline{u}(x)\gamma^{\mu}u(x) + e_d\,\overline{d}(x)\gamma^{\mu}d(x)$

AAV correlator

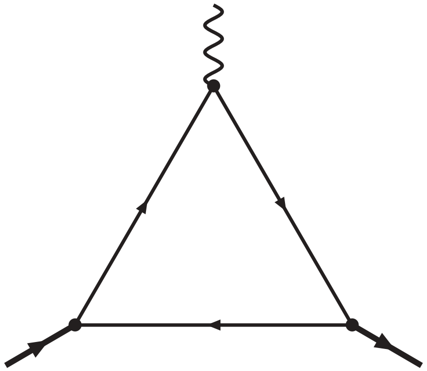
Axial-Axial-Vector correlator can be used for studying pion FF by QCD SR technique:

$$\int\!\!\int d^4x\, d^4y\, e^{i(qx-p_2y)}\langle 0|T\left[J^{+}_{5\beta}(y)J^{\mu}(x)J_{5\alpha}(0)\right]|0\rangle$$



where EM current $J^{\mu}(x) = e_u\,\overline{u}(x)\gamma^{\mu}u(x) + e_d\,\overline{d}(x)\gamma^{\mu}d(x)$ and axial-vector current: $J_{5\alpha}(x) = \overline{d}(x)\gamma_5\gamma_{\alpha}u(x)$.

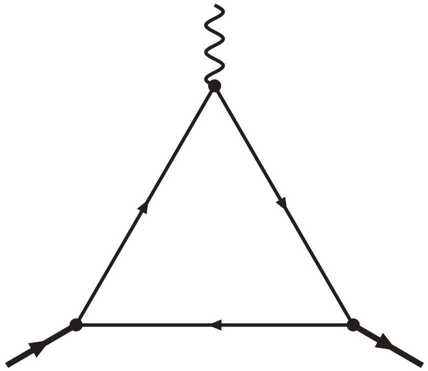
Diagramms for AAV-correlator



1 Perturbative LO term

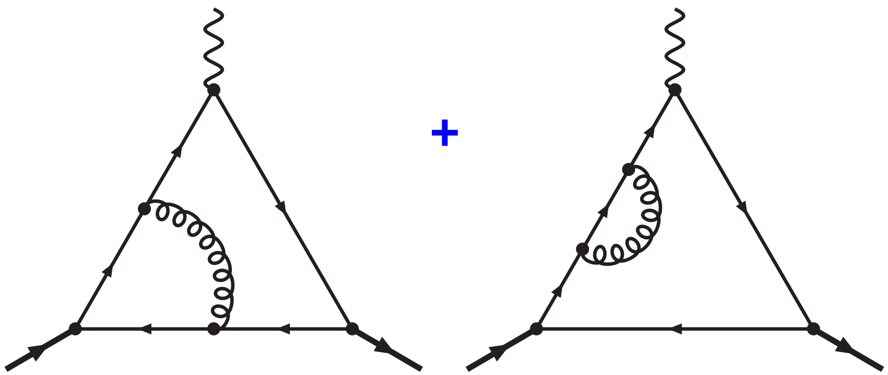
Nesterenko&Radyushkin
$\oplus$**Ioffe&Smilga [1982]**

Diagramms for AAV-correlator



1 Perturbative LO term

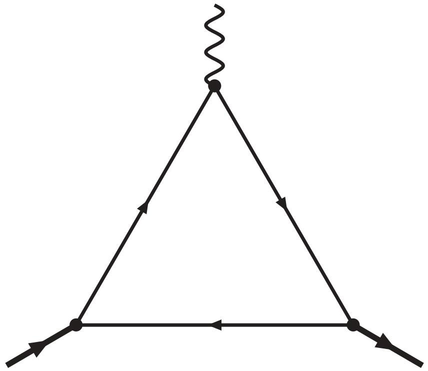
Nesterenko&Radyushkin
$\oplus$**Ioffe&Smilga [1982]**



Perturbative NLO terms

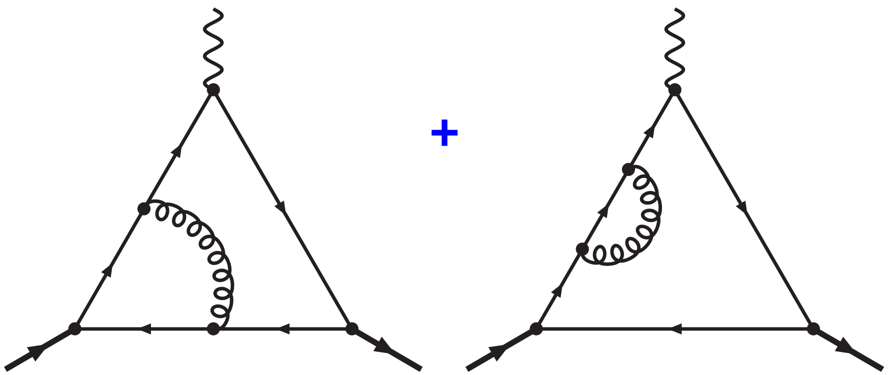
+ . . .
Braguta&Onishchenko [2004]

Diagramms for AAV-correlator



1 Perturbative LO term

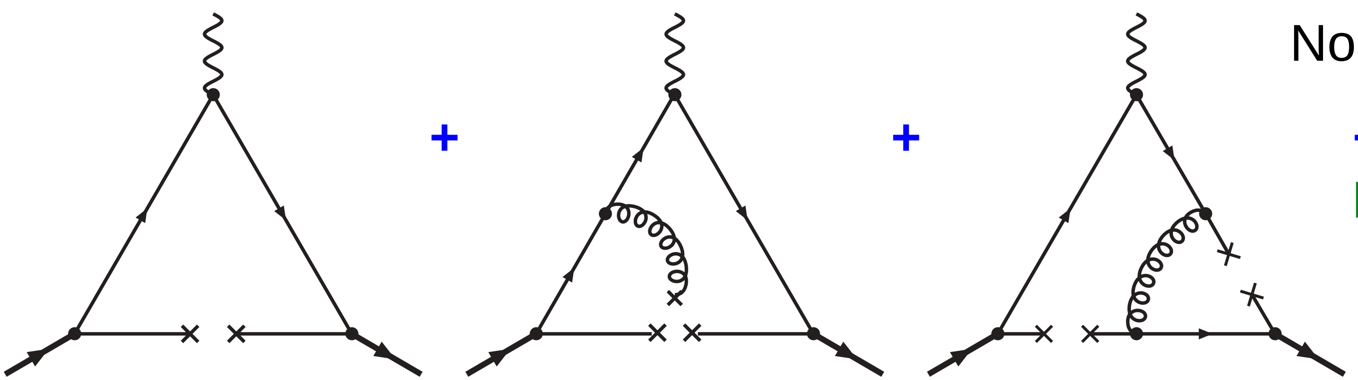
Nesterenko&Radyushkin
⊕Ioffe&Smilga [1982]



Perturbative NLO terms

+ . . .

Braguta&Onishchenko [2004]

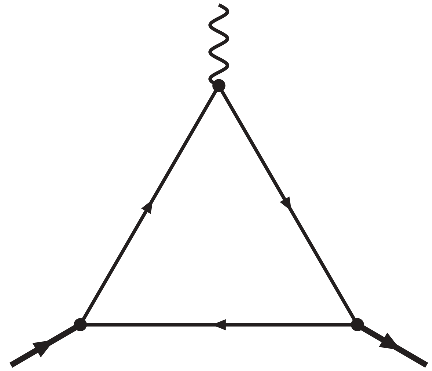


Nonperturbative terms

+ . . .

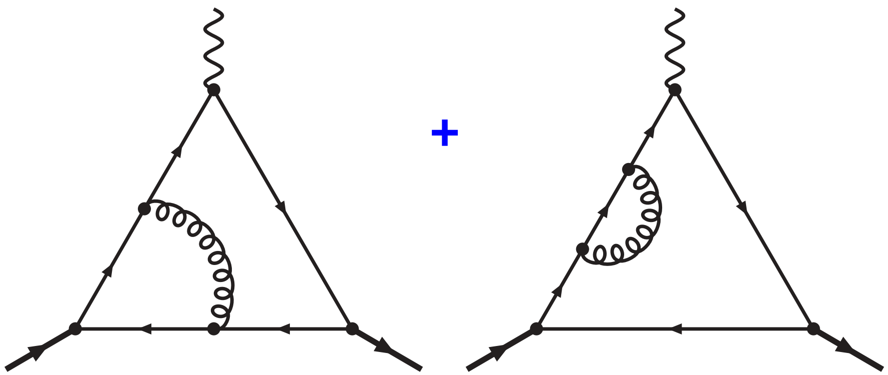
Nesterenko&Radyushkin
⊕Ioffe&Smilga [1982]
local condensates

Diagramms for AAV-correlator



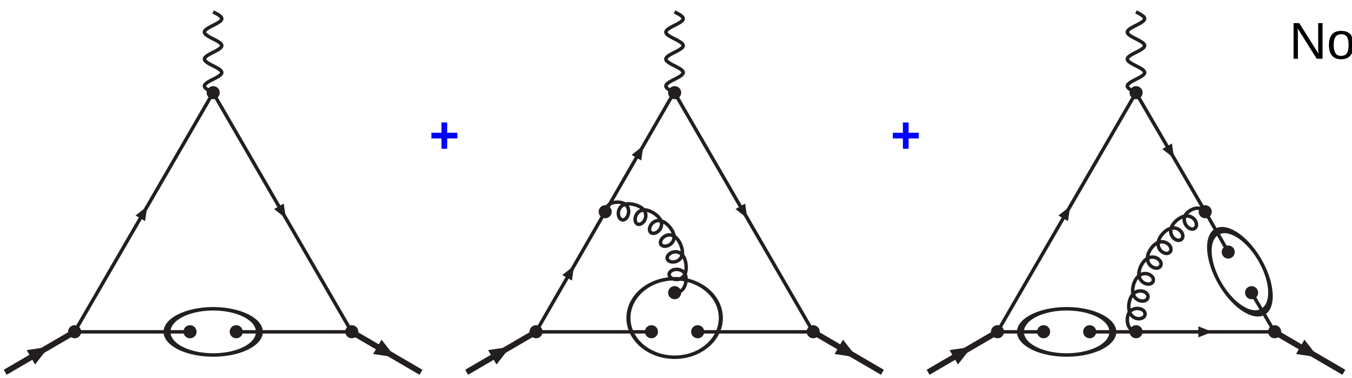
1 Perturbative LO term

Nesterenko&Radyushkin
⊕Ioffe&Smilga [1982]



Perturbative NLO terms

+ . . .
Braguta&Onishchenko [2004]



Nonperturbative terms

+ . . .
A. B.&Radyushkin [1991]

nonlocal condensates

Non-Local Condensates
in
QCD Sum Rule Approach

Introducing NLC in QCD calculations

$$T\left(\overline{\psi}\psi\right) = \overline{\psi}\psi \quad + \quad :\overline{\psi}\psi: \text{ (Wick theorem)}$$

$$\langle 0|T\left(\overline{\psi}\psi\right)|0\rangle = i^{-1}\hat{S}_0(x) \quad + \quad \boxed{?}$$

QCD PT

$$\langle :\overline{\psi}\psi: \rangle \stackrel{\text{def}}{=} 0$$

Introducing NLC in QCD calculations

$$T\left(\overline{\psi}\psi\right) = \overbrace{\overline{\psi}\psi} + :\overline{\psi}\psi: \text{ (Wick theorem)}$$

$$\langle 0|T\left(\overline{\psi}\psi\right)|0\rangle = i^{-1}\hat{S}_0(x) + \boxed{?}$$

QCD PT

$$\langle :\overline{\psi}\psi:\rangle \overset{\text{def}}{=} 0$$

QCD SR

$$\langle :\overline{\psi}(0)\psi(0):\rangle = \langle \overline{q}q\rangle$$

CONST $\neq 0$

[SVZ'79]

Condensate

↓

Decay constants,
masses of hadrons

Introducing NLC in QCD calculations

$$T\left(\bar{\psi}\psi\right) = \overline{\bar{\psi}\psi} + :\bar{\psi}\psi: \text{ (Wick theorem)}$$

$$\langle 0|T\left(\bar{\psi}\psi\right)|0\rangle = i^{-1}\hat{S}_0(x) + \boxed{?}$$

QCD PT

$$\langle :\bar{\psi}\psi: \rangle \stackrel{\text{def}}{=} 0$$

QCD SR

$$\langle :\bar{\psi}(0)\psi(0): \rangle = \langle \bar{q}q \rangle$$

CONST $\neq 0$

[SVZ'79]

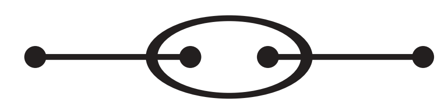
Condensate

Decay constants,
masses of hadrons

NLC QCD SR

$$\langle :\bar{\psi}(0)\psi(z): \rangle$$

$$F_S(z^2) + \hat{z}F_V(z^2)$$

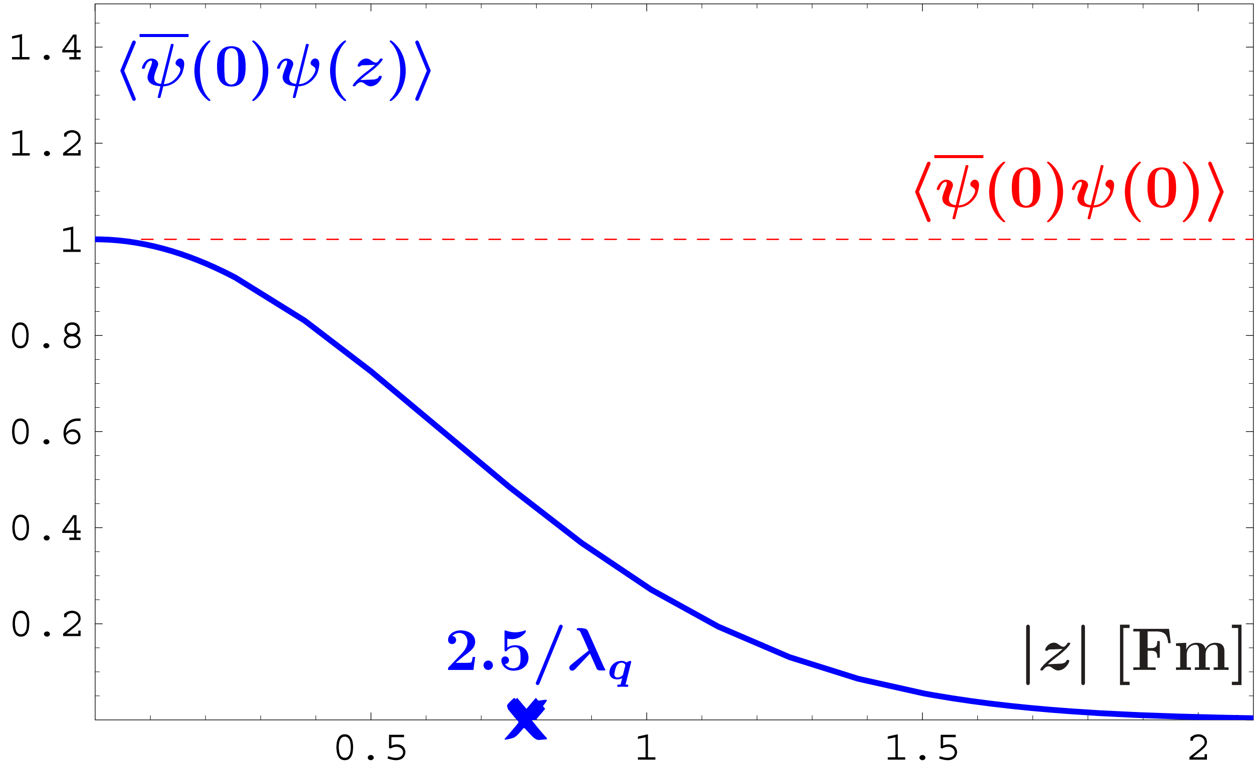


M&R '86

Nonlocal condensate

Distribution Amplitudes,
Form Factors

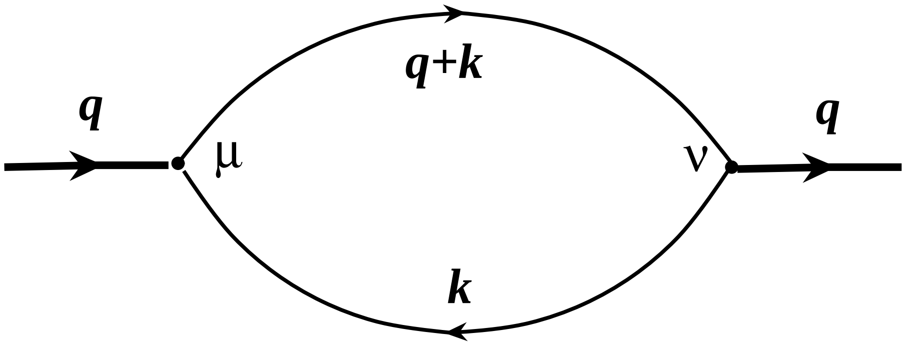
Lattice data of Pisa group



Nonlocality of quark condensates from lattice data of Pisa group in comparison with **local limit**.

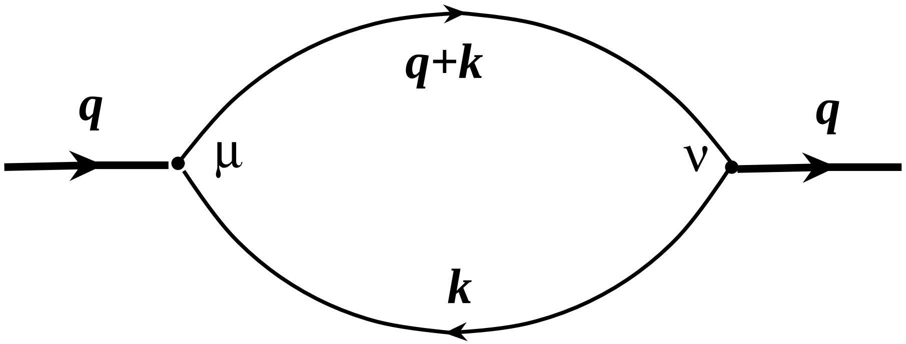
Even at $|z| \simeq 0.5$ **Fm** nonlocality is quite important!

Diagrams for $\langle T\left(J_1(z)J_2(0)\right)\rangle$

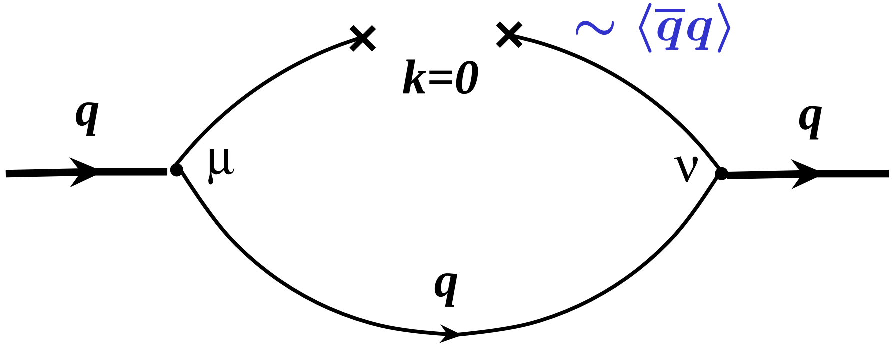


PT

Diagrams for $\langle T\left(J_1(z)J_2(0)\right)\rangle$

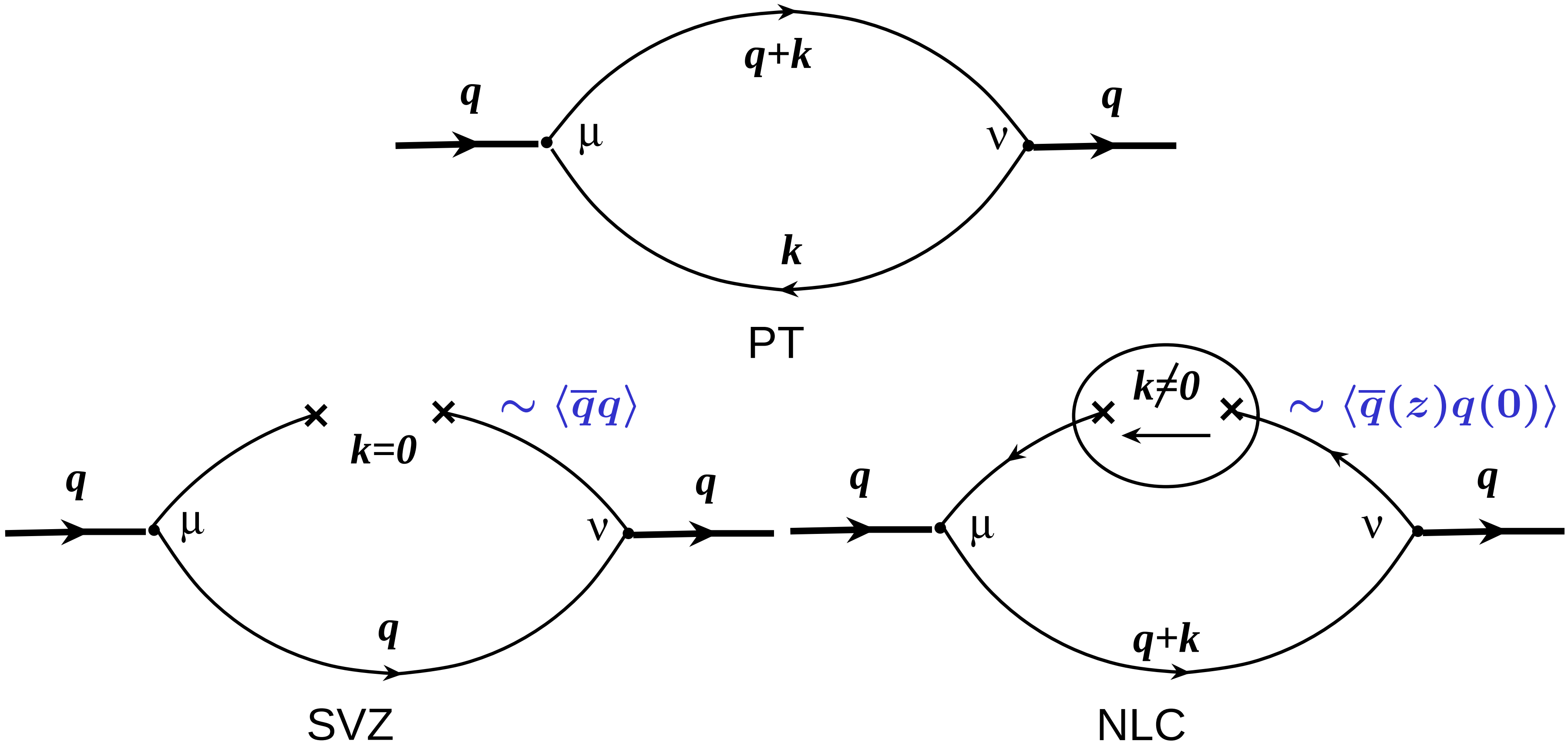


PT



SVZ

Diagrams for $\langle T\left(J_1(z)J_2(0)\right)\rangle$



Quarks run through vacuum with nonzero momentum $k \neq 0$:

$$\langle k^2\rangle = \frac{\langle\overline{\psi}D^2\psi\rangle}{\langle\overline{\psi}\psi\rangle} = \lambda_q^2 = 0.4 - 0.5\,\text{GeV}^2$$

Non-Local Condensates in QCD SR

- Illustration of NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$

Non-Local Condensates in QCD SR

- Illustration of NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \text{GeV}^2 & [\text{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05\ \text{GeV}^2 & [\text{ QCD SRs, 1991 }] \\ 0.4 - 0.5\ \text{GeV}^2 & [\text{ Lattice, 1998-2002 }] \end{cases}$$

Non-Local Condensates in QCD SR

- Illustration of NLC-model: $\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2\rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1\ \text{GeV}^2 & [\ \text{QCD SRs, 1987}\ ] \\ 0.5 \pm 0.05\ \text{GeV}^2 & [\ \text{QCD SRs, 1991}\ ] \\ 0.4 - 0.5\ \text{GeV}^2 & [\ \text{Lattice, 1998-2002}\ ] \end{cases}$$

- Correlation length $\lambda_q^{-1} \sim \rho$-meson size

Non-Local Condensates in QCD SR

- Illustration of NLC-model: $\langle \bar{q}(0)q(z)\rangle = \langle \bar{q}(0)q(0)\rangle e^{-|z^2|\lambda_q^2/8}$
- A **single scale** parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

$$\lambda_q^2 = \begin{cases} 0.4 \pm 0.1 \text{ GeV}^2 & [\text{ QCD SRs, 1987 }] \\ 0.5 \pm 0.05 \text{ GeV}^2 & [\text{ QCD SRs, 1991 }] \\ 0.4 - 0.5 \text{ GeV}^2 & [\text{ Lattice, 1998-2002 }] \end{cases}$$

- Correlation length $\lambda_q^{-1} \sim \rho$-meson size
- Possible to include second ($\Lambda \simeq 450$ MeV) scale with $\langle \bar{q}(0)q(z)\rangle\Big|_{|z|\gg 1 \text{ Fm}} \sim \langle \bar{q}q\rangle e^{-|z|\Lambda}$ (not included here)

Non-Local Condensates in QCD SR

Parameterization for scalar and vector condensates:

$$\langle\bar{\psi}(0)\psi(x)\rangle = \langle\bar{\psi}\psi\rangle \int_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,;$$

$$\langle\bar{\psi}(0)\gamma_\mu\psi(x)\rangle = -ix_\mu A_0 \int_0^\infty \boxed{f_V(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$

where $A_0 = 2\alpha_s\pi\langle\bar{\psi}\psi\rangle^2/81$.

Non-Local Condensates in QCD SR

Convenient to parameterize the 3-local condensate in fixed-point gauge by introduction of three scalar functions:

$$\begin{aligned}
\langle\bar{\psi}(0)\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= (x_\mu y_\nu - g_{\mu\nu}(xy))\overline{M}_1 \\
&+ (x_\mu x_\nu - g_{\mu\nu}x^2)\overline{M}_2\,; \\
\langle\bar{\psi}(0)\gamma_5\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle &= i\varepsilon_{\mu\nu xy}\overline{M}_3\,,
\end{aligned}$$

with

$$\overline{M}_i(y^2, x^2, (x-y)^2) =$$

$$A_i \iiint_0^\infty d\alpha_1\, d\alpha_2\, d\alpha_3 \boxed{f_i(\alpha_1, \alpha_2, \alpha_3)}\, e^{(\alpha_1 y^2 + \alpha_2 x^2 + \alpha_3 (x-y)^2)/4}\,.$$

where $A_i = \{-\frac{3}{2}, 2, \frac{3}{2}\}A_0$ **[Mikhailov&Radyushkin'89]**.

Non-Local Condensates in QCD SR

The minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\,(\alpha - \Lambda)\ ; \quad f_V(\alpha) = \delta\,'(\alpha - \Lambda)\ ; \quad \Lambda \equiv \lambda_q^2/2\,;$$

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1 - \Lambda)\ \delta\,(\alpha_2 - \Lambda)\ \delta\,(\alpha_3 - \Lambda)\ .$$

Only one parameter $\lambda_q^2 = 0.35 - 0.55$ **GeV^2**.

Non-Local Condensates in QCD SR

The minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\,(\alpha - \Lambda)\ ; \quad f_V(\alpha) = \delta\,'(\alpha - \Lambda)\ ; \quad \Lambda \equiv \lambda_q^2/2\,;$$

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1 - \Lambda)\ \delta\,(\alpha_2 - \Lambda)\ \delta\,(\alpha_3 - \Lambda)\ .$$

Only one parameter $\lambda_q^2 = 0.35 - 0.55$ **GeV**^{2}.

Problems:

- **QCD equations of motion are violated**
- **Vector current correlator is not transverse**
  $\Rightarrow$ **gauge invariance is broken**

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

What does it give us?

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

What does it give us?

- **If** $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\; x + y = 1\,,$

  than QCD equations of motion are satisfied;

Improved Gaussian model

We modify functions f_i: $f_i^{\text{imp}}(\alpha_1, \alpha_2, \alpha_3) =$

$$(1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z)\, \delta(\alpha_1 - x\Lambda)\, \delta(\alpha_2 - y\Lambda)\, \delta(\alpha_3 - z\Lambda)$$

What does it give us?

- **If** $12\,(X_2 + Y_2) - 9\,(X_1 + Y_1) = 1\,,\ x + y = 1\,,$

  than QCD equations of motion are satisfied;
- **We minimize nontransversity of polarization operator by special choice of model parameters:**

$$\begin{aligned} X_1 &= -0.082\,;\ Y_1 = Z_1 = -2.243\,;\ x = 0.788\,; \\ X_2 &= -1.298\,;\ Y_2 = Z_2 = -0.239\,;\ y = 0.212\,; \\ X_3 &= +1.775\,;\ Y_3 = Z_3 = -3.166\,;\ z = 0.212\,. \end{aligned}$$

Pion FF
in
NLC QCD SRs

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 F_\pi(Q^2) = \iint_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	**const** $+\, Q^2$

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 F_\pi(Q^2) = \iint_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	$\textbf{const} + Q^2$
B&R 91	LO	local $+$ nonlocal	$(\textbf{const} + Q^2)(e^{-Q^2\lambda_q^2} + \textbf{const})$

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2\, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	$\textbf{const} + Q^2$
B&R 91	LO	local $+$ nonlocal	$(\textbf{const} + Q^2)(e^{-Q^2\lambda_q^2} + \textbf{const})$
B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1 \, ds_2 \; \rho_3(s_1, s_2, Q^2) \, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2) \, .$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	$\textbf{const} + Q^2$
B&R 91	LO	local $+$ nonlocal	$(\textbf{const} + Q^2)(e^{-Q^2\lambda_q^2} + \textbf{const})$
B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)
Here	NLO	nonlocal	$(\textbf{const} + Q^2) \, e^{-Q^2\lambda_q^2}$

NLC QCD SR

The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1 \, ds_2 \; \rho_3(s_1, s_2, Q^2) \, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	$\textbf{const} + Q^2$
B&R 91	LO	local $+$ nonlocal	$(\textbf{const} + Q^2)(e^{-Q^2\lambda_q^2} + \textbf{const})$
B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)
Here	NLO	nonlocal	$(\textbf{const} + Q^2)\, e^{-Q^2\lambda_q^2}$

- Nonlocality improves Q^2 behavior of OPE $\Rightarrow$ widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.

NLC QCD SR

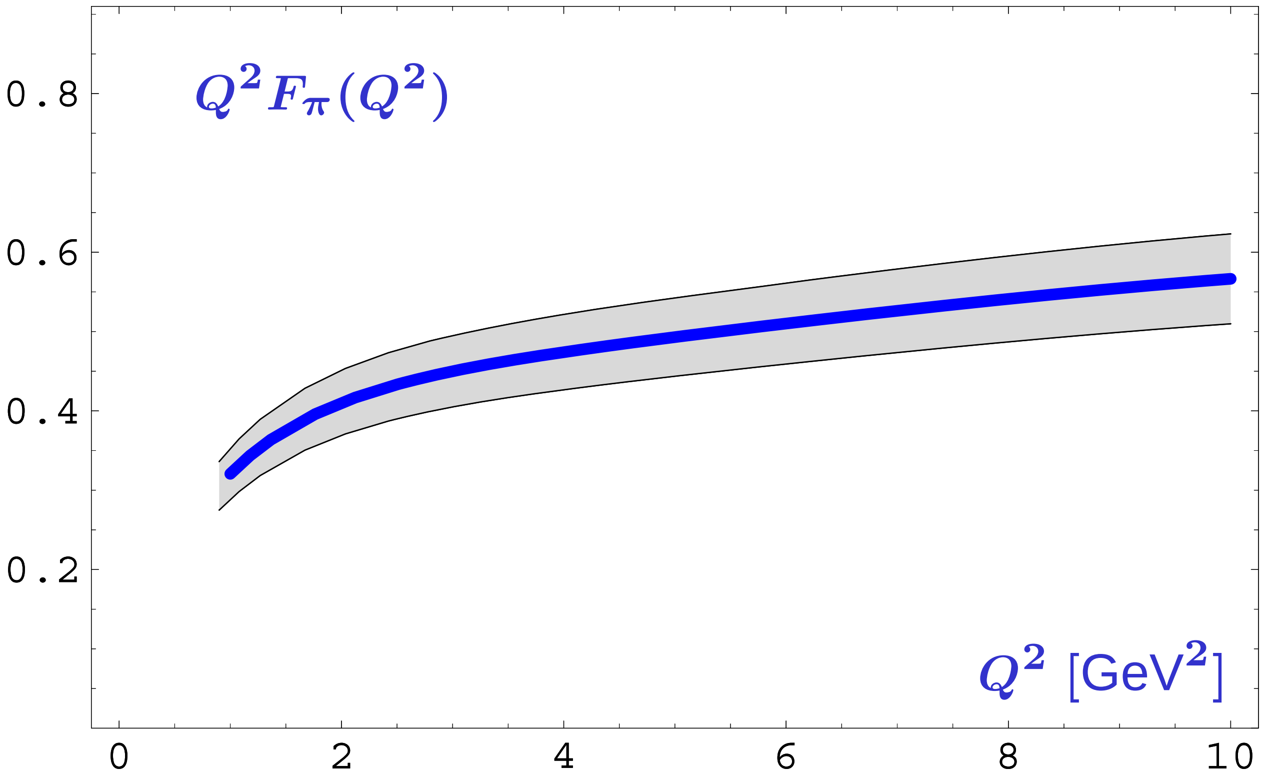
The Borel SR for the pion FF based on three-point AAV correlator:

$$f_\pi^2 F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Approach	Acc	Condensates	Q^2-behavior of Φ_{OPE}
N&R, I&S 82	LO	local	**const** $+ Q^2$
B&R 91	LO	local $+$ nonlocal	(**const** $+ Q^2)(e^{-Q^2\lambda_q^2} +$ **const**)
B&O 04 - LD	NLO	NO $M^2 \to \infty$	$\Phi_{\text{OPE}} \to 0$, $s_0 =$? (f_π LD SR)
Here	NLO	nonlocal	(**const** $+ Q^2)\, e^{-Q^2\lambda_q^2}$

- Nonlocality improves Q^2 behavior of OPE $\Rightarrow$ widens region of applicability up to $Q^2 \simeq 10$ **GeV**^{2}.
- We use model-independent expression for Φ_{OPE}-term obtained by **A. B.&Radyushkin**, but significantly different model of condensate's nonlocality.

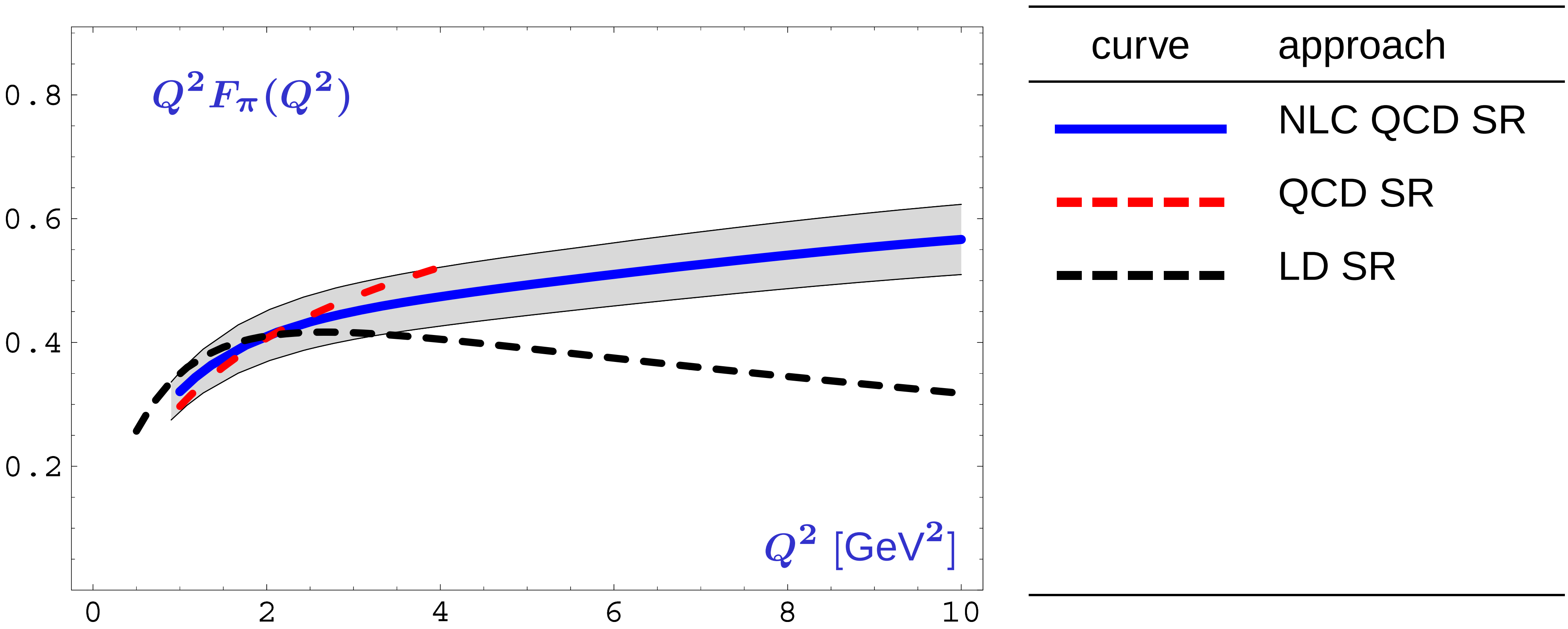
NLC QCD SR Result for $Q^2 F_\pi(Q^2)$



curve	approach
	NLC QCD SR

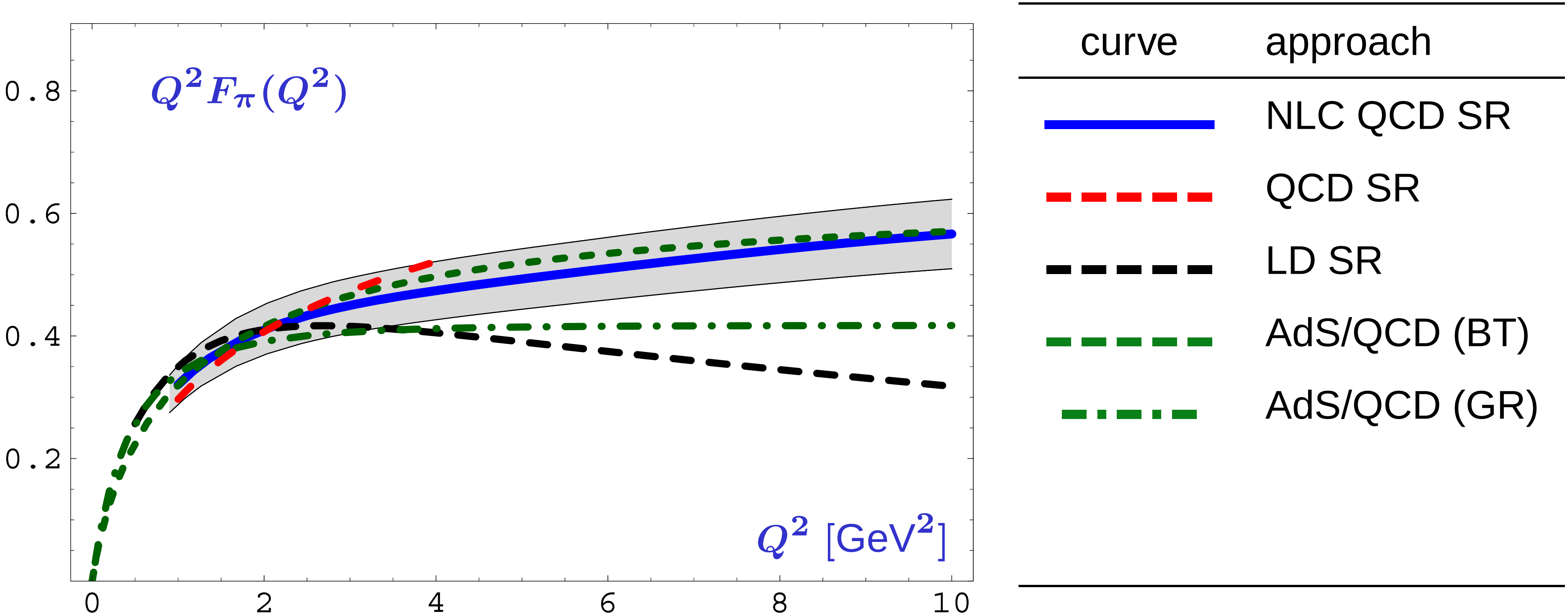
Pion FF from: SRs with NLC **(blue solid line)**,

NLC QCD SR Result for $Q^2 F_\pi(Q^2)$



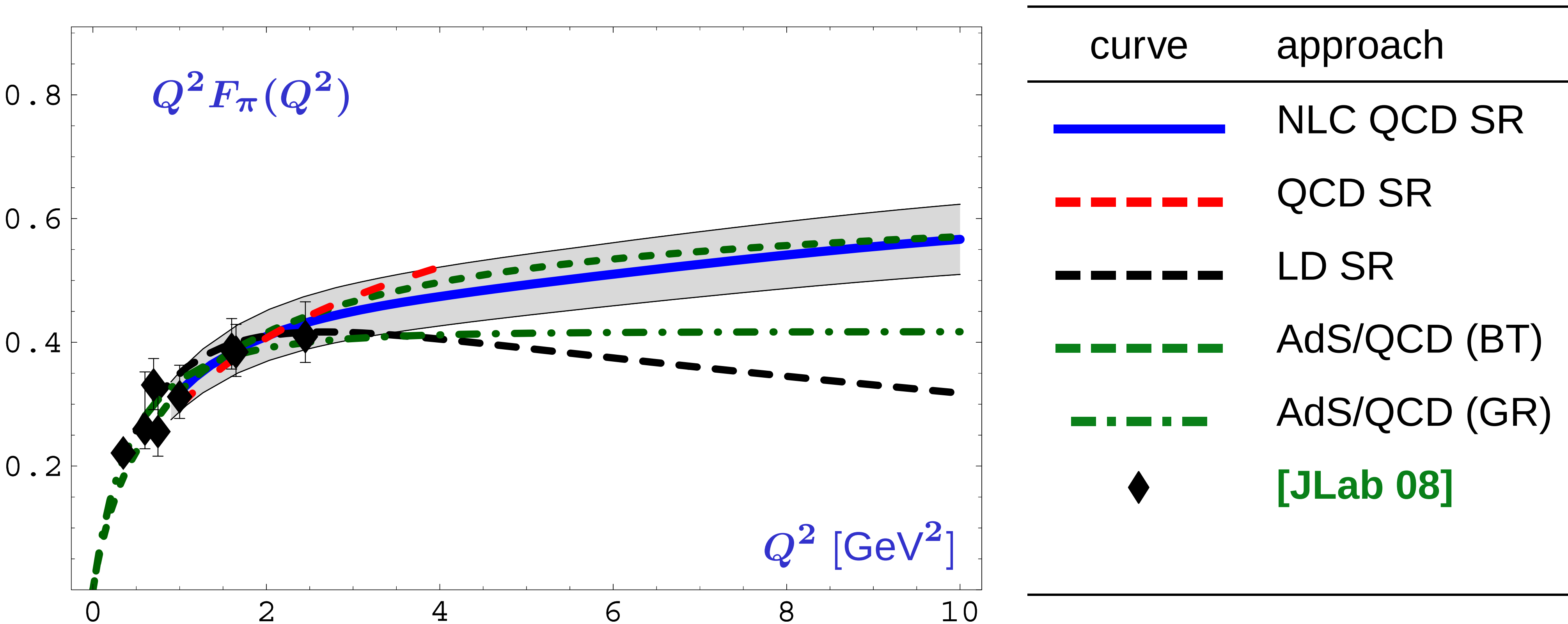
Pion FF from: SRs with NLC **(blue solid line)**,
standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**,
$O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**,

NLC QCD SR Result for $Q^2\,F_\pi(Q^2)$



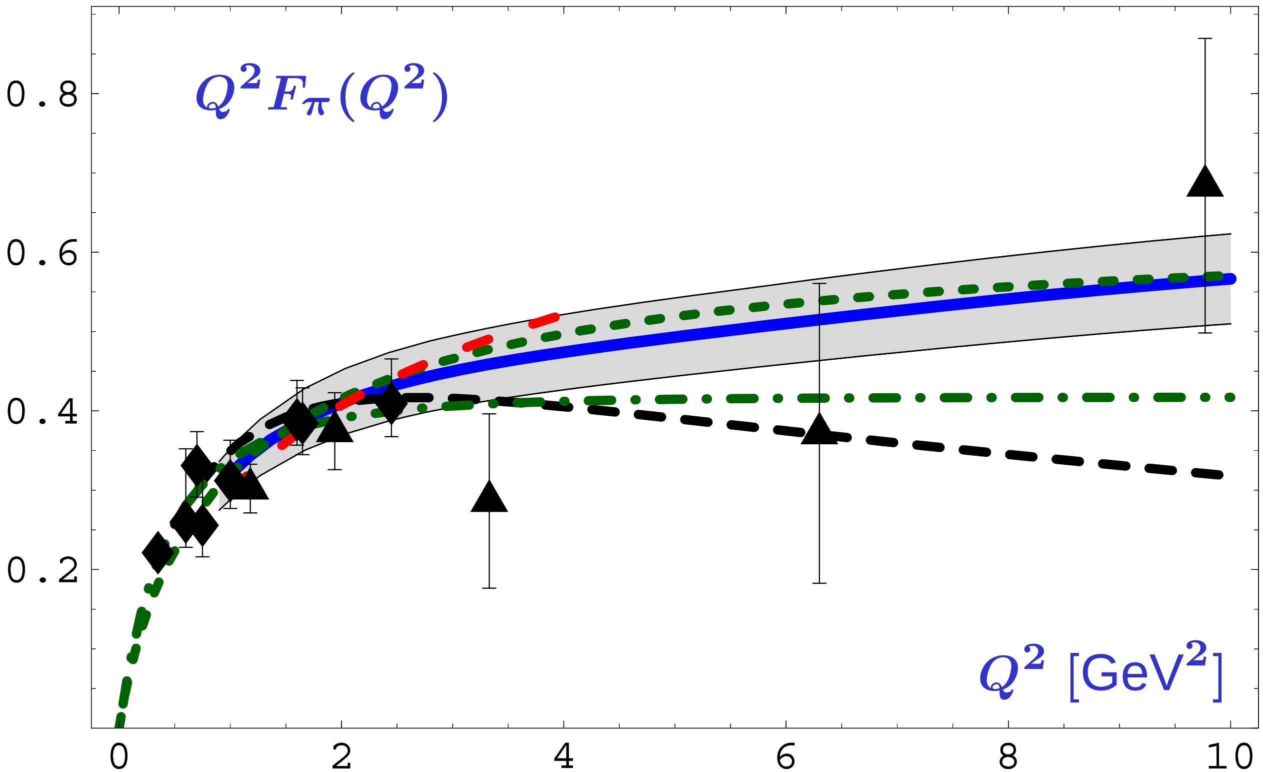
Pion FF from: SRs with NLC **(blue solid line)**, standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**, $O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**, AdS/QCD **(green dashed line)** **[B&T 07]** and **(green dot-dashed line)** **[G&R 07]**

NLC QCD SR Result for $Q^2\, F_\pi(Q^2)$



Pion FF from: SRs with NLC **(blue solid line)**, standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**, $O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**, AdS/QCD **(green dashed line)** **[B&T 07]** and **(green dot-dashed line)** **[G&R 07]** in comparison with **[JLab 08]** (♦) experimental data.

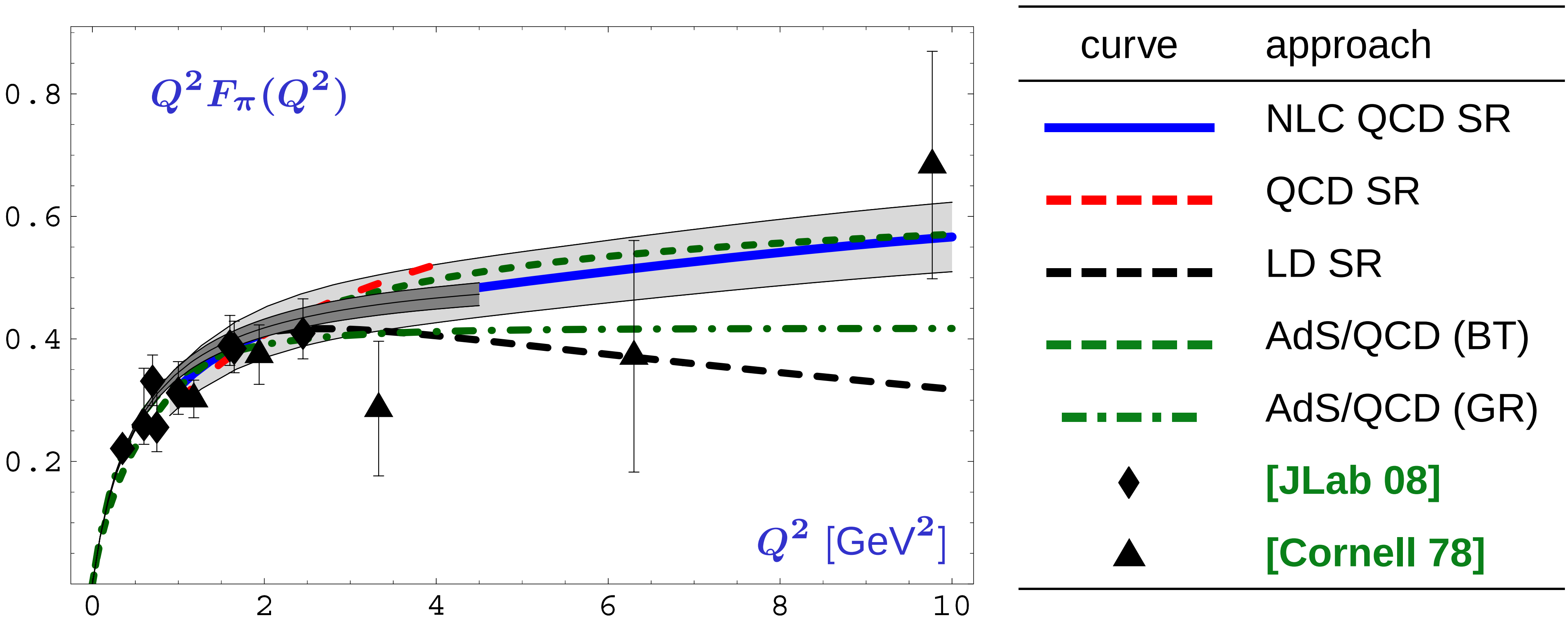
NLC QCD SR Result for $Q^2\,F_\pi(Q^2)$



curve	approach
	NLC QCD SR
	QCD SR
	LD SR
	AdS/QCD (BT)
	AdS/QCD (GR)
♦	**[JLab 08]**
▲	**[Cornell 78]**

Pion FF from: SRs with NLC **(blue solid line)**, standard QCD SRs **(red dashed line)** **[N&R⊕I&S 82]**, $O(\alpha_s)$ Local Duality **(black dashed line)** **[B&O 04]**, AdS/QCD **(green dashed line)** **[B&T 07]** and **(green dot-dashed line)** **[G&R 07]** in comparison with **[JLab 08]** (♦) and **[Cornell 78]** (▲) experimental data.

NLC QCD SR vs. Lattice QCD results



Pion FF from: SRs with NLC **(blue solid line)**,

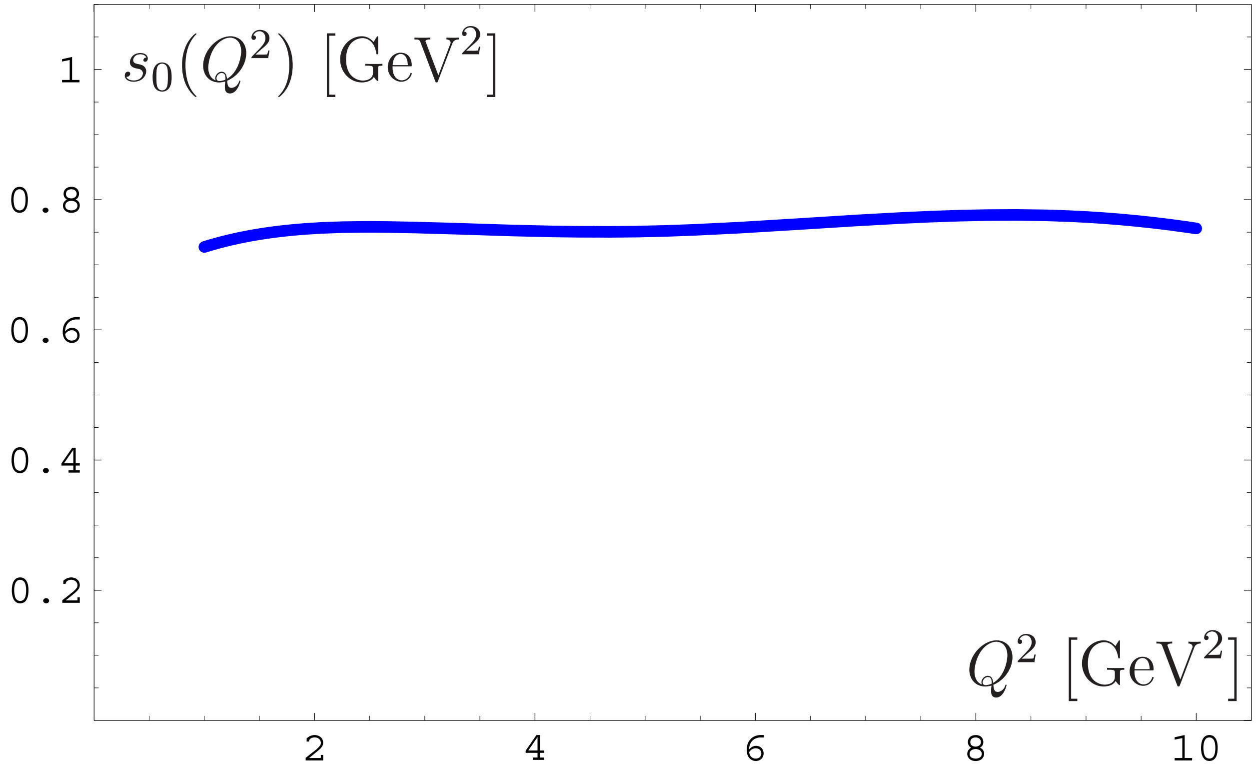
in comparison with recent lattice results by **D. Brommel et al. [Eur. Phys. J., C51 (2007) 335]**.

Pion FF
in
Local Duality Approach

Local Duality vs QCD SRs

Borel SR:

$$f_\pi^2 F_\pi(Q^2) = \iint_0^{s_0} ds_1\, ds_2\, \rho_3(s_1, s_2, Q^2)\, e^{-\frac{s_1+s_2}{M^2}} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$



Local Duality vs QCD SRs

Borel SR:

$$f_\pi^2\, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1\, ds_2\; \rho_3(s_1, s_2, Q^2)\, e^{-\frac{s_1+s_2}{M^2}} + \Phi_{\text{OPE}}(Q^2, M^2)\,.$$

Local Duality approximation:

$$f_\pi^2\, F_\pi(Q^2) = \iint\limits_0^{s_0^{\text{LD}}} ds_1\, ds_2\; \rho_3(s_1, s_2, Q^2)\,.$$

Local Duality vs QCD SRs

Borel SR:

$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0} ds_1 \, ds_2 \; \rho_3(s_1, s_2, Q^2) \, e^{-\frac{s_1+s_2}{M^2}} + \Phi_{\text{OPE}}(Q^2, M^2) \, .$$

Local Duality approximation:

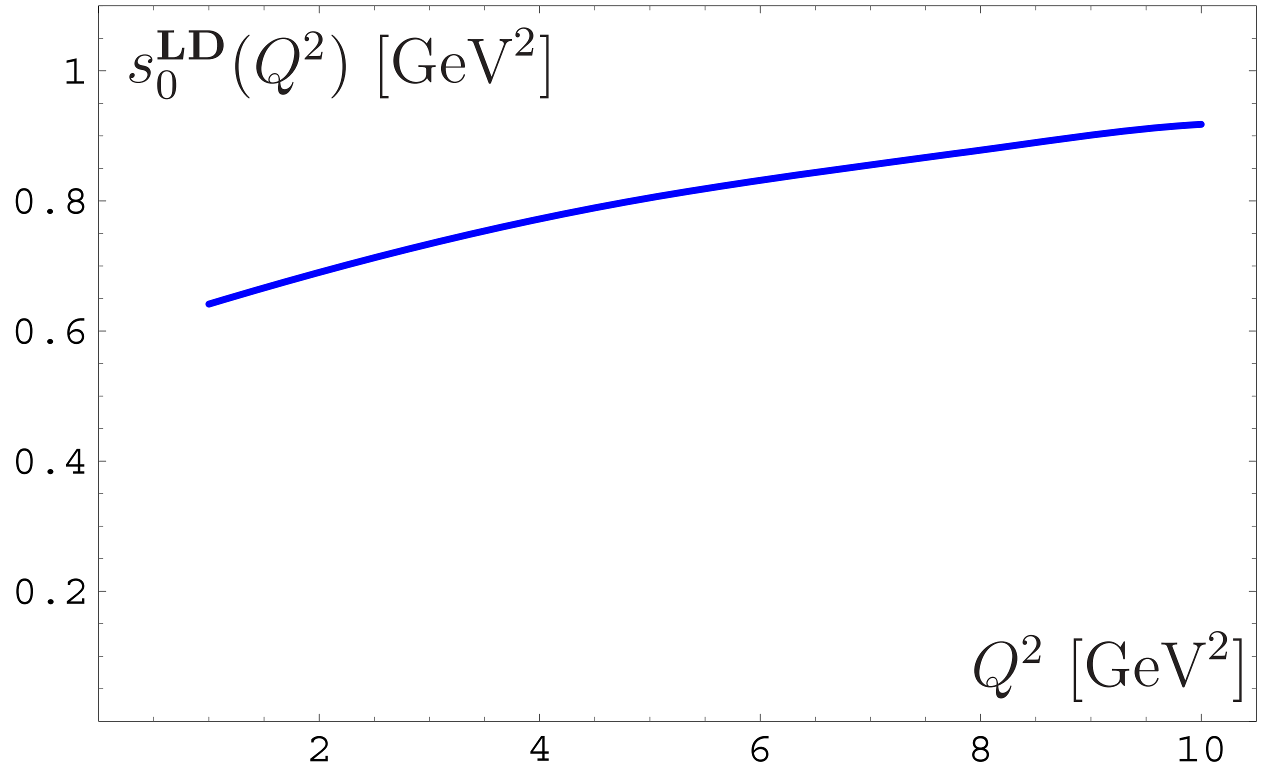
$$f_\pi^2 \, F_\pi(Q^2) = \iint\limits_0^{s_0^{\text{LD}}} ds_1 \, ds_2 \; \rho_3(s_1, s_2, Q^2) \, .$$

In general $s_0 = s_0^{\text{LD}}(Q^2) \neq s_0(Q^2)$.

Refinement of Local Duality Model

In general $s_0^{\text{LD}}(Q^2) \neq s_0(Q^2)$.

We define $s_0^{\text{LD}}(Q^2)$ to reproduce **NLC QCD SR** results in **LD** approach:

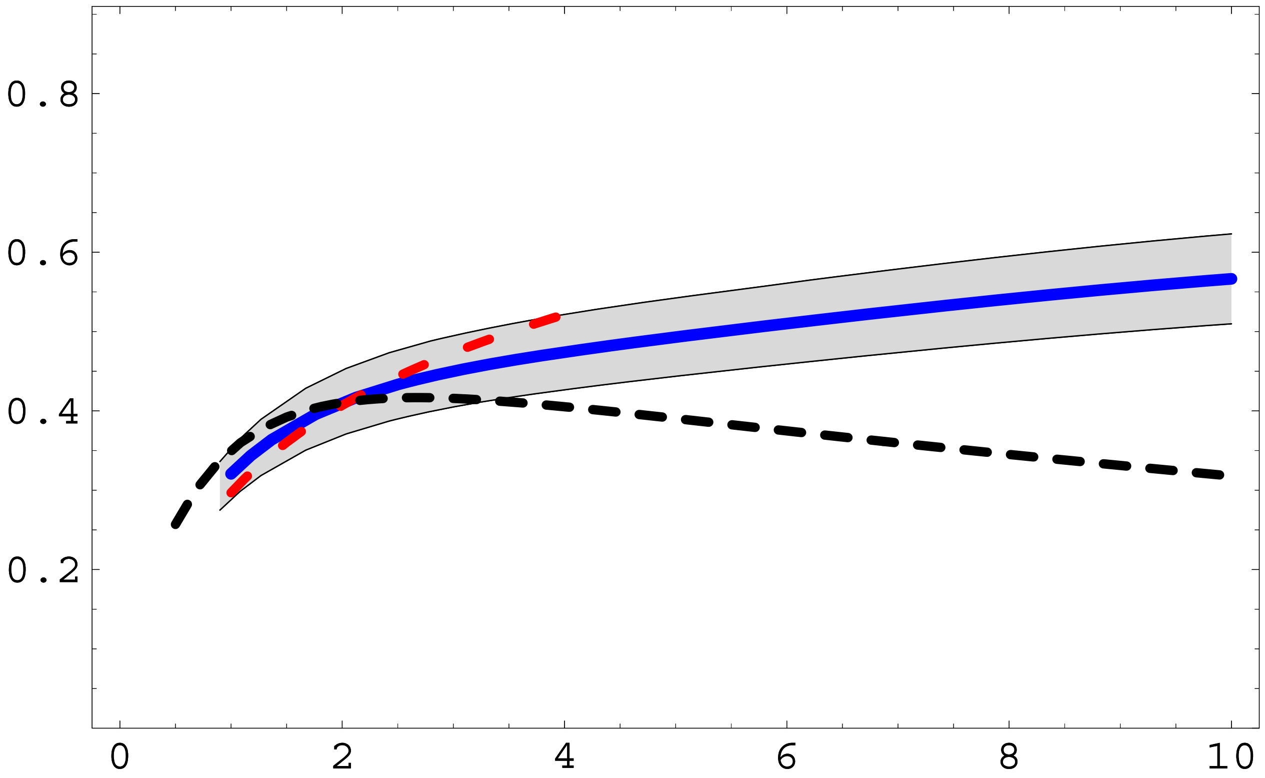


We see that $s_0^{\text{LD}}(Q^2)$ linearly grows with Q^2, whereas $s_0(Q^2) \approx$ ***const***.

Refinement of Local Duality Model

This is the reason for pion FF underestimation in

Braguta–Lucha–Melikhov (2008) LD approach :



Using (F)APT
in
order to estimate NNLO

Pion form factor in analytic pQCD scheme

- *Naive Analytization*

  [Stefanis, Schroers, Kim, PLB 449 (1998) 299]

$$F_\pi^{\text{Fact}}(Q^2;\mu_{\text{R}}^2) = \mathcal{A}_1^{(2)}(\mu_{\text{R}}^2)\,\mathcal{F}_\pi^{\text{LO}}(Q^2) \;+\; \frac{1}{\pi}\left[\mathcal{A}_1^{(2)}(\mu_{\text{R}}^2)\right]^2 \mathcal{F}_\pi^{\text{NLO}}(Q^2;\mu_{\text{R}}^2)$$

Pion form factor in analytic pQCD scheme

- *Naive Analytization*

  [Stefanis, Schroers, Kim, PLB 449 (1998) 299]

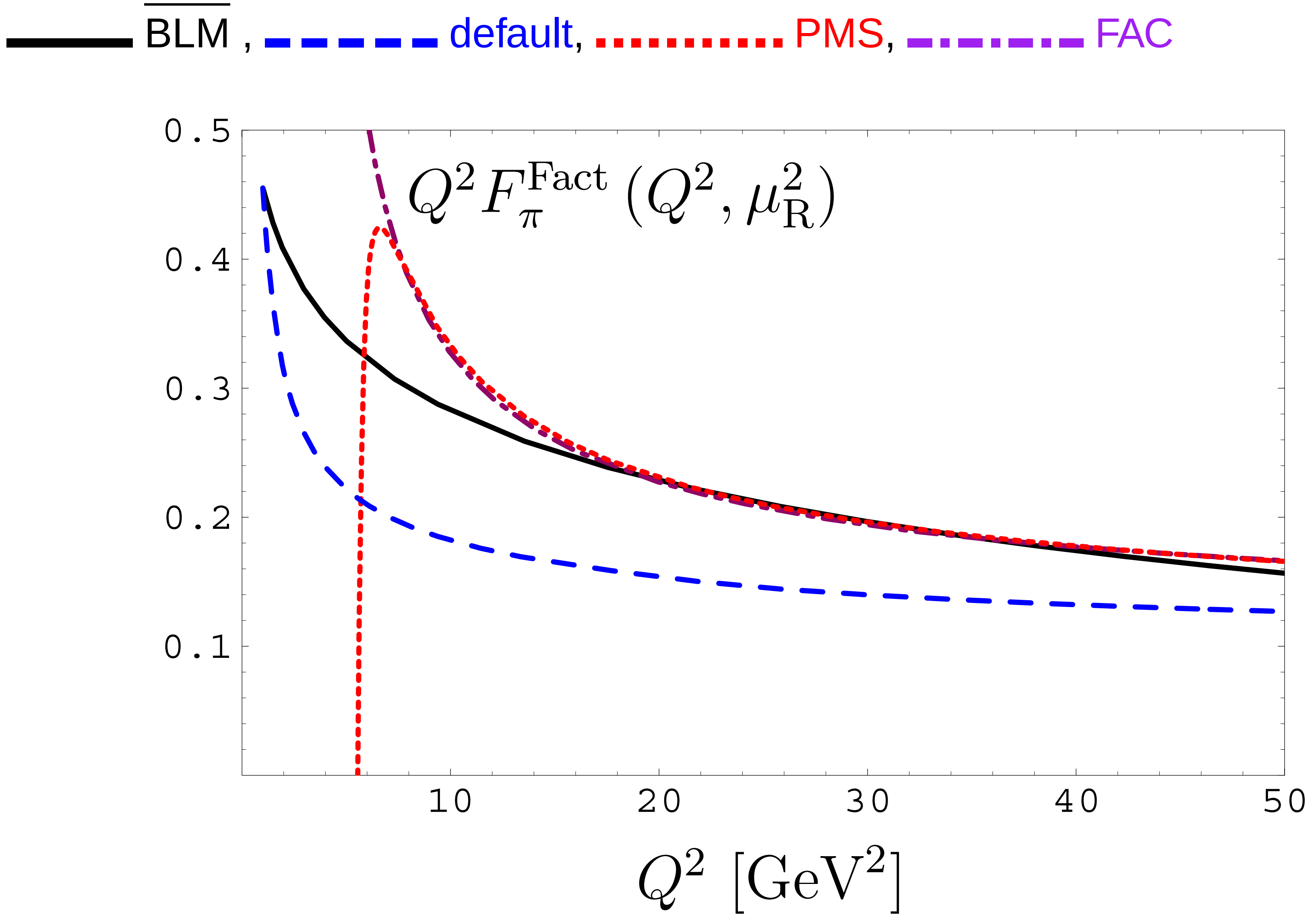
$$F_{\pi}^{\text{Fact}}(Q^2;\mu_{\text{R}}^2) = \mathcal{A}_1^{(2)}(\mu_{\text{R}}^2)\,\mathcal{F}_{\pi}^{\text{LO}}(Q^2) \;+\; \frac{1}{\pi}\left[\mathcal{A}_1^{(2)}(\mu_{\text{R}}^2)\right]^2 \mathcal{F}_{\pi}^{\text{NLO}}(Q^2;\mu_{\text{R}}^2)$$

- ***APT***

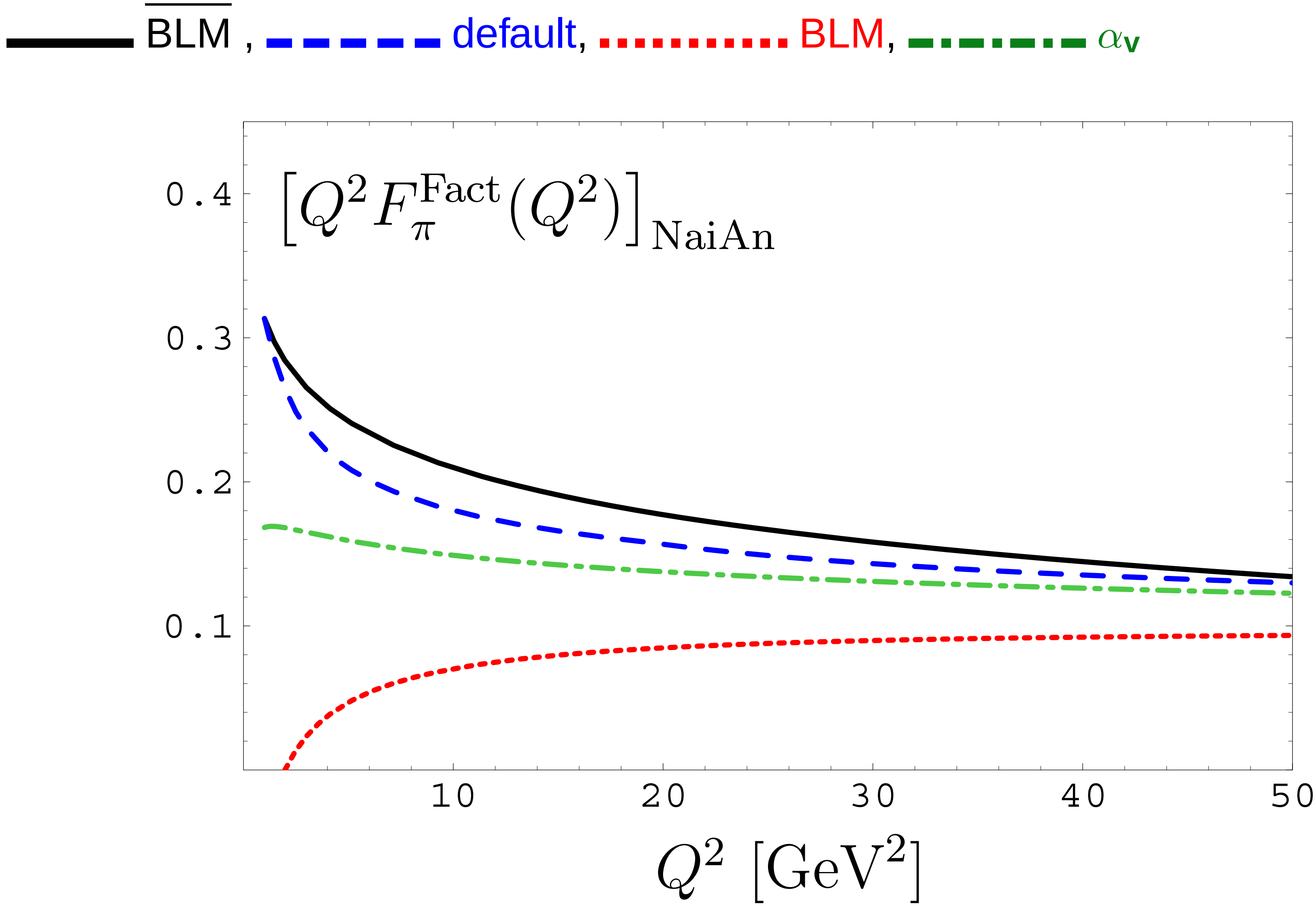
  [Bakulev, Passek, Schroers, Stefanis, Phys. Rev. D 70 (2004) 033014]

$$F_{\pi}^{\text{Fact}}(Q^2;\mu_{\text{R}}^2) = \mathcal{A}_1^{(2)}(\mu_{\text{R}}^2)\,\mathcal{F}_{\pi}^{\text{LO}}(Q^2) \;+\; \frac{1}{\pi}\left[\mathcal{A}_2^{(2)}(\mu_{\text{R}}^2)\right] \mathcal{F}_{\pi}^{\text{NLO}}(Q^2;\mu_{\text{R}}^2)$$

Factorized Pion FF in Standard $\overline{MS}$ scheme

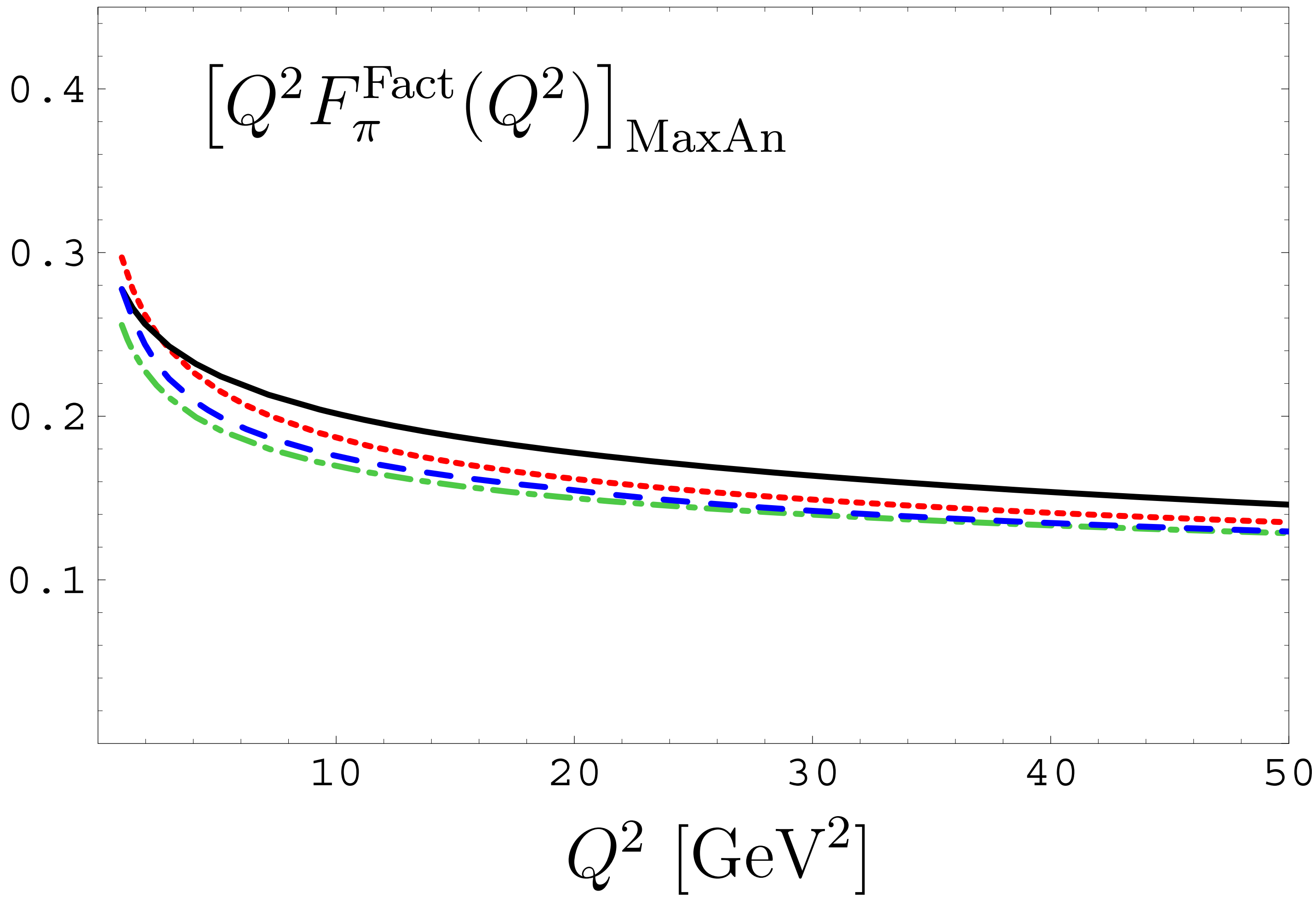


Factorized Pion FF in Naive Analyticization

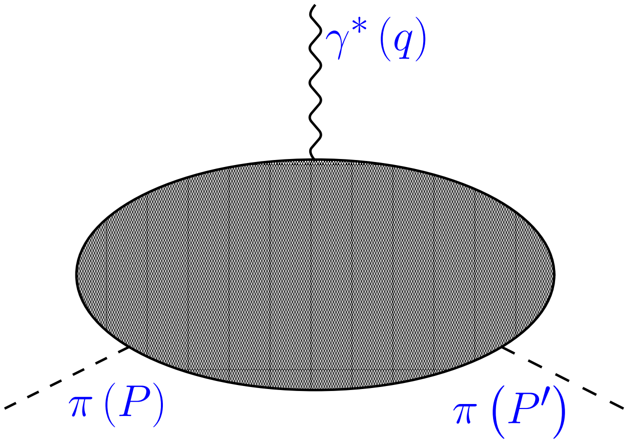


Factorized Pion FF in APT

$\overline{\text{BLM}}$, default, BLM, α_V



Matching soft and hard parts of FF



Definition by matrix element

$$\langle\pi\rangle^+(P')|J_\mu(0)|\pi^+(P) = (P+P')_\mu F_\pi(Q^2)$$

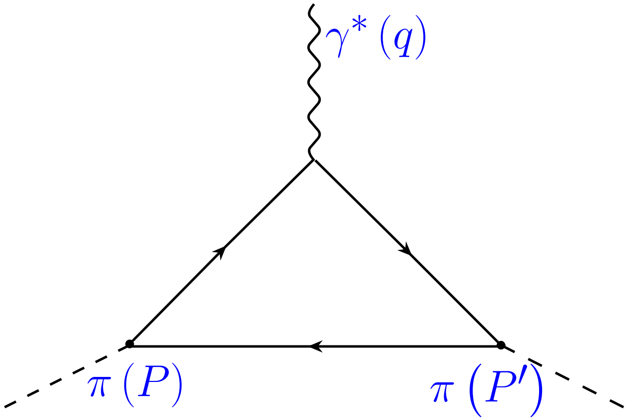
with $J_\mu =$ quark electromagnetic current and $(P'-P)^2 \equiv -Q^2 =$ large momentum transfer squared injected into pion.

Total form factor: $F_\pi(Q^2) = F_\pi^{\text{Soft}}(Q^2) + \Phi(Q^2) F_\pi^{\text{Fact}}(Q^2)$

- $F_\pi^{\text{Fact}}(Q^2)$ describes **gluon exchanges** via pQCD in terms of convolutions with **universal** pion DAs
- $F_\pi^{\text{Soft}}(Q^2)$ originates from **soft** Feynman mechanism

Switching function $\Phi(Q^2)$ is needed to match collinear regime $Q^2 \ll 1$ **GeV**^{2} results with **Soft** part.

Matching Soft and Hardf in Pion FF



Soft part is modelled via Local Duality

$$F_\pi^{\text{LD}}(Q^2) = \frac{1}{f_\pi^2} \iint_0^{s_0} \rho_3(s, s', Q^2)\, ds\, ds'$$

$$= 1 - \frac{1 + 6\, s_0/Q^2}{(1 + 4\, s_0/Q^2)^{3/2}}$$

and saturates Ward identity $F_\pi(0) = 1$ completely.

Switching function:

- $\Phi(Q^2) \to 1$ at $Q^2 \to \infty$ in order to reproduce collinear results;
- $\Phi(Q^2)/Q^2 \to 0$ at $Q^2 \to 0$ in order to fulfill Ward identity.

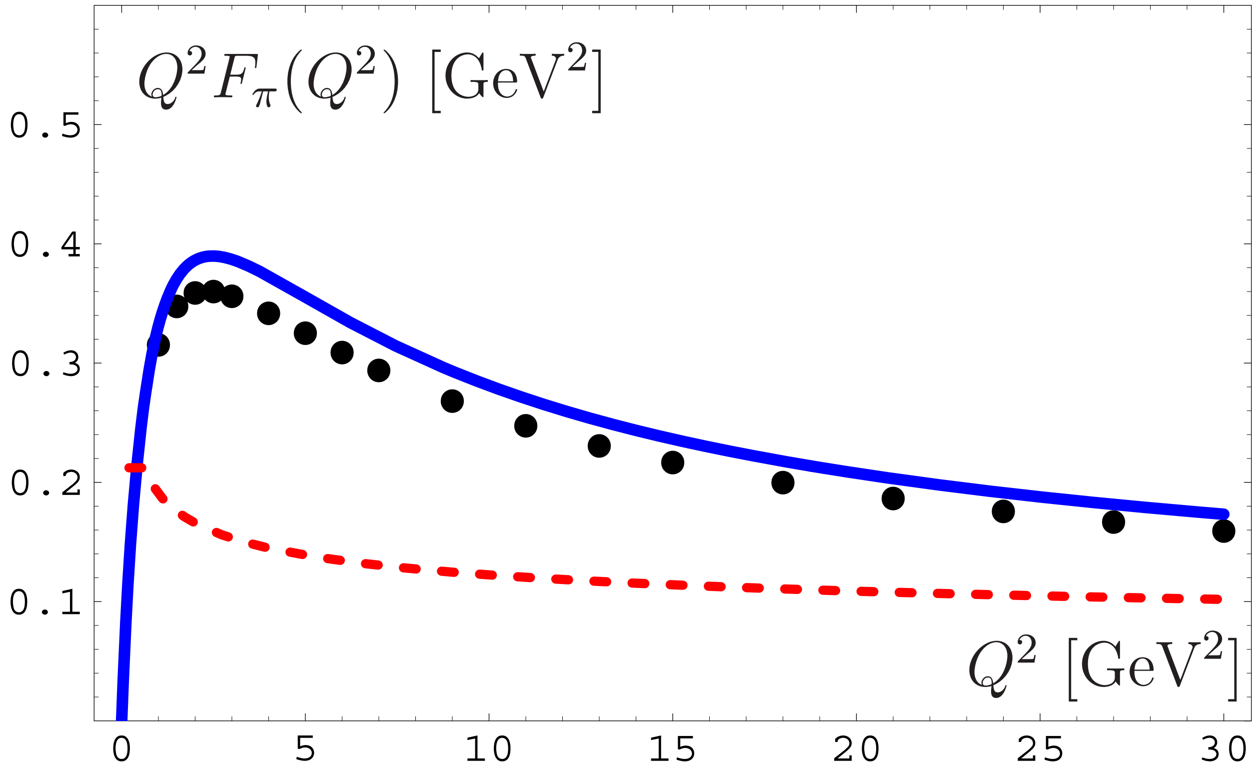
Simplest solution $\Phi(Q^2) = \left[\frac{Q^2}{Q^2 + 2s_0}\right]^2$.

Quality of Switching Function $\Phi(Q^2)$

$F_\pi(Q^2) = F_\pi^{\text{LD}}(Q^2) + \Phi(Q^2) F_\pi^{\text{Fact}}(Q^2)$ with **switching function**:

(i) $\Phi(Q^2) \to 1$ at $Q^2 \to \infty$; (ii) $\Phi(Q^2)/Q^2 \to 0$ at $Q^2 \to 0$.

Simplest solution $\Phi(Q^2) = \left[\frac{Q^2}{Q^2 + 2s_0}\right]^2$.

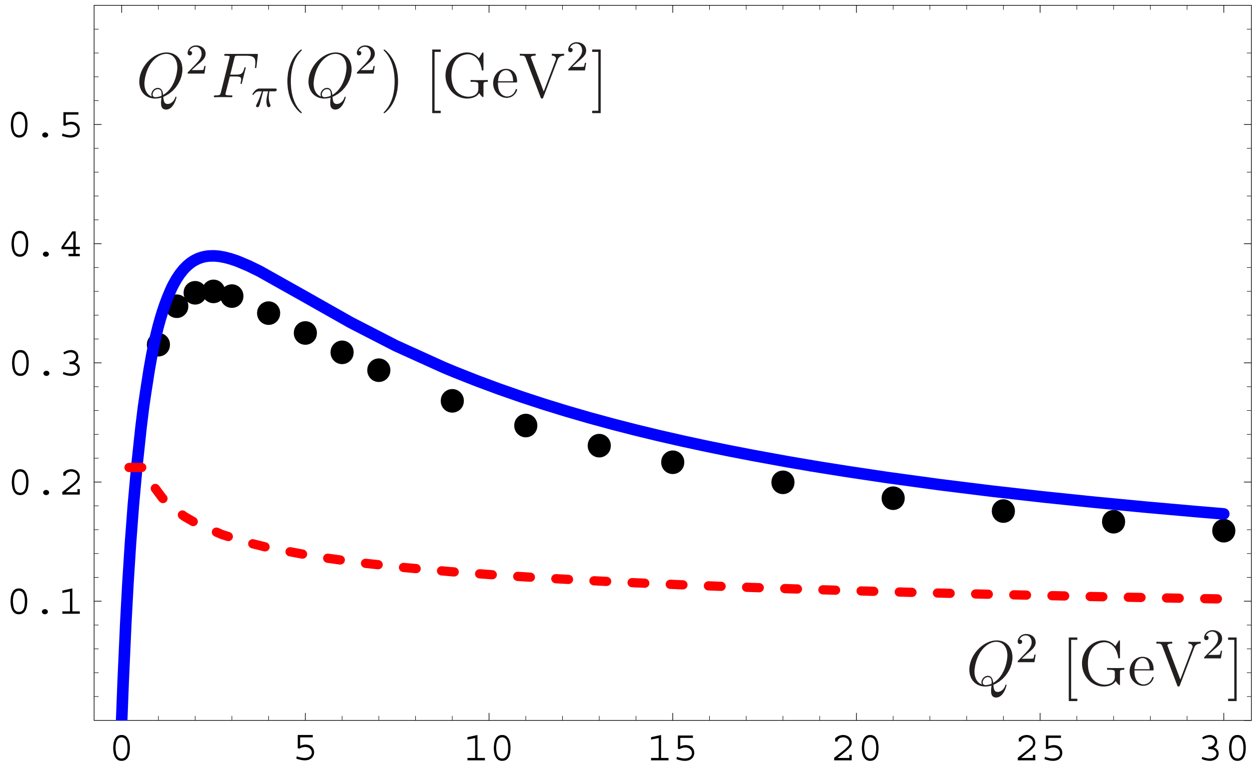


Quality of Switching Function $\Phi(Q^2)$

$F_\pi(Q^2) = F_\pi^{\text{LD}}(Q^2) + \Phi(Q^2) F_\pi^{\text{Fact}}(Q^2)$ with **switching function**:

$(i)\ \Phi(Q^2) \to 1$ at $Q^2 \to \infty$; $(ii)\ \Phi(Q^2)/Q^2 \to 0$ at $Q^2 \to 0$.

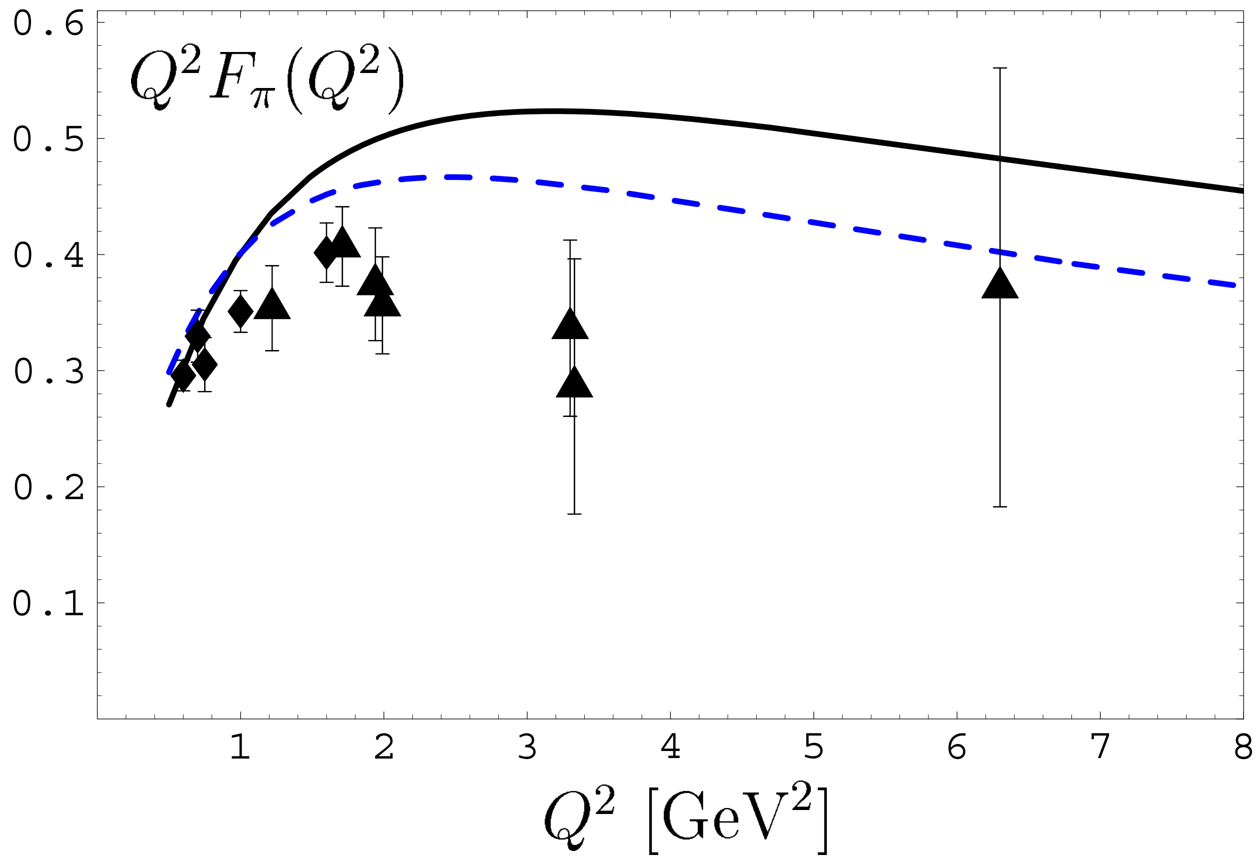
This $\Phi(Q^2) = \left[\frac{Q^2}{Q^2 + 2s_0}\right]^2$ delivers accuracy of the order of 10%.



Pion FF in Standard $\overline{MS}$-scheme

Total form factor: $F_\pi(Q^2) = F_\pi^{\text{LD}}(Q^2) + \left[\frac{Q^2}{Q^2+2s_0}\right]^2 F_\pi^{\text{Fact}}(Q^2)$.

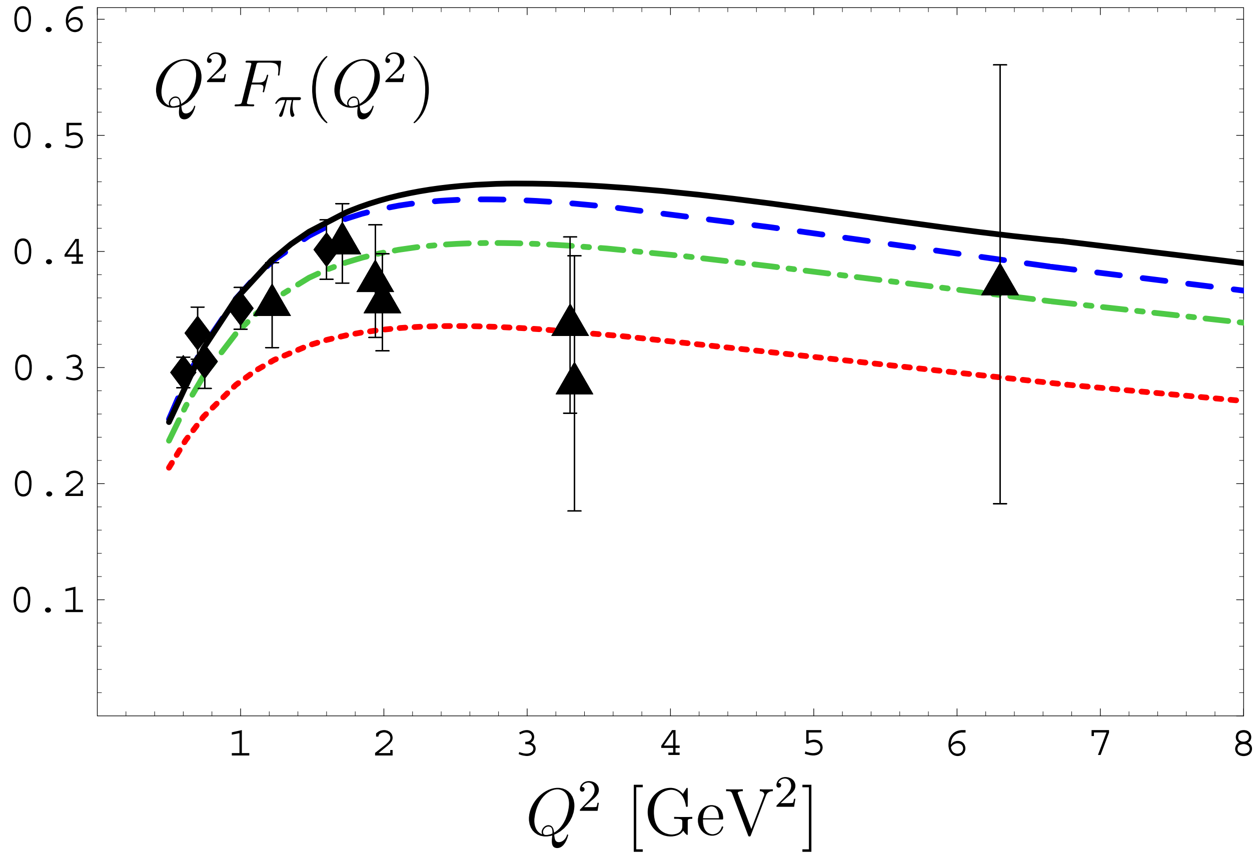
—— $\overline{\text{BLM}}$, - - - - default



Pion FF in Naive Analyticization

Total form factor: $F_\pi(Q^2) = F_\pi^{\text{LD}}(Q^2) + \left[\frac{Q^2}{Q^2+2s_0}\right]^2 F_\pi^{\text{Fact}}(Q^2)$.

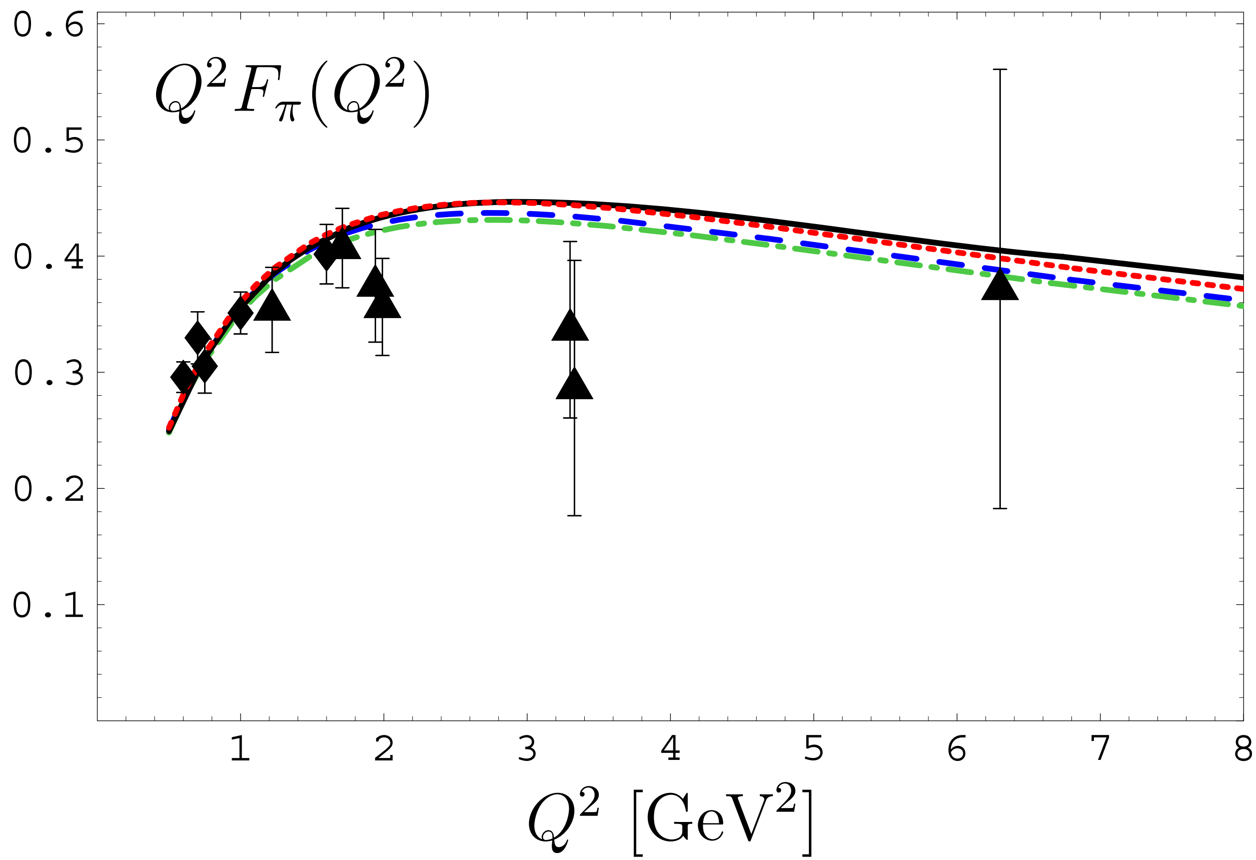
$\overline{\text{BLM}}$ (solid), default (dashed), BLM (dotted), α_V (dash-dotted)



Factorized Pion FF in APT

Total form factor: $F_\pi(Q^2) = F_\pi^{\text{LD}}(Q^2) + \left[\frac{Q^2}{Q^2+2s_0}\right]^2 F_\pi^{\text{Fact}}(Q^2)$.

$\overline{\text{BLM}}$ (solid), default (dashed), BLM (dotted), α_V (dash-dotted)



Using FAPT

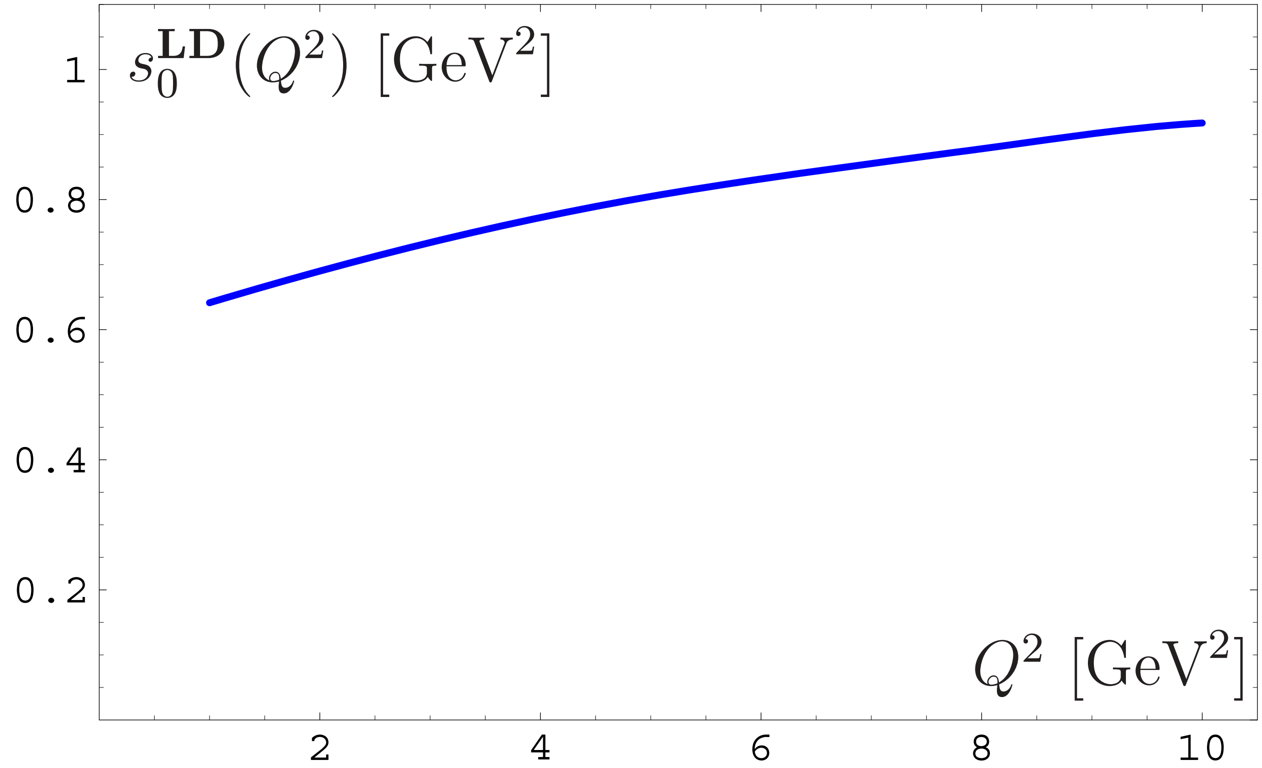
in order

to estimate NNLO

Refinement of Local Duality Model

We define $s_0^{\mathrm{LD}}(Q^2)$ to reproduce **NLC QCD SR** results in **LD** approach:

$$F_\pi(Q^2;S) = F_\pi^{\mathrm{LD};(0)}(Q^2,S) + \\ + \frac{S}{4\pi^2 f_\pi^2}\left\{\frac{\alpha_s(Q^2)}{\pi}\left(\frac{2S}{2S+Q^2}\right)^2 + F_\pi^{\mathrm{Fact},(1)}(Q^2)\left(\frac{Q^2}{2S+Q^2}\right)^2\right\}$$

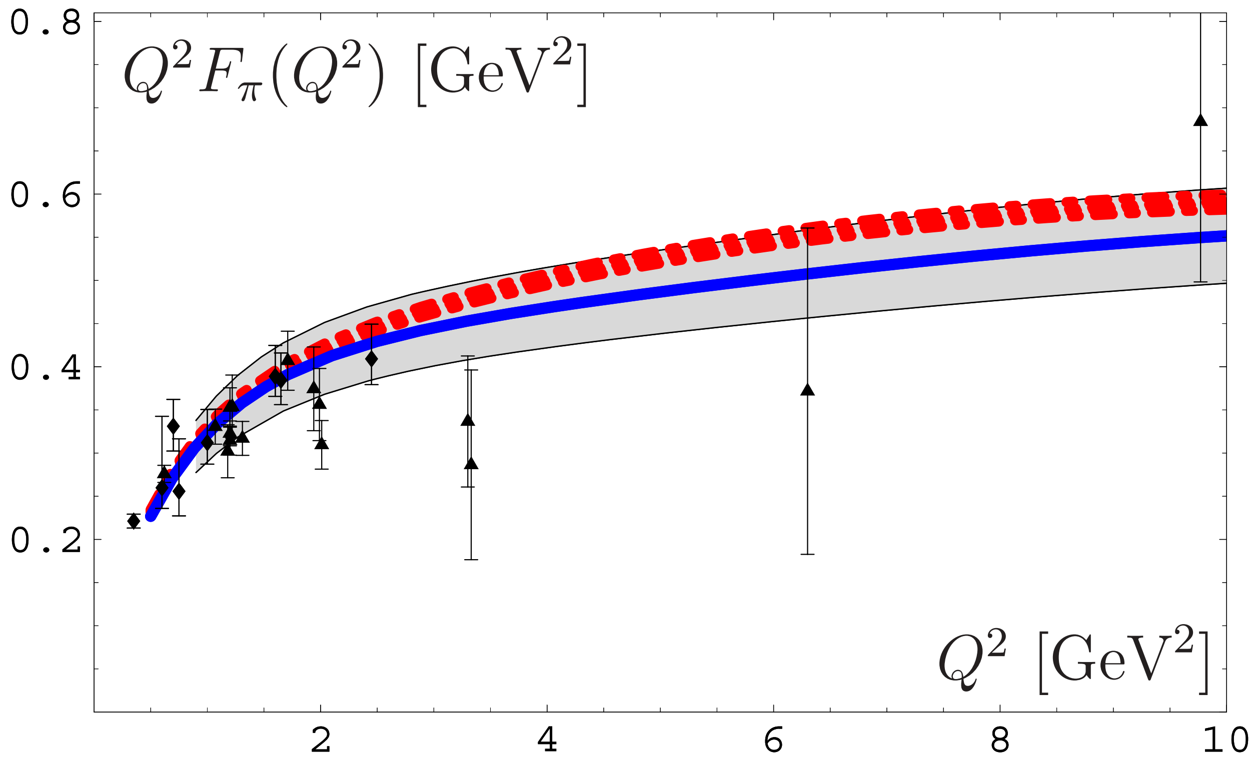


We see that $s_0^{\mathrm{LD}}(Q^2)$ linearly grows with Q^2, whereas $s_0(Q^2) \approx$ ***const***.

FAPT and 2nd Order Correction Estimation

We use this effective threshold to estimate $O(\alpha_s^2)$ correction using **FAPT**:

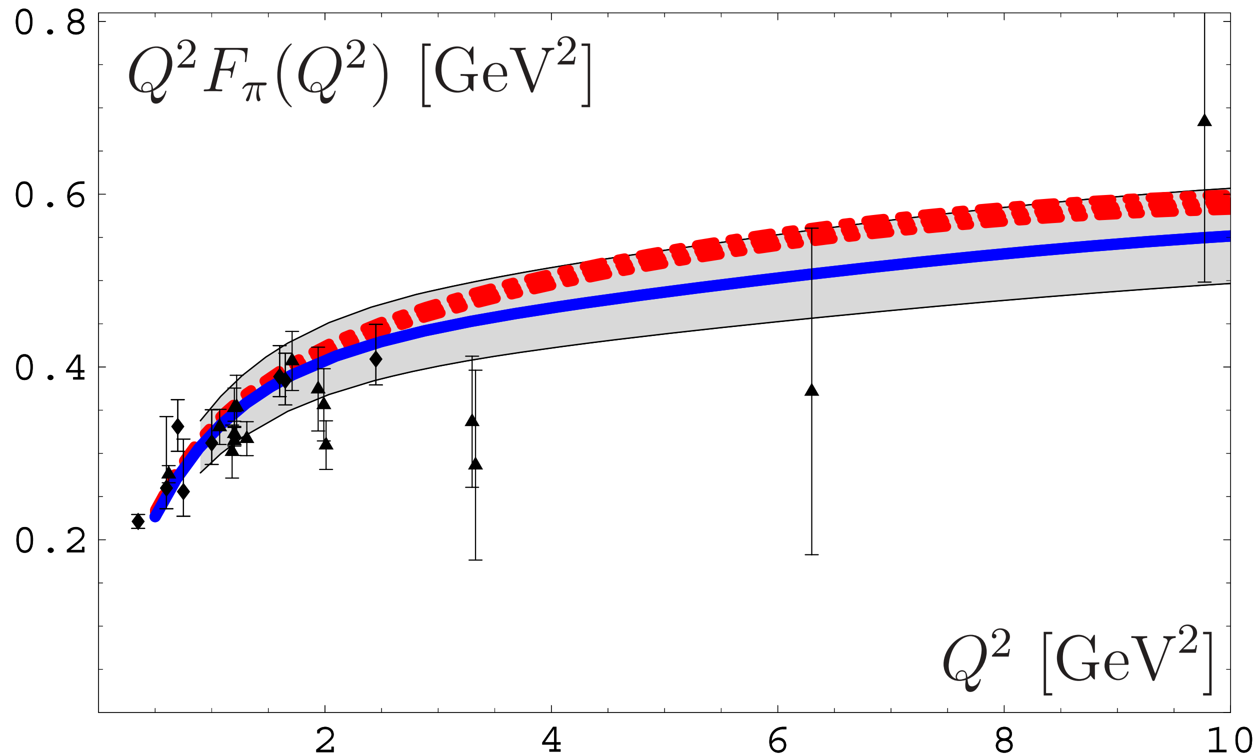
$$F_\pi(Q^2;S) = F_\pi^{\text{LD};(0)}(Q^2,S) + \\ + \frac{S}{4\pi^2 f_\pi^2}\left\{\frac{\alpha_s(Q^2)}{\pi}\left(\frac{2S}{2S+Q^2}\right)^2 + F_\pi^{\text{FAPT},(2)}(Q^2)\left(\frac{Q^2}{2S+Q^2}\right)^2\right\}$$



FAPT and 2nd Order Correction Estimation

We use this effective threshold to estimate $O(\alpha_s^2)$ correction using **FAPT**:

$$F_\pi(Q^2; S) = F_\pi^{\text{LD};(0)}(Q^2, S) + \\ + \frac{S}{4\pi^2 f_\pi^2}\left\{\frac{\alpha_s(Q^2)}{\pi}\left(\frac{2S}{2S+Q^2}\right)^2 + F_\pi^{\text{FAPT},(2)}(Q^2)\left(\frac{Q^2}{2S+Q^2}\right)^2\right\}$$



It appears to be of the order of **10%**.

Conclusion

- Taking into account nonlocality of condensates makes QCD SR stable and widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.

Conclusion

- Taking into account nonlocality of condensates makes QCD SR stable and widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.
- We use the model-independent expression for Φ_{OPE}-term obtained by **A. B.&Radyushkin [1991]**, but significantly different model of NLCs with nonlocality parameter $\lambda_q^2 = 0.4$ **GeV^2**.

Conclusion

- Taking into account nonlocality of condensates makes QCD SR stable and widens region of applicability up to $Q^2 \simeq 10$ GeV^2.
- We use the model-independent expression for Φ_{OPE}-term obtained by **A. B.&Radyushkin [1991]**, but significantly different model of NLCs with nonlocality parameter $\lambda_q^2 = 0.4$ GeV^2.
- NLO corrections to double spectral density, obtained by **Braguta&Onishchenko [2004]**, are large. Taking them into account in Local Duality approach suffers from underestimation of $s_0^{\text{LD}}(Q^2)$: our results show that $s_0^{\text{LD}}(Q^2)$ grows with Q^2.

Conclusion

- Taking into account nonlocality of condensates makes QCD SR stable and widens region of applicability up to $Q^2 \simeq 10$ **GeV^2**.
- We use the model-independent expression for Φ_{OPE}-term obtained by **A. B.&Radyushkin [1991]**, but significantly different model of NLCs with nonlocality parameter $\lambda_q^2 = 0.4$ **GeV^2**.
- NLO corrections to double spectral density, obtained by **Braguta&Onishchenko [2004]**, are large. Taking them into account in Local Duality approach suffers from underestimation of $s_0^{\text{LD}}(Q^2)$: our results show that $s_0^{\text{LD}}(Q^2)$ grows with Q^2.
- QCD SR method with NLCs for the pion FF gives us a strip of predictions. This strip appears to be in a good agreement with existing experimental data of **JLab** and **Cornell**, as well as with lattice data.