Nontrivial Relations between GPDs and TMDs

(A. Metz, Temple University, Philadelphia)

- Introduction

- Nontrivial relation between GPD E and TMD $f_{1T}^{\perp}$ (see also talk by Gamberg)
  (Burkardt, 2002, ... / Burkardt, Hwang, 2003)

- Overview of model-dependent nontrivial relations

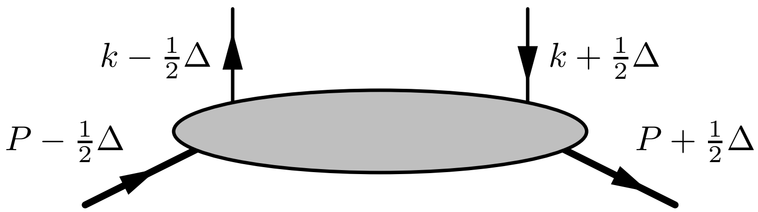
- GTMD analysis

- Summary

In collaboration with K. Goeke, S. Meißner, M. Schlegel
(hep-ph/0703176 ; arXiv:0805.3165 ; arXiv:0906.5323)

Definition of GPDs and TMDs

- GPDs
  - Appear in QCD-description of hard exclusive reactions
  - Kinematics



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

  - GPD-correlator

$$F^q = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p'; \lambda' | \bar{\psi}\Big(-\frac{z}{2}\Big) \gamma^+ \mathcal{W}_{GPD}\, \psi\Big(\frac{z}{2}\Big) |p; \lambda\rangle \Big|_{z^+ = z_T = 0}$$

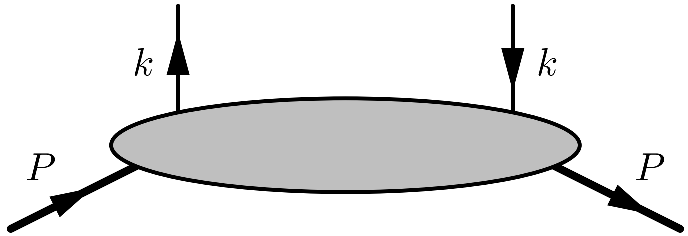
$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \Big(\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^q(x, \xi, t) \Big) u(p, \lambda)$$

$$x = \frac{k^+}{P^+} \qquad \xi = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

  - Leading twist for

$$\bar{\psi}\, \gamma^+ \psi \qquad \bar{\psi}\, \gamma^+ \gamma_5\, \psi \qquad \bar{\psi}\, i\sigma^{j+} \gamma_5\, \psi$$

- TMDs
  - Appear in QCD-description of hard semi-inclusive reactions
  - Kinematics



  - TMD-correlator

$$\Phi^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\, e^{ik\cdot z}\,\langle P;S|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|P;S\rangle\Big|_{z^+=0}$$

$$= f_1^q(x,\vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij}k_T^i S_T^j}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2})$$

$\longrightarrow$ Sivers function $f_{1T}^{\perp}$ induces distortion of TMD correlator

  - Leading twist for

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

- Leading twist GPDs and TMDs

	Quarks				Gluons			
Forward	f_1^q	g_1^q	h_1^q		g	Δg		
k_T-dependent	f_1^q	$f_{1T}^{\perp q}$	g_{1L}^q	g_{1T}^q	f_1^g	$f_{1T}^{\perp g}$	g_{1L}^g	g_{1T}^g
	h_{1T}^q	$h_{1L}^{\perp q}$	$h_{1T}^{\perp q}$	$h_1^{\perp q}$	h_{1T}^g	$h_{1L}^{\perp g}$	$h_{1T}^{\perp g}$	$h_1^{\perp g}$
Generalized	H^q	E^q	$\tilde{H}^q$	$\tilde{E}^q$	H^g	E^g	$\tilde{H}^g$	$\tilde{E}^g$
	H_T^q	E_T^q	$\tilde{H}_T^q$	$\tilde{E}_T^q$	H_T^g	E_T^g	$\tilde{H}_T^g$	$\tilde{E}_T^g$

  - Trivial relations:

$$H^q(x,0,0) = f_1^q(x) = \int d^2\vec{k}_T \, f_1^q(x,\vec{k}_T^{\,2}) \quad \text{etc.}$$

  - Nontrivial relations: 3 for quarks, 4 for gluons

Impact parameter representation of GPDs

- Fourier transform of GPD-correlator $(\xi = 0)$ (Burkardt, 2000)

$$F^q(x, \vec{\Delta}_T, S) = \frac{1}{2} \int \frac{dz^-}{2\pi} \, e^{ik\cdot z} \, \langle p'; S | \, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \gamma^+ \, \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) \, |p; S\rangle \, \Big|_{z^+ = z_T = 0}$$

$$\begin{aligned} \mathcal{F}^q(x, \vec{b}_T; S) &= \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} \, e^{-i\vec{\Delta}_T\cdot\vec{b}_T} \, F^q(x, \vec{\Delta}_T; S) \\ &= \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \Big(\mathcal{E}^q(x, \vec{b}_T^{\,2})\Big)' \\ &\quad \text{with} \quad \mathcal{H}^q(x, \vec{b}_T^{\,2}) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} \, e^{-i\vec{\Delta}_T\cdot\vec{b}_T} \, H^q(x, 0, -\vec{\Delta}_T^{\,2}) \end{aligned}$$

- Distortion of GPD-correlator in impact parameter space

$$d^{q,i} = \int dx \int d^2\vec{b}_T \, b_T^i \, \mathcal{F}^q(x, \vec{b}_T; S) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx \, E^q(x, 0, 0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

$\longrightarrow$ Flavor dipole moment of about $0.2\,\mathrm{fm}$

- Relation between distortion and Sivers effect (Burkardt, 2002)

  (Not obvious because QCD-description of, e.g., SIDIS does not know about GPD-correlator in b_T-space)

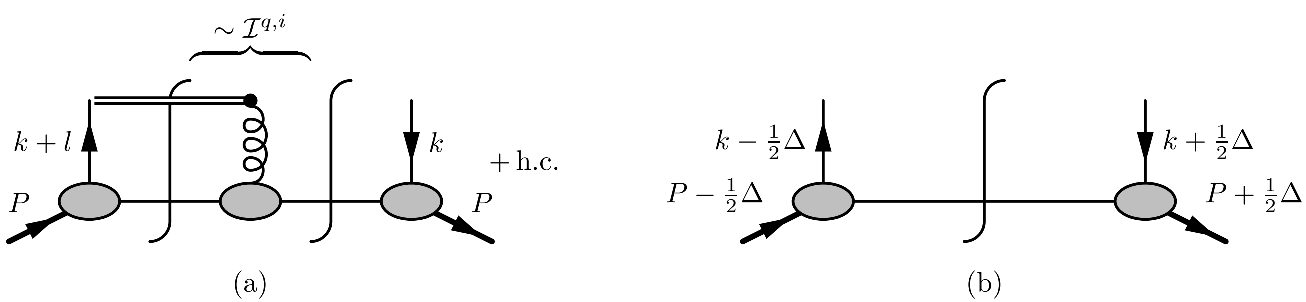
  – Quantitative nontrivial relation in spectator model (Burkardt, Hwang, 2003)

$$\langle k_T^{q,i}(x) \rangle_{UT} = - \int d^2\vec{k}_T \, k_T^i \, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M} \, f_{1T}^{\perp q}(x, \vec{k}_T^{\,2})$$

$$= \int d^2\vec{b}_T \, \mathcal{I}^{q,i}(x, \vec{b}_T) \, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x, \vec{b}_T^{\,2}) \right)'$$

  – Interpretation

Sivers effect = Distortion ⊗ FSI

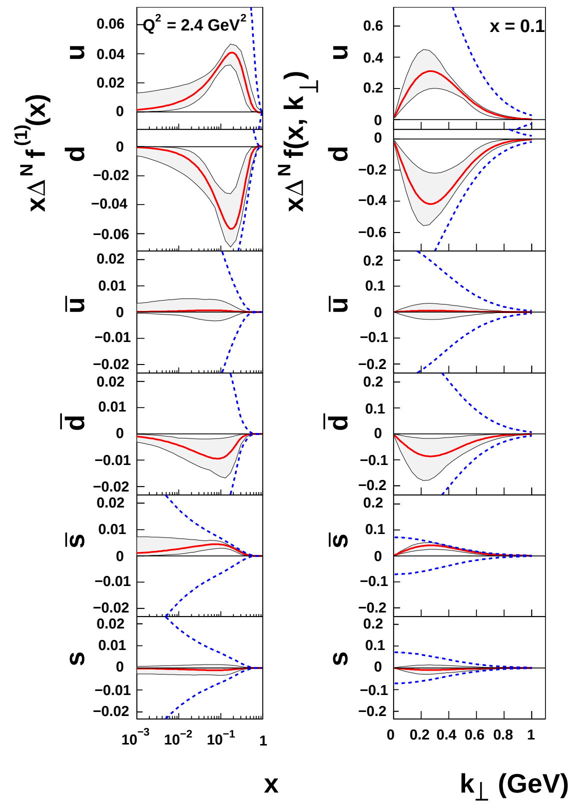


– Prediction

$$f_{1T}^{\perp u/p} \sim -0.8\, f_{1T}^{\perp d/p} < 0$$

$\longrightarrow$ Relative (but not absolute) sign also from large N_c-analysis (Pobylitsa, 2003)

$\longrightarrow$ Extraction of $\Delta^N f = -\frac{2k_T}{M} f_{1T}^{\perp}$ by Anselmino et al., 2008



$\longrightarrow$ Nice agreement between qualitative picture and extraction

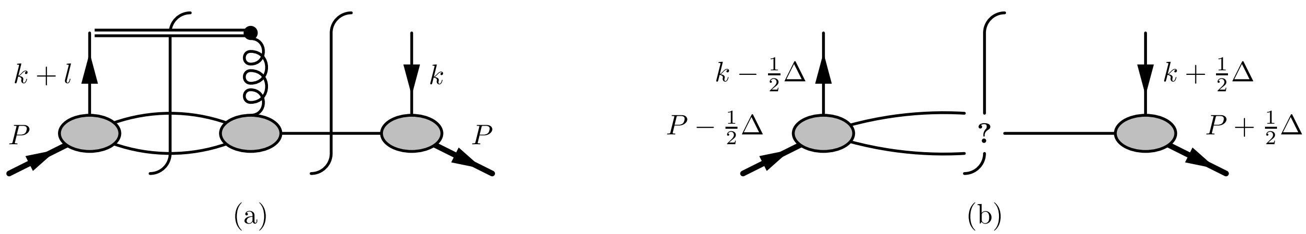
$\longrightarrow$ Agreement with qualitative picture and large N_c-analysis supports interpretation of HERMES signal for $A_{UT}^{\sin(\Phi-\Phi_S)}$ as Sivers effect

– Sign reversal of the Sivers function (Collins, 2002)

$$f_{1T}^{\perp}\Big|_{\mathrm{DY}} = -f_{1T}^{\perp}\Big|_{\mathrm{SIDIS}}$$

$\longrightarrow$ Sign reversal provided in qualitative picture:
lensing function $\mathcal{I}^q$ changes sign (attractive vs repulsive interaction)

– Higher order contributions should spoil picture (Meißner, Metz, Goeke, 2007)



(confirmed by Gamberg, Schlegel, 2009)

Comparing GPD- and TMD-correlator

$\longrightarrow$ Additional relations by comparing the GPD-correlator with the TMD-correlator (Diehl, Hägler, 2005)

$$\Phi^q(x, \vec{k}_T; S) \quad = \quad f_1^q(x, \vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} \, f_{1T}^{\perp q}(x, \vec{k}_T^{\,2})$$

$$\mathcal{F}^q(x, \vec{b}_T; S) \quad = \quad \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^{\,2}) \right)'$$

$\longrightarrow$ Comparison allows one to find analogy:

$$f_{1T}^{\perp q} \leftrightarrow -\left(\mathcal{E}^q \right)'$$

$\longrightarrow$ Comparison can be extended to other quark and gluon distributions

$\longrightarrow$ No relation for GPDs $\tilde{E}, \ \tilde{E}_T$ (drop out for $\xi = 0$) and TMDs $g_{1T}, \ h_{1L}^{\perp}$

- Relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \qquad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp q}\right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta\tilde{\mathcal{H}}_T^q\right)$$

- Relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g}\right)' \qquad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q\right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp g}\right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta\tilde{\mathcal{H}}_T^g\right)'$$

- Relations of third type

$$h_{1T}^{\perp q} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^q\right)'' \qquad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g\right)''$$

- Relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g\right)'''$$

- Some consequences
  - Relation for Boer-Mulders function $h_1^{\perp q}$ expected to match with the one for $f_{1T}^{\perp q}$

$$
\begin{aligned}
\langle k_T^{q,i}(x)\rangle_{UT} &= -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M}\, \Big(\mathcal{E}^q(x,\vec{b}_T^{\,2})\Big)'
\end{aligned}
$$

$$
\begin{aligned}
\langle k_T^{q,i}(x)\rangle_{TU}^j &= -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{kj} k_T^k}{M}\, h_1^{\perp q}(x,\vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{kj} b_T^k}{M}\, \Big(\mathcal{E}_T^q(x,\vec{b}_T^{\,2}) + 2\tilde{\mathcal{H}}_T^q(x,\vec{b}_T^{\,2})\Big)'
\end{aligned}
$$

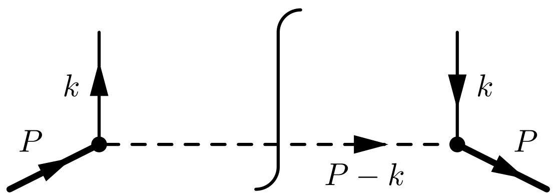
(Burkardt, 2005 / Meißner, Metz, Goeke, 2007)

$\longrightarrow$ Information on chiral odd GPDs (Pasquini, Pincetti, Boffi, 2005 / QCDSF, 2007)
$\longrightarrow$ Implies: $h_1^{\perp u/p} < 0 \qquad h_1^{\perp d/p} < 0$
$\longrightarrow$ Agrees, e.g., with spectator model calculations
(Gamberg, Goldstein, Schlegel, 2007 / Bacchetta, Conti, Radici, 2008)

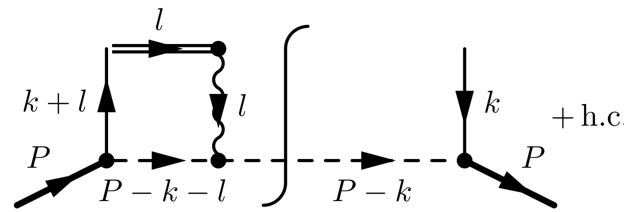
  - Relation for $h_{1T}^{\perp q}$ expected to be different

Model results, continued

- Scalar diquark spectator model of the nucleon

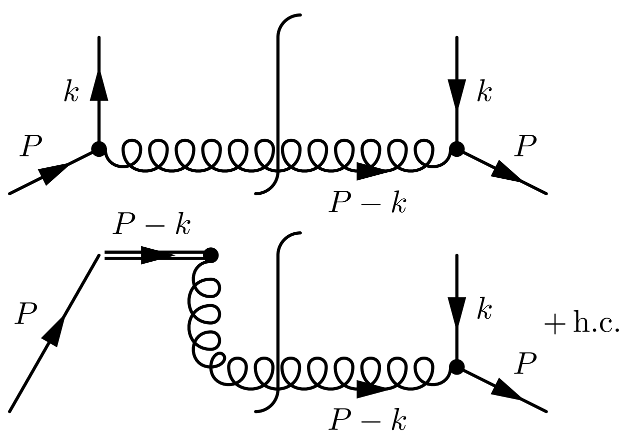


(a)

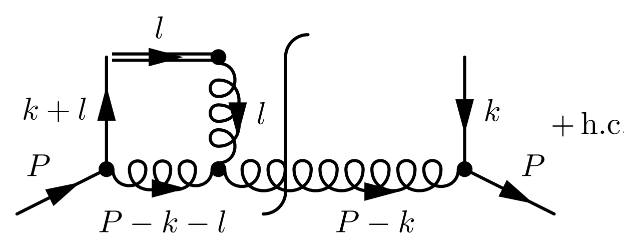


(b)

- Quark target model in QCD



(a)



(b)

- Moments of GPDs and TMDs

$$X^{(n)}(x) = \frac{1}{2M^2}\int d^2\vec{\Delta}_T \left(\frac{\vec{\Delta}_T^2}{2M^2}\right)^{n-1} X\left(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2}\right)$$

$$Y^{(n)}(x) = \int d^2\vec{k}_T \left(\frac{\vec{k}_T^2}{2M^2}\right)^{n} Y(x, \vec{k}_T^2)$$

- Relations of second type

$$f_{1T}^{\perp q\,(n)}(x) = H_2(n)\,\frac{1}{1-x}\,E^{q\,(n)}(x) \qquad (0 \le n \le 1)$$

$\longrightarrow$ $H_2(n)$ depends on model

$\longrightarrow$ Formula holds for all the relations of second type

$\longrightarrow$ Particular cases

$$f_{1T}^{\perp q\,(0)}(x) = \frac{\pi e_q e_s}{48(1-x)}\,E^q(x, 0, 0) \qquad \text{(Lu, Schmidt, 2006)}$$

$$f_{1T}^{\perp q\,(1)}(x) = \frac{e_q e_s}{4(2\pi)^2\,(1-x)}\,E^{q\,(1)}(x) \qquad \text{(Burkardt, Hwang, 2003)}$$

- Relations of third type

$$h_{1T}^{\perp q\,(n)}(x) = H_3(n)\,\frac{1}{(1-x)^2}\,\tilde{H}_T^{q\,(n)}(x) \qquad\qquad (0 \le n \le 1)$$

$\longrightarrow$ $H_3(n)$ is the same in both models

$\longrightarrow$ Formula holds for all the relations of third type

$\longrightarrow$ Particular cases

$$h_{1T}^{\perp q\,(0)}(x) \;=\; \int d^2\vec{k}_T\, h_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) = \frac{3}{(1-x)^2}\,\tilde{H}_T^q(x,0,0)$$

$$h_{1T}^{\perp q\,(1)}(x) \;=\; \int d^2\vec{k}_T\,\frac{\vec{k}_T^{\,2}}{2M^2}\,h_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) = \int d^2\vec{b}_T\,\frac{\vec{b}_T^{\,2}}{2M^2}\,2\Big(\tilde{\mathcal{H}}_T^q(x,\vec{b}_T^{\,2})\Big)''$$

$\longrightarrow$ No immediate evidence for breakdown of relations of third type

- Relation of fourth type

$\longrightarrow$ Trivially satisfied because

$$h_{1T}^{\perp g} = \tilde{\mathcal{H}}_T^g = 0$$

GTMD analysis

- GTMD-correlator (e.g., Belitsky, Ji, Radyushkin, Yuan, 2003, 2005)

$$W^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle p';\lambda'|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{GTMD}\,\psi\Big(\frac{z}{2}\Big)\,|p;\lambda\rangle\,\Big|_{z^+=0}$$

- Projection onto GPDs and TMDs

$$\begin{aligned} F^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\,e^{ik\cdot z}\,\langle p';\lambda'|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{GPD}\,\psi\Big(\frac{z}{2}\Big)\,|p;\lambda\rangle\,\Big|_{z^+=z_T=0} \\ &= \int d^2\vec{k}_T\,W^q \end{aligned}$$

$$\begin{aligned} \Phi^q &= \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle p;\lambda'|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|p;\lambda\rangle\,\Big|_{z^+=0} \\ &= W^q\Big|_{\Delta=0} \end{aligned}$$

$\longrightarrow$ GPDs and TMDs appear as certain limits of GTMDs (mother distributions)

$\longrightarrow$ Which GPDs and TMDs have the same mother distributions ?

- Parameterization of GTMD-correlator

  Example:

$$W^{q\,[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1}+\frac{i\sigma^{i+}k_T^i}{P^+}\,F_{1,2}+\frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{1,3}+\frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}\,F_{1,4}\right]u(p,\lambda)$$

  $\longrightarrow$ GTMDs are complex functions: $F_{1,n} = F_{1,n}^e + iF_{1,n}^o$

- Implications for potential nontrivial relations
  - Relations of second type

$$\begin{aligned} E(x,0,\vec{\Delta}_T^2) &= \int d^2\vec{k}_T\left[-F_{1,1}^e + 2\left(\frac{\vec{k}_T\cdot\vec{\Delta}_T}{\vec{\Delta}_T^2}\,F_{1,2}^e + F_{1,3}^e\right)\right] \\ f_{1T}^{\perp}(x,\vec{k}_T^2) &= -F_{1,2}^o(x,0,\vec{k}_T^2,0,0) \end{aligned}$$

  $\longrightarrow$ No model-independent nontrivial relation between E and $f_{1T}^{\perp}$ possible

  $\longrightarrow$ Relation in spectator model due to simplicity of the model

  $\longrightarrow$ No information on numerical violation of relation

  $\longrightarrow$ Likewise for nontrivial relation involving $h_1^{\perp}$

– Relation of third type

$$\tilde{H}_T(x, 0, \vec{\Delta}_T^2) = \int d^2\vec{k}_T \left[\left(\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} H^e_{1,1} + H^e_{1,2} \right) - 2 \left(\frac{2(\vec{k}_T \cdot \vec{\Delta}_T)^2 - \vec{k}_T^2 \vec{\Delta}_T^2}{(\vec{\Delta}_T^2)^2} H^e_{1,4} + \frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} H^e_{1,5} + H^e_{1,6} \right) \right]$$

$$h_{1T}^{\perp}(x, \vec{k}_T^2) = H^e_{1,4}(x, 0, \vec{k}_T^2, 0, 0)$$

$\longrightarrow$ No model-independent nontrivial relation between $\tilde{H}_T$ and $h_{1T}^{\perp}$ possible

$\longrightarrow$ Relation in spectator model

$$H^e_{1,1}(\xi = 0) = H^e_{1,5}(\xi = 0) = 0$$

$$2\tilde{H}^e_{1,2}(x) - 4\tilde{H}^e_{1,6}(x) = (1-x)^2\, \tilde{H}^e_{1,4}(x)$$

$\longrightarrow$ Relation in light-front constituent quark model

(Pasquini, Cazzaniga, Boffi, 2008) (see also talk by Pasquini)

$$\int d^2\vec{k}_T\, h_{1T}^{\perp}(x, \vec{k}_T^2) = \frac{2}{(1-x)^2}\, \tilde{H}_T(x, 0, 0) \qquad \text{instead of}$$

$$\int d^2\vec{k}_T\, h_{1T}^{\perp}(x, \vec{k}_T^2) = \frac{3}{(1-x)^2}\, \tilde{H}_T(x, 0, 0) \qquad \text{in spectator model}$$

Summary

- Various nontrivial relations between GPDs and TMDs established in low order spectator model calculations

- Neglecting various diagrams relations of second type also hold in higher orders (Gamberg, Schlegel, 2009)

- Relation between E^q and $f_{1T}^{\perp q}$ nicely agrees with phenomenology

- Relations (of second type) break down in spectator models in higher orders

- GTMD analysis implies that no model-independent nontrivial relations possible (analysis also for subleading twist)

- So far not much definite known about the numerical violation of the relations