

Pion transition form factor:
Factorization,
LCSR and higher-order corrections
vs experimental data

S. V. Mikhailov¹ N. G. Stefanis²

¹Bogoliubov Laboratory of Theoretical Physics,
Dubna, Russia

²Ruhr-Universität Bochum, Germany

21 июля 2009 г.

Plan of Presentation

$\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$, factorization, structure of $F^{\gamma^*\gamma^*\pi}$

- ▶ Meson Distribution Amplitudes (DA)
- ▶ NLO and NNLO hard-scattering amplitudes

Light Cone Sum Rules (LCSR)

- ▶ Why Light Cone Sum Rules (LCSR)?
- ▶ Dispersion relations for $F^{\gamma^*\gamma^*\pi}$ at $q_2^2 \downarrow 0$
- ▶ NLO Spectral density
- ▶ β_0 -part of NNLO density

$F_{\text{LCSR}}^{\gamma^*\gamma^*\pi}$ analysis vs CELLO and CLEO data

- ▶ CLEO data on $F_{\text{LCSR}}^{\gamma^*\gamma^*\pi} \Rightarrow$ endpoint-suppressed Pion DA
- ▶ Recent Lattice data confirm CLEO LCSR analysis and endpoint-suppressed Pion DA obtained from NLO QCD SR
- ▶ Independent estimate of Pion DA from inverse moment also conform

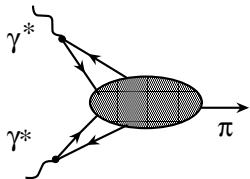
Conclusions

Is new BaBar data a “pion puzzle”?

$$\gamma^*(\mathbf{q}_1)\gamma^*(\mathbf{q}_2) \rightarrow \pi^0(\mathbf{p}),$$

factorization, and structure of $F^{\gamma^*\gamma^*\pi}$

$\gamma^*(q_1)\gamma^*(q_2) \rightarrow \pi^0(p)$ in pQCD



$$\int d^4x e^{-iq_1 \cdot z} \langle \pi^0(p) | T\{j_\mu(z)j_\nu(0)\} | 0 \rangle =$$

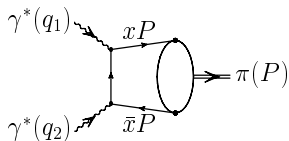
$$i\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \cdot F^{\gamma^* \gamma^* \pi}(Q^2, q^2),$$

$$-q_1^2 = Q^2 > 0, \quad -q_2^2 = q^2 \geq 0$$

Collinear factorization at $Q^2, q^2 \gg (\text{hadron scale} \sim m_p)^2$

$$F^{\gamma^* \gamma^* \pi}(Q^2, q^2) = T(Q^2, q^2, \mu_F^2; \mathbf{x}) \otimes \varphi_\pi^{(2)}(\mathbf{x}; \mu_F^2) + O\left(\frac{1}{Q^4}\right)$$

For leading twist 2 and at parton level



$$F^{\gamma^* \gamma^* \pi} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 dx \frac{1}{Q^2 x + q^2 \bar{x}} \varphi_\pi^{(2)}(\mathbf{x})$$

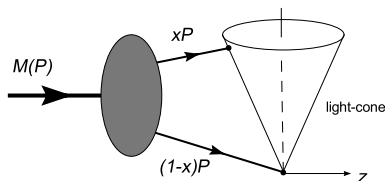
$$Q^2 F^{\gamma^* \gamma^* \pi}(Q^2, \mathbf{0}) = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{dx}{x} \varphi_\pi^{(2)}(\mathbf{x})$$

$$\equiv \frac{\sqrt{2}}{3} f_\pi \langle \mathbf{x}^{-1} \rangle_\pi$$

Distribution amplitudes in exclusive reactions

$$\langle 0 | \bar{q}(z) \gamma_\mu \gamma_5 E(z, 0) q(0) | \pi(P) \rangle \Big|_{z^2=0} = iP_\mu \int dx e^{ix(zp)} \varphi_\pi^{(2)}(x, \mu_F^2)$$

$$E(z, 0) = P \exp\left(ig \int_0^z A_\mu(\tau) d\tau^\mu\right)$$



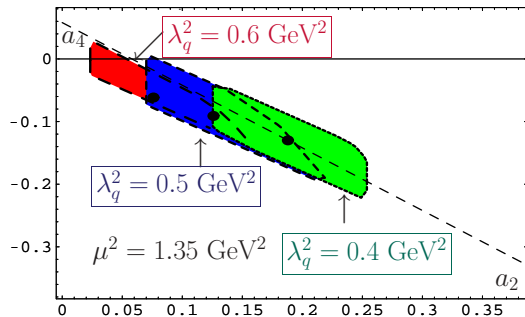
Distribution amplitudes can be obtained in:

- ▶ QCD SR [CZ 1984],
NLC QCD SR [M&Radyushkin 1986–92; Bakulev&M&Stefanis 1998, 2001–04]
- ▶ Instanton vacuum approaches,
[Polyakov et al. 1998, 2009; Dorokhov et al. 2000]
- ▶ Lattice QCD, [Braun et al. 2006; Donnellan et al. 2007]
- ▶ from experimental data [Schmedding&Yakovlev 2000, BMS 2003–2006]
- ▶ **not within pQCD**, but with μ_F^2 -evolution according to ERBL [79–80] in pQCD

Pion distribution amplitudes in NLC QCD SR

$$\varphi_\pi^{(2)}(x; \mu_F^2) = \psi_0(x) + a_2(\mu_F^2) \psi_2(x) + a_4(\mu_F^2) \psi_4(x) + \dots$$

$$\varphi_\pi^{(2)} \Leftrightarrow \{a_n\}; \quad \psi_n(x) = 6x\bar{x} C_n^{(3/2)}(x - \bar{x}) \quad \text{-- Gegenbauer harmonics}$$



λ_q^2 – average virtuality of vacuum quarks – the single parameter of NLC approach

BMS [PLB 508(2001)279]

BMS

[Ann.Phys.13(2004)629]

- ▶ Green rectangle forms the BMS bunch of DA, $\psi_0 + a_2\psi_2 + a_4\psi_4$
- ▶ ψ_0 – Asymptotic (As) DA
- ▶ Chernyak-Zhitnitsky (CZ) DA, QCD SR, $\psi_0 + a_2\psi_2$, $a_2(1) \approx 0.56$

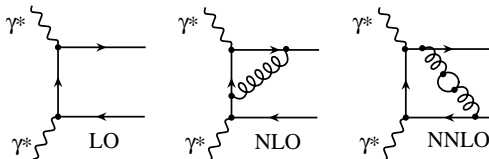
NLO and NNLO amplitudes.

Collinear factorization is Theorem [Efremov&Radyushkin 1978]

$$\begin{aligned}
 F^{\gamma^* \gamma^* \pi} \sim & \left(T_0(Q^2, q^2; x) + a_s^1 T_1(Q^2, q^2; \mu_F^2; x) \right. \\
 & \left. + a_s^2 T_2(Q^2, q^2; \mu_F^2; \mu_R^2; x) + \dots \right) \otimes \varphi_\pi^{(2)}(x; \mu_F^2) \\
 & - \delta_{tw4}^2 \cdot T_0^2(Q^2, q^2; x) \otimes \varphi_\pi^{(4)}(x)
 \end{aligned}$$

— calculable in pQCD, $a_s(\mu_R^2) = \alpha_s/(4\pi)$. Usually $\mu_R^2 = \mu_F^2 = \langle Q^2 \rangle$ to simplify and to minimize rad. corrections.

$\delta_{tw4}^2 = (0.19 \pm 0.02)$ GeV² – twist-4 scale parameter.



LO:
$$T_0(Q^2, q^2; x) = \frac{1}{x Q^2 + \bar{x} q^2}$$

NLO amplitudes and NLO evolution

NLO (last editions):

[Bakulev&MS&Stefanis(2003)],

[Melić&Müller&Passek-Kumerički(2003)]

$$\begin{aligned} T_1(x; Q^2, q^2) \otimes \varphi(x) &= T_0(Q^2, q^2; y) \otimes \left\{ C_F T^{(1)}(y, x) + \mathbf{L}(y) \cdot \mathbf{V}^{(0)}(y, x) \right\} \otimes \varphi(x) \\ T^{(1)} &= \left[-3\mathbf{V}^b + \mathbf{g} \right] (x, y)_+ - 3\delta(x - y), \quad \mathbf{L}(y) \equiv \ln \left[\left(Q^2 y + q^2 \bar{y} \right) / \mu_F^2 \right] \end{aligned}$$

$\varphi(x; \mu^2) \rightarrow \varphi(x; Q^2)$ evolves according to NLO ERBL [79-80] equation:

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} \varphi(x; \mu^2) &= \left(a_s \mathbf{V}^{(0)}(x, y) + a_s^2 \mathbf{V}^{(1)}(x, y) \right) \otimes \varphi(y; \mu^2) \\ \mathbf{V}^{(0)} \otimes \psi_n &= \mathbf{v}(n) \cdot \psi_n \end{aligned}$$

NNLO amplitude and coefficient functions

β_0 -part of NNLO: $T_2 \otimes \varphi \rightarrow \beta_0 \cdot T_\beta \otimes \varphi$
[Melić&Müller&Passek-Kumerički(2003)]

$$T_\beta \otimes \varphi = \ln \left(\frac{\mu_R^2}{\mu_F^2} \right) T_1 \otimes \varphi + T_0 \otimes \left\{ C_F T_\beta^{(2)} - C_F L(\mathbf{y}) \cdot T^{(1)} \right. \\ \left. + L(\mathbf{y}) \cdot \left(V_\beta^{(1)} \right)_+ - \frac{1}{2} L^2(\mathbf{y}) \cdot V^{(0)} \right\} \otimes \varphi.$$

The origins of these terms:

$\sim V^{(0)}$ – 1-loop ERBL together with \mathbf{a}_s RG evolution, while $\sim \left(V_\beta^{(1)} \right)_+$ – as the β_0 -part of 2-loop ERBL kernel; $T^{(1)}$ – from \mathbf{a}_s RG evolution.
These terms together form the exponential RG-solution:

$$\exp \left\{ \int^L V(\mathbf{a}_s(L)) dL \right\}$$

$T_\beta^{(2)}$ – the coefficient function – original, the most cumbersome part

This contribution gives the **sign and size of NNLO effect** following to BLM prescription

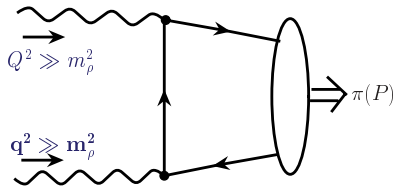
Light Cone Sum Rules (LCSR)

Why Light Cone Sum Rules (LCSR)?

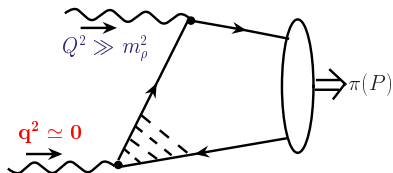
The experimental conditions require $q^2 \rightarrow 0$

For $Q^2 \gg m_\rho^2$, $q^2 \ll m_\rho^2$ pQCD factorization valid only in leading twist and higher twists are of importance [Radyushkin–Ruskov, NPB (1996)].

Reason: if $q^2 \rightarrow 0$ one needs to take into account interaction of real photon at long distances of order of $O(1/\sqrt{q^2})$



pQCD is OK



photons behaves like a hadron

[Khodjamirian, EJPC (1999)]: LCSR effectively accounts for long-distances effects of real photon using dispersion relation in variable q^2 and quark-hadron duality in vector channel.

Dispersion relation for $F^{\gamma^* \gamma \pi}$

The main further goal – spectral density ρ

$$F^{\gamma^* \gamma \pi}(Q^2, q^2) = \int_0^\infty ds \frac{\rho^{\text{ph}}(Q^2, s)}{s + q^2}$$

$$\rho^{\text{ph}} = \theta(s_0 - s) \rho^{\text{phen}}(Q^2, s) + \theta(s - s_0) \rho^{\text{PT}}(Q^2, s)$$

$$\rho^{\text{PT}}(Q^2, s) = \frac{\text{Im}}{\pi} \left[F^{\gamma^* \gamma^* \pi}(Q^2, -s) \right]$$

$$\rho^{\text{phen}}(Q^2, s) = \sqrt{2} f_\rho F^{\gamma^* V \pi}(Q^2) \cdot \delta(s - m_V^2) \Big|_{V=\rho, \omega}$$

using quark-hadron duality in vector channel for $F^{\gamma^* V \pi}$ [Khodjamirian 1999]:

$$\begin{aligned} F^{\gamma \gamma^* \pi}(Q^2, q^2 \rightarrow 0) &= \frac{1}{\pi} \int_{s_0}^\infty \frac{\text{Im} F^{\gamma^* \gamma^* \pi}(Q^2, -s)}{s} ds \\ &+ \frac{1}{\pi} \int_0^{s_0} \frac{\text{Im} F^{\gamma^* \gamma^* \pi}(Q^2, -s)}{m_\rho^2} e^{(m_\rho^2 - s)/M^2} ds \end{aligned}$$

$s_0 \simeq 1.5 \text{ GeV}^2$ – effective threshold in vector channel,

M^2 – Borel parameter (0.5 – 0.9 GeV^2).

NLO Spectral density $\rho^{(1)}$

$$\rho^{(1)}(Q^2, s) = \frac{\text{Im}}{\pi} \left[(T_1 \otimes \varphi_\pi)(Q^2, -s) \right], \quad s \geq 0$$

$\rho_n^{(1)}(x, \mu_F^2)$ for Gegenbauer harmonic ψ_n , $x = Q^2/(s + Q^2)$

The general case [M&Stefanis NPB(2009)] in press:

$$\begin{aligned} \bar{\rho}_n^{(1)}(x; \mu_F^2) &= C_F \left\{ -3 [1 - \mathbf{v}^a(n)] + \frac{\pi^2}{3} - \ln^2 \left(\frac{\bar{x}}{x} \right) + 2\mathbf{v}(n) \ln \left(\frac{\bar{x}}{x} \frac{Q^2}{\mu_F^2} \right) \right\} \psi_n(x) \\ &\quad - C_F 2 \left[\sum_{l=0,2,\dots}^n (\mathbf{G}_{nl} + \mathbf{v}(n) \cdot \mathbf{b}_{nl}) \psi_l(x) \right] \end{aligned}$$

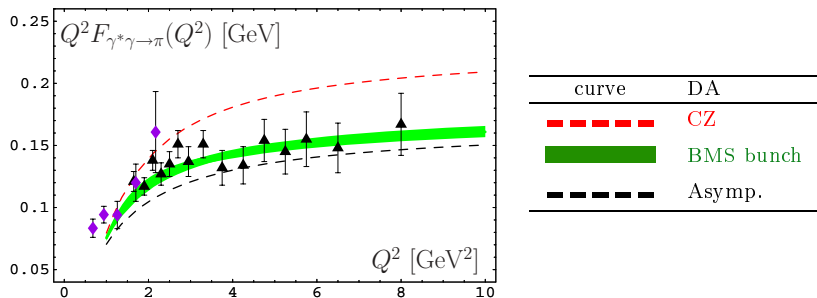
\mathbf{G}_{nl} (originates from \mathbf{g}), \mathbf{b}_{nl} – calculable triangular matrices

The partial case $\bar{\rho}_0^{(1)}$ [Schmedding&Yakovlev (2000)]:

$$\bar{\rho}_0^{(1)}(x) = C_F \left[-5 + \frac{\pi^2}{3} - \ln^2 \left(\frac{\bar{x}}{x} \right) \right] \psi_0(x)$$

Conclusion: The spectral density and $F^{\gamma\gamma^* \pi}$ can be obtained for any numbers of Gegenbauer harmonics

NLO LCSR vs. CELLO (\blacklozenge) & CLEO (\blacktriangle) data



Radiative corrections contribute up to **-17%** at low/moderate Q^2

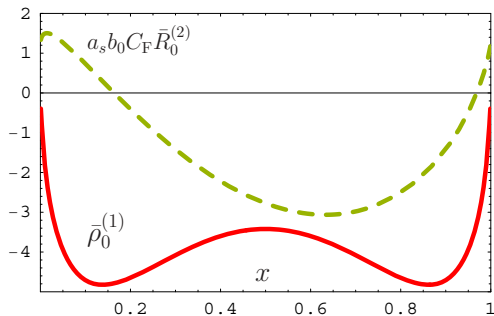
- ▶ BMS bunch describes rather well all data above $Q^2 \gtrsim 1.5 \text{ GeV}^2$;
- ▶ Low- Q^2 CELLO data (only statistical errors shown) excludes As DA and high- Q^2 CLEO data excludes CZ DA .

NNLO $_{\beta_0}$ Spectral density

$$\rho^{(2)}(Q^2, s) = \frac{\text{Im}}{\pi} \left[(T_2 \otimes \varphi_\pi)(Q^2, -s) \right], \quad s \geq 0$$

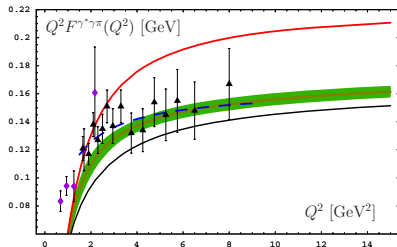
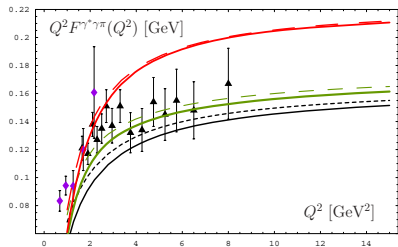
$$x = Q^2/(s + Q^2), \quad \text{put } \mu_{\mathbf{R}}^2 = \mu_{\mathbf{F}}^2$$

$$\bar{\rho}_n^{(2)} \rightarrow b_0 C_F \left[\bar{R}_n^{(2)} \left(x; \frac{\bar{x}}{x} \frac{Q^2}{\mu_{\mathbf{F}}^2} \right) \right].$$



The dashed green line shows $a_s(\mu_{\mathbf{F}}^2) \bar{\rho}_0^{(2)} = a_s(\mu_{\mathbf{F}}^2) b_0 C_F \bar{R}_0^{(2)}(x, \bar{x}/x)$ at the typical CLEO reference scale $Q^2 = \mu_{\mathbf{F}}^2 = \mu_{\text{SY}}^2 = (2.4 \text{ GeV})^2$, whereas the solid red line represents $\bar{\rho}_0^{(1)}(x)$

NNLO $_{\beta_0}$ LCSR vs. NLO LCSR



Dashed lines – NLO form factor

Solid lines – $[\text{NNLO}_{\beta_0}] \oplus [\text{Improved Breit-Wigner } \rho^{\text{phen}} \text{ model}]$

- ▶ Combined effect: -7% at $Q^2 \simeq 2 \text{ GeV}^2$, -2% at $Q^2 \geq 6 \text{ GeV}^2$
- ▶ Effect of BLM prescription can be stronger \Rightarrow
we expect the size of the total NNLO contribution is less -10%

$F_{\text{LCSR}}^{\gamma^* \gamma \pi}$, CLEO data,
and
inverse problem for $\varphi_{\pi}^{(2)}$

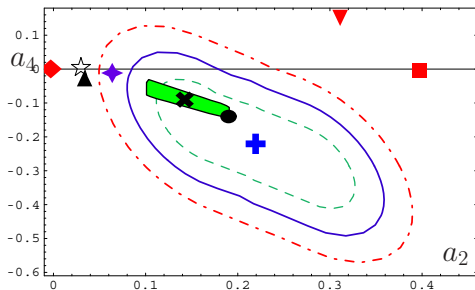
CLEO data on $F_{LCSR}^{\gamma^* \gamma \pi} \Rightarrow$ Pion DA in plane (a_2, a_4)

NLO Light-Cone SR \oplus Twist-4 $\oplus (\mu^2 = Q^2)$

with **20%** uncertainty of $\delta_{\text{Tw-4}}^2$: $\delta_{\text{Tw-4}}^2 = \mathbf{0.19}$ (4) GeV^2

BMS [PLB 578 (2004) 91]: $\lambda_q^2 = \mathbf{0.4}$ GeV^2

1 σ – green line; 2 σ – blue line; 3 σ –dashed-dotted red line.



at $\mu_{\text{SY}}^2 = \mathbf{5.76}$ GeV^2

+ = best-fit BMS, **●** = SY points

◆ = Asymptotic DA

■ = CZ DA, **▼** = Braun-Filyanov DA

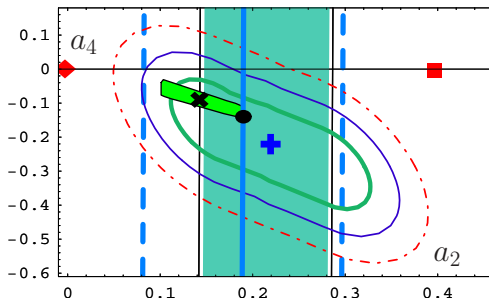
× = BMS model

Even with 20% uncertainty in twist-4
CZ, BF DA excluded at least at 4 σ -level!

As DA — at **3 σ** -level.

BMS DA and most of BMS bunch — inside 1 σ -domain.

Recent Lattice data vs. CLEO analysis and Pion DA



Comparison of the CLEO-data constraints on $F_{\gamma^* \gamma \pi}(Q^2)$ in the (a_2, a_4) plane at the scale $\mu_{\text{SY}}^2 = 5.76 \text{ GeV}^2$ in terms of error regions around the BMS best-fit point \oplus

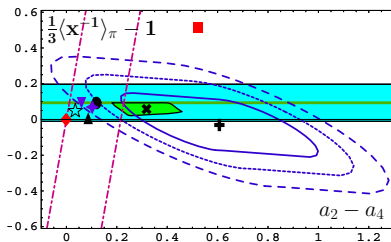
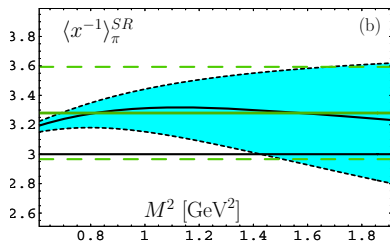
1σ — green line; 2σ — blue line; 3σ —dashed-dotted red line.

Recent lattice simulations, denoted by vertical dashed lines [LAT2006] and solid ones [PoS LAT2007] are also shown together with predictions obtained from NLC QCD SR (slanted green rectangle)

Independent estimate of Pion DA inverse moment $\langle x^{-1} \rangle_\pi$

BMS [Ann. Phys.(Leipzig)13(2004) 629]:

processing of CLEO data is evolved to normalization scale $\mu^2 \simeq 1 \text{ GeV}^2$



$$\lambda_q^2 = 0.4 \text{ GeV}^2, \quad \frac{1}{3}\langle x^{-1} \rangle_\pi^{SR} - 1 = 0.1 \pm 0.1$$

Good agreement of a theoretical “tool” of different origin with CLEO data

Conclusion

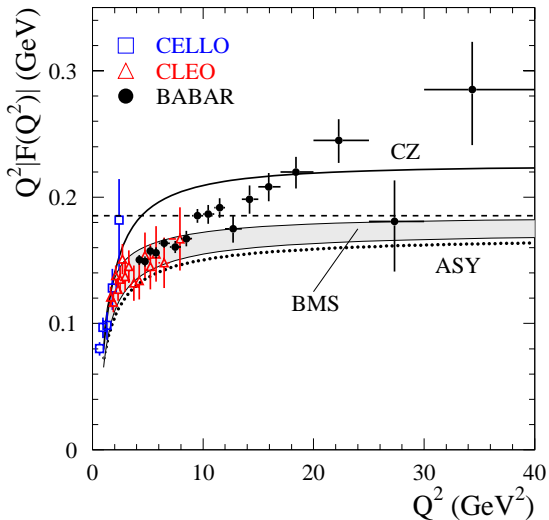
- ▶ **NLO LCSR** produces **constraints** on pion DA parameters ($\mathbf{a}_2, \mathbf{a}_4$) in conjunction with CLEO data. The same can be done for any number of parameters $\{\mathbf{a}_n\}$.
- ▶ **QCD SR** with **NLC** for pion DA gives **admissible sets** of DAs and, independently, the inverse moment $\langle \mathbf{x}^{-1} \rangle_{\pi}^{SR}$
- ▶ **This NLO LCSR constraints** agree well with: **the bunch** of pion DAs and $\langle \mathbf{x}^{-1} \rangle_{\pi}^{SR}$, and recent **lattice** data.
- ▶ **NNLO LCSR** correction does no change $F^{\gamma^* \gamma \pi}(Q^2)$ significantly. The common effect of NNLO_{β_0} and improved hadron model leads: **from -7% at $Q^2 \simeq 2 \text{ GeV}^2$, to -2% at $Q^2 \geq 6 \text{ GeV}^2$**

We thank **A. P. Bakulev** for collaboration and help in the preparation of this report.

Do the new BaBar data pose a “pion puzzle”?

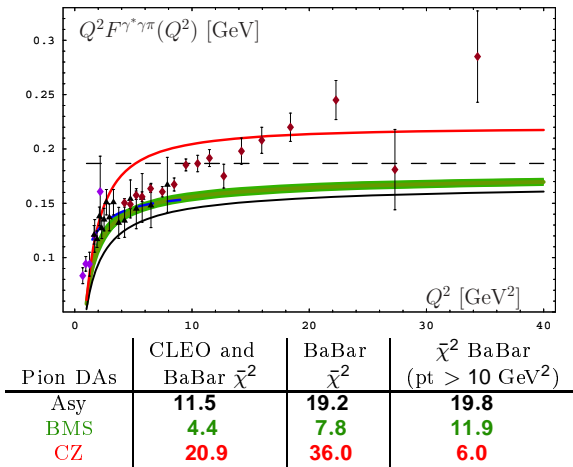
New data on transition form factor

by BaBar Collaboration [B. Aubert et al., arXiv:0905.4778[hep-exp]]:



“BaBar data contradicts most models for the pion DA ...”

Is the new BaBar data a “pion puzzle”?



The high- Q^2 BaBar data points—growing with Q^2 —contradict all pion DA models that have a finite projection on Gegenbauer polynomials, i.e., vanish at the endpoints 0, 1.

Is the new BaBar data a “pion puzzle”?

- ▶ Moreover, they contradict the
collinear factorization formula per se.
- ▶ Even more, they contradict the “counting rules” – the most reliable method up to now.
- ▶ Provided the BaBar data are correct, they constitute a challenge for QCD

First attempts to overcome the “pion puzzle”

A possible scenario to explain the BaBar data was proposed by [Dorokhov arXiv:0905.3577], [Radyushkin arXiv:0906.0323] and [Polyakov arXiv:0906.0538]: “practically flat DA” that generates the logarithmic growth in Q^2 and transfers the **start of the collinear factorization far** from the BaBar region

	Cello&CLEO	BaBar
Cello&CLEO	1.22	15.8
BaBar	3.5	1.8

σ, m^2 – adjusted parameters

$$Q^2 F^{\gamma^* \gamma \pi} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{1}{x} \left[1 - \exp\left(-\frac{xQ^2}{\bar{x}2\sigma}\right) \right] dx$$

$$\text{or} = \frac{\sqrt{2}}{3} f_\pi \int_0^1 \frac{1}{x + m^2/Q^2} dx$$

$$\sim \log(1 + Q^2/m^2)$$

	Cello&CLEO	BaBar
Cello&CLEO	0.48	7.8
BaBar	10.8	1.8

‘QCD like’ Dipole fit, Λ, b – adjusted parameters

$$Q^2 F^{\gamma^* \gamma \pi} = \frac{Q^2}{2\sqrt{2}f_\pi} \left[\frac{\Lambda^2}{\Lambda^2 + Q^2} + b \left(\frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^2 \right]$$

Preliminary conclusions about BaBar data

- ▶ This is a hint that the “old data” – Cello and CLEO and the “new data” – BaBar cannot be fitted simultaneously.
- ▶ From the theoretical side, it is not possible to describe these data with the inclusion of higher radiative corrections—as we have shown at the NNLO level.