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Insights on non-perturbative aspects of TMDs from models

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based on works with Anatoli Vasilievich,
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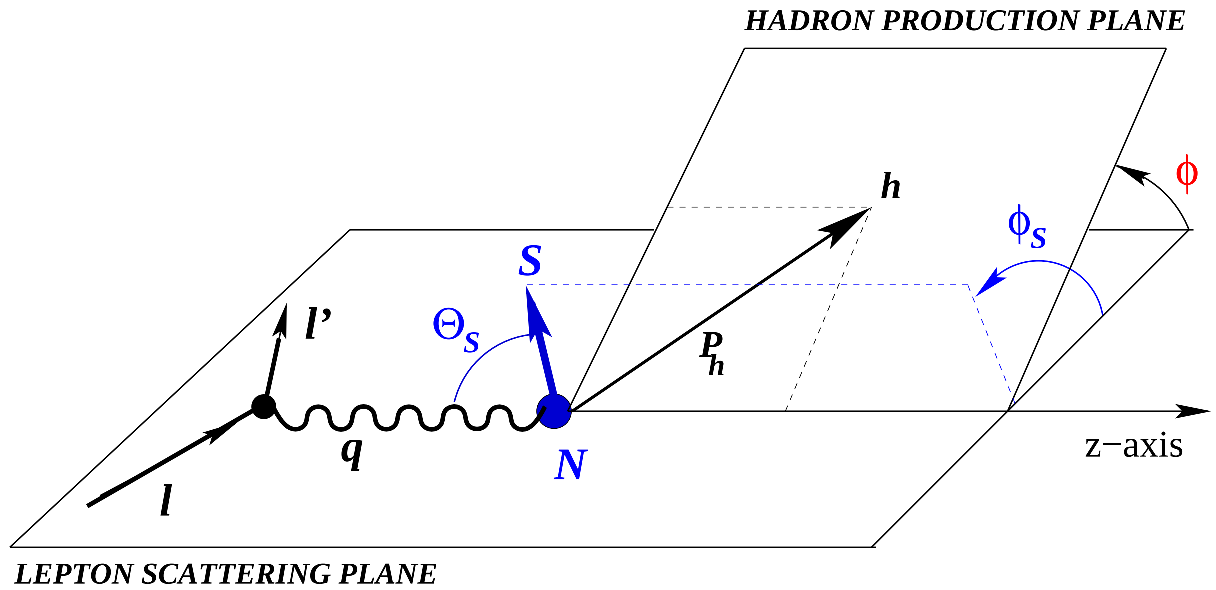
Overview:

- Motivation: Many TMDs (many new functions)! Question: Are there . . .
- . . . *exact* relations among TMDs, in QCD? No.
- . . . *approximate* relation in QCD? Possibly yes.
- . . . relations in *models*, part I? There must be!
- . . . relations in *models*, part II? There can be!
- Are these *model* relations useful? They are!
- Conclusions

Motivation



e.g. semi-inclusive DIS:

$$\frac{d\sigma}{d\phi} = \underbrace{\left[\frac{d\sigma}{d\phi}\right]_{\text{leading}}}_{8} + \underbrace{\left[\frac{d\sigma}{d\phi}\right]_{\text{subleading}}}_{8} + \underbrace{\left[\frac{d\sigma}{d\phi}\right]_{\text{subsubleading}}}_{2} = 18 \text{ structure functions}$$

leading: $\{\underbrace{f_1, g_1, h_1, g_{1T}^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp}}_{\text{6 T-even}}, \underbrace{f_{1T}^{\perp}, h_1^{\perp}}_{\text{2 T-odd}}\} \otimes \{D_1, H_1^{\perp}\}$ factorization ✓

(Ji, Ma, Yuan 2004)

sublead.: $\{\underbrace{g_T, e, h_L, f^{\perp}, g_L^{\perp}, g_T^{\perp}, h_T^{\perp}, h^{\perp}}_{\text{8 T-even}}, \underbrace{e_T^{\perp}, e_L, e_T, f_T, f_L^{\perp}, f_T^{\perp}, g^{\perp}, h}_{\text{8 T-odd}}\} \otimes \{\dots\}$ fact. ?

Will all be measured! And also 48 structure functions in Drell-Yan!

- In order to be prepared:
  need to know as much as possible about the 8 twist-2, 16 twist-3 TMDs.

- Valuable source of information: models.

- Use TMD models for phenomenology:
  range of reliability, evolution, model dependence, ...
  Hard, but possible $\rightarrow$ see talk by Barbara Pasquini.

- particularly 'elegant':
  explore relations among TMDs supported in several models!
  Less model-dependent, and more robust prediction.

TMDs

- define fully "unintegrated" quark-quark correlator of nucleon (depends on nucleon momentum P and spin S, quark momentum p)

$$\Phi_{ij}(P,S,p,\text{ path}) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P,S\,|\,\bar{\psi}_j(0)\,\mathcal{W}(0,\xi,\text{ path})\,\psi_i(\xi)\,|\,P,S\rangle$$

- define p_T-dependent quark-quark correlator

$$\phi_{ij}(x,\vec{p}_T,\text{ path}) = \int \mathrm{d}p^-\,\Phi_{ij}(P,S,p,\text{ path})\Big|_{p^+=xP^+}$$

- define **TMD** in certain process (factorization fixes path):

$$\frac{1}{2}\,\mathrm{tr}\;\phi(x,\vec{p}_T,\text{ path}) \times (\gamma\text{-matrix}) = \sum_i c_i\;\mathrm{TMD}_i(x,\vec{p}_T,\text{ process})$$

- still "universal": (T-even TMD, DIS) = + (T-even TMD, DY),
  (T-odd TMD, DIS) = − (T-odd TMD, DY)

For completeness: definition of TMDs (review Bacchetta et al, 2006)

- leading twist

$$\frac{1}{2}\,\mathrm{tr}\big[\gamma^+\,\phi(x,\vec{p}_T)\big] = f_1 - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N}\,f_{1T}^\perp$$

$$\frac{1}{2}\,\mathrm{tr}\big[\gamma^+\gamma_5\,\phi(x,\vec{p}_T)\big] = S_L\,g_1 + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N}\,g_{1T}^\perp$$

$$\frac{1}{2}\,\mathrm{tr}\big[i\sigma^{j+}\gamma_5\,\phi(x,\vec{p}_T)\big] = S_T^j\,h_1 + S_L\,\frac{p_T^j}{M_N}\,h_{1L}^\perp + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2}\,h_{1T}^\perp + \frac{\varepsilon^{jk}p_T^k}{M_N}\,h_1^\perp$$

- subleading twist

$$\frac{1}{2}\,\mathrm{tr}\big[\,1\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[\;e - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N}\,e_T^\perp\Big]$$

$$\frac{1}{2}\,\mathrm{tr}\big[i\gamma_5\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[S_L e_L + \frac{\vec{p}_T\vec{S}_T}{M_N}\,e_T\Big]$$

$$\frac{1}{2}\,\mathrm{tr}\big[\,\gamma^\alpha\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[\frac{p_T^j}{M_N}\,f^\perp + \varepsilon^{jk}S_T^k f_T + S_L\varepsilon^{jk}S_T^k f_L^\perp + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})\varepsilon^{kl}S_T^l}{M_N^2}\,f_T^\perp\Big]$$

$$\frac{1}{2}\,\mathrm{tr}\big[\gamma^j\gamma_5\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[S_T^j\,g_T + S_L\,\frac{p_T^j}{M_N}\,g_L^\perp + \frac{(p_T^j p_T^k - \frac{1}{2}\,\vec{p}_T^{\,2}\delta^{jk})S_T^k}{M_N^2}\,g_T^\perp + \frac{\varepsilon^{jk}p_T^k}{M_N}\,g^\perp\Big]$$

$$\frac{1}{2}\,\mathrm{tr}\big[i\,\sigma^{jk}\gamma_5\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[\frac{S_T^j p_T^k - S_T^k p_T^j}{M_N}\,h_T^\perp - \varepsilon^{jk}\,h\Big]$$

$$\frac{1}{2}\,\mathrm{tr}\big[\,i\,\sigma^{+-}\gamma_5\,\phi(x,\vec{p}_T)\big] = \frac{M_N}{P^+}\Big[S_L\,h_L + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N}\,h^\perp\Big]$$

1. Can there be exact relations? No!

look at fully unintegrated quark-quark correlation function:

$$\Phi_{ij}(P,S,p|n_-) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P,S\,|\,\bar\psi_j(0)\,\mathcal{W}(0,\xi|n_-)\,\psi_i(\xi)\,|\,P,S\rangle$$

depends on:

- nucleon momentum $P = P^+ n_+ + (M^2/2P^+) n_-$ ($n_\pm^2 = 0$, with $n_+ \cdot n_- = 1$)
- spin of the nucleon S
- quark momentum p with $p^+ = xP^+$
- light-cone direction n_- (in path of $\mathcal{W}$)

General Lorentz-decomposition of $\Phi_{ij}(P,S,p|n_-)$
in terms of P, S, p and n!

There are **32 independent** amplitudes $\underbrace{A_1\text{–}A_{12}}_{\text{from } P,S,p}$ and $\underbrace{B_1\text{–}B_{20}}_{\text{and } n_-}$

For completeness I: general decomposition (Goeke, Metz and Schlegel, 2005):

$$\Phi(P,p,S|n_-) = M A_1 + \not{P} A_2 + \not{p} A_3 + \frac{i}{2M}\,[\not{P},\not{p}]\, A_4 + i(p\cdot S)\gamma_5\, A_5 + M\not{S}\gamma_5\, A_6$$
$$+\frac{p\cdot S}{M}\not{P}\gamma_5\, A_7 + \frac{p\cdot S}{M}\not{p}\gamma_5\, A_8 + \frac{[\not{P},\not{S}]}{2}\gamma_5\, A_9 + \frac{[\not{p},\not{S}]}{2}\gamma_5\, A_{10} + \frac{p\cdot S}{2M^2}[\not{P},\not{p}]\gamma_5\, A_{11} + \frac{\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho S_\sigma}{M}\, A_{12}$$
$$+\frac{M^2}{P\cdot n_-}\not{n}_-\, B_1 + \frac{iM}{2P\cdot n_-}[\not{P},\not{n}_-]\, B_2 + \frac{iM}{2P\cdot n_-}[\not{p},\not{n}_-]\, B_3 + \frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu p_\rho n_{-\sigma}\, B_4$$
$$+\frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}P_\mu p_\nu n_{-\rho}S_\sigma\, B_5 + \frac{iM^2}{P\cdot n_-}(n_-\cdot S)\gamma_5\, B_6 + \frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu n_{-\rho}S_\sigma\, B_7$$
$$+\frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu p_\nu n_{-\rho}S_\sigma\, B_8 + \frac{p\cdot S}{M(P\cdot n_-)}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma}\, B_9 + \frac{M(n_-\cdot S)}{(P\cdot n_-)^2}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma}\, B_{10}$$
$$+\frac{M}{P\cdot n_-}(n_-\cdot S)\not{P}\gamma_5\, B_{11} + \frac{M}{P\cdot n_-}(n_-\cdot S)\not{p}\gamma_5\, B_{12} + \frac{M}{P\cdot n_-}(p\cdot S)\not{n}_-\gamma_5\, B_{13}$$
$$+\frac{M^3}{(P\cdot n_-)^2}(n_-\cdot S)\not{n}_-\gamma_5\, B_{14} + \frac{M^2}{2P\cdot n_-}[\not{n}_-,\not{S}]\gamma_5\, B_{15} + \frac{p\cdot S}{2P\cdot n_-}[\not{P},\not{n}_-]\gamma_5\, B_{16}$$
$$+\frac{p\cdot S}{2P\cdot n_-}[\not{p},\not{n}_-]\gamma_5\, B_{17} + \frac{n_-\cdot S}{2P\cdot n_-}[\not{P},\not{p}]\gamma_5 B_{18} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{P},\not{n}_-]\gamma_5 B_{19} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{p},\not{n}_-]\gamma_5 B_{20}\,.$$

A_4, A_{12} : 'naive T-odd' ↔ Sivers, Boer-Mulders function

the B_i: 'explicitly due to gauge link' (n_- in Lorentz-decomposition)

We see: • **32** independent amplitudes *

• **32** (8 twist-2, 16 twist-3, $\underbrace{\text{8 twist-4}}_{\text{academical}}$) quark TMDs

$$\text{TMD}(x, \vec{p}_T) = \int \mathrm{d}p^- \Big\{ \text{ linear combination of } A_i,\ B_i \Big\}$$

but A_i, B_i linearly independent

$\Longrightarrow$ hence all twist-2 and twist-3 TMDs independent of each other

$\Longrightarrow$ there exist no relations!

* Dieter Müller, private communication:
"twist-2 and twist-4 talk to each other"
But we do not talk about twist-4 here.

2. Can there be approximate relations? Maybe

Wandzura-Wilczek approximation: $g_T^q(x) = \int_x^1 \frac{\mathrm{d}y}{y} g_1^q(y) + \tilde{g}_T^q(x)$
(Wandzura, Wilczek, 1977)

$\tilde{g}_T^q(x) =$ quark-gluon-quark correlators + current quark mass-terms

In experiment $\tilde{g}_T^q(x)$ seems small!

Approximations in this spirit possible also for TMDs?

Maybe, . . .

. . . see talk by Tobias Teckentrup

3. Can there be some model relations?

We have to specify what 'model' means.

In the following: **quark models**, i.e. no gluons!

e.g.

- spectator models (Jakob, Mulders, Rodrigues, 1997; ...)
- light-cone constituent quark models (Barbara Pasquini et al)
- parton model with intrinsic angular motion (Petr Zavada, and et al)
- non-relativistic limit (Efremov, PS, Teryaev, Zavada, 2008)
- chiral quark soliton model (Diakonov et al, Wakamatsu)
- bag model (...)
- other models

So, can we expect relations among TMDS in these models? **Yes!**

There must be!

Model-independent relations in quark models

Recall the decomposition of general unintegrated correlator $\Phi(P, p, S|n_-)$:
32 independent amplitudes $\underbrace{A_1\text{–}A_{12}}_{\text{from } P,S,p}$ and $\underbrace{B_1\text{–}B_{20}}_{}$ and n_- due to gauge link

But no gluons in quark models, hence: no dependence on n_- $\Rightarrow$ no B_i!

Conclusion: **more TMDs than amplitudes $\longrightarrow$ relations!**
What kind of relations come out of that? Job already done:

'Lorentz-invariance relations' (LIRs)!

'History':
In early works presence of n_- and B_i not recognised (Tangerman, Mulders 1994)
Then hints from models with gauge fields that LIRs violated (Kundu, Metz 2001)
Finally model-independent understanding (Goeke, Metz, Pobylitsa, Polyakov 2003)

By the way, in models without gluons: 'naive T-odd' A_4, A_{12} do not appear. Sivers, Boer-Mulders effects absent (need at least 'one-gluon-exchange')

Examples for LIRs: $g_T(x) = g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)}(x)$

$h_L(x) = h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x)$

etc.

We learn:

A consistent quark model MUST satisfy LIRs!

... independently of details of the model.

- Application: Valuable cross check for models!

- Can LIRs be useful approximations in nature? We do not know.
  What we know: LIRs $\Leftrightarrow$ 'WW approximation', only tilde-terms spoil them so **maybe** as good (or bad) as other WW-type approximations? Will see!!!

  See talk by Tobias Teckentrup

4. Can there be other model relations? **Yes**...

but depends
on model

We established:
in quark models unintegrated correlator $\Phi(P, p, S|n_-)$ has
10 amplitudes $\boldsymbol{A_i}$ with $\boldsymbol{i = 1\text{–}3,\ 5\text{–}11}$ (recall $i = 4$, 12 T-odd, absent)

The **10** amplitudes A_i with $i = 1$–3, 5–11 could be all different.
Or, some of them could happen to be equal. **Depends on model!**

If some of the A_i equal $\rightarrow$ further relations in quark models!

Some relations have already been found!
And they are **fascinating!**

Three examples.

Example 1: $$h_{1L}^{\perp q}(x,p_T) = -\,g_{1T}^{\perp q}(x,p_T)$$

satisfied in

- spectator model of Jakob et al, 1997
- spectator model of Meissner, Metz, Goeke 2007
- MIT bag model of Avakian, Efremov, PS, Yuan, 2008
- light front constituent model Pasquini, Cazzaniga, Boffi 2008
- covariant parton model with quark orbital motion Efremov et al 2008
- some spectator model versions of Bacchetta, Conti, Radici 2008

not satisfied in

- other spectator model versions of Bacchetta, Conti, Radici 2008

meaning:

(polarization of transv. pol. quark in long. pol. nucleon)

opposite to

(polarization of long. pol. quark in transv. pol. nucleon)

and in nature?

Do $h_{1L}^{\perp q}$ and $g_{1T}^{\perp q}$ have similar magnitude and opposite sign? Will see!

Example 2: $$g_1^q(x,p_T) - h_1^q(x,p_T) = h_{1T}^{\perp(1)q}(x,p_T)$$

satisfied in

- bag model Avakian et al, 2008
- spectator model of Jakob et al, 1997
- spectator model of She, Zhu, Ma 2009
- spectator model of Meissner, Metz, Goeke 2007
- light front constituent model Pasquini, Cazzaniga, Boffi 2008
- covariant parton model with quark orbital motion Efremov et al 2008
- some spectator model versions of Bacchetta, Conti, Radici 2008

not satisfied in

- other spectator model versions of Bacchetta, Conti, Radici 2008

meaning: One motivation to access h_1^q:
its difference from g_1^q measures relativistic effects in nucleon! (Jaffe, Ji 1991)
Now we know this difference is nothing but the pretzelosity*: $h_{1T}^{\perp(1)q}(x,p_T)$!!!

Draw back of these relations: connect chirally even and chirally odd functions.

*Popular but not generally approved name.

Example 3: remarkable(?) non-linear(!) relation

$$\frac{1}{2}\left[h_{1L}^{\perp q}(x,p_T)\right]^2 = -\,h_1^q(x,k_\perp)\,h_{1T}^{\perp q}(x,p_T)$$

satisfied in

- covariant parton model with quark orbital motion Efremov et al 2008
- light front constituent model Pasquini, Cazzaniga, Boffi 2008
- spectator model of Meissner, Metz, Goeke 2007
- spectator model of Jakob et al, 1997
- some spectator model versions of Bacchetta, Conti, Radici 2008

not known, whether valid or not in

- other spectator model versions of Bacchetta, Conti, Radici 2008

meaning:

presently unclear

but it has use!

5. Are these relations useful?

Of course: need numerical results, 'hard numbers' for predictions.

Typically difficult task, because it is hard to consider evolution effects, as good as we can. See talk by Barbara Pasquini.

So 'safest predictions' do not use particular quantitative model results but explore model relations, especially when supported in many models!

Example 1: $\frac{1}{2}\left[h_{1L}^{\perp q}\right]^2 = -\,h_1^q\,h_{1T}^{\perp q}$ $\Rightarrow$ h_1^q and $h_{1T}^{\perp q}$ have opposite signs!

compare: $A_{UT}^{\sin(\phi+\phi_S)} \propto h_1^q\,H_1^{\perp q}$ vs. $A_{UT}^{\sin(3\phi-\phi_S)} \propto h_{1T}^{\perp q}\,H_1^{\perp q}$

sign of $A_{UT}^{\sin(\phi+\phi_S)} \propto h_1^q\,H_1^{\perp q}$ known from HERMES, COMPASS

prediction: $A_{UT}^{\sin(3\phi-\phi_S)}(\pi^+)$ negative, $A_{UT}^{\sin(3\phi-\phi_S)}(\pi^-)$ positive. Will see!

Example 2:

- we know h_1^q and $h_{1T}^{\perp q}$ have opposite signs!
- we know $h_1^u > 0$, $h_1^d < 0$ (lattice QCD)
- we have relation $g_1^q - h_1^q = h_{1T}^{\perp(1)q}$

$\Rightarrow \quad h_1^u = g_1^u - h_{1T}^{\perp(1)u} > g_1^u$

$h_1^d = g_1^d - h_{1T}^{\perp(1)d} < g_1^d \qquad \Rightarrow$ **prediction**: $|\boldsymbol{h_1^q}| > |\boldsymbol{g_1^q}|$ in any case

(Question: at which scale? In bag model, Barbara et al model scale very low. But Petr Zavada's model refers to a high scale.)

Prediction presently at variance with extraction by Anselmino et al, but we are at an early stage of art. Will see!

6. How many such model relations exist?

Answer in bag model: altogether 9 linear relations!

reason is:

in any quark model there are: (6 twist-2) + (8 twist-3) = (14 T-even TMDs)

in bag model the correlator has (5 linearly independent amplitudes A_i)

$\Rightarrow$ (14 TMDs) − (5 Amplitudes) = (9 relations)

Avakian, Efremov, PS, Yuan, forthcoming

In other model there could be fewer or more relations. **Depends on model!**

Suspicion: counting could be valid in a larger class of models.

What is physical reason for **number 5** (independent structures in bag)?
Not clear at present, but the results may stimulate. Let us have a look.

Bag model

3 non-interacting quarks inside a cavity, s- and p-wave
Let us skip technicalities. Only one important detail:

We use SU(6) spin-flavour-symmetry:
- unpolarized functions $f_1^q = N_q\, f_1$, etc. with $N_u = 2$, $N_d = 1$
- & polarized functions $g_1^q = P_q\, g_1$, etc. with $P_u = \frac{4}{3}$, $P_d = -\frac{1}{3}$

Let us introduce the 'dilution factor' $D^q = \dfrac{P_q}{N_q}$

First 3 relations: 'weak' connect unpolarized and polarized TMDs

$$\begin{aligned}
&(I) && \mathcal{D}^q\, f_1^q(x,k_\perp) + g_1^q(x,k_\perp) = 2h_1^q(x,k_\perp) && \text{Jaffe, Ji, 1991} \\
&(II) && \mathcal{D}^q\, e^q(x,k_\perp) + h_L^q(x,k_\perp) = 2g_T^q(x,k_\perp) && \text{Signal, 1995} \\
&(III) && \mathcal{D}^q\, f^{\perp q}(x,k_\perp) = h_T^{\perp q}(x,k_\perp) && \text{new!}
\end{aligned} \quad (1)$$

'weak' = valid only in restricted class of models (bag, Barbara's model), but not in spectator model which has also SU(6) Jakob et al (only when $m_a \to m_s$), not in Zavada model

Next 3 relations: connect two polarized TMDs

$$(IV) \quad g_{1T}^{\perp q}(x,k_\perp) = -\,h_{1L}^{\perp q}(x,k_\perp) \qquad \text{Jakob et al 1997}$$

$$(V) \quad g_{T}^{\perp q}(x,k_\perp) = -\,h_{1T}^{\perp q}(x,k_\perp) \qquad \text{new!}$$

$$(VI) \quad g_{L}^{\perp q}(x,k_\perp) = -\,h_{T}^{q}(x,k_\perp) \qquad \text{new!}$$

Last 3 relations: connect three polarized TMDs

$$(VII) \quad g_{1}^{q}(x,k_\perp) - h_{1}^{q}(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp) \qquad \text{pretzelosity-relation}$$

$$(VIII) \quad g_{T}^{q}(x,k_\perp) - h_{L}^{q}(x,k_\perp) = h_{1T}^{\perp(1)q}(x,k_\perp) \qquad \text{new!}$$

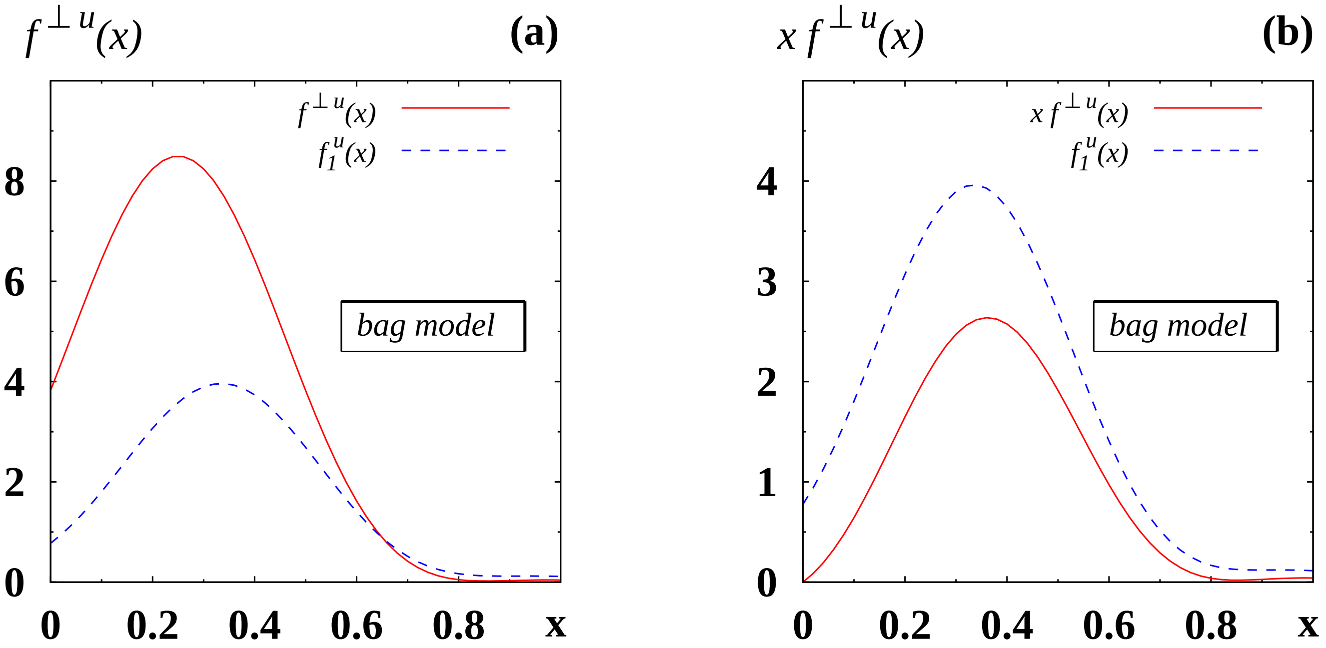
$$(IX) \quad h_{T}^{q}(x,k_\perp) - h_{T}^{\perp q}(x,k_\perp) = h_{1L}^{\perp q}(x,k_\perp) \qquad \text{new!}$$

- concerning 'new!': all 6 relations valid in Jakob et al 1997!

- stronger than first 3 (no dilution factor, valid in larger class of models!) but (IV-VIII) connect chirally even and chirally odd (different evolution) maybe (IX) strongest as only chirally odd?

Test: WW approx.

twist-3 $f^{\perp q}(x)$
at low scale



- exact eom-relation $xf^{\perp q}(x) = x\tilde{f}^{\perp q}(x) + f_1^q(x)$
- approximation $xf^{\perp q}(x) \approx f_1^q(x)$ a bit rough, interesting because:
- one ingredient for intepretation $A_{UU}^{\cos\phi} \propto$ Cahn,
  $\rightarrow$ information on p_T EMC data 1986, Anselmino et al 2005

Question:
$\tilde{f}^{\perp q}(x) \neq 0$, but no gluons in bag model. So why non-zero?
If quarks were free: it would be zero. But quarks not free!
There is bag surface, in other models 'potential'!
Binding, 'mimics' gluons! Jaffe, Ji 1991

8. Warning:

After all excitement about the relations,
they hold only in no-gluon (quark) models!

- **Quark target model** (explicit gluons!) Meissner, Metz and Goeke, 2007
  none of these quark model relations holds!

  Quark target model brings us step closer to real world (there are gluons),
  though unrealistic in the sense that: no free quarks!

  $\Rightarrow$

  indication that relations not conserved under evolution
  (which we, of course, do not expect)

- **QCD**
  Of course, no relations at all!
  (all amplitudes A_i, B_i independent)

Conclusions

- exist no exact relations among TMDs
- possibly reasonable (WW-type) approximations, need to test
- for sure relations ('LIRs') in quark models
- further model relations in many (not all) quark models,
- which yield robust predictions (supported by many models).
- None of these model relations is exact/fundamental. Quark models!
- Quark models have limitations, but successfully catch key features.
- Also in the case of TMDs? **Data will show. Exciting future! Will see!**

Thank you !