Scale evolution of kt-distributions

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Personal path to TMD's

- During the analysis of Semi-inclusive π^+ production at CLAS ('05)

CLAS Collaboration (M. Osipenko et al.) e-Print: arXiv:0809.1153

we faced the problem to give a QCD description of SIDIS five-fold differential cross-sections:

$$\frac{d^5\sigma}{dx dQ^2 dz_h d^2\mathbf{p}_\perp}$$

- We re-discovered k_t-dependent time-like evolution equations first introduced in the '80 by

A. Bassetto, M. Ciafaloni, G. Marchesini *Nucl. Phys.* **B163**, 477 (1980)

and successfull used for determining NLL soft gluon resummation coefficients.

J. Kodaira, L. Trentadue, *Phys. Lett.* **B112**, 66 (1982).

- We therefore generalize them to space-like evolution and also to the target fragmentation region in terms of k_t-dependent fracture functions.

F.A.C, L. Trentadue, *Phys. Lett.* **B636**, 310 (2006).

♦ The initial goal of giving a fully-differential description of CLAS cross-sections was achived.
In the mean-time we realized that a large community was involved in this research field...

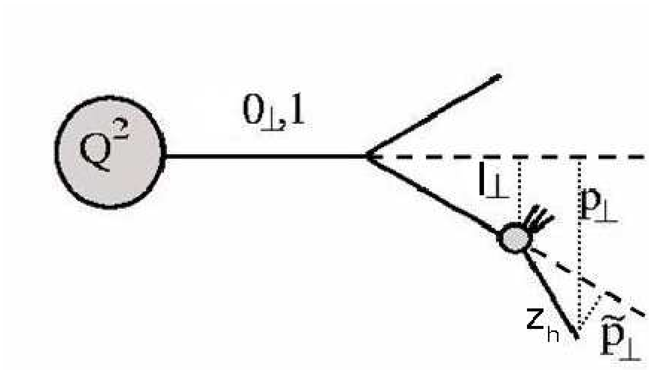
BCM equations

The originally proposed k_t-dependent evolution equations read:

$$Q^2 \frac{\partial \mathcal{D}_i^h(z_h, Q^2, \boldsymbol{p}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{du}{u} P_{ij}(u, \alpha_s(Q^2)) \cdot$$

$$\cdot \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \, \delta(\, u(1-u)\; Q^2 - l_\perp^2\,)\, \mathcal{D}_j^h\Big(\frac{z_h}{u}, Q^2, \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{l}_\perp\Big)$$

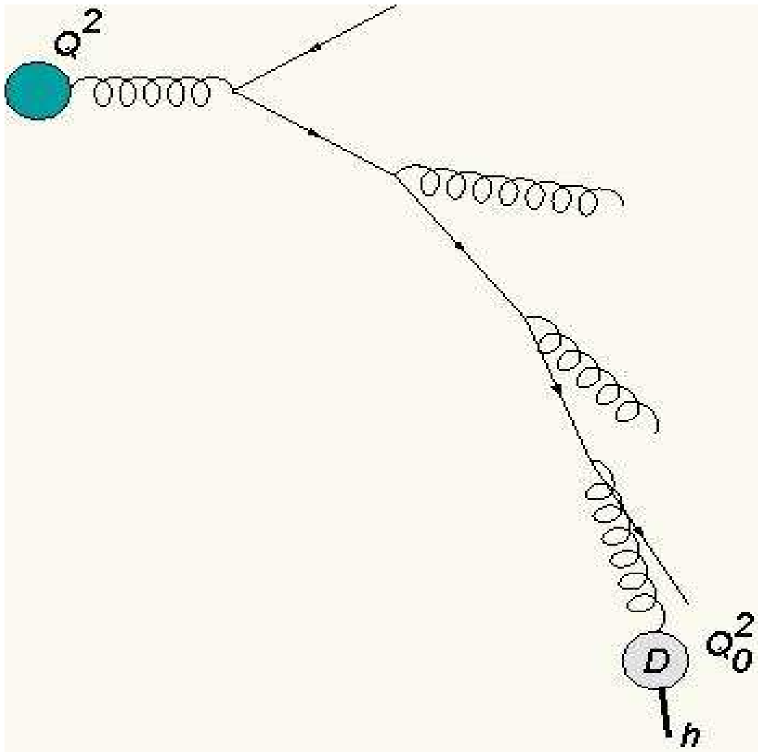
- light-like gauge;
- transverse boost : $\widetilde{\boldsymbol{p}}_\perp = \boldsymbol{p}_\perp - \frac{z_h}{u}\boldsymbol{l}_\perp$;
- P_{ij} ordinary AP splitting functions;
- $\int d^2\mathbf{p}_\perp \mathcal{D}_i^h(z, Q^2, \mathbf{p}_\perp) = D_i^h(z, Q^2)$
- The δ-function imposes energy-momentum conservation at the vertex



A. Bassetto, M. Ciafaloni, G. Marchesini, *Nucl. Phys.* **B163**, 477 (1980)

Logic of k_t-evolution

- In the collinear limit, at each branching, the active parton acquires a small relative $\widetilde{\mathbf{p}}_\perp$
- Collinear emissions give however leading logarithmic corrections
- Such contributions are resummed by k_t-evolution equations
- The radiative process in the collinear limit generates therefore an appreciable $\mathbf{p}_\perp$ which adds to the non-perturbative fragmentation one
- Notably even logs due to soft gluon emission can be taken into account in the formalism



Much like as in `HERWIG` Monte Carlo:

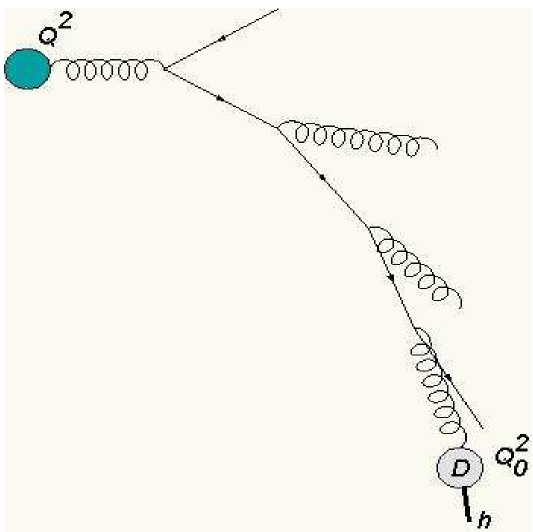
S.Gieseke, P.Stephens, B.Webber, *JHEP* **0312:045** (2003)

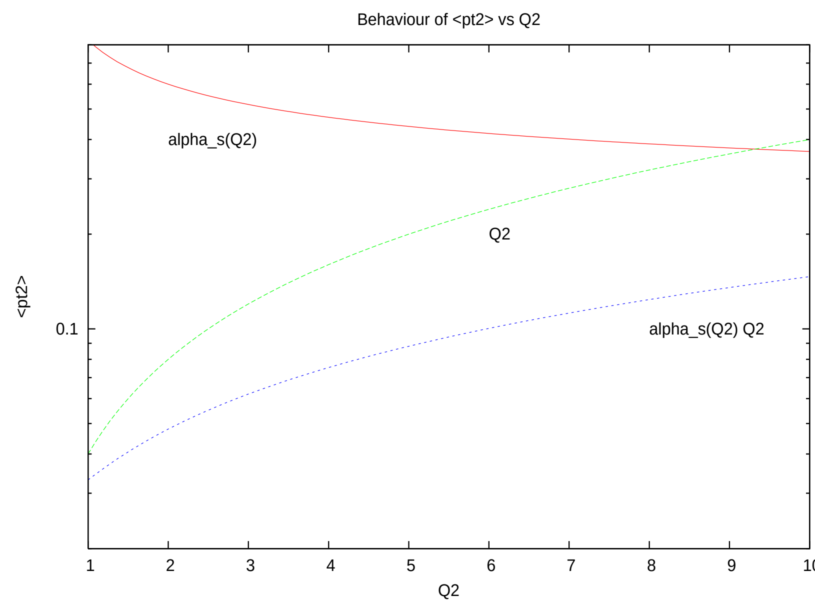
Expectations

A quantity sensitive to dynamics is the average $p_\perp^2$ of final state hadrons: (jet broadening):

$$\langle p_\perp^2(z,Q^2)\rangle = \frac{\pi \int_0^\infty dp_\perp^2\, p_\perp^2\, D_i^h(z,Q^2,\mathbf{p}_\perp)}{D_i^h(z,Q^2)}$$

- Time-like cascade by an off-shell parton (for example in e^+e^-) of virtuality Q^2:

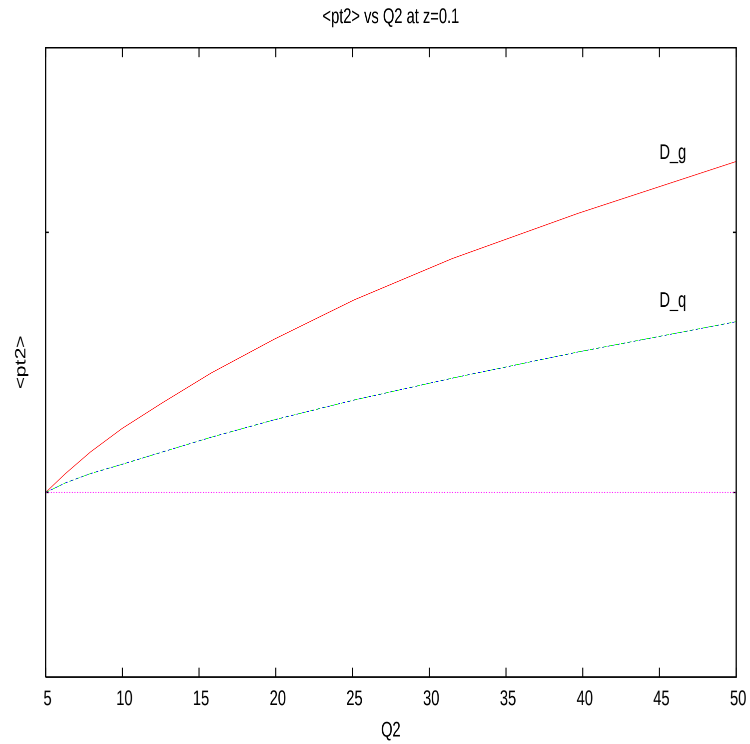




▶ $\langle p_\perp^2(z,Q^2)\rangle \propto \alpha_s(Q^2)Q^2$ due to competing effects : phase space (Q^2) vs running coupling ($\alpha_s(Q^2)$).

Numerical Solutions

- Solution : finite difference method on a grid $[n_z \otimes n_q \otimes n_{pt}] = 100 \times 10 \times 100$;
- $5 < Q^2 < 50$ GeV^2, $0.1 < z < 1$, $0 < p_\perp^2 < 50$ GeV^2, $n_f = 3$, HQ in β-function;



- intial conditions $D_i^h(z, Q_0^2)$, $Q_0^2 = 5$ GeV^2;
- $h = h^+ + h^-$, from LO Kretzer set(*);
- $D_i^h(z, Q_0^2, \mathbf{p}_\perp) = \frac{1}{\pi \langle p^2_{\perp,i} \rangle} D_i^h(z, Q_0^2) \exp^{-p_\perp^2 / \langle p^2_{\perp,i} \rangle}$
- $\langle p^2_{\perp,q} \rangle = \langle p^2_{\perp,\bar{q}} \rangle = \langle p^2_{\perp,g} \rangle = 0.2$ GeV^2,

$\langle p_\perp^2(z = 0.1, Q^2) \rangle$ obtained by simulations is in qualitative agreement with expectations.

(*) S. Kretzer, *Phys. Rev.* **D62**, 054001 (2000)

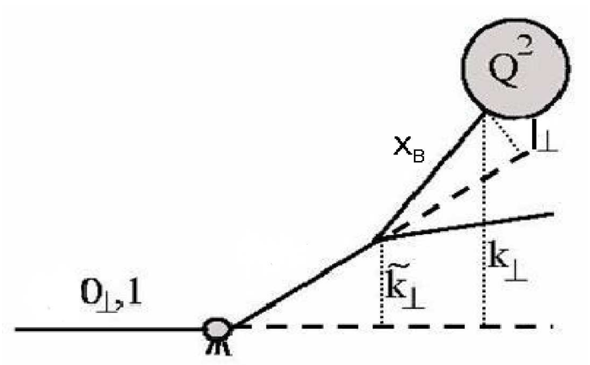
Space-like evolution

Space-like evolution is obtained "rotating" in phase-space the BCM equations:

$$Q^2 \frac{\partial \mathcal{F}_P^i(x_B, Q^2, \boldsymbol{k}_\perp)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} P_{ji}(u, \alpha_s(Q^2)) \cdot$$

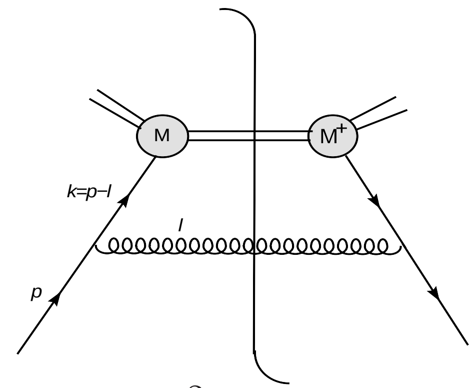
$$\cdot \int \frac{d^2 \boldsymbol{l}_\perp}{\pi} \, \delta(\, (1-u)Q^2 - l_\perp^2\,) \, \mathcal{F}_P^j\Big(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{l}_\perp}{u}\Big) \, .$$

- transverse boost : $\widetilde{\boldsymbol{k}}_\perp = (\boldsymbol{k}_\perp - \boldsymbol{l}_\perp)/u$;
- P_{ji} ordinary AP splitting functions;
- $\int d^2\mathbf{k}_\perp \mathcal{F}_P^i(x, Q^2, \mathbf{k}_\perp) = F_P^i(x, Q^2)$
- The δ-function imposes energy-momentum conservation at the vertex



F.A.C, L. Trentadue, *Phys. Lett.* **B636**, 310 (2006).

Sketch of the derivation (1)



- Radiative corrections in the collinear limit with transverse gluon.
- Sudakov decompositon : $l = (1-u)p + \boldsymbol{l}_\perp + \beta\eta$
- The phase space for the emission of this extra gluon reads:

$$d\Phi = \frac{d^4 l}{(2\pi)^4} 2\pi\,\delta(l^2) = \frac{dl_0 d^2\boldsymbol{l}_\perp dl_3}{(2\pi)^3}\delta(l^2) = \frac{1}{2u}\frac{dk^2 d^2\boldsymbol{l}_\perp du}{(2\pi)^3}\,\delta\Big[k^2\Big(1-\frac{1}{u}\Big) - \frac{l_\perp^2}{u}\Big]\,.$$

- The one-gluon emission amplitude squared in the collinear limit is

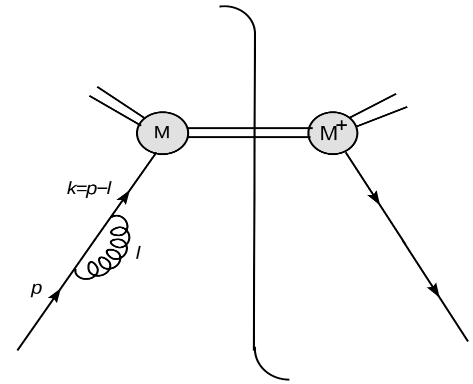
$$|M|^2_{coll} \simeq g_s^2 \widehat{P}(u)\frac{1}{l_\perp^2}\,,$$

where $\widehat{P}(u) = C_F(1+u^2)/(1-u)$ is the unregularized $q \to q(g)$ splitting function and $k^2 < 0$.

- Combining these results one gets

$$d\sigma^{(1,R)} \simeq |M|^2_{coll}\,d\Phi = \frac{\alpha_s}{2\pi}\,\widehat{P}(u)\,du\,\frac{d^2\boldsymbol{l}_\perp}{\pi}\frac{dk^2}{k^2}\delta\Big((1-u)k^2 + l_\perp^2\Big)\,.$$

Sketch of the derivation (2)



- Virtual corrections obtained expanding at one loop the Sudakov form factor.

$$\frac{d\sigma^{(1,V)}}{dud^2\boldsymbol{p}_\perp dk^2} = -\delta(1-u)\delta^{(2)}(\boldsymbol{p}_\perp - \boldsymbol{k}_\perp)\frac{\alpha_s}{2\pi}\frac{1}{k^2}\int_0^1 du\widehat{P}(u)\,.$$

- Relabeling $\boldsymbol{p}_\perp - \boldsymbol{k}_\perp = \boldsymbol{l}_\perp$ and using $\delta^{(2)}(\boldsymbol{l}_\perp) = \delta(l_\perp^2)/\pi$,

$$d\sigma^{(1,V)} = -\delta(1-u)du\frac{\delta(l_\perp^2)}{\pi}d^2\boldsymbol{l}_\perp\frac{\alpha_s}{2\pi}\frac{dk^2}{k^2}\int_0^1 du\widehat{P}(u)$$

- Combining real and virtual corrections one gets

$$d\sigma^{(1)} = d\sigma^{(1,R)} + d\sigma^{(1,V)} = \frac{\alpha_s}{2\pi}\,\widehat{P}(u)\,du\,\frac{d^2\boldsymbol{l}_\perp}{\pi}\frac{dk^2}{k^2}\delta\Big((1-u)k^2 + l_\perp^2\Big) +$$

$$-\,\delta(1-u)du\frac{\delta(l_\perp^2)}{\pi}d^2\boldsymbol{l}_\perp\frac{\alpha_s}{2\pi}\frac{dk^2}{k^2}\int_0^1 du\widehat{P}(u) = \frac{\alpha_s}{2\pi}\,P(u)\,du\,\frac{d^2\boldsymbol{l}_\perp}{\pi}\frac{dk^2}{k^2}\delta\Big((1-u)k^2 + l_\perp^2\Big)$$

- $[f(x)]_+ = f(x) - \delta(1-x)\int_0^1 dx\,f(x)$;
- IR singularity ($u \to 1$) ok but collinear one ($k^2 \to 0$) is still present. Perform Mass Factorization

Observation

One loop calculation in coordinate(*) space ($y = (0, y^-, y_\perp)$) gives for a quark distribution $\tilde{f}(y)$:

$$\tilde{f}(y) = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} p^+ \int_0^1 dv \frac{v}{1-v} \Big\{ [e^{ip\cdot yv} - e^{ip\cdot y}] \Gamma\Big(2-\frac{d}{2}\Big)\Big(\frac{\mu^2}{\rho^2}\Big)^{2-d/2} + e^{ip\cdot yv} 4^{d/2-2} \Gamma\Big(\frac{d}{2}-2\Big)(-y^2\mu^2)^{2-d/2} + \mathcal{O}(y^2)^k \Big\}$$

- space-time dimension $d = 4 - 2\epsilon$, n light-like, μ the dimensional regularization scale, $\rho^2 = (1-v)m^2 + v\lambda^2$, with m and λ the quark and gluon mass respectively.

- Long-distance contributions are proportional to $\ln(\mu^2/\rho^2)$ and $v \to 1$ singularities do cancel;

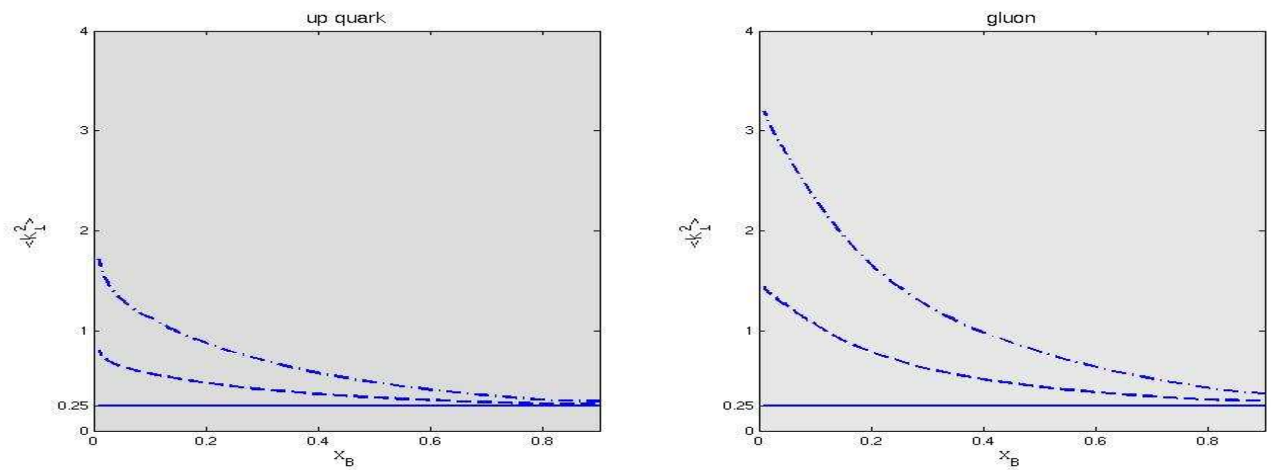
- Short-distance contributions are proportional to $\ln(y^2\mu^2)$ and $v \to 1$ singularities do not cancel.

F. Hautmann, *Phys. Lett.* **B655**,26 (2007)

Now if one imagine to apply renormalization group technique and take a ρ^2-derivative, the evolution of unintegrated distributions seems to be driven by ordinary AP splitting functions.

Features of space-like evolution

- $\langle \boldsymbol{k}_\perp^2 \rangle$ vs Q^2 at x=0.1; $\langle \boldsymbol{k}^2_{\perp(0)q,g} \rangle = 0.25$ GeV^2 at $Q_0^2 = 5GeV^2$. GRV94 PDF set
- Evolution of quark and gluon from $Q_0^2 = 5GeV^2$ to $Q^2 = 50GeV^2$.



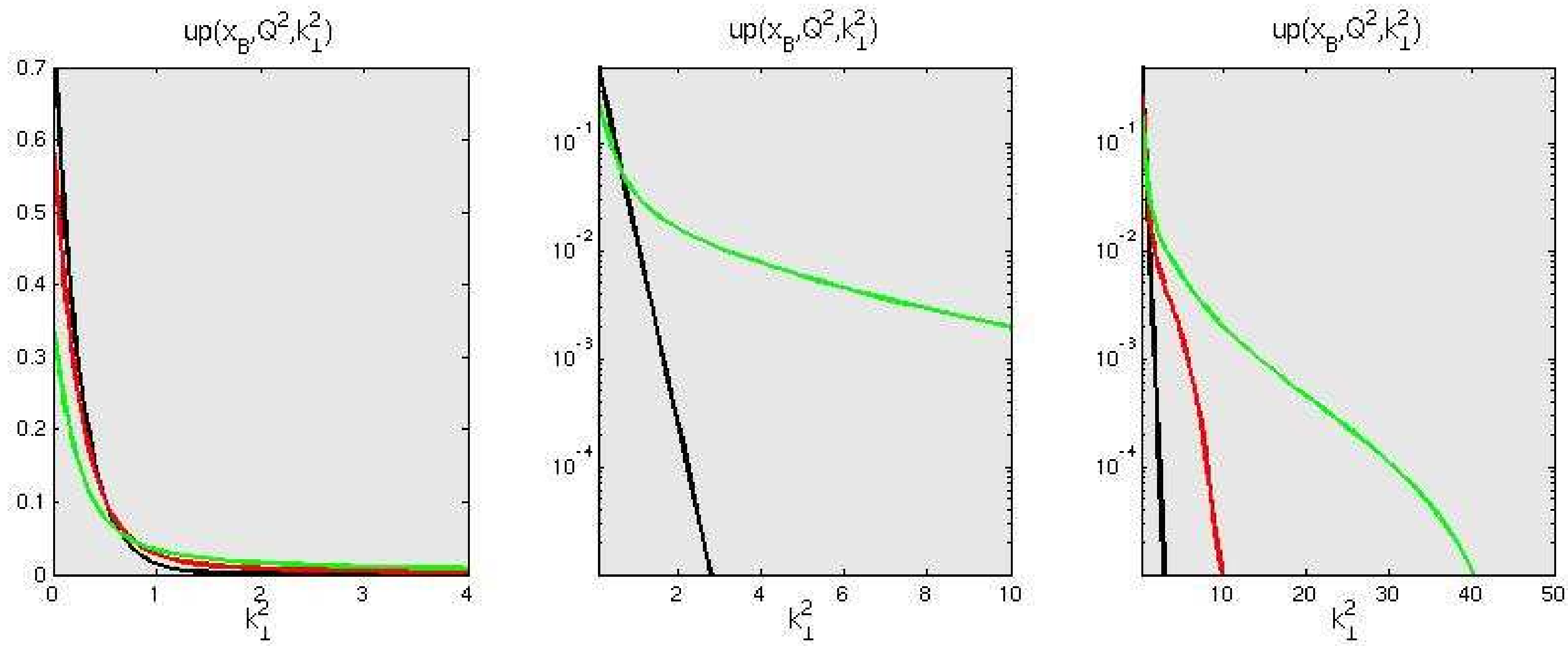
- The evolution generates an average transverse momentum which increases with decreasing x, due to more available phase space for large k_t-emissions.

$$\langle \boldsymbol{k}^2_\perp(x) \rangle = \langle \boldsymbol{k}^2_{\perp(0)} \rangle x^\gamma, \gamma < 0$$

- In the soft ($x \longrightarrow 1$) limit one may overserve that $\langle \boldsymbol{k}_\perp^2 \rangle \longrightarrow \langle \boldsymbol{k}^2_{\perp(0)} \rangle$ for all Q^2.

Features of space-like evolution

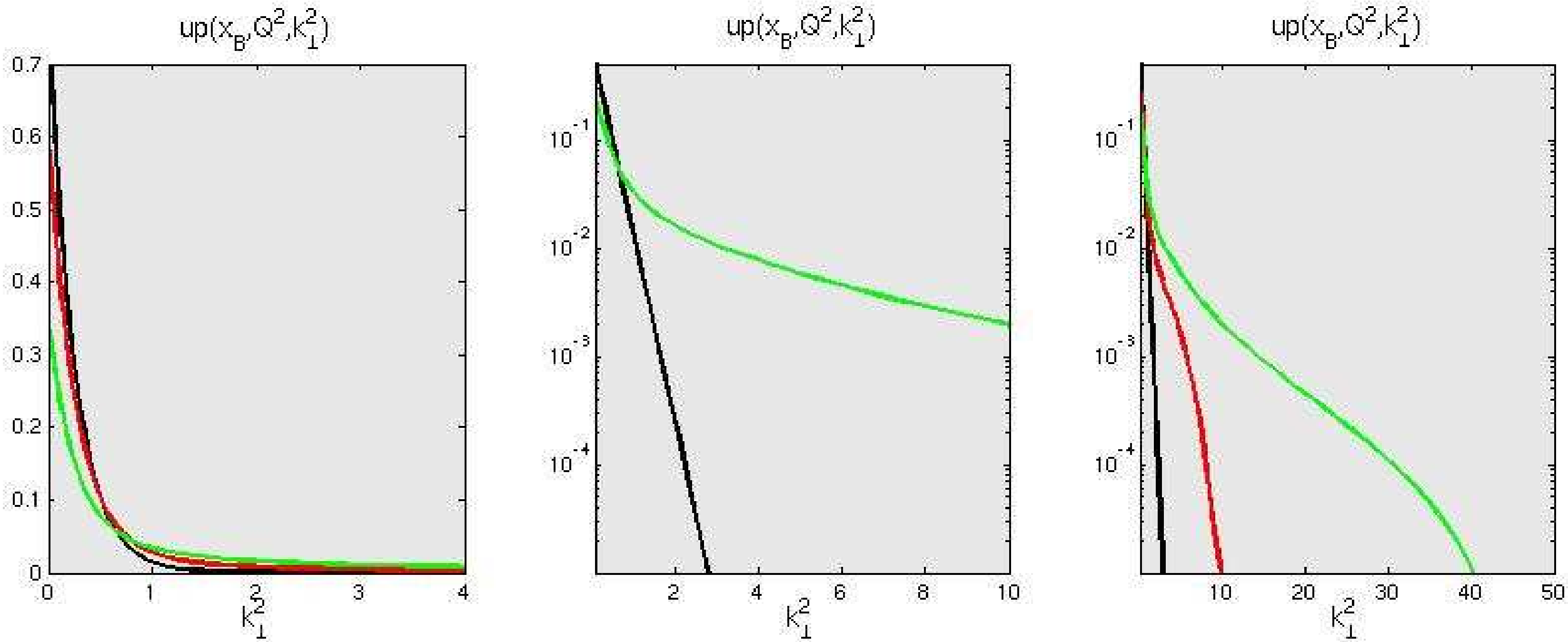
- Evolution from $Q_0^2 = 5GeV^2$ to $Q^2 = 50GeV^2$ of up-quark distribution vs $k_\perp^2$ at fixed $x = 0.1$.



- left panel: smooth broadening of TMD distributions in transverse plane.

Features of space-like evolution

- Evolution from $Q_0^2 = 5GeV^2$ to $Q^2 = 50GeV^2$ of up-quark distribution vs $k_\perp^2$ at fixed $x = 0.1$.

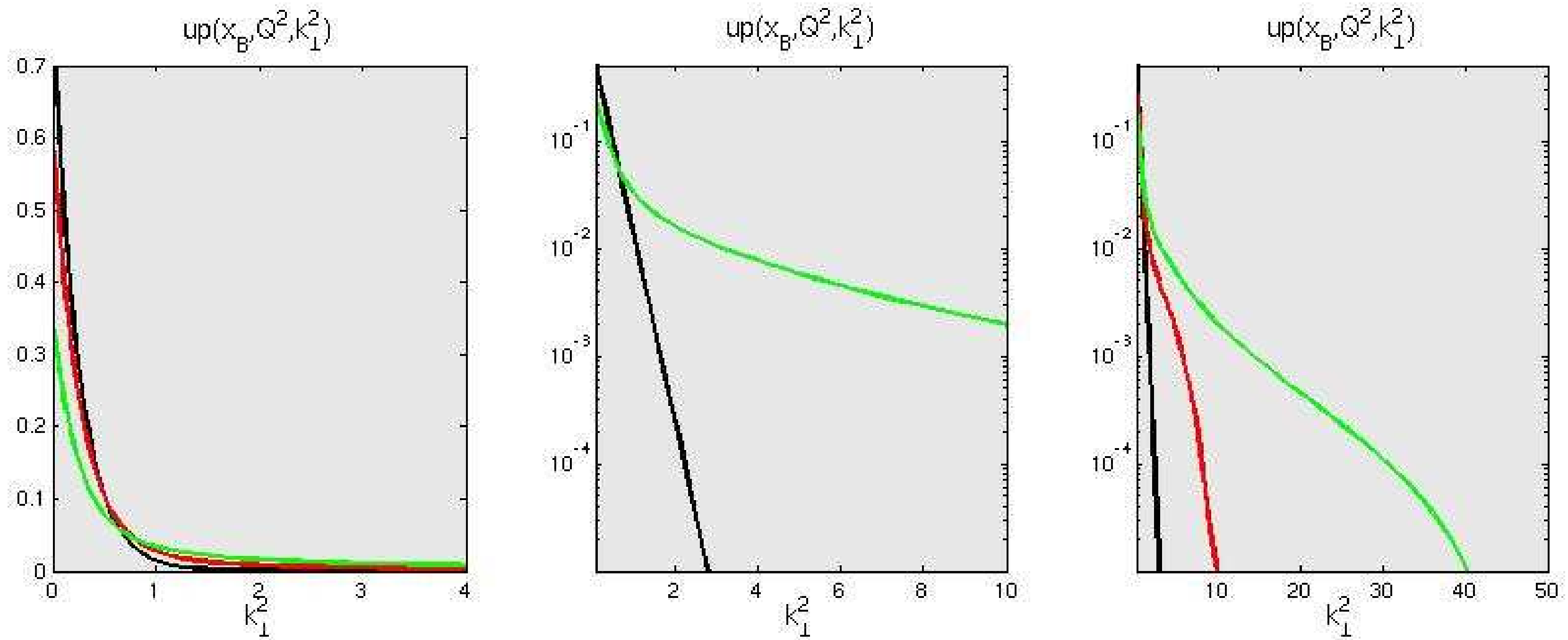


- middle panel: gaussian not preserved under evolution;

- The tail of the evolved distributions behaves as $a/((k_\perp^2)^b + c)$ with a,b,c depending on x and Q^2

Features of space-like evolution

- Evolution from $Q_0^2 = 5GeV^2$ to $Q^2 = 50GeV^2$ of up-quark distribution vs $k_\perp^2$ at fixed $x = 0.1$.



- With this setup, the very high $k_\perp^2$ region is not still populated.
- According to leading log : $k_{\perp,max}^2 \leq Q^2$

Comparison with DIS data

- The cross-sections for producing charged hadrons h with transverse momentum $\boldsymbol{P}_{h,\perp}$:

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^{h=h^+ + h^-}}{d\boldsymbol{P}^2_{h,\perp}}$$

is constructed starting from the expression for the semi-inclusive structure functions H_2:

$$H_2(x_B, z_h, \boldsymbol{P}_{h,\perp}, Q^2) = \sum_{i=q,\bar{q}} \int d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_\perp \delta^{(2)}(z_h \boldsymbol{k}_\perp + \boldsymbol{p}_\perp - \boldsymbol{P}_{h,\perp}) \mathcal{F}^i_P(x_B, Q^2, \boldsymbol{k}_\perp) \mathcal{D}^h_i(z_h, Q^2, \boldsymbol{p}_\perp)$$

- Such factorized form is in spirit of factorization theorem at low $\boldsymbol{P}_{h,\perp}$

Ji & al., *Phys. Rev.* **D71**, 034005 (2005)

- Values of intrinsic transverse momentum are set to $\langle k^2_\perp \rangle$=0.25 Ge$V^2$, $\langle p^2_\perp \rangle$=0.20 Ge$V^2$

Anselmino & al., *Phys. Rev.* **D71**, 074006 (2005)

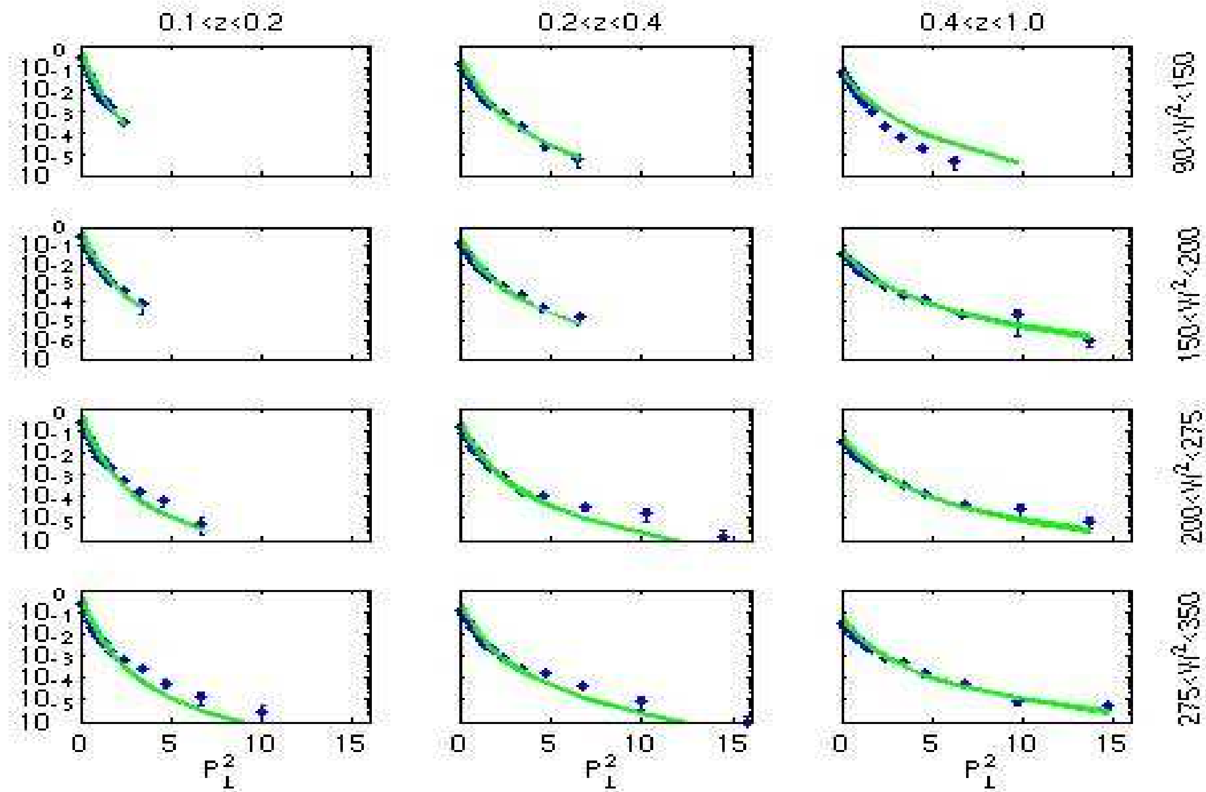
- The structure functions is then integrated over experimental kinematics and normalized to σ_{tot}

Comparison with DIS data

μP DIS at 280 GeV

$$\frac{1}{\sigma_{tot}} \frac{d\sigma^{h=h^{+}+h^{-}}}{d\boldsymbol{P}^2_{h,\perp}}$$

- No matching required at intermediate $\boldsymbol{P}_{h,\perp}$, but what is the size of NLO?



EMC, *Z. Phys.* **C52**,361 (1991);
F.C., L. Trentadue, *Phys. Lett.* **B660**, 43 (2008).

New Developments

- DY and Z-boson p_t-spectrum with an eye to LHC (Higgs via k_t-gluons?)

- Spin physic program : Sivers Evolution

Comparison with DY data

- The differential cross-sections for the production of a Drell-Yan pair of invariant mass M^2, at rapidity y, and transverse momentum $\vec{p}_T$ is

$$\frac{d^4\sigma}{dM^2 dy d^2\vec{p}_T} = \frac{\sigma_0}{N_C\ s} \sum_q e_q^2 \int \frac{d^2 \vec{k_{T1}}}{\pi} \frac{\int d^2\vec{k}_{T2}}{\pi} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{p}_T) \cdot$$

$$\cdot \left[\mathcal{F}_q(x_1, M^2, \vec{k}_{T1}) \mathcal{F}_{\bar{q}}(x_2, M^2, \vec{k}_{T2}) + (1 \leftrightarrow 2) \right]$$

- Parton momentum fractions are evaluated through

$$x_{1,2} = m_T/\sqrt{s}\exp(\pm y)$$

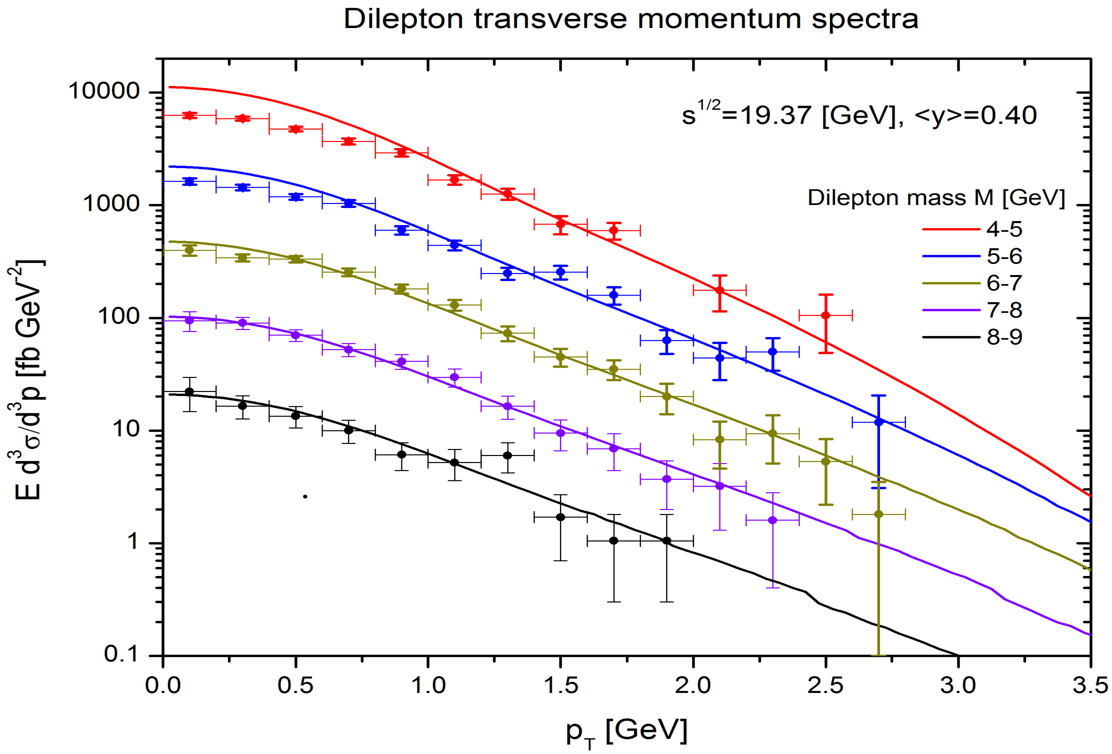
with $m_T = \sqrt{M^2 + p_t^2}$, s the hadronic collision energy squared and $\sigma_0 = \frac{4\pi\alpha_{em}^2}{3M^2}$.

- Parton distributions are evaluated at $\mu^2 = M^2$.

J. Kwiecinski, A. Szczurek, *Nucl. Phys.* **B680**, 164 (2004).

Comparison with low energy DY data

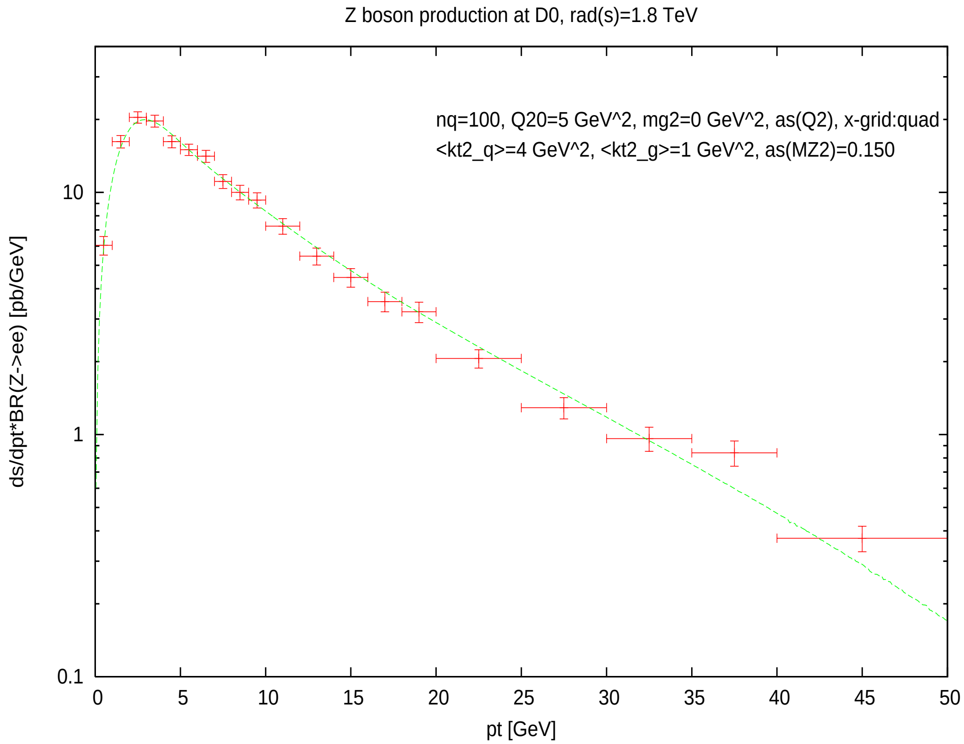
- Dilepton production in proton-nucleus at $\sqrt{s} = 19.37$ GeV
- $\langle k^2_{\perp,0,q}\rangle$=0.5 Ge$V^2$
- $\langle k^2_{\perp,0,g}\rangle$=0.5 Ge$V^2$



A.S. Ito et al., *Phys .Rev.* **D23**, 604 (1981).

Z production at Tevatron

- Almost insensitive to gluon intrinsic transverse momentum;
- $\langle k^2_{\perp,0,q} \rangle$=4 GeV^2, $\langle k^2_{\perp,0,g} \rangle$=1 GeV^2
- Large values of the LL running coupling $\alpha_s(M_Z^2) = 0.150$;
- Full vertex kinematics implemented



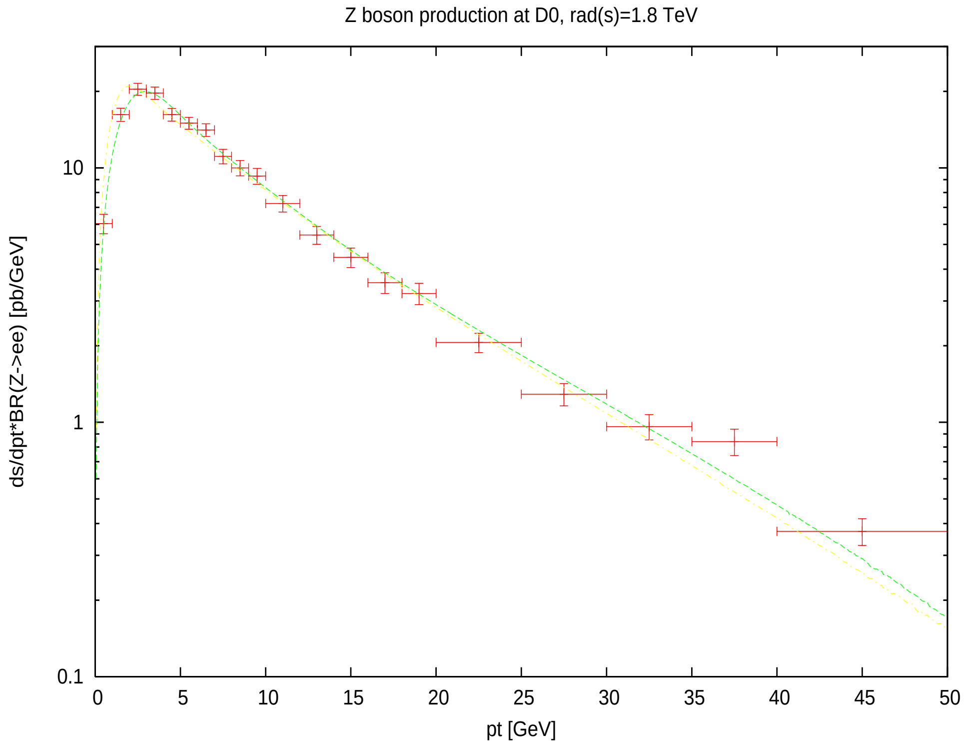
Abbott & al. (D0 Collab.), hep-ex/99090290

Main features observed:

- Changes in $\langle k^2_{\perp,0,q} \rangle$ shift the position of the maximum;
- Changes in $\alpha_s(M_Z^2)$ influence the height of the maximum;

Z production at Tevatron

- Anomalous intrinsic transverse momentum;
- $\langle k^2_{\perp,0,q}\rangle$=0.25 GeV^2, $\langle k^2_{\perp,0,g}\rangle$=0.25 GeV^2
- $\alpha_s(M_Z^2) = 0.150$;
- Full vertex kinematics implemented



Abbott & al. (D0 Collab.), hep-ex/99090290

- x-dependent gaussian input (in yellow):

$$\mathcal{F}_i(x, Q_0^2, \boldsymbol{k}_\perp) = F_i(x, Q_0^2)\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}, \quad \langle k_\perp^2\rangle(x) = \langle k_{\perp,0}^2\rangle x^\gamma, \quad \gamma = -0.5$$

Soft Gluon resummation

- SG Resummation is obtained rescaling the argument of α_S:

$$\begin{aligned}\alpha_S(Q^2) \to \alpha_S(p_\perp^2) &= \alpha_S(Q^2(1-x)) \\ &\simeq \alpha_S(Q^2) - \ln(1-x)\beta_0\alpha_S^2(Q^2) + \ln^2(1-x)\beta_0^2\alpha_S^3(Q^2) + \ldots \\ &= \alpha_S(Q^2)\left[1 - \ln(1-x)\beta_0\alpha_S(Q^2) + \ln^2(1-x)\beta_0^2\alpha_S^2(Q^2) + \ldots\right] \\ &= \frac{\alpha_S(Q^2)}{1+\beta_0\alpha_S(Q^2)\ln(1-x)}\end{aligned}$$

- DGLAP evolution in the soft ($x \longrightarrow 1$) limit (f=non-singlet distribution)

$$\frac{df(x,Q^2)}{d\ln Q^2} = \int_x^1 \frac{dz}{z}\frac{C_F}{\pi}\left[\frac{\alpha_S(Q^2(1-z))}{1-z}\right]_+ f\left(\frac{x}{z}, Q^2\right)$$

- Double soft logs appearing coefficient functions can be resummed via k_t evolution by setting $\alpha_S(p_\perp^2)$.

$$C_q^{DY}(z) = C_F\left[P_{qq}(z)\ln\frac{Q^2}{\mu_F^2} + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z + \delta(1-z)\left(\frac{2\pi^2}{3} - 8\right)\right]$$

J. Kodaira, L. Trentadue, *Phys. Lett.* **B112**, 66 (1982).

Evolution of Sivers function

- Sivers function captures the correlation between parton transverse momentum $\boldsymbol{k}_\perp$ and the spin $\boldsymbol{S}$ of the parent hadron:

$$f_{q/p^\uparrow}(x, Q^2, \boldsymbol{k}_\perp) - f_{q/p^\uparrow}(x, Q^2, -\boldsymbol{k}_\perp) = \Delta^N f_{q/p^\uparrow}(x, Q^2, k_\perp^2) \frac{(\boldsymbol{P} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \, .$$

- A partonic interpretation can be assigned to the left hand side. Apply evolution equations
- One may construct the difference by using unpolarized evolution with ϕ-modulated ansatz
- Or, by using the evolution equation separately for $f_{q/p^\uparrow}(x, Q^2, \boldsymbol{k}_\perp)$ and $f_{q/p^\uparrow}(x, Q^2, -\boldsymbol{k}_\perp)$

$$Q^2 \, \frac{\partial [f^i_{q/p^\uparrow}(x, Q^2, \boldsymbol{k}_\perp) - f^i_{q/p^\uparrow}(x, Q^2, -\boldsymbol{k}_\perp)]}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \, P_{ji}(u, \alpha_s(Q^2)) \cdot$$

$$\cdot \int \frac{d^2 \boldsymbol{q}_\perp}{\pi} \, \delta\big((1-u)Q^2 - q_\perp^2\big) \Big[f^j_{q/p^\uparrow}\Big(\frac{x_B}{u}, Q^2, \frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}\Big) - f^j_{q/p^\uparrow}\Big(\frac{x_B}{u}, Q^2, -\frac{\boldsymbol{k}_\perp - \boldsymbol{q}_\perp}{u}\Big) \Big]$$

- So, if a correlation is present at some initial scale Q_0^2, it will be possible to propagate it to higher scales.

Evolution of Sivers function

- However a more compact form can be obtained:

  working out the tensor structure and introducing the function $g_S^i(x_B, Q^2, k_\perp^2) \equiv k_\perp^2 \Delta^N f_{q/p^\uparrow}(x, Q^2, k_\perp^2)$

$$Q^2 \frac{\partial g_S^i(x_B, Q^2, k_\perp^2)}{\partial Q^2} =$$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^2} P_{ji}(u, \alpha_s(Q^2)) \int \frac{d^2 \boldsymbol{q}_\perp}{2\pi} \delta\big((1-u)Q^2 - q_\perp^2\big) \left(1 + \frac{k_\perp^2 - q_\perp^2}{(\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2}\right) g_S^j\Big(\frac{x_B}{u}, Q^2, \frac{(\boldsymbol{k}_\perp - \boldsymbol{q}_\perp)^2}{u^2}\Big).$$

Oleg Teryaev, F.C.

- Details of the simulations: Unpolarized code with input parametrizations from Ref.

  Anselmino & al., *Eur. Phys. J.* **A39**, 89 (2009).

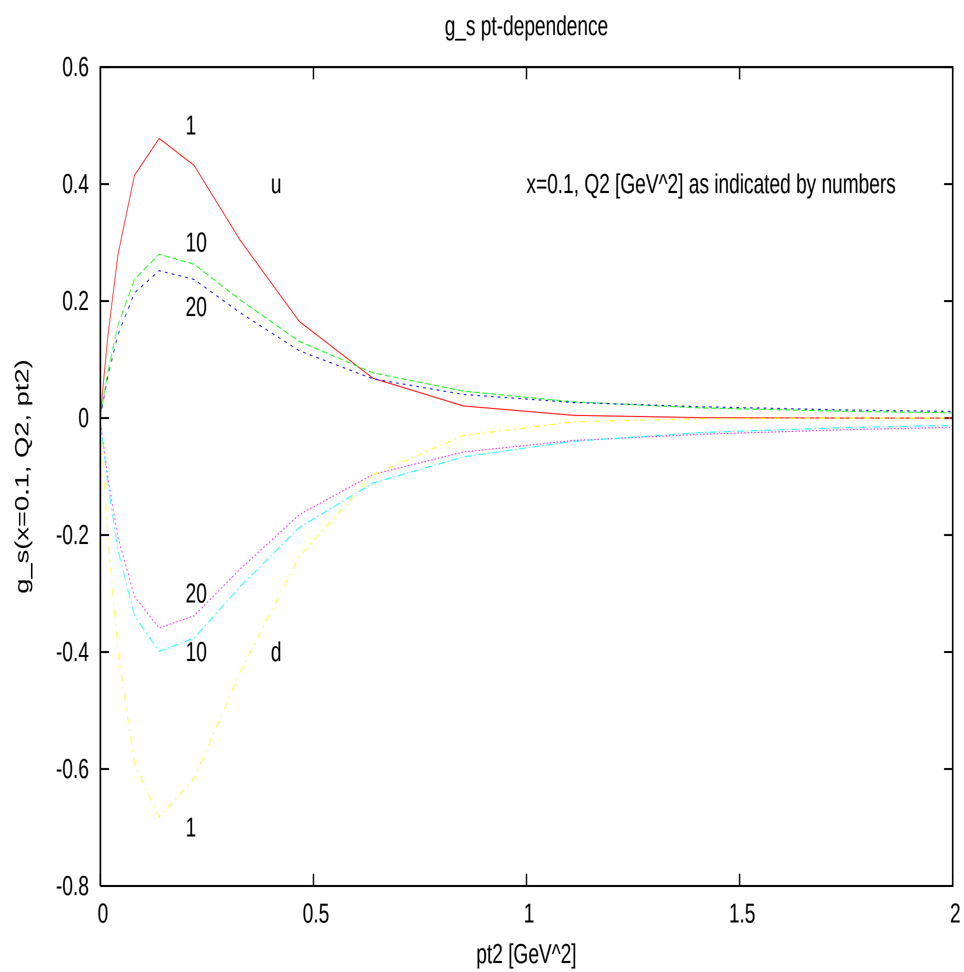
- Light flavours only, $\alpha(M_Z^2) = 0.120$, $\langle \mathbf{p}_\perp^2 \rangle = 0.25$ GeV^2 for all flavours.

- Gluon g_s distributions is set to zero at $Q_0^2 = 1$ GeV^2.

- $1 < Q^2 < 20$ GeV^2, $0 < \mathrm{p}_\perp^2 < 10$ GeV^2, $0.01 < x_B < 1$.

Evolution of Sivers function

- Evolution of u and d Sivers distrubutions at fixed $x = 0.1$ and $Q^2 = 1, 10, 20$ GeV^2
- Goal : evaluate k_t-evolution effects in a asymmetry.



Conclusions

Predictions based on the k_t-evolution equations formalism are in qualitative agreement with SIDIS and DY data. However a promising phenomenology does not implies necessarely a correct result...

Thank you!