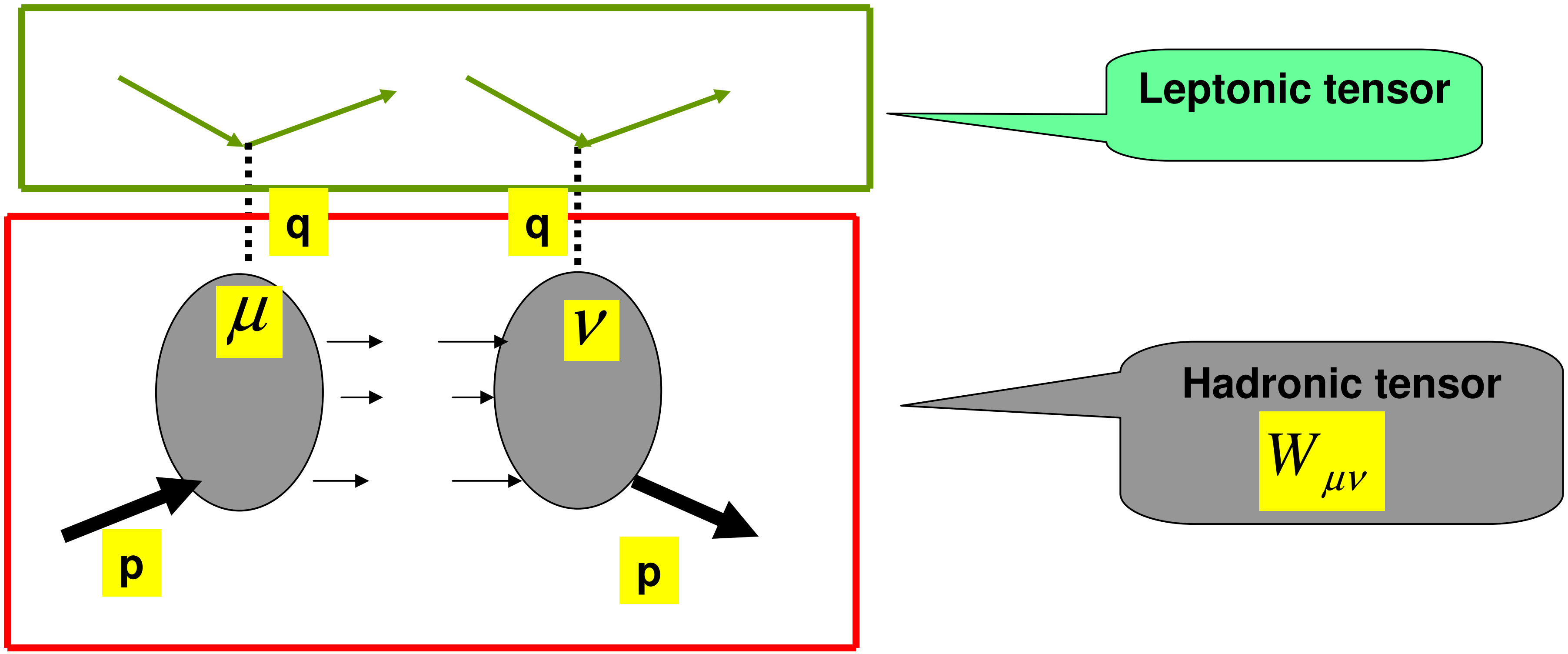
ECT, Trento 20-25 July 2009

B. Ermolaev

Evolution of our understanding Polarized DIS: from large to small x and Q^2

talk based on results obtained in collaboration with
M. Greco and S.I. Troyan



Does not depend on spin

Spin-dependent

$$W_{\mu\nu} = W_{\mu\nu}^{unpolarized}(p,q) + W_{\mu\nu}^{spin}(p,q)$$

symmetric

antisymmetric

Spin-dependent part of W is parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_\lambda \left[S_\rho g_1(x,Q^2) + \left(S_\rho - \frac{Sq}{pq} p_\rho \right) g_2(x,Q^2) \right]$$

where m, p and S are the mass, momentum and spin of the hadron ;
q is the virtual photon momentum ($Q^2 = - q^2 > 0$). Both functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$. They measure asymmetries

g_1 is related to the longitudinal spin asymmetry

$$g_1 \propto \sigma_{L\uparrow\downarrow} - \sigma_{L\uparrow\uparrow}$$

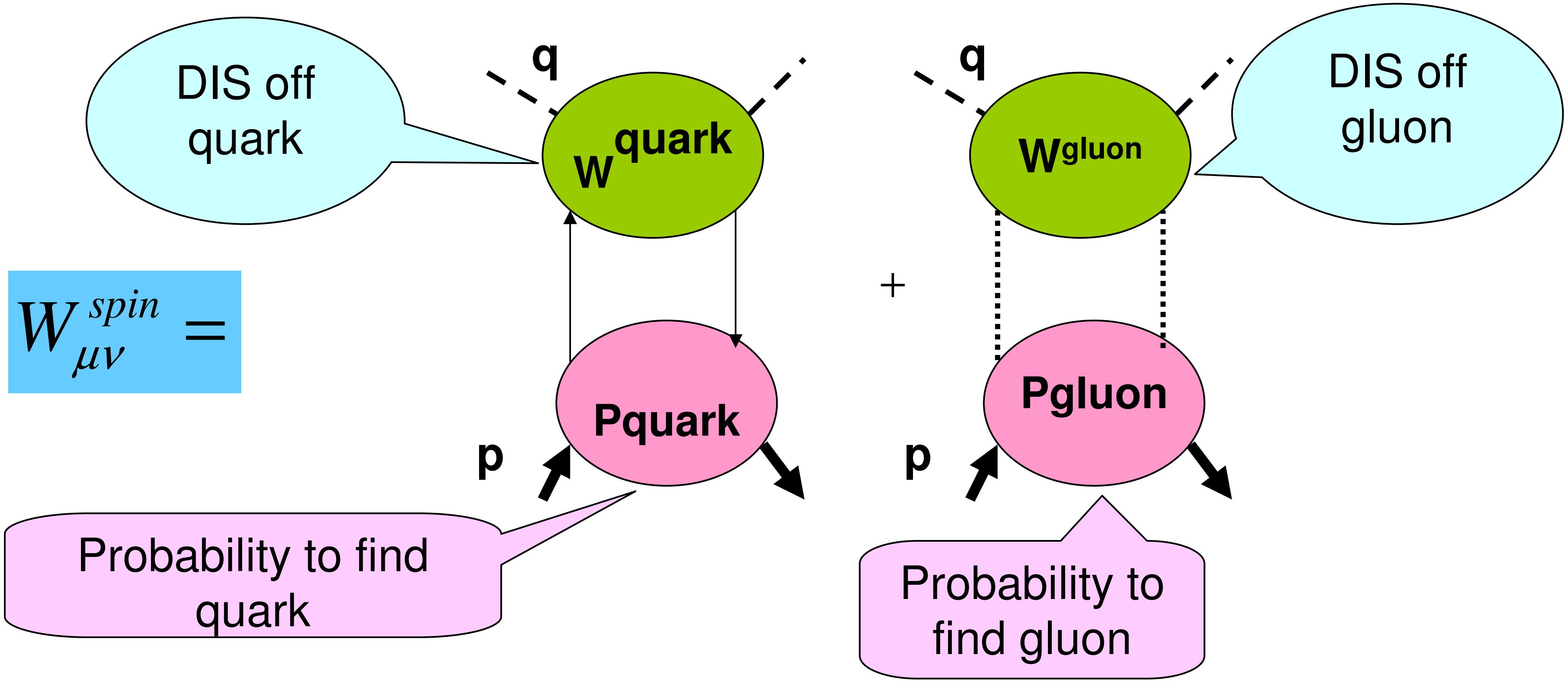
$g_1 + g_2$ is related to the transverse spin asymmetry

So, they can be positive or negative

$$g_1 + g_2 \propto \sigma_{T\uparrow\downarrow} - \sigma_{T\uparrow\uparrow}$$

FACTORISATON: $W_{\mu\nu}$ **is a convolution of the the partonic tensor and probabilities to find a polarized parton (quark or gluon) in the hadron :**

$$W^{spin}_{\mu\nu} =$$



FACTORISATON is the only way to apply **Perturbative QCD** to inclusive and exclusive processes in Hadronic Physics

Efremov, Efremov-Radyushkin;
Amati-Petronzio-Veneziano;
Libby-Sterman;
Duncan-Mueller

KINEMATICS

Hard kinematics
all invariants of the same size

Form factors;
Scattering at large angles;
DIS at large x

Regge kinematics
some invariants are small

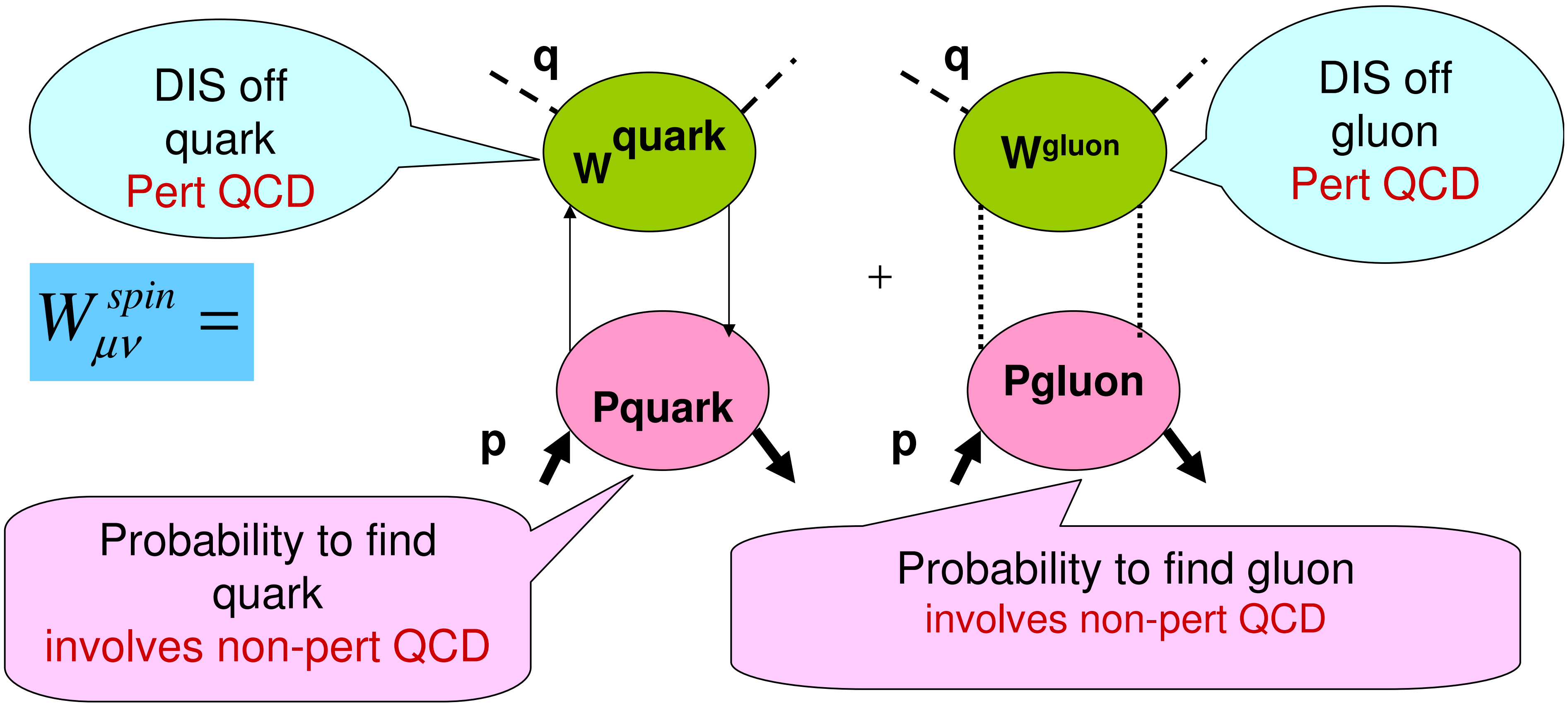
Forward and backward scattering;
DIS at small x

2->2 scattering

$$s \approx -t \approx -u$$

$$s \approx -u >> -t; \;\; s \approx -t >> -u$$

Historically, Hard kinematics was the first one to discuss. Calculations for Hard kinematics are often simpler than in Regge one.



$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

Initial quark density

Initial gluon density

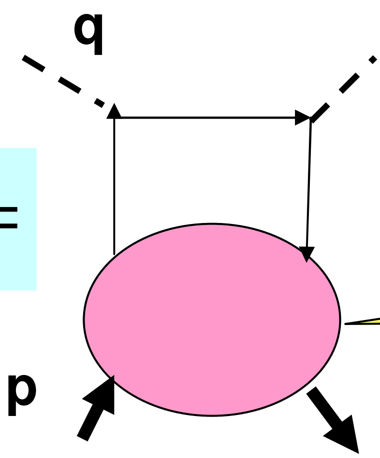
DIS off the quark,

DIS off the gluon

are calculated with methods of Pert QCD

Born approximation

$$W_{\mu\nu}^{spin} =$$



initial quark density

In general, any structure function depends on two independent arguments: $2pq$ and Q^2

However in the Born approximation:

$$g_1(x,Q^2) = F_1(x,Q^2) = \frac{e_q^2}{2}\delta(1-x)\otimes\delta q$$

$$F_2(x,Q^2) = 2xF_1(x,Q^2); \qquad g_2(x,Q^2) = 0$$

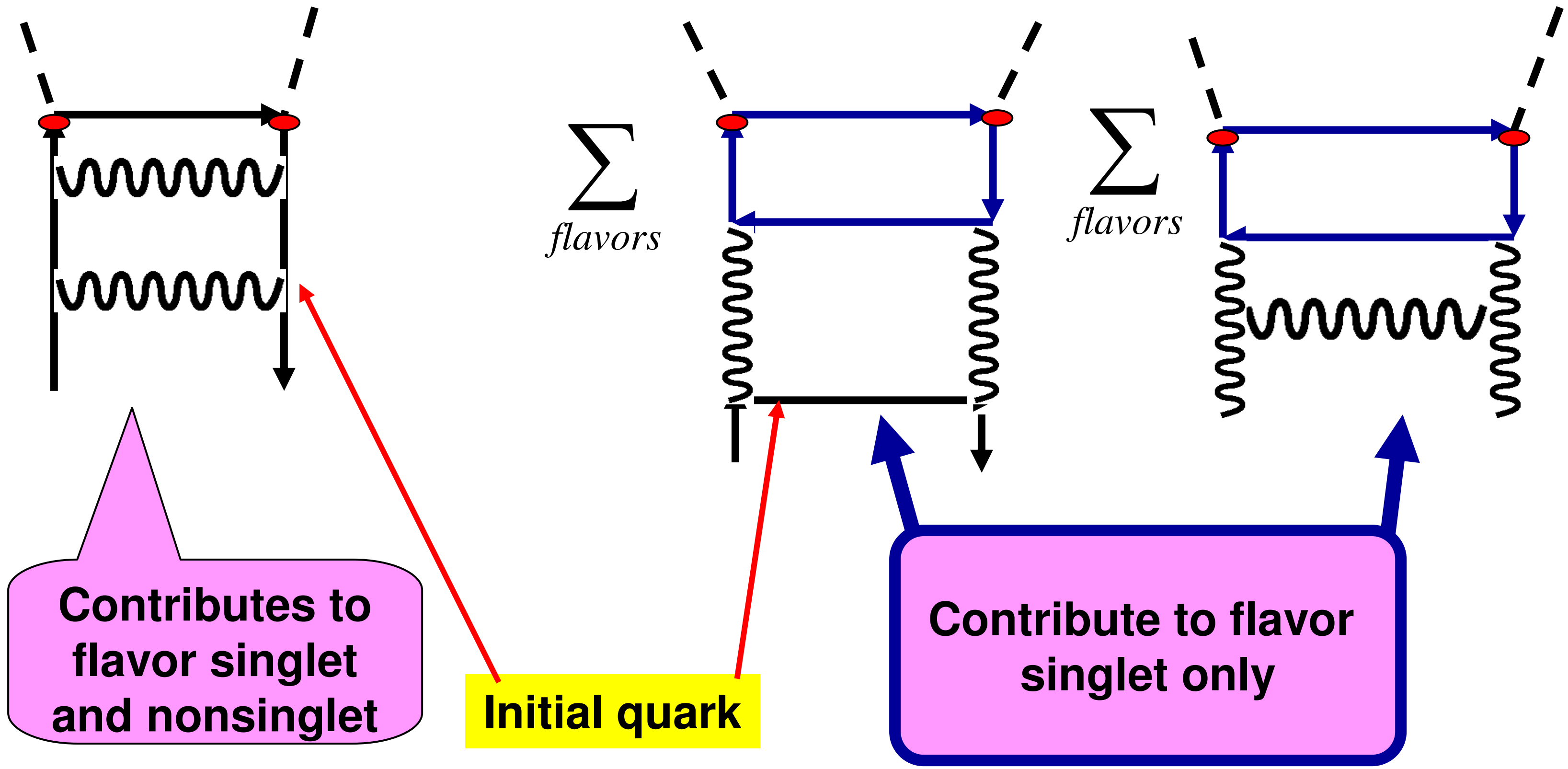
No separate dependences on $2pq$ and Q^2

$x = Q^2/2pq$ → SCALING

When $2pq$ is increasing, radiative corrections should be taken into account

The kind of most important corrections strongly depends on values of x

Piece of terminology



Each structure function has both the non-singlet and singlet components:

$$F_1(x,Q^2) = F_1^S(x,Q^2) + F_1^{NS}(x,Q^2) \quad g_1(x,Q^2) = g_1^S(x,Q^2) + g_1^{NS}(x,Q^2)$$

In order to measure the non-singlets, one needs several targets:

$$F(\text{proton}) - F(\text{neutron}) \sim F(u\text{-quark}) - F(d\text{-quark})$$

Polarized DIS at large x. Standard Approach

Standard Approach includes DGLAP Evolution Equations and Standard Fits for initial paron densities

Evolution Equations: Gribov-Lipatov, Altarelli-Parisi, Dokshitzer

$$g_1(x,Q^2) = C_q(x/y) \otimes \Delta q(y,Q^2) + C_g(x/y) \otimes \Delta g(y,Q^2)$$

Coefficient function

Evolved quark distribution

Coefficient function

Evolved gluon distribution

Standard Approach involves both Pert QCD and Non-Pert QCD.

Pert QCD (SCIENCE)

LO splitting functions	Ahmed-Ross, Altarelli-Parisi, Sasaki,
NLO splitting functions	Floratos, Ross, Sachradja, Gonzale- Arroyo, Lopes, Yandurain, Kounnas, Lacaze, Curci, Furmanski, Petronzio, Zijlstra, Mertig, van Neerven, Vogelsang
Coefficient functions $C^{(1)}_k$, $C^{(2)}_k$	Bardeen, Buras, Muta, Duke, Altarelli, Kodaira, Efremov, Anselmino, Leader, Zijlstra, van Neerven

Non-Pert QCD (ART)

Fits for initial parton densities	Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov- Stamenov, Hirai et al

For example,

$$\delta q = N x^{-\alpha} [(1 - x)^{\beta} (1 + \gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

Altarelli-Ball-Forte-Ridolfi

Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

DGLAP evolution equations

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$ are splitting functions

Mellin transformation of the splitting functions = anomalous dimensions

In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy:

LO

NLO

$$C\ (\omega) = 1 + (\alpha_s(Q^2)/2\pi)\, C^{(1)}\ (\omega) + \ldots$$

$$\gamma\ (\omega) = (\alpha_s(Q^2)/4\pi)\gamma^{(0)}\ (\omega) + (\alpha_s(Q^2)/2\pi)^2\gamma^{(1)}\ (\omega) + \ldots$$

LO

NLO

It is convenient to express structure functions in terms of the Mellin integrals

For example, non-singlet g_1 is

Initial quark density

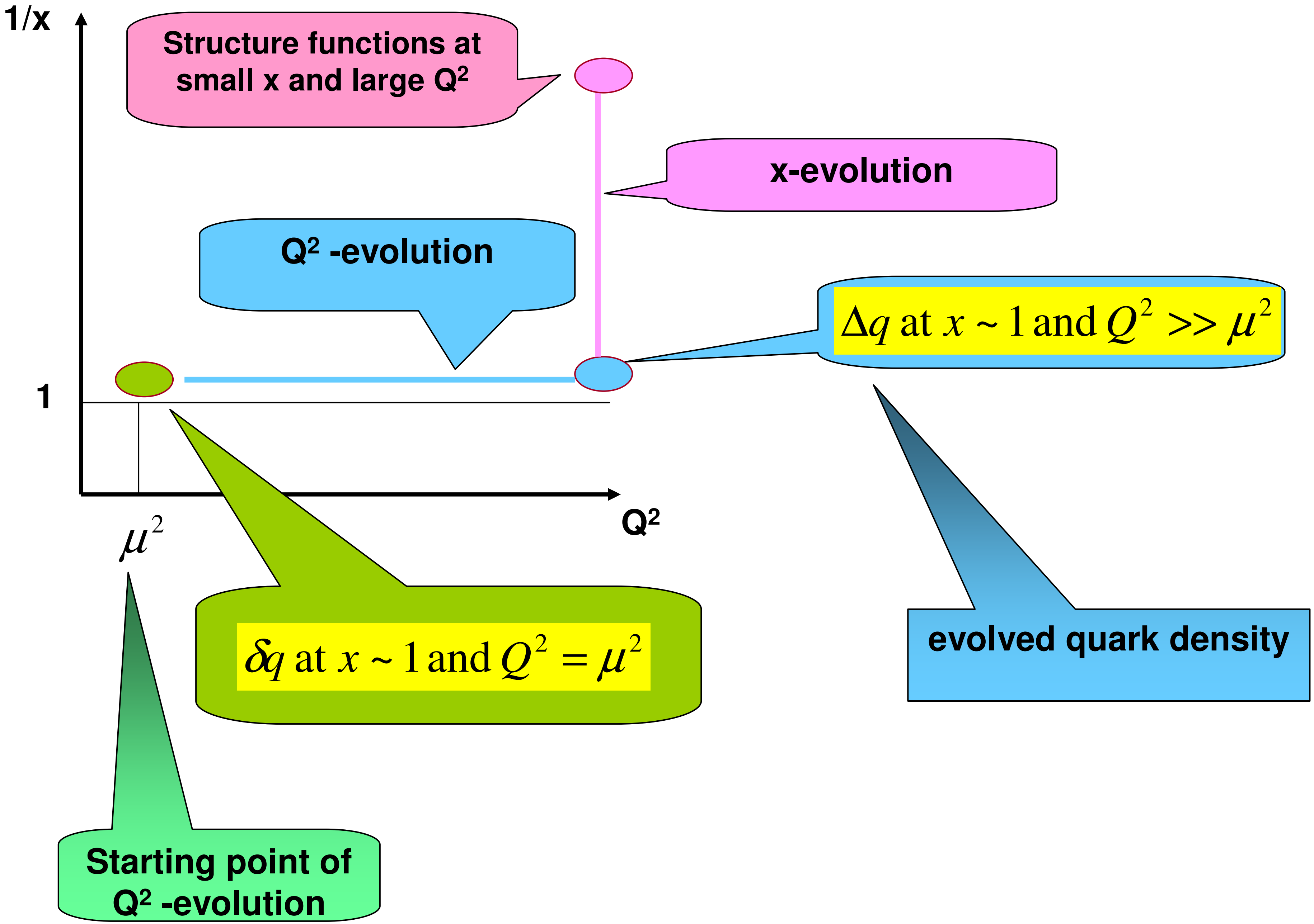
$$g_1^{NS}(x,Q^2) = (e_q^2/2) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} C(\omega)\delta q(\omega) \exp\left[\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \gamma(\omega, \alpha_s(k_\perp^2))\right]$$

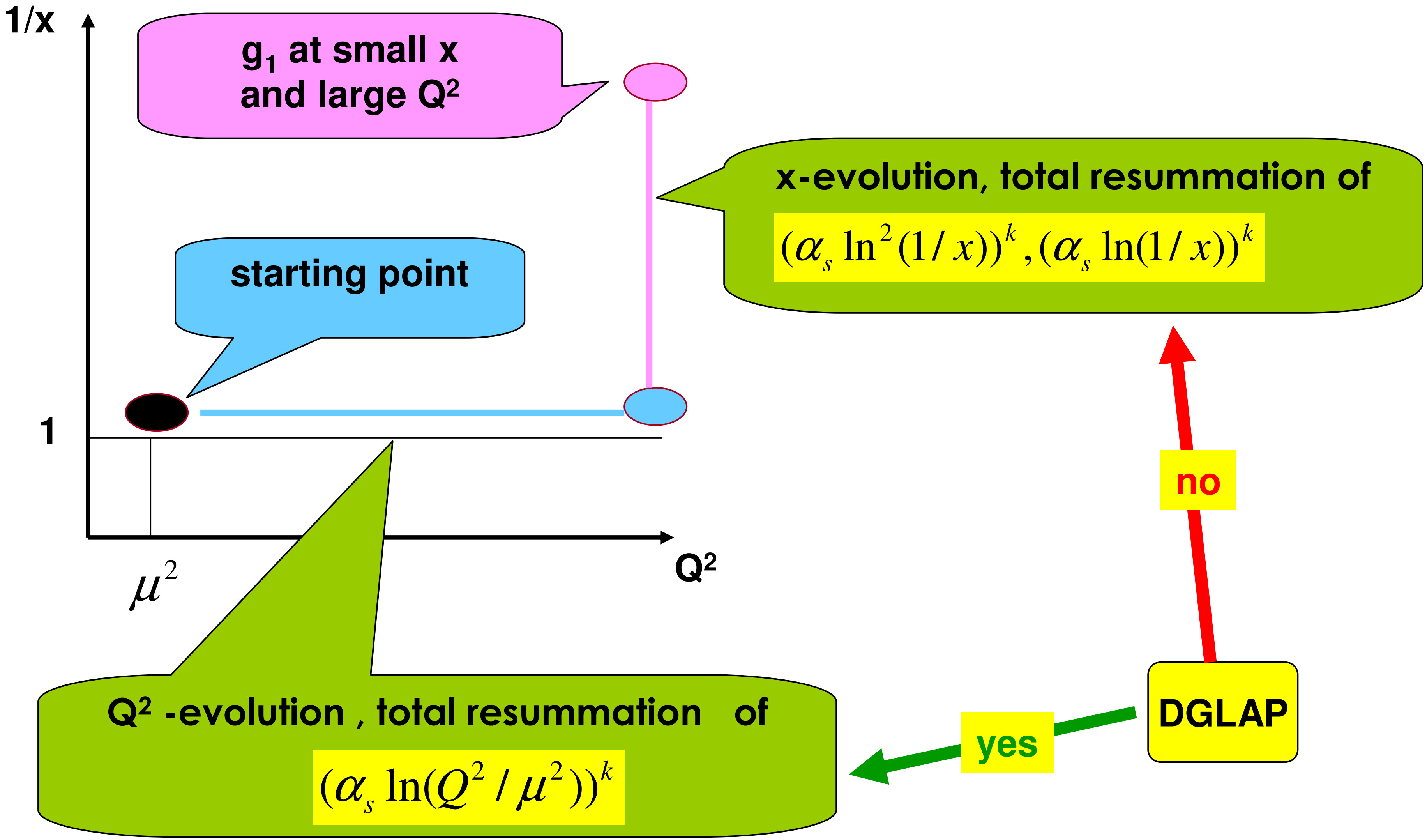
Coefficient function

Anomalous dimension

from fits

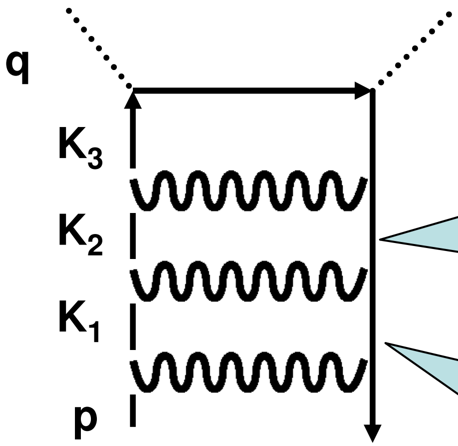
Pert QCD


1/x
Structure functions at small x and large Q²
x-evolution
Q² -evolution
Δq at x ~ 1 and Q² >> μ²
1
Q²
μ²
δq at x ~ 1 and Q² = μ²
evolved quark density
Starting point of Q² -evolution



DGLAP cannot do total resummation of logs of x because of
DGLAP-ordering – KEYSTONE of DGLAP

DGLAP –ordering: $\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < k_{3\perp}^2 < Q^2$



$$k_\lambda = \alpha(q + xp)_\lambda + \beta p_\lambda + k_{\lambda\perp}$$

Sudakov parameterization

$\alpha_s = \alpha_s(k_{2\perp}^2)$

$\alpha_s = \alpha_s(k_{1\perp}^2)$

$\alpha_s = \alpha_s(Q^2)$

DGLAP small-x asymptotics of g_1 is well-known:

$$g_1^{DGLAP} \sim \exp \sqrt{\ln(1/x) \ln \frac{\ln(Q^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)}}$$

providing the initial parton densities are not singular at small x

DGLAP –ordering is good approximation for large x when logs of x can be neglected. At x << 1 the ordering has to be replaced by

$$\mu^2 < \frac{k_{1\perp}^2}{\beta_1} < \frac{k_{2\perp}^2}{\beta_2} < \frac{k_{3\perp}^2}{\beta_3} < s = 2pq(1-x)$$

Gribov-Gorshkov -Frolov-Lipatov

which makes possible to account for the leading logarithms of x

Integration limits do not include Q^2 any more

DL contributions

SL contributions

$$(\alpha_s \ln^2(1/x))^k,$$

$$(\alpha_s \ln(1/x)\ln(Q^2/\mu^2))^k \qquad (\alpha_s \ln(1/x))^k,$$

$$k = 1,2..\infty$$

So, new evolution equations should combine evolutions in x and Q²

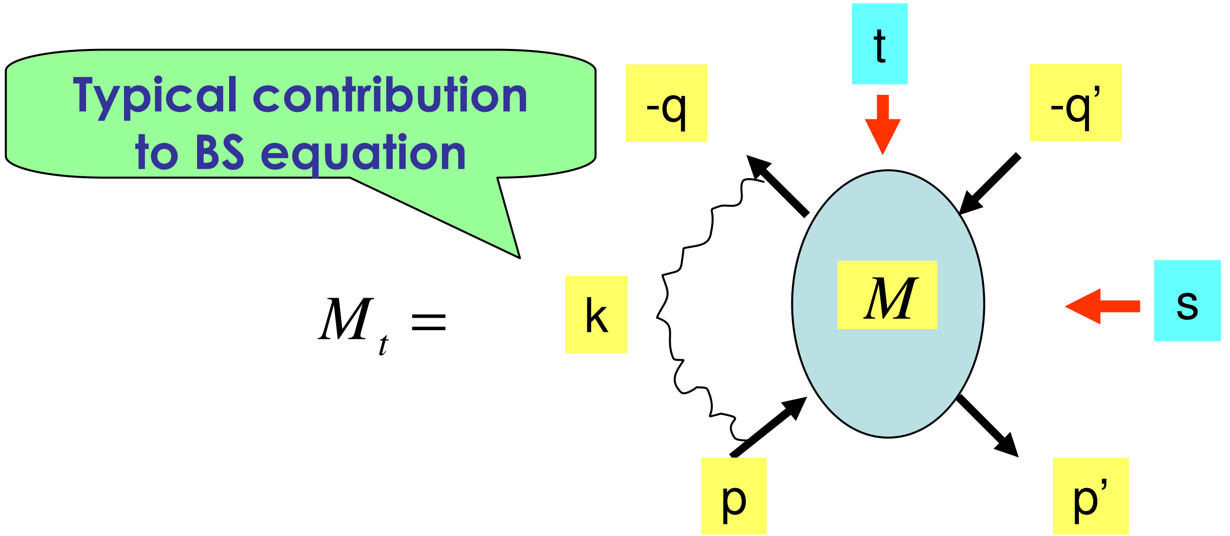
$$\frac{\partial g_1}{\partial \ln(Q^2)} \rightarrow \frac{\partial g_1}{\partial \ln(Q^2)} + \frac{\partial g_1}{\partial \ln x}$$

Before it: DGLAP parameterization of QCD coupling fails at small x and therefore should be modified

$$\alpha_s = \alpha_s(Q^2)$$

Parameterization of α_s in Hard kinematics

$$s = (p+q)^2 \approx -u = -(p - q')^2 \approx -t = -(p - p')^2$$



$$M_t = -i\int \frac{d^4k}{(2\pi)^4} \frac{2s}{[(q+k)^2+i\varepsilon][(p-k)^2+i\varepsilon]}$$

$$M(s,t,q^2,q'^2,(p-k)^2,(q+k)^2)\, 4\pi \frac{\alpha_s(k^2)}{k^2+i\varepsilon}$$

$$M_t = i\frac{s^2}{4\pi^2}\int \frac{d\alpha d\beta dk_\perp^2 \; M(s,t,q^2,q'^2,s\alpha,s\beta,k_\perp^2)}{[s\beta - Q^2 - s\alpha\beta - k_\perp^2 + i\varepsilon][s\alpha - s\alpha\beta - k_\perp^2 + i\varepsilon]} \frac{\alpha_s(-s\alpha\beta - k_\perp^2)}{(-s\alpha\beta - k_\perp^2 + i\varepsilon)}$$

M is unknown, so all the integrations are impossible to perform

Leading Log accuracy → Gribov bremsstrahlung theorem:

$$M = M(s,t,Q^2,Q'^2,k_\perp^2)$$

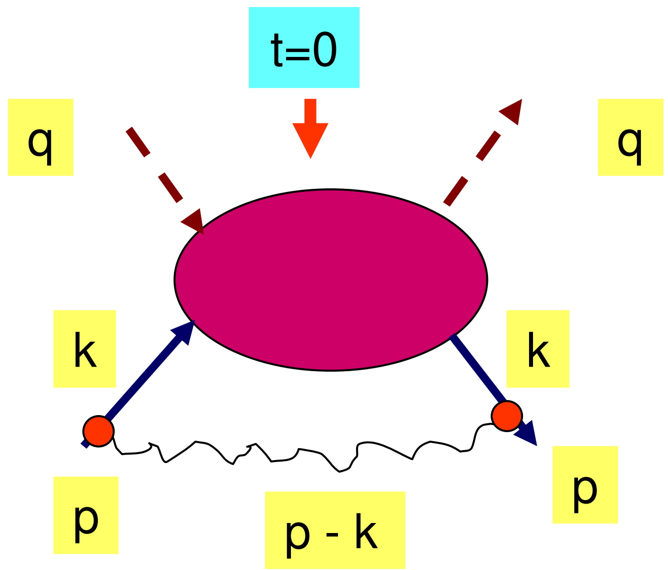
It allows to integrate over α, β

$$M_t = -\frac{1}{2\pi}\int_{\mu^2}^{s} \frac{dk_\perp^2}{k_\perp^2}\int_{\beta_0}^{1} \frac{d\beta}{\beta}(1-\beta) M(s,t,Q^2,Q'^2,k_\perp^2)\, \alpha_s\left(\frac{-k_\perp^2}{(1-\beta)}\right)$$

$$\beta_0 = x + k_\perp^2 / s$$

argument of α_s

Parameterization of α_s in Regge kinematics



$$M_s = \frac{i}{4\pi^2}\int d\alpha d\beta dk_\perp^2 M(2qk, Q^2, k^2)\frac{sk_\perp^2}{(k^2+i\varepsilon)^2}\frac{\alpha_s((p-k)^2)}{(p-k)^2+i\varepsilon}$$

$$k = -\alpha(q+xp)+\beta\, p + k_\perp$$

$$k^2 = -s\alpha\beta - k_\perp^2, \qquad (p-k)^2 = s\alpha - s\alpha\beta - k_\perp^2$$

$$m^2 \equiv (p-k)^2 \quad \rightarrow \quad s\alpha = \frac{m^2 + k_\perp^2}{1-\beta}$$

$$M_s = \frac{i}{4\pi^2} \int d\beta dk_\perp^2 (1-\beta) I(s, Q^2, \beta, k_\perp^2)$$

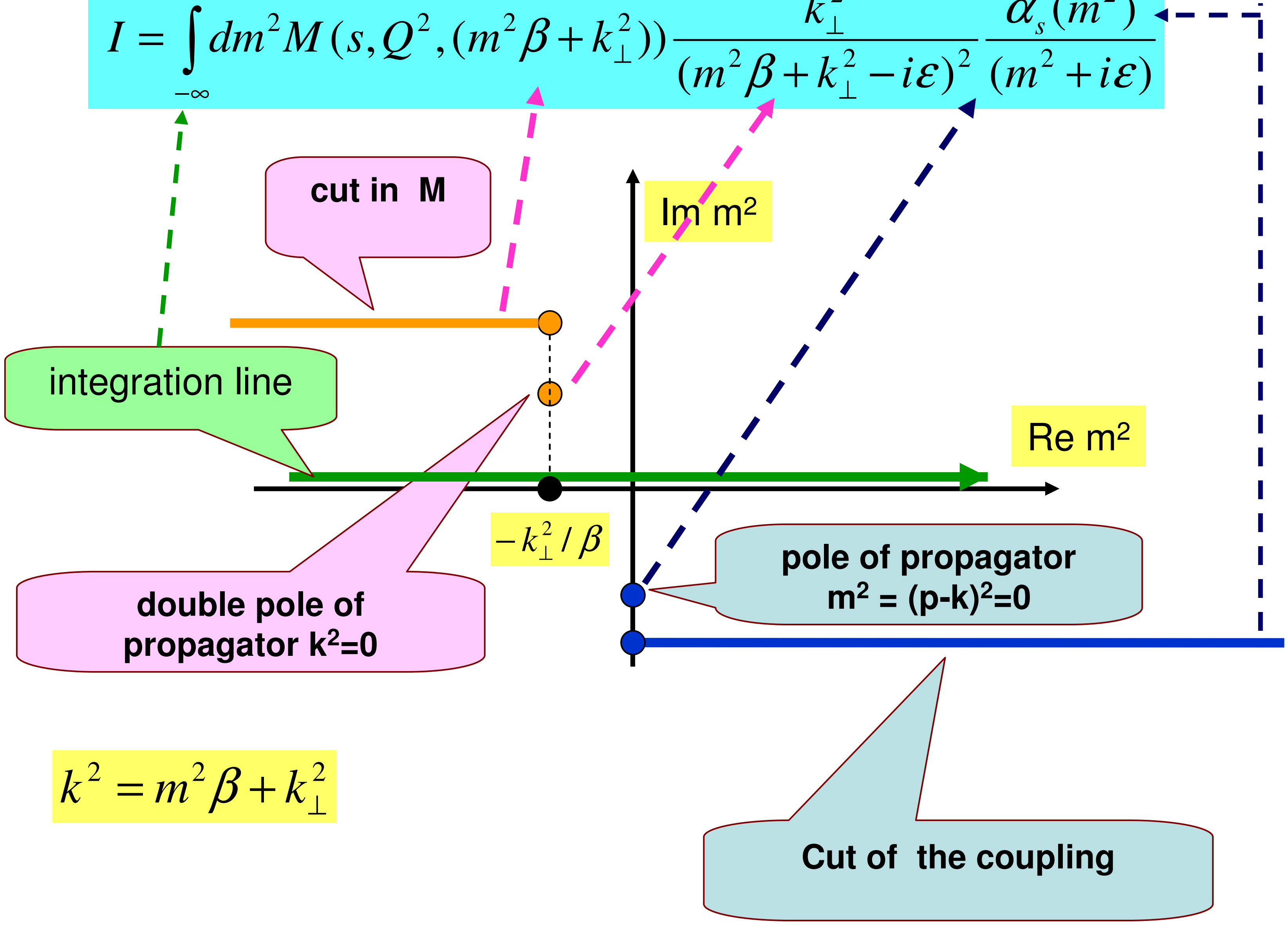
where

$$I = \int_{-\infty}^{\infty} dm^2 M(s, Q^2, (m^2\beta + k_\perp^2)) \frac{k_\perp^2}{(m^2\beta + k_\perp^2 - i\varepsilon)^2} \frac{\alpha_s(m^2)}{(m^2 + i\varepsilon)}$$

Before integrating over m^2, let us study singularities of the integrand

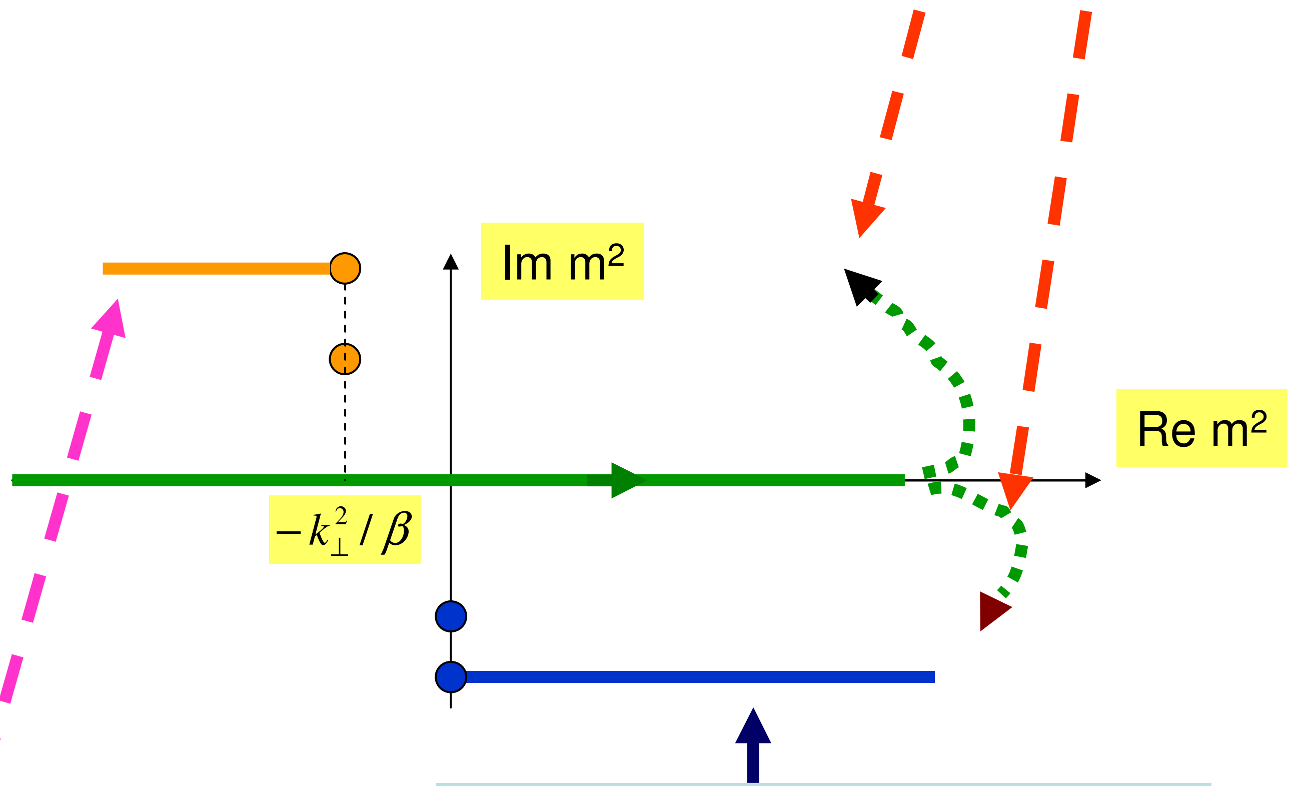
singularities of the integrand in m^2

$$I = \int_{-\infty}^{\infty} dm^2 M(s, Q^2, (m^2\beta + k_\perp^2)) \frac{k_\perp^2}{(m^2\beta + k_\perp^2 - i\varepsilon)^2} \frac{\alpha_s(m^2)}{(m^2 + i\varepsilon)}$$



$$k^2 = m^2\beta + k_\perp^2$$

It is convenient to perform integration over m^2 with using Cauchy theorem.
the integration contour has to be closed either up or down



Closing up involves dealing with the cut of M

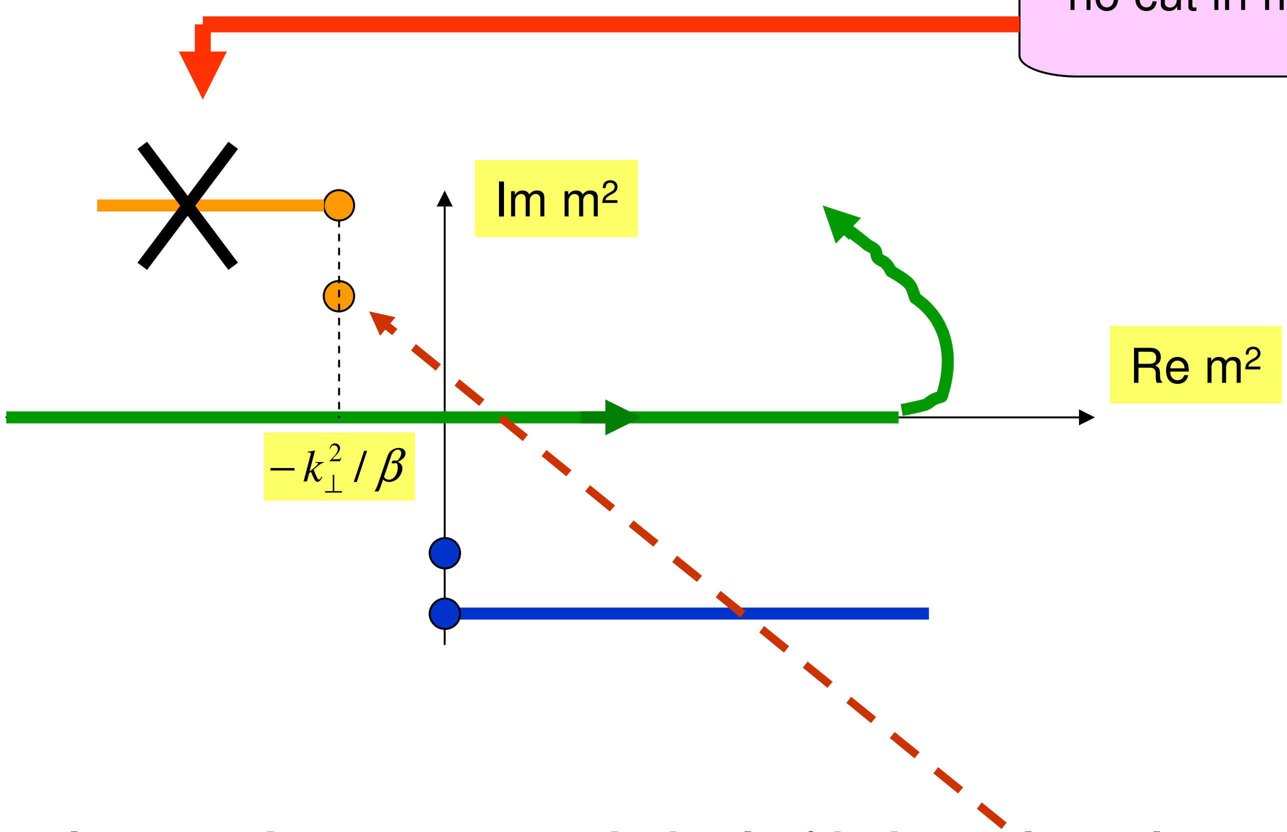
Closing down involves dealing with the cut of the coupling

The most detailed analysis of the QCD coupling in DGLAP: Dokshitzer-Shirkov

Approximation: $M(s,..,k^2) = M(s,..,m^2\beta + k_\perp^2) \approx M(s,..,k_\perp^2)$

Assumption: $m^2\beta << k_\perp^2$

no cut in m²



allows to close up the contour and deal with the pole only:

$$m^2 = -k_\perp^2 / \beta$$

$$\alpha_s = \alpha_s(-k_\perp^2 / \beta)$$

CONTRADICTION between the assumption and calculation:

$$m^2 \beta << k_\perp^2$$

assumption

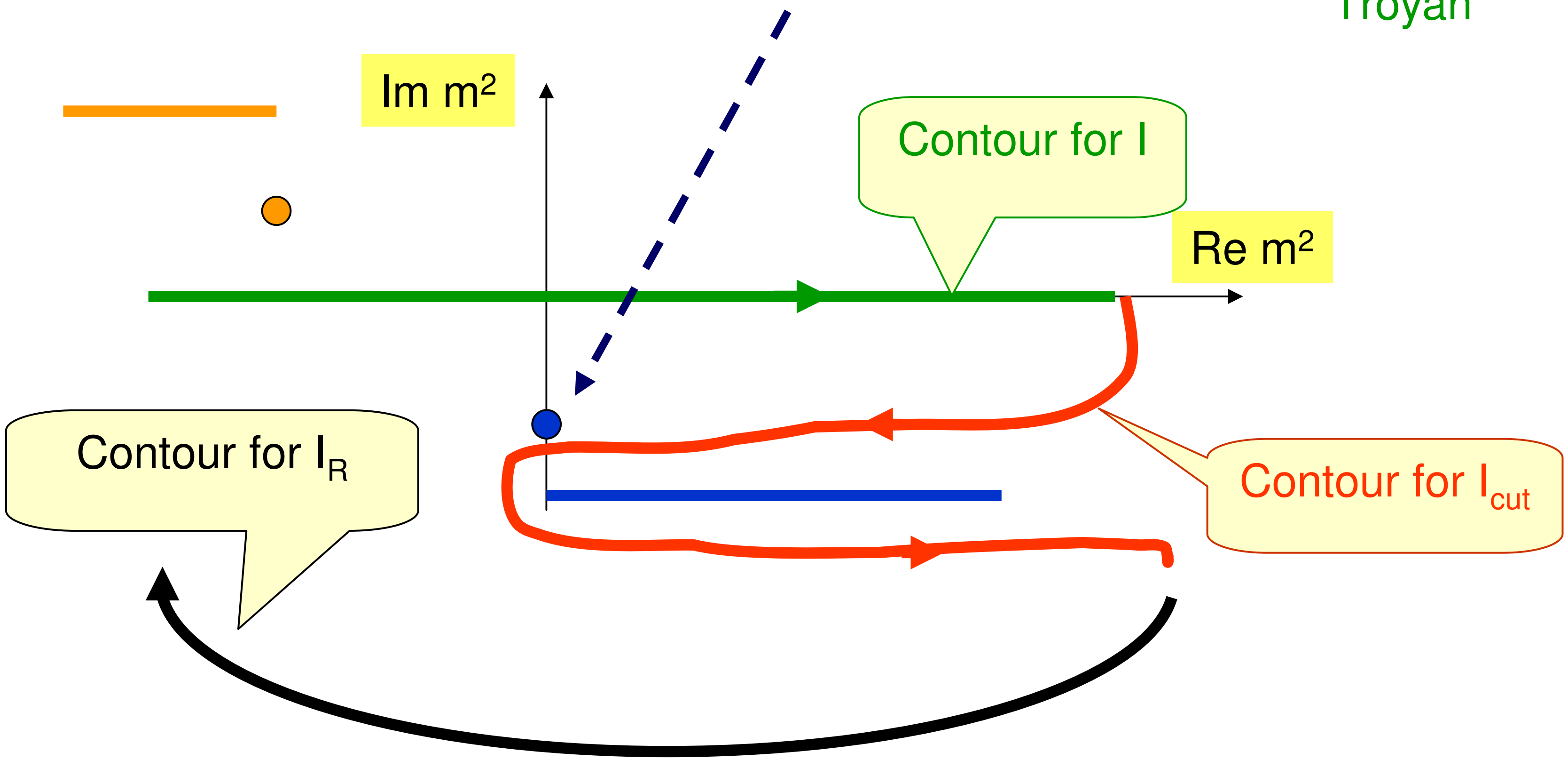
$$m^2 \beta + k_\perp^2 = 0$$

residue in the pole

Therefore the result $\alpha_s = \alpha_s(-k_\perp^2 / \beta)$ **should be revised**

we close the contour down to avoid dealing with singularities of M

Ermolaev-
Troyan



Cauchy theorem:

$$I_C = I + I_R + I_{cut} = -2\pi i \; res \quad \text{in the pole at } m^2 = 0$$

$$I_C = I + I_R + I_{cut} = -2\pi i \frac{(1-\beta)}{k_\perp^2} M(s, Q^2, -k_\perp^2/(1-\beta))\alpha_s(\mu^2)$$

Introduced to regulate IR region

$$\mu^2 \gg \Lambda^2$$

$$I_R \to 0 \text{ when } R \to \infty$$

$$I_{cut} = -2i \int_{\mu^2}^{\infty} dm^2 M(s, Q^2, (m^2\beta + k_\perp^2)) \frac{(1-\beta)k_\perp^2}{(m^2\beta + k_\perp^2 + i\varepsilon)^2} \frac{\mathrm{Im}\,\alpha_s(m^2)}{(m^2 + i\varepsilon)}$$

Assumption: if we assume that the essential region is $k_\perp^2 \gg m^2\beta$

$$I_{cut} \approx -2i \frac{\pi}{b} \frac{(1-\beta)}{k_\perp^2} M(s, Q^2, k_\perp^2) \int_{\mu^2}^{k_\perp^2/\beta} \frac{dm^2}{m^2} \frac{1}{[\ln(m^2/\Lambda^2) + \pi^2]}$$

$$I = -2\pi i \frac{(1-\beta)}{k_\perp^2} M(s, Q^2, k_\perp^2)\alpha_s^{eff},$$

$$\alpha_s^{eff} = \alpha_s(\mu^2) + \frac{1}{\pi b}\left[\arctan\left(\frac{\pi}{\ln(k_\perp^2/\beta\Lambda^2)}\right) - \arctan\left(\frac{\pi}{\ln(\mu^2/\Lambda^2)}\right)\right]$$

$$= \alpha_s(\mu^2) + \frac{1}{\pi b}\left[\arctan\left(\pi b\, \alpha_s(k_\perp^2/\beta)\right) - \arctan\left(\pi b\, \alpha_s(\mu^2)\right)\right]$$

$\ln(\mu^2/\Lambda^2) >> \pi$ → $\alpha_s^{eff} \approx \alpha_s(k_\perp^2/\beta)$

When additionally $x \sim 1$ → $\beta \sim 1$ → $\alpha_s(k_\perp^2/\beta) \approx \alpha_s(k_\perp^2)$

DGLAP region

DGLAP parameterization

Therefore $\alpha_s^{eff} \approx \alpha_s(k_\perp^2)$ only when $x \sim 1$ and $\mu^2 >> \Lambda^2 e^{\pi}$

More practical estimate for condition $\mu^2 >> \Lambda^2 e^{\pi} \approx 25\,\Lambda^2$

$$R(\mu) = \frac{(1/\pi)\arctan(\pi/l) - 1/l}{\arctan(1/l)}, \qquad l = \ln(\mu^2/\Lambda^2)$$

minimal option

at $\Lambda = 0.1\,\mathrm{GeV}$

$R = 5\%$	$\mu^2 = \Lambda^2\, 2.8 10^3 \approx 28\,\mathrm{GeV}^2$
$R = 10\%$	$\mu^2 = \Lambda^2\, 243 \quad \approx 2.4\,\mathrm{GeV}^2$
$R = 50\%$	$\mu^2 = \Lambda^2\, 8.72 \quad \approx 0.87\,\mathrm{GeV}^2$

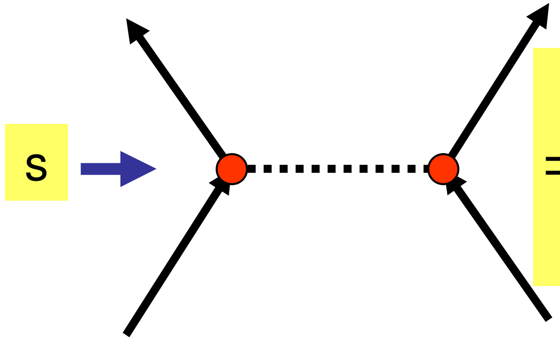
close to conventional option 1 GeV^2 !!

More realistic would be to change 0.1 GeV for 0.5 which causes increase of μ

Alternatively, one can use Mellin transform for QCD coupling

When argument is time-like, coupling participates in Mellin transform:

Ermolaev-Greco-Troyan



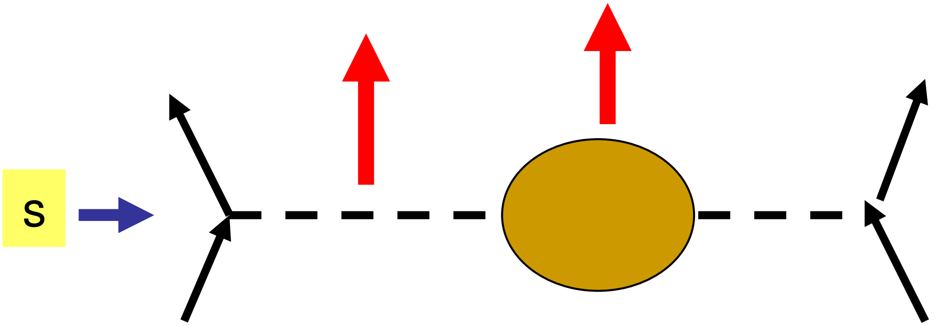
$$= -4\pi\alpha_s(s)C_F\left[\frac{j_\mu j_\mu}{s+i\varepsilon}\right] = \frac{j_\mu j_\mu}{s} M^{Born}(s)$$

$$M^{Born}(s) \rightarrow f^{Born}(\omega) = -4\pi\frac{A(\omega)C_F}{\omega}$$

$$A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty d\rho \frac{\exp(-\omega\rho)}{(\rho+\eta)^2+\pi^2}\right]$$

$$\eta = \ln(\mu^2/\Lambda_{QCD}^2)$$

$$b = (11N - 2n_f)/12\pi$$



Pert QCD

$$\mu^2 >> \Lambda_{QCD}^2$$

In both cases coupling does not depend on Q^2 in contrast to DGLAP

There is no straightforward way to describe the IR region $|m^2| \le \Lambda^2$

One of the most popular approaches is

Analytic Perturbation Theory Shirkov-Solovtsov,

Essence of APT: the aim is to get rid of the singularity at $m^2 = -\Lambda^2$

$$\alpha_s^{APT}(m^2) \equiv \alpha_s(m^2) + \frac{\Lambda^2}{b(m^2+\Lambda^2)}$$

α_s^{APT} is regular at $m^2 = -\Lambda^2$

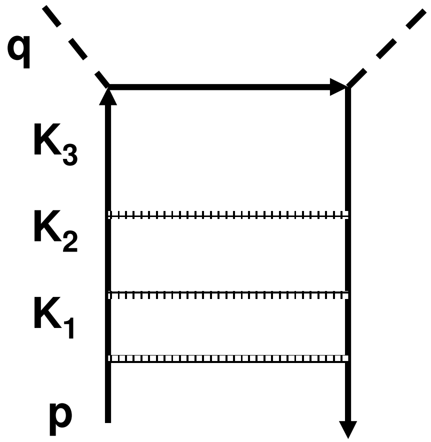
Applications:
calculations at fixed orders of Pert QCD

Bakulev-Mikhailov-Stefanis;
Baikov-Chetyrkin-Kuhn;
Kataev-Kim;....

Alternative to DGLAP for Polarized DIS:
Infrared Evolution Equations (IREE)

Essence: Evolution with respect to the infrared cut-off μ

Term was invented by M. Krawczyk

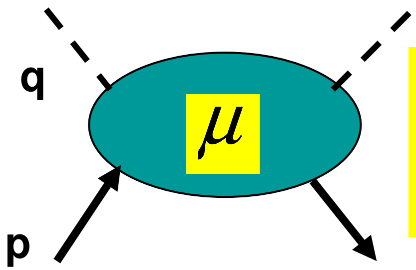


$$k_\perp^i > \mu$$

Lipatov

Pert QCD can be applied when $\mu >> \Lambda_{QCD}$

$$w \equiv \ln(2pq/\mu^2), \qquad y \equiv \ln(Q^2/\mu^2)$$



$$-\mu^2 \frac{\partial M}{\partial \mu^2} = \frac{\partial M}{\partial w} + \frac{\partial M}{\partial y}$$

Two-dimensional equations contrary to DGLAP and BFKL

IREE proved to be simple and efficient instrument for resumming double logarithmic contributions to various processes: form factors, scattering amplitudes in Hard and Regge kinematics, structure functions

Highlights of the history of the method

- Analyses of two-particle cuts in Regge kinematics — Gribov
- Factorization of photons with small transverse momenta — Gribov
- Infrared cut-off in the transverse momentum space — Lipatov
- Quark-quark scattering amplitudes — Kirschner-Lipatov
- Generalization of Gribov bremsstrahlung theorem to QCD , inelastic quark form factors — Ermolaev-Fadin -Lipatov
- QCD inelastic processes in Regge kinematics — Ermolaev-Lipatov
- Applications to Polarized Deep-Inelastic scattering — Bartels-Ermolaev -Manaenkov-Ryskin- Greco-Troyan
- Applications to electroweak reactions — Fadin-Lipatov-Martin-Melles, Ermolaev- Greco-Troyan

Expression for the non-singlet g_1 at large Q^2: $Q^2 >> 1$ GeV^2

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega}\left(\frac{\omega}{\omega - H(\omega)}\right)\delta q(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

New coefficient function and anomalous dimension sum up leading logarithms to all orders in QCD coupling

Compare our non-singlet anomalous dimension to the LO DGLAP one:

expand C and H into series in $1/\omega$

$$H = \frac{A(\omega) C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] + \ldots$$

coincide, save the treatment of coupling

$$\gamma_{NS}^{LO\,DGLAP} = \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n(n+1)} + \frac{3}{2} - S_2(n)\right] \approx \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n} + \frac{1}{2}\right]$$

when n << 1

where $S_k(n) = \sum_{j=1}^{n} \frac{1}{j^k}$

small/large x ⟷ small/large n

Small –x symptotics of g_1: when $x \to 0$, the saddle-point method leads to

$$g_1^{NS} \sim \frac{e_q^2}{2} (1/x)^{\Delta_{NS}} \left(Q^2/\mu^2\right)^{\Delta_{NS}/2} \delta q$$

$$g_1^{S} \sim \frac{\langle e_q^2 \rangle}{2} S(\Delta_S) \, (1/x)^{\Delta_S} \left(Q^2/\mu^2\right)^{\Delta_S/2}$$

intercepts $\Delta_{NS} = 0.42$ $\Delta_S = 0.86$

$$S(\Delta_S) = -\delta q - 0.064\ \delta g$$

Interplay between the quark and gluon densities can lead to different sign of g_1 singlet at x<<1

Intercept= the rightmost singularity. Such a singularity is the square-root branching point:

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}(\omega)}, \quad H_{NS}(\omega) = (1/2)\sqrt{\omega^2 - B_{NS}(\omega,\mu)}$$

$$B_{NS}(\omega,\mu) = A(\omega,\mu)C_F/(2\pi) + D(\omega,\mu)$$

$$A(\omega,\mu) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty d\rho \frac{\exp(-\omega\rho)}{(\rho+\eta)^2+\pi^2}\right]$$

$$\eta = \ln(\mu^2/\Lambda^2_{QCD})$$

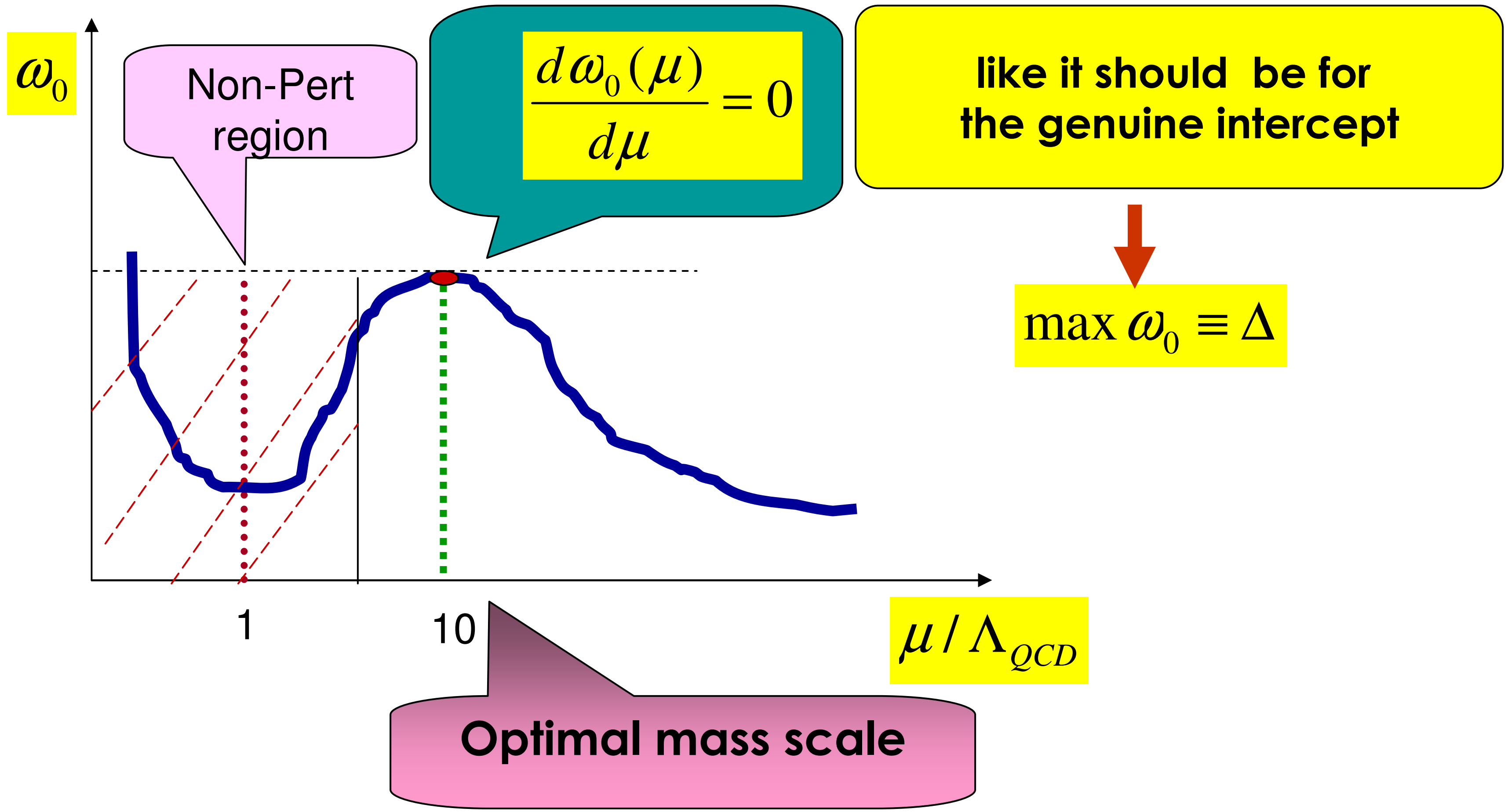
$$b = (11N - 2n_f)/12\pi$$

Therefore, non-singlet intercept is solution ω_0 to

$$\omega^2 - B_{NS}(\omega,\mu) = 0$$

and therefore depends on μ

Actually the solution is plot $\omega_0 = \omega_0(\mu)$



Values of the intercepts perfectly agree with results of several groups who fitted experimental data.

non-singlet intercept	Soffer-Teryaev, Kataev-Sidorov-Parente, Kotikov-Lipatov-Parente-Peshekhonov-Krivokhijine-Zotov,
singlet intercept	Kochelev-Lipka-Vento-Novak-Vinnikov

Conclusion: the impact of non-leading and non-pert contributions is small

Anatomy of the singlet intercept

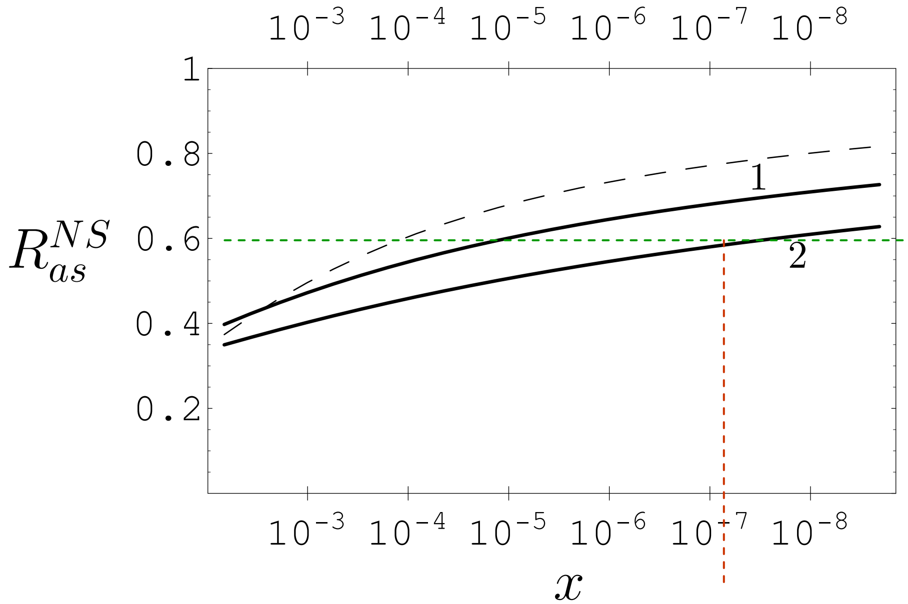
A. Graphs with gluons only: $\Delta_S = 1.1$ → **violates unitarity** → **similar to BFKL**

B. All graphs $\Delta_S = 0.86$ → **No violation of unitarity**

Warning: asymptotic expressions for g_1 are reliable at very small x: $x<10^{-6}$

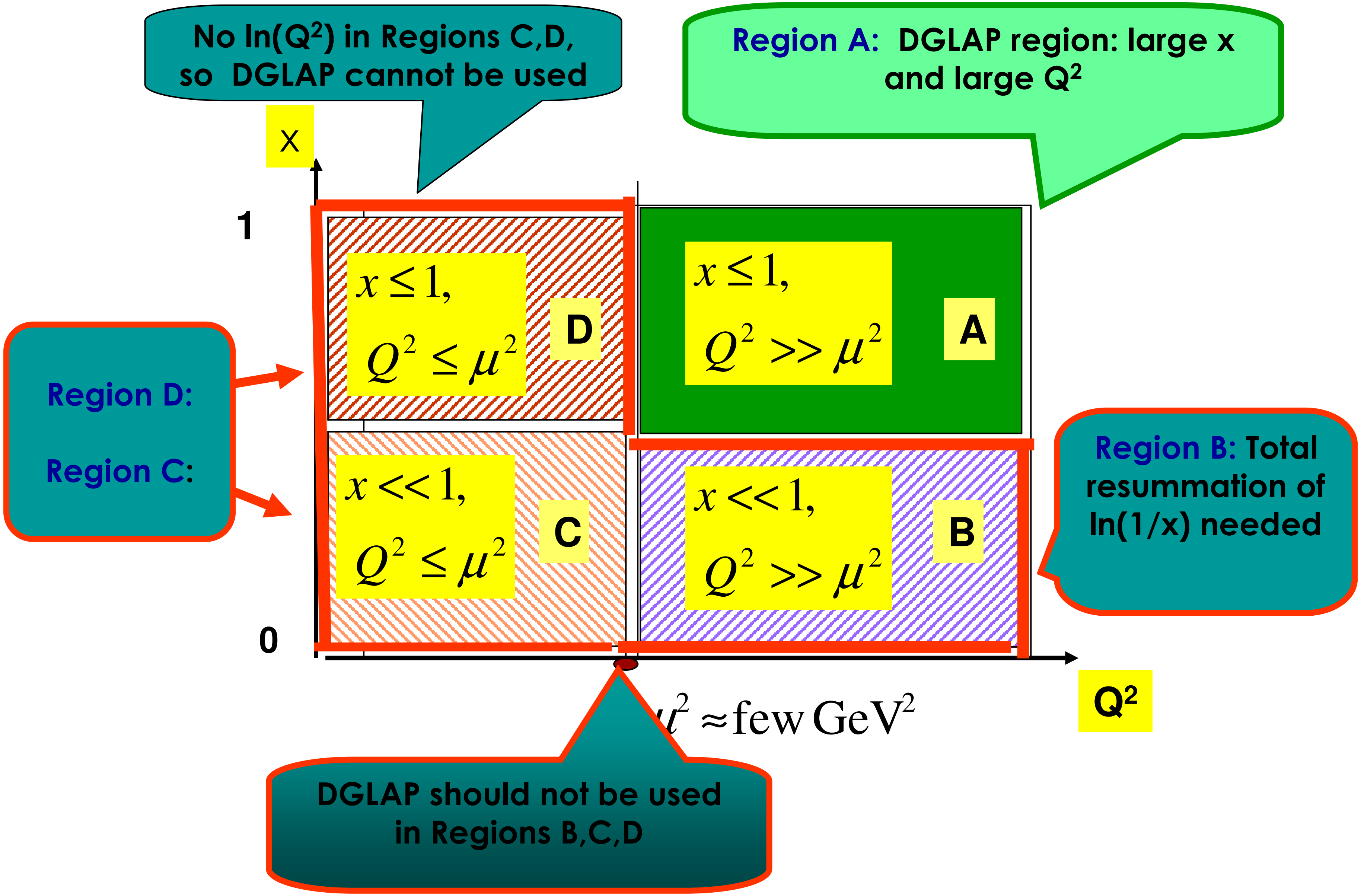
Applicability region for asymptotics

$$R_{as}^{NS} = \frac{(g_1^{NS})_{\text{asympt}}}{g_1^{NS}}$$



WARNING: It is definitely unreliable to use asymptotics at $x < 10^{-7}$

from theoretical grounds:



Comparison of our results to DGLAP at small but finite x

Without using asymptotic formulae

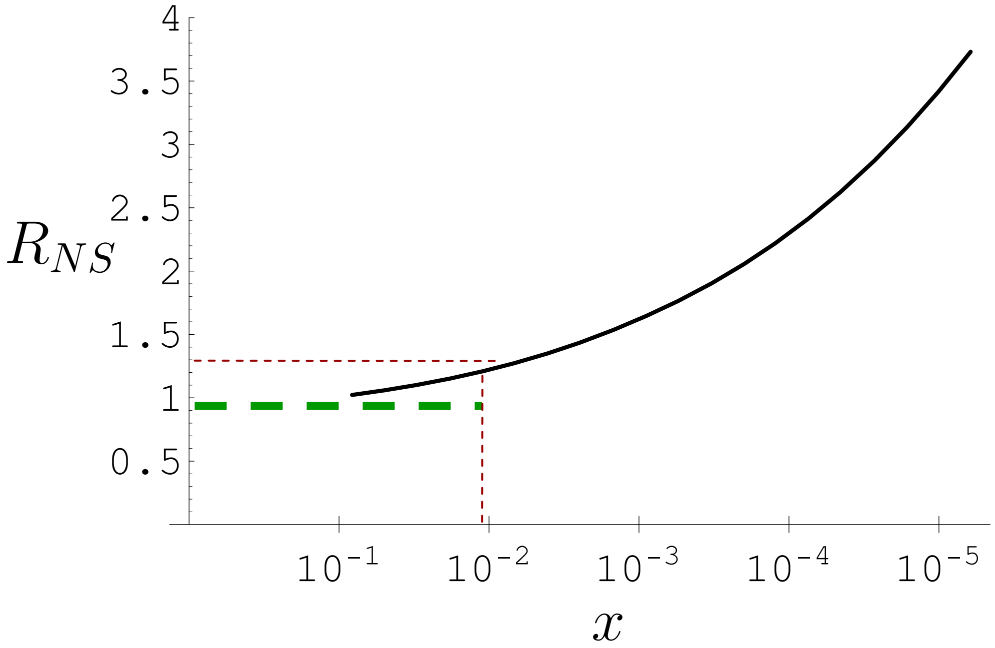
Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input

$$\delta q(x) = \delta(\mathrm{x}) \quad \Longleftrightarrow \quad \delta q(\omega) = 1$$

in x- space | in Mellin space

Numerical study of $R^{NS}(x) = \dfrac{g_1^{NS}(x)}{g_1^{NS\ DGLAP}(x)}$ at fixed Q^2



Deviation of DGLAP from the results with total resummation is seen as early as at x = 0.05

PUZZLE: DGLAP should have Failed at x < 0.05.
However, actually it works

SOLUTION TO PUZZLE: consider in more detail standard DGLAP fit for initial parton densities

$$\delta q(x) = N\, x^{-\alpha}[(1+\gamma x^{\delta})(1-x)^{\beta}]$$

normalization

singular factor

regular factors

parameters $\alpha \approx 0.58,\ \beta \approx 2.7, \gamma \approx 34.3,\ \delta \approx 0.75$

are fixed from fitting experimental data at large x and $Q^2 = Q_0^2 \approx 1$ GeV^2

↓

All parameters depend on Q_0^2

In the Mellin space this fit is

Leading pole

$$\delta q(\omega) = N\left[(\omega-\alpha)^{-1} + \sum_{k=1}^{\infty} c_k\left((\omega+k-\alpha)^{-1} + \gamma(\omega+k+1-\alpha)^{-1}\right)\right]$$

Non-leading poles

$$g_1^{DGLAP} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} \delta q(\omega)\, C_q(\omega)\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)^{\gamma(\omega)/(2b\pi)}$$

$g_1^{DGLAP} = C\,(1/x)^{\alpha} + \ldots$ **instead of** $g_1^{DGLAP} \sim \exp\left(\sqrt{\ln(1/x)}\right)$

obviously $(1/x)^{\alpha} \gg \exp\left(\sqrt{\ln(1/x)}\right)$ at $x \ll 1$

Therefore, the small-x DGLAP asymptotics of g_1 used in practice is the Regge asymptotics:

$$g_1^{DGLAP} \sim (1/x)^{\alpha}$$

contrary to the well-known DGLAP -expression existing in textbooks only: $g_1^{DGLAP} \sim \exp \sqrt{c \ln(1/x)}$

it coincides with our asymptotics: $g_1 \sim (1/x)^{\Delta}$

phenomenology

$$g_1^{DGLAP} \sim (1/x)^{\alpha}$$

Asymptotics depends on Q_0^2

calculations

$$g_1 \sim (1/x)^{\Delta}$$

number

CONCLUSION: the singular factors in the DGLAP fits mimic the total resummation of ln(1/x) .

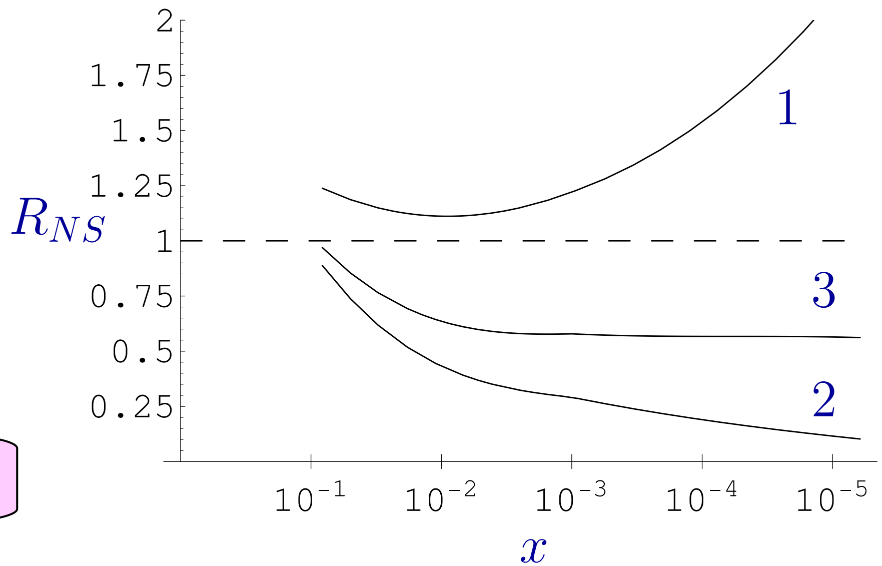
Misconception: the total resummation is not relevant at available x
Actually: the resummation has always been accounted for through the singular factors in standard fits, however without realizing it

Misconception: Numerical comparison (Blumlein-Vogt) demonstrates that non-logarithmic terms in DGLAP expresions have big impact at available small x
Actually: this interpretation is wrong

Resummation of LL

$$R^{NS}(x) = \frac{g_1^{NS}(x)}{g_1^{NS\ DGLAP}(x)}$$

NLO DGLAP



Curve 1: only regular part of the fit in numerator and denominator
Curve 2: whole fits in numerator and denominator
Curve 3: whole fit in denominator and regular part in numerator

Misconception: fits for initial parton densities are defined at large x, then convoluting them with coefficient functions weakens the singularity

$$C(x, y) \otimes \delta q(y) = \Delta q(x)$$

initial — x-evolved

Obviously, it is not true:
They both are singular equally

Structure of DGLAP fit once again:

$$\delta q(x) = N\, x^{-\alpha}[(1+\gamma x^{\delta})(1-x)^{\beta}]$$

Can be dropped when ln(x) are resummed

x-dependence is weak at x<<1 and can be dropped

Therefore at x << 1

$$\delta q(x) \approx \mathrm{N} \text{ or } \mathrm{N}(1+\mathrm{ax})$$

DGLAP Reggeons

DGLAP fit once again:

$$\delta q(x) = N\, x^{-\alpha}[(1+\gamma x^{\delta})(1-x)^{\beta}]$$

$$\alpha \approx 0.58,\;\; \beta \approx 2.7,\, \gamma \approx 34.3,\;\; \delta \approx 0.75$$

$$j_0 = \alpha, \quad j_k = \alpha - k, \quad \tilde{j}_k = \alpha - k - 1$$

In the Mellin space this fit is

$$\delta q(\omega) = \frac{N}{(\omega - j_0)} + \sum_{k=1}^{\infty}\left[\frac{c_k}{(\omega - j_k)} + \frac{\tilde{c}_k}{(\omega - \tilde{j}_k)}\right]$$

Substituting it into

$$g_1^{DGLAP} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} C_q(\omega)\, \delta q(\omega)\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)^{\gamma(\omega)/(2b\pi)}$$

arrive at the sum of Reggeon contributions:

$$g_1^{DGLAP} = (e_q^2/2)\left[S_0(x,Q^2) + \sum_{k=1}^{\infty} S_k(x,Q^2) + \tilde{S}_k(x,Q^2) \right]$$

It is identical representation, without approximations at all

Leading Reggeon

where

$$S_0 = \beta(j_0)\left(\frac{1}{x}\right)^{j_0}\left(\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)^{\gamma(j_0)/(2b\pi)}$$

$$S_k = \beta(j_k)\left(\frac{1}{x}\right)^{j_k}\left(\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)^{\gamma(j_k)/(2b\pi)} \qquad \tilde{S}_k = \beta(\tilde{j}_k)\left(\frac{1}{x}\right)^{\tilde{j}_k}\left(\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\right)^{\gamma(\tilde{j}_k)/(2b\pi)}$$

Non-leading Reggeons

As is known DGLAP structure functions successfully describe experimental data, so a truncated set of DGLAP Reggeons can also describe the data.

$$g_1^{DGLAP} \approx (e_q^2/2)\left[S_0(x,Q^2) + \sum_{k=1}^{k=N} S_k(x,Q^2) + \tilde{S}_k(x,Q^2) \right]$$

intercepts: $j_0 = \alpha = 0.58, \quad j_k = \alpha - k, \quad \tilde{j}_k = \alpha - k - 1$

$j_0 > 0$

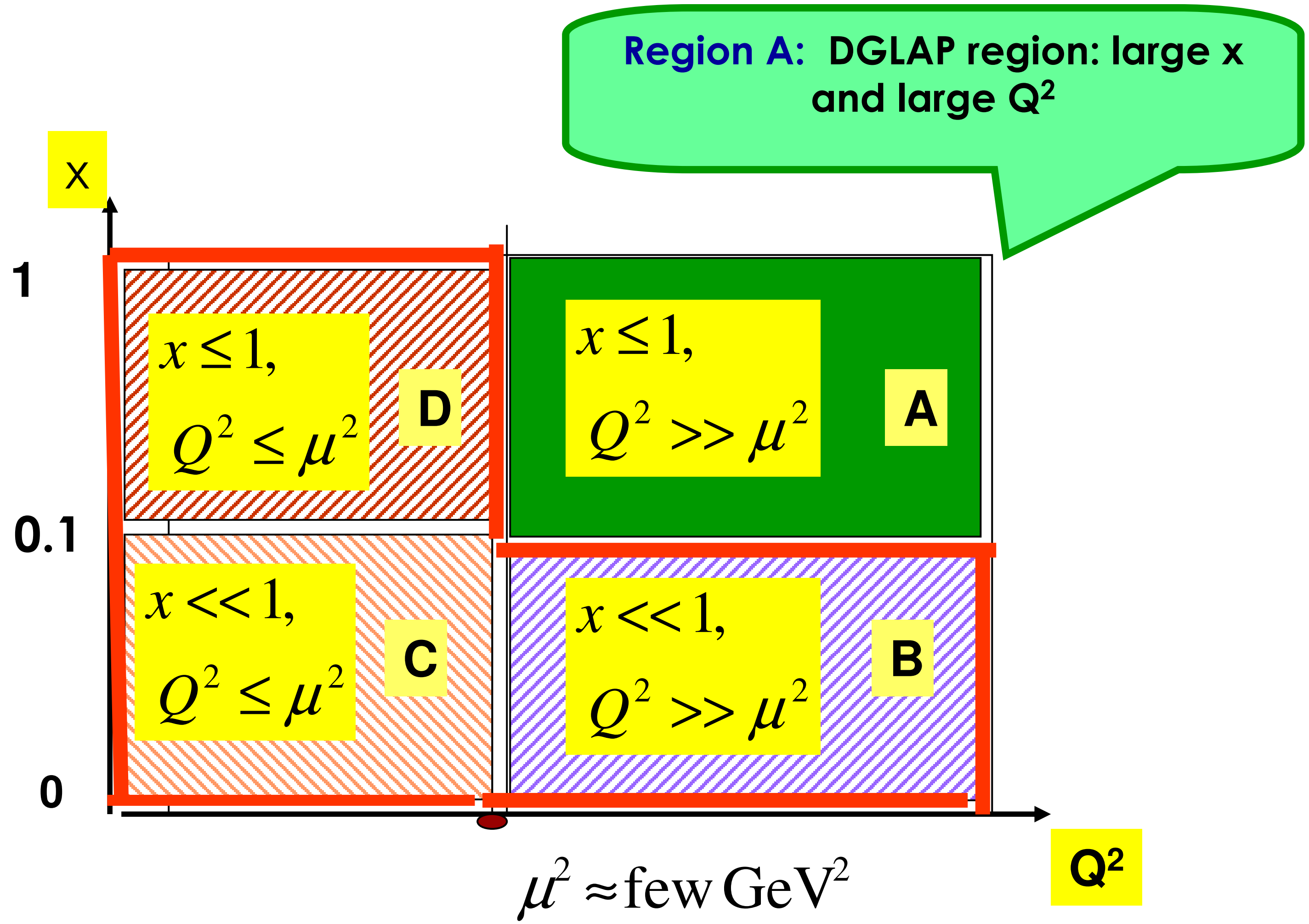
$j_k, \tilde{j}_k < 0$

All these Reggeons are artificial: introduced by hands in the fits to match exp data: they have nothing to do with QCD radiative corrections and exist not only at small **x** but also at **x ~1**, i.e. at low energies : ~ few GeV

Describing exp data in terms of Reggeons at GeV- scale has nothing to do with genuine QCD Reggeons.

It represents the data in the Regge-like form $f(s,t) = \sum c_k(t)(s/s_0)^{a_k + tb_k}$

with a_k, b_k, c_k, s_0 taken from experiment



Region A: DGLAP region: large x and large Q2
x
1
0.1
0
$x \le 1,$
$Q^2 \le \mu^2$
D
$x \le 1,$
$Q^2 >> \mu^2$
A
$x << 1,$
$Q^2 \le \mu^2$
C
$x << 1,$
$Q^2 >> \mu^2$
B
$\mu^2 \approx \mathrm{few\ GeV}^2$
Q2

Technology of getting universal description of g_1:

Step 1: Resummation of leading $\ln(1/x)$ and $\ln(Q^2)$ → g_1 in Region B: large Q^2 and small x

Step 2: Combining above results and DGLAP → g_1 in Region A&B: large Q^2 and arbitrary x

Step 3: Shift $Q^2 \rightarrow Q^2 + \mu^2$ → g_1 in Region C&D: small Q^2 and arbitrary x

Thus, we arrive at universal and model-independent description of g_1 at arbitrary Q^2 and x without singular fits:

$$g_1^{NS} = \frac{e_q^2}{2}$$

$$\int \frac{d\omega}{2\pi i}\left(\frac{1}{z+x}\right)^{\omega} C_C(\omega)\, \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2}\right)^{H_C(\omega)}$$

$$z = \frac{\mu^2}{2pq}, \quad x = \frac{Q^2}{2pq}$$

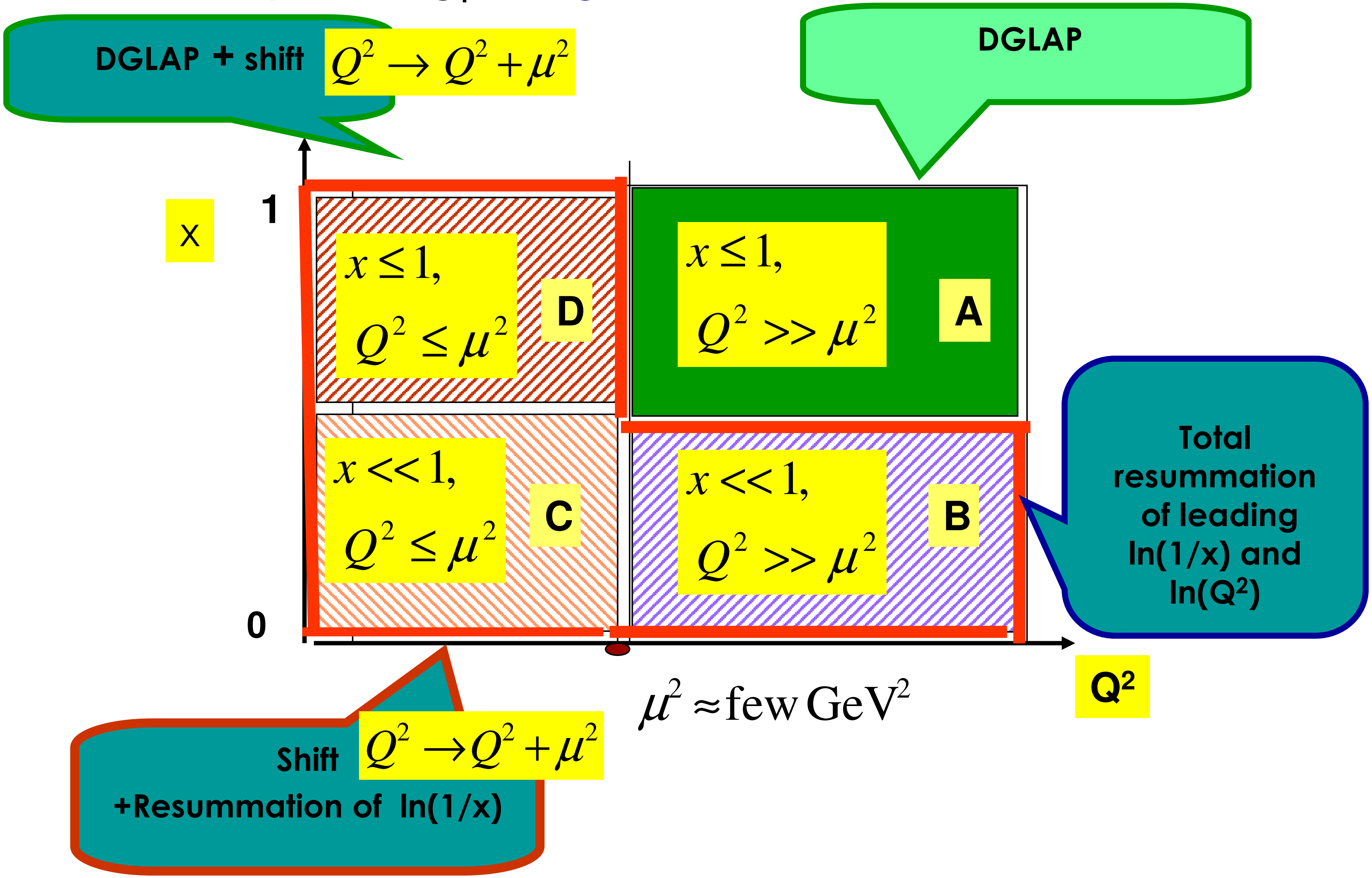
Combined coefficient function

Non-singular quark density

Combined anomalous dimension

expression for the singlet g_1 is written quite similarly

Main impact on g_1 in Regions A,B,C,D comes from:



Recent applications of our approach to:

1. **COMPASS results**

2. **Power Q^2-corrections**

Taken from wwwcompass.cern.ch

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.

On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.

Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

Expression for the singlet g_1 at small Q^2: $Q^2 < \mu^2;\ \mu \approx 5\ \mathrm{GeV}$

$Q^2 \to Q^2 + \mu^2 \Rightarrow x \to x + z$

$x = Q^2 / 2pq, \ \ z = \mu^2 / 2pq$

$$g_1^S = \frac{< e_q^2 >}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x + z} \right)^{\omega} \left[F_q \delta q + F_g \delta g \right]$$

$$F_q = C_q^{(+)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$F_g = C_g^{(+)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left(\frac{Q^2 + \mu^2}{\mu^2} \right)^{\Omega^{(+)}}$$

$C_q^{(\pm)}(\omega),\ C_q^{(\pm)}(\omega),\ \Omega^{(\pm)}(\omega)$ contain resummations of LL

COMPASS: 10^{-1} GeV^2 < Q^2 < 3 GeV^2 → DGLAP cannot be used:

Our approach is not sensitive to values of Q^2, so we can use it

Prediction 1: very weak dependence g_1 on x at the COMPASS range of Q^2 even at very small x

x-dependence is very weak

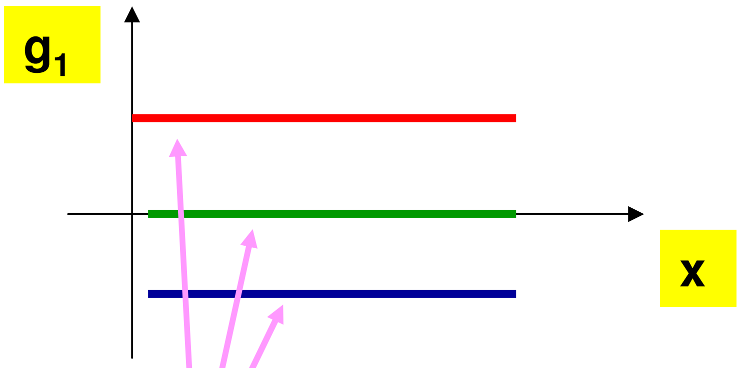
$$Q^2 << \mu^2$$

$$x << z \Rightarrow g_1(x+z) \approx g_1(z) + x d g_1(z)/dz + \ldots$$

$$z = \mu^2/(2pq)$$

$$g_1(z) = \left(\frac{< e_q^2 >}{2}\right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{z}\right)^{\omega} \left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

so x- dependence is flat, even for x<<1.



Status of this prediction:

Confirmed by COMPASS

Location of the line cannot be predicted from theoretical grounds because depends on the interplay between the unknown initial quark and gluon densities: at small Q^2

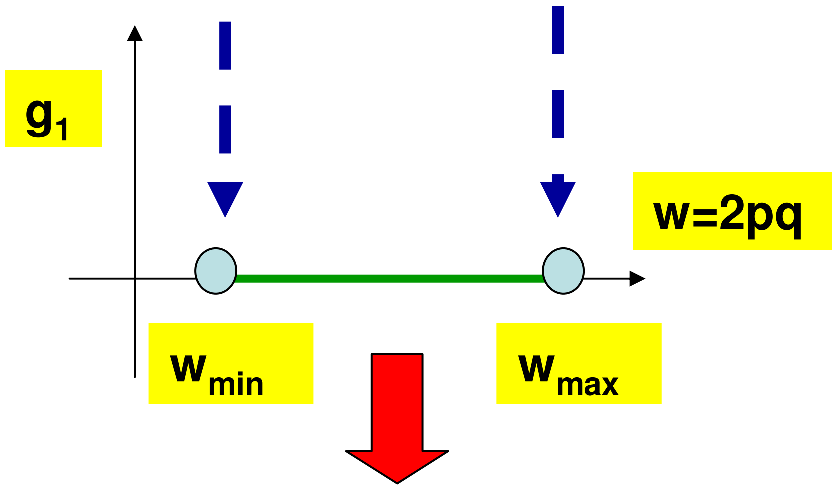
$$g_1(z) = \left(\frac{< e_q^2 >}{2}\right)\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{z}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

are calculated

COMPASS data: g_1 =0 at small Q^2 but values of 2pq and Q^2 are not specified. It opens two possibilities:

Case A: g_1=0 at any value of 2pq (=w) in the COMPASS 2pq- range:

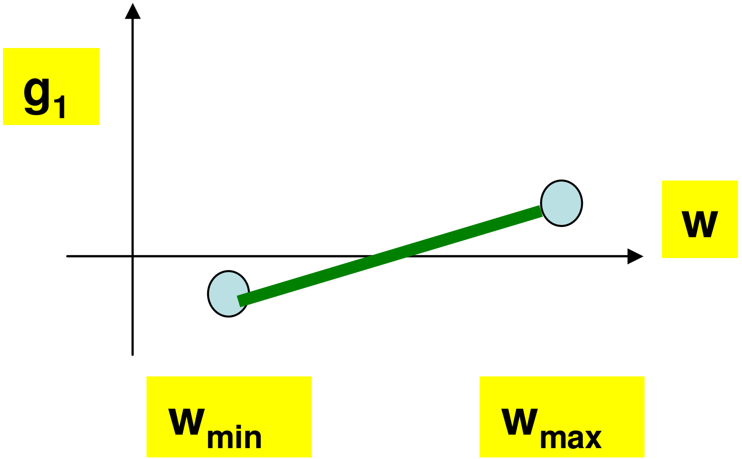
30 GeV^2 < w <270 GeV^2

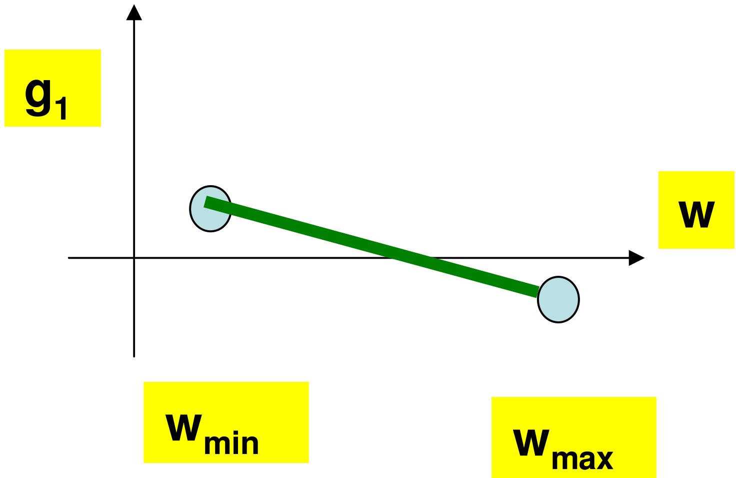


$$\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\left(\frac{\mu^2}{w}\right)^{\omega}\left[C_q(\omega)\delta q + C_g(\omega)\delta g\right] \approx 0$$

for all w from the interval $w_{\min} \leq w \leq w_{\max}$

Case B: g1=0 only after averaging over w





Problem: What expressions for the Initial parton densities to use?

Usually they are fixed from phenomenological considerations

For example, in DGLAP

$$\delta q = 0.4 z^{0.5}(1-z)^3(1+3z), \quad \delta g = 1.7 z^{-0.5}(1-z)^4(1+3z)$$

(Altarelli-Ball-Forte-Ridolfi)

singular at z->0

DGLAP needs singular factors to mimic the total resummation of logs. When the resummation is accounted for, they should be dropped, so the fits can be chosen as

$$\delta q = N_q z^{a_q} (1 - z)^{b_q}, \quad \delta g = N_g z^{a_g} (1 - z)^{b_g},$$

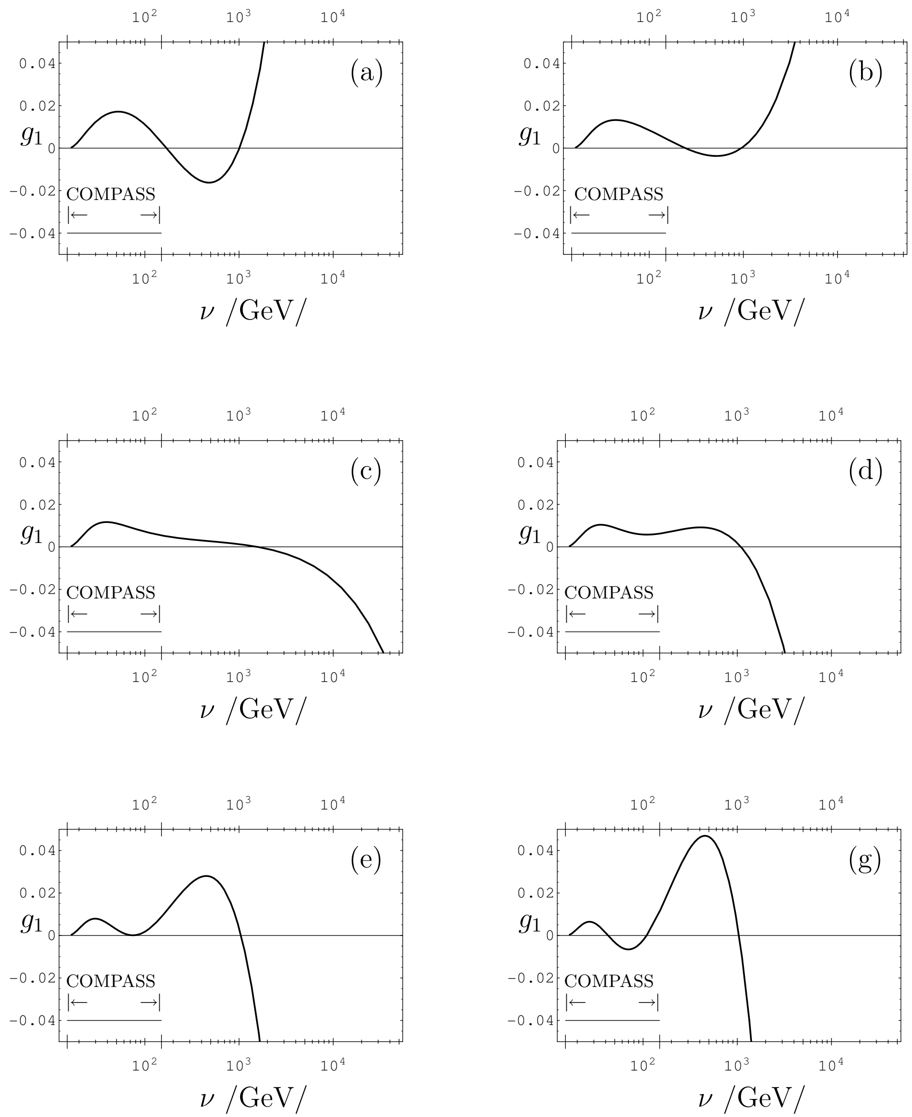
with all parameters >0, so there are no singularities in the fits. They are supposed to mimic the hadron structure

PROBLEM: in the COMPASS range of w, z is not small: 0.5 > z > 0.17, so non-logarithmic contributions to the coefficient functions are essential and must be accounted for. We do it in the one-loop approximation, like it is in NLO DGLAP

SUGGESTION: Let us choose the DGLAP-like fits for the initial parton densities, then play with parameters in them

(a),(b): $N_g<0$
(c): Ng = 0,
(d),(e),(g): $N_g>0$
(g): $N_g = 3.5$

Positive N_g better agree with COMPASS than negative



Power Corrections to non-singlet g_1

Leading twist contribution

mass scale: $Q^2 > M^2$

$$g_1(x,Q^2) = g_1^{LT}(x,Q^2)\left[1+\sum_k C_k\left(\frac{M^2}{Q^2}\right)^k\right]$$

Power corrections

PC are supposed to come from higher twists.
No satisfactory theory
is known for the higher twists

Standard way of obtaining PC from experimental
data at small x: Leader-Stamenov- Sidorov

Compare experimental data to predictions of the Standard Approach
and assign the discrepancy to the impact of PC

$$g_1^{LT} = g_1^{DGLAP}$$

Counter-argument:

1. DGLAP, the main ingredient of SA, is theoretically unreliable at small x, so comparing experiment to it is not so productive: it proves nothing
2. SA cannot explain why PC appear at $Q^2 > 1$ GeV^2 only and predict what happens at smaller Q^2

Our approach can do it:

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{\mathrm{w}}{\mu^2 + Q^2} \right)^{\omega} C(\omega)\, \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

where w = 2pq and Q^2 can be large or small, ❍ = 1 GeV

At $Q^2 > 1$ GeV^2 expansion into series is

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{w}{Q^2}\right)^{\omega} C(\omega)\,\delta q(\omega)\left(\frac{\mu^2}{Q^2}\right)^{H(\omega)} \left[1 + \sum_{k=1} T_k(\omega)\left(\frac{\mu^2}{Q^2}\right)^{k}\right]$$

Conventionally looking Power Corrections

Leading contribution for g_1^{NS}

These Power Corrections have perturbative origin and should be accounted in the first place. Only AFTER THAT one can reliable estimate a genuine impact of higher twist contributions

Conclusion

DGLAP is theoretically based for describing g_1 only in Region A: **large x and large Q^2**

Theoretical situation with description of spin-independent DIS is unclear: there is no model-independent alternatives to DGLAP. The most popular models include BFKL Pomeron, however applicability region for the Pomeron is unknown

Situation about Polarized DIS is more clear

Conventional extrapolating DGLAP into Region B (**small x and large Q^2**) is done with introducing singular fits, which has no theoretical grounds

Use DGLAP beyond its applicability region leads to many misconceptions:

LIST OF MOST SERIOUS MISCONCEPTIONS

Misconception: Standard fits mimic non-perturbative (basically unknown) physics

Actually: the singular factor in the fits mimic the lack of total resummation of ln(1/x) in DGLAP. Their only role is to give fast growth to g_1 at small x. They should be dropped when resummation is accounted for and therefore the fits are becoming simpler. It turns out that non-perturbative contributions are small at small x

Misconception: Total resummation of logs of x brings only small impact on the small-x behavior of g_1

Actually: It happens when both Resummation and Standard singular fits are used together. In this
In this case the same logs of x are accounted twice:
first implicitly through the fits and secondly explicitly trough Resummation. Besides, this approach predicts incorrect intercepts

Misconception: Conventional Q^2-corrections are believed to correspond to non-perturbative QCD, so they are attributed to higher twists.

Actually: At least a part of these corrections, if not all of them, have the perturbative origin. Impact of higher twists should be determined only after accounting for the perturbative Q^2-corrections

ALTERNATIVES TO DGLAP

Unpolarized DIS

At the moment there is no practical alternatives to DGLAP

Polarized DIS

The straightforward way to consider g_1 at small x and arbitrary Q^2 (beyond the DGLAP region) involves total resummation of leading logs of x and the shift $Q^2 \to Q^2 + \mu^2$

Combining those expressions and DGLAP formulae for anomalous dimensions and coefficient functions leads to universal description of g_1, however with much simpler fits for initial parton densities