# Renormalization-group anatomy of transverse-momentum dependent parton distribution functions in QCD

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## Publications

♣ "Wilson lines and transverse-momentum dependent parton distribution functions: A Renormalization-group analysis"
I.O. Cherednikov, N.G. Stefanis
Nucl. Phys. B 802 (2008) 146, arXiv:0802.2821 [hep-ph]

♣ "Renormalization, Wilson lines, and transverse-momentum dependent parton distribution functions"
I.O. Cherednikov, N.G. Stefanis
Phys. Rev. D 77 (2008) 094001, arXiv:0710.1955 [hep-ph]

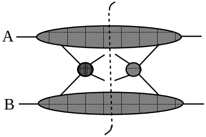
♣ Mandelstam-Leibbrandt gauge, arXiv:0904.2727 [hep-ph];
arXiv:0811.4357 [hep-ph]
arXiv:0811.0969 [hep-ph]
arXiv:0809.5235 [hep-ph]
arXiv:0809.1315 [hep-ph]
arXiv:0808.3390 [hep-ph]; arXiv:0711.1278 [hep-ph]

- Color gauge invariance of universal matrix elements, like parton distributions, fragmentation functions, etc., demands insertion of appropriate gauge links (Wilson lines).
- Gauge links can give rise to dynamical effects, like single-spin asymmetries (**SSA**), because they encode initial and/or final state interactions (Brodsky, Hwang, Schmidt).
- Applications considered by many authors, e.g., Boer, Boffi, Collins, Efremov, Feng, Ji, Hautmann, Ma, Metz, Mulders, Pasquini, Pijlman, Schweitzer, Teryaev, Wang, Yuan...
- Belitsky+Ji+Yuan (BJY) [NPB656(2003)165] have shown that transverse gauge link at light cone infinity indispensable for restoration of gauge invariance of transverse-momentum dependent (TMD) PDFs in light cone gauge.

- Cherednikov+Stefanis (CS) considered in PRD77 (2008) 094001 and NPB802 (2008) 146 interplay between gauge invariance and renormalization properties of TMD PDFs.
- It was shown at one-loop that in light-cone gauge with $q^-$-independent pole prescriptions, the anomalous dimension of TMD PDF contains contribution $\sim \ln p^+$, coinciding with universal cusp anomalous dimension (Korchemsky + Radyushkin, 1986) ensuing from renormalization effect on non-smooth junction point of transverse gauge contours.
- Multiplication rule for gauge links joined non-smoothly in transverse configuration space modified by extra eikonal phase.
- Origin of this phase similar to "intrinsic Coulomb phase" in QED (Jakob+Stefanis, Ann. Phys. (NY) 210 (1991) 112) and peculiar to decomposed contours extending to infinity.

- To compensate associated anomalous-dimension defect and recover known result in covariant gauges, modified definition of TMD PDF was proposed that contains soft counter term (Collins+Hautmann), i.e., eikonal factor along particular cusped gauge contour off-the-light-cone.
- Adopting $q^-$-dependent Mandelstam-Leibbrandt pole prescription in light-cone gauge (*"ML gauge"*), no cusp anomalous dimension appears and compensating soft factor reduces to unity.
- Evolution behavior of modified TM PDF, studied in connection with real-gluon contributions, found [Cherednikov + Stefanis, NPB802 (2008) 146] to be controlled by simple evolution equation with the same anomalous dimension as in covariant gauges.

Factorization of hadron collisions:




Cross section can be written as [modulo corrections $\sim (m/P_\perp)^n$]

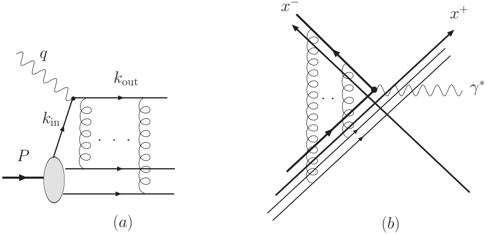
$$\frac{d\sigma}{dP_\perp} \sim \sum_{a,b} \int dx_{\rm A} f_{a/A}(x_{\rm A},\mu) \int dx_{\rm B} f_{b/B}(x_{\rm B},\mu) \frac{d\hat{\sigma}}{dP_\perp}$$

Parton-level cross section has expansion in powers of $\alpha_s$:

$$\frac{d\hat{\sigma}}{dP_\perp} \sim \sum_N \left[\frac{\alpha_s(\mu)}{\pi}\right]^N H_N(x_{\rm A}, x_{\rm B}, P_\perp; a, b; \mu)$$

Coefficients $H_N$ calculable in perturbative QCD

- Consider single parton distribution of quark with fractional longitudinal momentum **x** and flavor **i** in hadron **H** in DIS




(a) Feynman graph for DIS (b) Spacetime picture at amplitude level

$$f_{i/H}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi} \exp^{-ik^+\xi^-} \langle H(P)|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)\gamma^+\psi_i(0^-, \mathbf{0}_\perp)|H(P)\rangle$$

nonperturbative content separated by virtue of **factorization theorem**

- $k^+ = xP+$ with $x = Q^2/(2P\cdot q)$
- **thick line** in (b) denotes struck quark after hard collision with $\gamma^*$

## Kinematics

- Use null-plane coordinates with momentum of initial hadron

$$P^{\mu} = (P^{+}, P^{-}, \mathbf{P}_{\perp}) \ , \ P^{\pm} = (P^{0} \pm P^{3})/\sqrt{2} \ , \ P^{2} = 2P^{+}P^{-} - \mathbf{P}_{\perp}^{2}$$

- $P^{\mu} = n^{*\mu} + \frac{M^2}{2} n^{\mu}\ , \quad P^2 = M^2 \quad n^{*\mu} = \Omega(1, 1, \mathbf{0}_{\perp})\ , \quad n^{\mu} = \frac{1}{2\Omega}(1, -1, \mathbf{0}_{\perp})$
- $n^{*+} = \sqrt{2}\Omega\ , \ n^{*-} = 0\ , \ n^{+} = 0\ , \ n^{-} = \frac{1}{\sqrt{2}\Omega}\ , \quad n^{*}n = 1\ , \ (n^{*})^2 = n^2 = 0$
- Momentum of struck quark before being probed by photon is
  $k_{\rm in}^{\mu} = xP^{\mu} + k_{\perp}^{\mu} \quad \rightarrow \quad k_{\rm in}^{+} = xP^{+} = \sqrt{2}x\Omega$ [$\Omega$: arbitrary mass parameter]
- $k_{\rm in}^{-} = \frac{xM^2}{2\sqrt{2}\Omega}\ , \ k_{\perp}^{\mu} = (0^{+}, 0^{-}, \mathbf{k}_{\perp})\ , \ \mathbf{k}_{{\rm in}\perp} = \mathbf{k}_{\perp}$
- After interaction with highly virtual photon, struck quark has momentum
  $k_{\rm out}^{\mu} = (k_{\rm in}^{\mu} + q^{\mu}) = (x - x')n^{*\mu} + n^{\mu}(xx'M^2 + Q^2)/(2x')$
- whereas off-shell photon has momentum
  $q^{\mu} = -x'n^{*\mu} + \frac{Q^2}{2x'}n^{\mu} \quad \rightarrow \quad q^{+} = -\sqrt{2}x'\Omega\ , \quad q^{-} = Q^2/(2\sqrt{2}x'\Omega)$

In Lorentz frame with $\Omega = Q/(2x')$, one has almost lightlike hadron moving along $x^{+}$. In Bjorken limit $Q^2 \rightarrow \infty$, $x'$ and $x$ coincide up to $O(M^2/Q^2)$ terms.

• $q^{+} = -Q/\sqrt{2}, \quad q^{-} = Q/\sqrt{2}, \quad k^{+} = 0, \quad k^{-} = Q^2/2\sqrt{2}x\Omega = Q/\sqrt{2} = q^{-}$

To preserve gauge invariance, a *path-ordered* gauge link (Wilson line)

$$[\xi^-, 0^-] = \mathcal{P} \exp \left[ -ig \int_{0^-}^{\xi^-} dz^- A_a^+(0, z^-, \mathbf{0}_\perp) t_a \right]$$

has to be included, so that gauge-invariant integrated PDF of quark **i** in quark **a** reads

$$f_{i/a}(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} \exp^{-ik^+\xi^-} \langle P | \bar{\psi}_i(\xi^-, \mathbf{0}_\perp) \gamma^+ [\xi^-, 0^-] \psi_i(0^-, \mathbf{0}_\perp) | P \rangle$$

- If points 0 and $\xi$ **not** on purely light-like contour, then $[\xi^-, 0^-]$ **cannot be gauged away** by choosing light-cone gauge $\boxed{A^+ = 0}$
- Gauge link contributes anomalous dimension $\gamma_{\rm link}$ owing to its local obstructions: *endpoints, cusps, self-intersections*, i.e.,

$$\gamma_{\rm link} = \sum_{\rm i=obstructions} \gamma_i$$

- $f_{i/a}(x)$ gauge invariant, but implicitly gauge-contour dependent

## Splitting the gauge contour

Insert complete set of intermediate states and split $[\xi^-, 0^-]$ into two gauge links connecting points $0^-$ and $\xi^-$ through infinity, so that each of them can be associated with a quark field operator (Mandelstam fields):

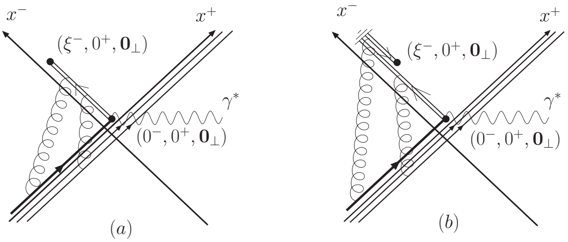
$$\Psi(x|\mathcal{C}_2) = \mathcal{P}\exp\left[-ig\int_{\infty[\mathcal{C}_2]}^{x} d\xi_\mu A_a^\mu(\xi)t_a\right]\psi(x) \qquad \textit{fermion}$$

$$\bar{\Psi}(x|\mathcal{C}_1) = \psi^\dagger(x)\mathcal{P}\exp\left[ig\int_{\infty[\mathcal{C}_1]}^{x} d\xi_\mu A_a^\mu(\xi)t_a\right]\gamma^0 \qquad \textit{antifermion}$$

Then, "eikonalized" quark PDF over split gauge contour reads

$$\begin{aligned} f_{i/a}^{\rm split}(x) &= \frac{1}{2}\sum_n \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-}\langle P|\bar{\psi}_i(\xi^-, \mathbf{0}_\perp)[\xi^-, \infty^-]^\dagger|n\rangle \\ &\quad \times\gamma^+\langle n|[\infty^-, 0^-]\psi_i(0^-, \mathbf{0}_\perp)|P\rangle \\ &= \frac{1}{2}\sum_n \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-}\langle P|\bar{\Psi}_i\left(\xi^-, \mathbf{0}_\perp|\mathcal{C}_1\right)|n\rangle\gamma^+\langle n|\Psi_i\left(0^-, \mathbf{0}_\perp|\mathcal{C}_2\right)|P\rangle \ , \end{aligned}$$

## Spacetime picture of DIS process with direct and split contour




- (a) direct contour (gauge link is a *connector*—Stefanis, 1984)
- (b) split contour—joined at light-cone infinity

★ Smooth connection of contours $\mathcal{C}_1$ and $\mathcal{C}_2$ entails trivial renormalization of junction point, ensuring validity of algebraic identity

$$[x_2, z \mid \mathcal{C}_1] \, [z, x_1 \mid \mathcal{C}_2] = [x_2, x_1 \mid \mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2]$$

## One-loop anomalous dimension of TMD PDF in light-cone gauge

Operator definition of (unpolarized) TMD distribution of quark with momentum $k_\mu = (k^+, k^-, \mathbf{k}_\perp)$ in quark with momentum $p_\mu = (p^+, p^-, \mathbf{0}_\perp)$ reads

$$f_{q/q}(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \, \mathrm{e}^{-ik^+\xi^-} \, \mathrm{e}^{+i\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp} \left\langle q(p) | \bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \right.$$
$$\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]$$
$$\times \left. \psi(0^-, \mathbf{0}_\perp) | q(p) \right\rangle \ |_{\xi^+=0} \ ,$$

$$[\infty^-, \mathbf{z}_\perp; z^-, \mathbf{z}_\perp] \equiv \mathcal{P} \exp \left[ ig \int_0^\infty d\tau \ n^-_\mu \ A^\mu_a t^a (z + n^- \tau) \right] \quad \text{lightlike link}$$

$$[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp] \equiv \mathcal{P} \exp \left[ ig \int_0^\infty d\tau \ \mathbf{l} \cdot \mathbf{A}_a t^a (\boldsymbol{\xi}_\perp + \mathbf{l}\tau) \right] \quad \text{transverse link}$$

Note that two-dimensional vector $\mathbf{l}$ arbitrary with no influence on (local) anomalous dimensions (drops out from final results).

## Light-cone gauge

$$A^+ = (A \cdot n^-) = 0 \ , \ (n^-)^2 = 0 \ ,$$

Gluon propagator has additional pole, related to plus light-cone component of gluon momentum:

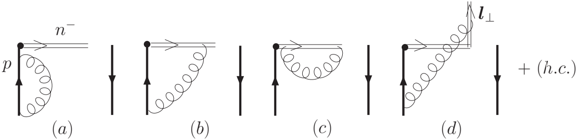
$$D_{\mu\nu}^{\rm LC}(q) = \frac{-i}{q^2 - \lambda^2 + i0}\left(g_{\mu\nu} - \frac{q_\mu n_\nu^- + q_\nu n_\mu^-}{[q^+]}\right)$$

Pole-prescription dependence appears (absent in integrated case)

$$\left.\frac{1}{[q^+]}\right|_{\rm Ret/Adv} = \frac{1}{q^+ \pm i\eta} \quad , \quad \left.\frac{1}{[q^+]}\right|_{\rm PV} = \frac{1}{2}\left[\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right] \ ,$$

- $\eta$ has dimension of mass and is kept small but finite
- Collinear poles controlled by quark virtuality $p^2 < 0$; IR singularities regularized by auxiliary gluon mass $\lambda$

## One-loop gluon radiative corrections




Remarks:

1) Curly lines: gluon contributions
2) Double lines: gauge links
3) Distance between quark fields spacelike, i.e., $\xi_\mu \xi^\mu = -\boldsymbol{\xi}_\perp^2 \neq 0$.
   Hence, UV-divergent contributions arise only from virtual gluon corrections
4) Diagrams (a) (b) and (c) contribute in covariant gauges
5) Diagrams (a) and (d) contribute in light-cone gauge
6) Diagram (d) associated with transverse gauge link
7) Hermitian-conjugate (*h.c.*) contributions are generated by corresponding "mirror" diagrams (not shown)

## Quark self-energy diagram (a) — $g_{\mu\nu}$ term

$$\Sigma^{(a)}(p, \alpha_s; \mu, \eta, \epsilon) = -g^2 C_{\mathrm{F}} \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \frac{\gamma_\mu(\hat{p} - \hat{q})\gamma_\nu}{(p-q)^2(q^2 - \lambda^2 + i0)} d_{\mathrm{LC}}^{\mu\nu}(q) \frac{i\hat{p}}{p^2}$$

with

$$d_{\mathrm{LC}}^{\mu\nu}(q) = g^{\mu\nu} - \frac{q^\mu (n^-)^\nu}{[q^+]} - \frac{q^\nu (n^-)^\mu}{[q^+]}$$

♣ Dependence on auxiliary mass scale $\eta$ "hidden" in pole prescription $[q^+]$

**$g^{\mu\nu}$ term** $\Rightarrow$ "Feynman"-like contribution:

$$\Sigma^{(a)}_{\mathrm{Feynman}}(p, \alpha_s, \mu, \epsilon) = -\frac{\alpha_s}{4\pi} C_{\mathrm{F}} \Gamma(\epsilon)(1-\epsilon) \left(-4\pi \frac{\mu^2}{p^2}\right)^\epsilon \times \left[1 + \epsilon\left(2 + \frac{\lambda^2}{p^2} \ln \frac{\lambda^2 - p^2}{\lambda^2} - \ln \frac{p^2 - \lambda^2}{p^2}\right) + O(\epsilon^2)\right]$$

♣ "Mirror" diagram gives precisely same contribution, doubling this result

## Quark self-energy diagram (a) — $[q^+]$ term

$$\Sigma_{\rm pole}^{(a)}(p,\alpha_s,\mu,\eta;\epsilon) = g^2 C_{\rm F}\mu^{2\epsilon}\int \frac{d^\omega q}{(2\pi)^\omega}\frac{1}{(p-q)^2(q^2-\lambda^2)} \times \left[\frac{\hat{q}(\hat{p}-\hat{q})\gamma^+}{[q^+]} + \frac{\gamma^+(\hat{p}-\hat{q})\hat{q}}{[q^+]}\right]\frac{i\hat{p}}{p^2}$$

♠ Depends on pole prescription applied to regularize light cone singularities

Noting that $\int \frac{d^\omega q}{(q^2-\lambda^2)[q^+]}$ vanishes, we find

$$\Sigma_{\rm pole}^{(a)} = g^2 C_{\rm F}\mu^{2\epsilon}(\hat{p}\gamma_\mu\gamma^+ + \gamma^+\gamma_\mu\hat{p})\left[p^\mu\sigma_1(p) + n^\mu\sigma_2(p)\right]\frac{i\hat{p}}{p^2} \ ,$$

where

$$\sigma_1(p) = \frac{i}{(4\pi)^{\omega/2}}\frac{\Gamma(\epsilon)}{(-p^2)^\epsilon}\int_0^1 dx\,\frac{(1-x)}{[xp^+]}\left[x(1-x)\left(1-\frac{\lambda^2}{xp^2}\right)\right]^{-\epsilon} \ ,$$

while term $\sigma_2(p)$ does not contribute by virtue of $\gamma^+\gamma^+ = (n^-)^2 = 0$

## Quark self-energy diagram (a) — $g^{\mu\nu}$ and $[q^+]$ term I

$$\left[\Sigma^{(a)}_{\rm Feynman} + \Sigma^{(a)}_{\rm pole}\right](p, \alpha_s, \mu, \eta, \epsilon) = \frac{\alpha_s}{4\pi} C_{\rm F} \left(-4\pi \frac{\mu^2}{p^2}\right)^{\epsilon} \Gamma(\epsilon) \left\{ (1-\epsilon) \times \left[1 + \epsilon\left(2 + \frac{\lambda^2}{p^2} \ln \frac{\lambda^2 - p^2}{\lambda^2} - \ln \frac{p^2 - \lambda^2}{p^2}\right)\right] - \frac{2\gamma^+ \hat{p}}{p^+} \int_0^1 dx \frac{(1-x)}{[x]} \left\{1 - \epsilon \ln \left[x(1-x)\left(1 - \frac{\lambda^2}{xp^2}\right)\right]\right\} + O(\epsilon^2) \right\}$$

To evaluate $\int dx(1-x)/[x]$, one has to use specific pole prescription for $[x]$.
Three different $q^-$-independent pole prescriptions considered ($\bar{\eta} = \eta/p^+$):

Retarded

$$\frac{1}{[x]_{\rm Ret}} = \frac{1}{x + i\bar{\eta}},$$

Advanced

$$\frac{1}{[x]_{\rm Adv}} = \frac{1}{x - i\bar{\eta}},$$

Principal Value:

$$\frac{1}{[x]_{\rm PV}} = \frac{1}{2}\left(\frac{1}{x + i\bar{\eta}} + \frac{1}{x - i\bar{\eta}}\right)$$

## Quark self-energy diagram (a) — $g^{\mu\nu}$ and $[q^+]$ term II

Taking the limit of small $\bar{\eta}$ and keeping only logarithmic terms,

♣ UV-divergent part (in $\overline{\mathrm{MS}}$-scheme) reads

$$\Sigma^{(a)}_{\mathrm{UV}} = -\frac{\alpha_s}{4\pi} C_{\mathrm{F}} \frac{1}{\epsilon} \left[ 1 - \ln 4\pi + \gamma_E - \frac{2\gamma^+ \hat{p}}{p^+} \left( 1 + \ln \frac{\eta}{\mathbf{p}^+} - \frac{i\pi}{2} - i\pi \mathbf{C}_\infty \right) \right]$$

♣ Finite part of pole-prescription dependent gluon radiative corrections is

$$\Sigma^{(a)}_{\mathrm{finite}}(p, \alpha_s, \mu, \eta, \epsilon) = -\frac{\alpha_s}{4\pi} C_{\mathrm{F}} \left( 1 + \ln \frac{\mu^2}{p^2} - \frac{2\gamma^+ \hat{p}}{p^+} \left\{ \left( 1 + \ln \frac{\eta}{\mathbf{p}^+} - \frac{i\pi}{2} - i\pi \mathbf{C}_\infty \right) \ln \frac{\mu^2}{p^2} \right.\right.$$
$$\left.\left. +2 + (1 \mp i\eta) \left[ \mathrm{Li}_2 \left( \frac{1}{\pm i\eta} \right) - \mathrm{Li}_2 \left( \frac{1}{1 \mp i\eta} \right) \right] \right\} \right)$$

where the pole prescription is contained in the factor

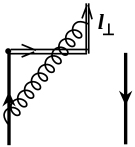
$$C_\infty = \begin{cases} 0 \ , & \text{Advanced} \quad \frac{1}{[q^+]} = \frac{1}{q^+ - i\eta} \\ -1 \ , & \text{Retarded} \quad \frac{1}{[q^+]} = \frac{1}{q^+ + i\eta} \\ -\frac{1}{2} \ , & \text{Principal Value} \quad \frac{1}{[q^+]} = \frac{1}{2} \left( \frac{1}{q^+ - i\eta} + \frac{1}{q^+ + i\eta} \right) \end{cases}$$

## Cancelation of pole-prescription dependent terms — I

♣ Show that contributions from interactions with gluon field of transverse gauge link

$$[\infty^-, \mathbf{l}_\perp \tau + \boldsymbol{\xi}_\perp)]^\dagger [\infty^-, \mathbf{l}_\perp \tau] = \mathcal{P} \exp\left[+ig \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\infty^-, 0^+; \mathbf{l}_\perp \tau + \boldsymbol{\xi}_\perp)\right] \times \mathcal{P} \exp\left[-ig \int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\infty^-, 0^+; \mathbf{l}_\perp \tau)\right]$$

in




cancel all terms proportional to $C_\infty$ in $\Sigma^{(a)}_{\mathrm{UV}}$ and $\Sigma^{(a)}_{\mathrm{finite}}(p, \alpha_s, \mu, \eta, \epsilon)$.

## Cancelation of pole-prescription dependent terms — II

Transverse components $\mu = i = 1, 2$ of gauge field in $A^+ = (A \cdot n^-) = 0$ gauge read

$$
\begin{aligned}
A^i_\perp(\xi) &= -g\ n^+_\nu \int \frac{d^4k}{2(2\pi)^4}\ \mathrm{e}^{-ik\cdot\xi}\ \tilde{D}^{\mu\nu}(k) \int dy^+ dy^- d^2y_\perp \mathrm{e}^{ik\cdot y}\delta(y^-)\delta^{(2)}(\mathbf{y}_\perp) \\
&= -\frac{1}{2}\, g \int \frac{dk^+}{2\pi}\ \frac{\mathrm{e}^{-ik^+\xi^-}}{[k^+]} \int \frac{d^2k_\perp}{(2\pi)^2}\, \frac{k^i_\perp}{\mathbf{k}^2_\perp}\, \mathrm{e}^{i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp}
\end{aligned}
$$

from which we find

$$
\int_0^\infty d\tau \mathbf{l}_\perp \cdot \mathbf{A}_\perp(\infty^-, 0^+; \mathbf{l}_\perp\tau) = \int \frac{dq^+}{2\pi} \mathrm{e}^{-iq^+\infty^-} \int \frac{d^2q_\perp}{(2\pi)^2}\, \mathbf{l}_\perp \cdot \mathbf{A}_\perp(q) \frac{i}{(\mathbf{q}_\perp \cdot \mathbf{l}_\perp) + i0}
$$

Propagator of longitudinal and transverse gluons reads

$$
\langle A^\mu(q) \mathbf{A}^i_\perp(q') \rangle = -\frac{q^i n^{-\mu}}{(q^2 - \lambda^2)[q^+]}(-i)(2\pi)^4\delta^{(4)}(q + q')
$$

## Cancelation of pole-prescription dependent terms — III

$$\Sigma_{\perp}^{(d)}(p,\mu,g;\epsilon) = g^2 C_{\mathrm{F}} \mu^{2\epsilon} 2\pi i C_{\infty} \int \frac{d^\omega q}{(2\pi)^\omega} \delta(q^+) \, \frac{\gamma^+(\hat{p}-\hat{q})}{(p-q)^2(q^2-\lambda^2)}$$

$$= i C_{\infty} \alpha_s C_{\mathrm{F}} \left(-4\pi \frac{\mu^2}{p^2}\right)^{\epsilon} \Gamma(\epsilon) \frac{\gamma^+ \hat{p}}{2p^+} \int_0^1 \frac{(1-x)\delta(x)}{[x(1-x)]^{\epsilon}} \left(1 - \frac{\lambda^2}{x p^2}\right)^{-\epsilon}$$

Notice that

- ♣ $$\frac{e^{-iq^+\infty^-}}{[q^+]} = 2\pi i C_{\infty} \delta(q^+)$$
- ♣ $$\frac{1}{[x]} = \lim_{\eta \to 0} \frac{1}{x \pm i\eta} = \mathrm{PV}\frac{1}{x} \mp i\pi\delta(x)$$

As a result, the complete UV-divergent part of TMD PDF $f_{q/q}(x,\mathbf{k}_\perp)$ becomes

$$\Sigma_{\mathrm{UV}}^{(a+d)}(p,\mu,\alpha_s;\epsilon) = -\frac{\alpha_s}{\pi} \, C_{\mathrm{F}} \frac{1}{\epsilon} \left[\frac{1}{4} - \frac{\gamma^+\hat{p}}{2p^+}\left(1 + \ln\frac{\eta}{p^+} - \frac{i\pi}{2} - i\pi \, C_{\infty} + i\pi C_{\infty}\right)\right]$$

$$= -\frac{\alpha_s}{\pi} \, C_{\mathrm{F}} \, \frac{1}{\epsilon} \left[1 - \frac{\gamma^+\hat{p}}{2p^+}\left(1 + \ln\frac{\eta}{p^+} - \frac{i\pi}{2}\right)\right]$$

To complete the argument, take into account

$$\frac{\gamma^+ \hat{p} \gamma^+}{2p^+} = \gamma^+$$

and recall that we have to include *mirror* ("right") counterparts of evaluated diagrams. These yield complex-conjugated contributions, so that imaginary terms mutually cancel. Hence, UV-divergent part of diagrams (a) and (d) contains only contributions due to $p^+$-dependent term:

$$\Sigma_{\mathrm{UV}}^{(\mathrm{a+d})}(\alpha_s, \epsilon) = 2\frac{\alpha_s}{\pi} C_{\mathrm{F}} \left[ \frac{1}{\epsilon} \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) - \gamma_E + \ln 4\pi \right]$$

thus, there is an extra anomalous dimension associated with $p^+$-dependent term which at one-loop level reads ($\gamma = \frac{\mu}{2} \frac{1}{Z} \frac{\partial \alpha_s}{\partial \mu} \frac{\partial Z}{\partial \alpha_s}$)

$$\boxed{\gamma_{1-\mathrm{loop}}^{\mathrm{LC}} = \frac{\alpha_s}{\pi} C_{\mathrm{F}} \left( \frac{3}{4} + \ln \frac{\eta}{p^+} \right) = \gamma_{\mathrm{smooth}} - \delta\gamma}$$

♣ $\delta\gamma$ induced by additional divergence to be compensated by suitable redefinition of TMD PDF

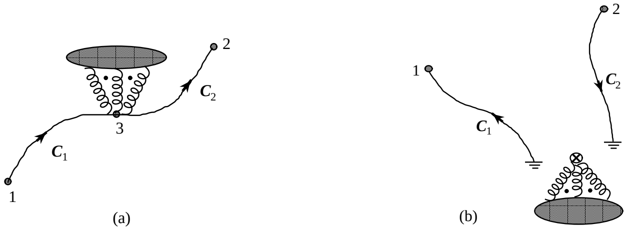
## Meaning of anomalous-dimensions defect

- In a covariant gauge, the gluon field vanishes at infinity ($\mathbf{A}_\perp = 0$); hence, anomalous dimensions ensuing from gauge link stem **only** from its endpoints that are joined by a smooth direct contour (a connector).
- In light cone gauge, $p^+ = (p \cdot n^-) \sim \cosh \chi$ defines angle $\chi$ between direction of quark momentum $p_\mu$ and lightlike vector $n^-$.
- In the large $\chi$ limit, one has $\ln p^+ \to \chi, \ \chi \to \infty$.
- As a result, "defect" of anomalous dimension, $\delta\gamma$, can be identified with well-known cusp anomalous dimension at one-loop order (Korchemsky + Radyushkin, 1986).

$$\gamma_{\rm cusp}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_{\rm F} \ (\chi \coth \chi - 1) \ ,$$

$$\frac{d}{d \ln p^+} \ \delta\gamma = \lim_{\chi \to \infty} \frac{d}{d\chi} \gamma_{\rm cusp}(\alpha_s, \chi) = \frac{\alpha_s}{\pi} C_{\rm F} \ .$$

## Renormalization effect on junction point of contours




♣ Renormalization effect on junction point due to gluon corrections illustrated by shaded oval with gluon lines attached to it

(a) Smoothly joined gauge contours $\mathcal{C}_1$ and $\mathcal{C}_2$ at point 3: $\gamma_{\mathcal{C}} = \gamma_{\mathcal{C}_1 \cup \mathcal{C}_2}$ .
(b) Contours joined by a cusp (indicated by the symbol $\otimes$) at infinite transverse distance (marked by the earth symbol) off the light cone:

$$\gamma_{\mathcal{C}} = \gamma_{\mathcal{C}_1^\infty \cup \mathcal{C}_2^\infty} + \gamma_{\text{cusp}} \quad \Longleftrightarrow \quad [2, 1|\mathcal{C}] = [2, \infty|\mathcal{C}_2^\infty]^\dagger [\infty, 1|\mathcal{C}_1^\infty] \mathrm{e}^{i\Phi_{\text{cusp}}}$$

## Soft counter term in TMD PDF — I

♣ To obtain in light-cone gauge the same gauge-invariant definition of $f_{q/q}(x, \mathbf{k}_\perp)$ as in covariant gauges, we have to dispense with anomalous-dimension artefact that has been generated by cusped-like junction point at transverse light-cone infinity. To achieve this goal, redefine TMD PDF by including soft counter term (Collins + Hautmann):

$$R \equiv \Phi(p^+, n^- | 0) \Phi^\dagger(p^+, n^- | \xi)$$

with eikonal factors given by

$$\Phi(p^+, n^- | 0) = \left\langle 0 \left| \mathcal{P} \exp \left[ ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^\mu \ t^a A^a_\mu(\zeta) \right] \right| 0 \right\rangle ,$$

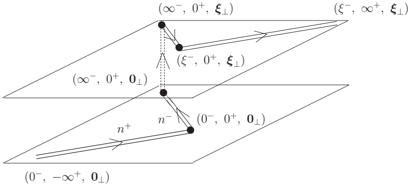
$$\Phi^\dagger(p^+, n^- | \xi) = \left\langle 0 \left| \mathcal{P} \exp \left[ - ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^\mu \ t^a A^a_\mu(\xi + \zeta) \right] \right| 0 \right\rangle$$

## Soft counter term in TMD PDF — I

Evaluate $R$ along the cusped integration contour ($n_\mu^-$: minus light-cone vector)

$$\mathcal{C}_{\rm cusp} : \zeta_\mu = \left\{ [p_\mu^+ s, -\infty < s < 0] \cup [n_\mu^- s', 0 < s' < \infty] \cup [\mathbf{l}_\perp \tau, 0 < \tau < \infty] \right\}$$

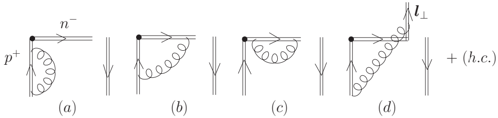
shown below




Jump in four-velocity $v_1 = p^+$ ($\|$ to plus light-cone ray) at origin to $v_2 = n^-$ (parallel to minus light-cone ray), creates angle-dependence via $(v_1 \cdot v_2) = p^+$. Hence, $\mathcal{C}$ **cusped** with angle $\chi \sim \ln p^+ = \ln(p \cdot n^-)$.

## Soft counter term in TMD PDF — II

♣ Calculate one-loop virtual gluon corrections contributing to UV divergences of $R$ in light-cone gauge from the diagrams




- Diagrams (a) and (d) give rise to anomalous dimension that compensates anomalous-dimensions defect generated by cusp-like junction point of contours.
- By virtue of light-cone gauge $A^+ = (n^- \cdot A) = 0$, diagrams (b) and (c) vanish.

## Soft counter term in TMD PDF — III

♣ $$\Phi_{\rm UV}^{(\rm a)}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - i\frac{\pi}{2} - i\pi C_\infty \right)$$

♣ $$\Phi^{(\rm d)} = -\alpha_s C_{\rm F} i\pi C_\infty \Gamma(\epsilon) \left( -4\pi \frac{\mu^2}{\lambda^2} \right)^\epsilon$$

Combining UV terms, we find

$$\Phi_{\rm UV}^{(\rm a+d)}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - i\frac{\pi}{2} - i\pi C_\infty + i\pi C_\infty \right) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{1}{\epsilon} \left( \ln \frac{\eta}{p^+} - i\frac{\pi}{2} \right)$$

Taking into account the *h.c.* ("mirror") contributions, we obtain total UV-divergent part of soft factor $R$ in one-loop order:

$$\Phi_{\rm UV}^{(1-\rm loop)}(\eta) = -\frac{\alpha_s}{\pi} C_{\rm F} \frac{2}{\epsilon} \ln \frac{\eta}{p^+}$$

♣ No dependence on pole prescription, (all $C_\infty$ dependent terms canceled) ✓
Only cusp-dependent term $\sim \ln p^+$ present yielding -$\gamma_{\rm cusp}$ ✓

We are now able to redefine the TMD PDF and get our **final result**

$$
\clubsuit \quad f_{q/q}^{\mathrm{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = \frac{1}{2} \int \frac{d\xi^- d^2 \boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp}
\times \Big\langle q(p) | \bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger
\times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]
\times [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]
\times \psi(0^-, \mathbf{0}_\perp) | q(p) \Big\rangle
\times \Big[ \Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \boldsymbol{\xi}_\perp) \Big]
$$

I.O. Cherednikov & N.G. Stefanis, PRD 77 (2008) 094001 [arXiv:0710.1955]; NPB 802 (2008) 146 [arXiv:0802.2821]

## Physical interpretation of soft counter term

Rewrite gauge link as

$$\begin{aligned} \Phi^{\dagger}(p^{+}, n^{-}|\xi) &= \left\langle 0 \middle| \mathcal{P} \exp\left[ - ig \int_{\mathcal{C}_{\rm cusp}} d\zeta^{\mu} \; t^{a} A^{a}_{\mu}(\xi + \zeta) \right] \middle| 0 \right\rangle \\ &= \exp\left[ \sum_{n=1}^{\infty} \alpha_s^{n} \Phi_{n}(u, n^{-}) \right] \end{aligned}$$

Leading term reads

$$\Phi_1(u, n^-) = -4\pi C_{\rm F} \int_{\mathcal{C}_{\rm cusp}} dx_{\mu} dy_{\nu} \theta(x-y) D^{\mu\nu}(x-y)$$

which by virtue of the current $j^{b}_{\nu}(z) = t^{b} v_{\nu} \int_{\mathcal{C}_{\rm cusp}} d\tau \delta^{(4)}(z - v\tau)$ assumes the form

$$\Phi_1(u, n^-) = -t^{a} 4\pi \int_{\mathcal{C}_{\rm cusp}} dx_{\mu} \int d^{4}z \delta^{ab} D^{\mu\nu}(x-z) j^{b}_{\nu}(z)$$

## Connection to intrinsic Coulomb phase

We make the following important observations

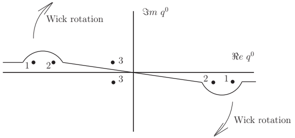
- Formal similarity of $\Phi_1(u, n^-)$ to "intrinsic" Coulomb phase found by Jakob + Stefanis [Ann. Phys. NY 210 (1991) 112] in QED for charged Mandelstam fields involving a timelike straight-line gauge contour.
- Phase is "intrinsic" because it exists even if external charge distributions absent. Reason is that each charged particle, though primordially separated from its counterpart, still in harness with it via a phase.
- In TMD PDF case, this phase accumulates effects due to interactions of struck quark with its target spectators [Belitsky, Ji, Yuan (2003)].
- Cusp-like junction point of two individual gauge contours plays similar role as so-called *"particle behind the moon"*. Both quantities *"hidden"* at infinity, revealing themselves only in terms of (path-dependent) phases, being independent of external charge distributions (QED case) and unrelated to boundary conditions to avert light-cone singularities (TMD PDF case in QCD).

## $q^-$-dependent Mandelstam-Leibbrandt pole prescription

Consider now light-cone gauge and apply Mandelstam-Leibbrandt pole prescription to gluon propagator (Mandelstam, 1983; Leibbrandt, 1984):

$$\frac{1}{[q^+]_{\mathrm{ML}}} = \begin{cases} \dfrac{1}{q^+ + i0q^-} \\ \dfrac{q^-}{q^+ q^- + i0} \end{cases} .$$

Both possible forms of this prescription are equivalent to each other. Prescription depends on both variables $q^+$ and $\mathbf{q}^-$ and has following pole structure in $(\mathbf{Re}q^0, \mathbf{Im}q^0)$ plane:



## Main advantages of ML-gauge

- Poles of gluon propagator (position 1) and those in a covariant gauge (position 2) belong to the same, i.e., second and fourth, quadrants. This is in contrast to the poles pertaining to the principal-value prescription (position 3).
- Thus Wick rotation can be performed without changing the position of the poles.
- Standard power counting rules can be applied to determine UV divergences.
- Quark self-energy and quark-quark-gluon vertex (Bassetto, 1996; Leibbrandt + Nyeo, 1984), and also DGLAP kernel in NLO (Bassetto et al., 1996), have no undesirable singularities.

## Transverse gauge field in ML-gauge

- Transverse gauge field at light-cone infinity in ML gauge

$$\frac{1}{[q^+]_{\rm ML}} = \begin{cases} \frac{1}{q^+ + i0 q^-} \\ \frac{q^-}{q^+ q^- + i0} \end{cases}$$

is a total transverse derivative, bearing no dependence on boundary conditions:

$$\mathbf{A}^\perp(\infty^-; \boldsymbol{\xi}_\perp) = -\frac{g}{4\pi} \boldsymbol{\nabla}^\perp \ln \, \Lambda |\boldsymbol{\xi}_\perp| \, .$$

- In $q^-$-independent pole prescriptions, one has instead

$$\mathbf{A}^\perp(\infty^-; \boldsymbol{\xi}_\perp) = \frac{g}{4\pi} C_\infty \, \boldsymbol{\nabla}^\perp \ln \, \Lambda |\boldsymbol{\xi}_\perp|$$

with

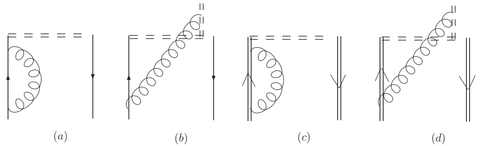
$$C_\infty = \begin{cases} 0 \, , & \text{Advanced} \\ -1 \, , & \text{Retarded} \\ -\frac{1}{2} \, , & \text{Principal Value} \end{cases} \, .$$

## RG analysis of TMD PDFs in ML-gauge I

Consider UV divergences of TMD PDF in LO of $\alpha_s$:

$$f_{q/q}^{\rm LO} = f^{(0)} + f^{(1)} + O(\alpha_s^2) \quad , \quad f^{(1)} = f^{(1)}_{\rm virt.} + f^{(1)}_{\rm real} \ ,$$

♣ Virtual gluon corrections derive from following diagrams



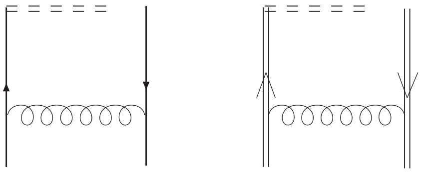

graphs in light-cone gauge

corresponding graphs from soft factor $R$

$$f^{(1)}_{\rm virt.} = \delta(1-x)\delta^{(2)}(\mathbf{k}_\perp)\ \Sigma^{(1)}_{\rm virt.}(p)\ \gamma^+ \ , \qquad \Sigma^{(1)}_{\rm virt.} = \Sigma^{(a)} + \Sigma^{(b)}$$

## RG analysis of TMD PDFs in ML-gauge II

Real gluon contributions to the TMD PDF in the light-cone gauge and using the ML pole prescription



$(a)$ $(b)$

Gluon propagator in ML gauge reads

$$d_{\mathrm{LC}}^{\mu\nu}(q) = g^{\mu\nu} - \frac{q^{\mu}(n^{-})^{\nu} + q^{\nu}(n^{-})^{\mu}}{[q^{+}]_{\mathrm{ML}}} \ .$$

## RG analysis of TMD PDFs in ML-gauge III. Quark self energy

Quark self-energy diagram (a) gives

$$\begin{aligned}\Sigma^{(a)}(p, \alpha_s; \mu, \epsilon) &= \Sigma^{(a)}_{\rm Feynman} + \Sigma^{(a)}_{\rm ML} \\ &= -g^2 C_{\rm F} \mu^{2\epsilon} \int \frac{d^\omega q}{(2\pi)^\omega} \frac{\gamma_\mu (\hat{p} - \hat{q}) \gamma_\nu}{(p-q)^2 (q^2 + i0)} \, d^{\mu\nu}_{\rm ML}(q) \frac{i \hat{p}}{p^2}\end{aligned}$$

Extracting the UV divergent terms in the $\overline{\rm MS}$-scheme, one gets (after adding the conjugated diagrams):

$$\Sigma^{\rm UV}_{(a)}(p, \alpha_s, \mu, \epsilon) = -\frac{\alpha_s}{4\pi} C_{\rm F} \left[ \frac{1}{\epsilon}(1-4) - \gamma_E + 4\pi \right] = -\frac{3\alpha_s}{4\pi} C_{\rm F} \left[ \frac{1}{\epsilon} - \gamma_E + 4\pi \right] .$$

♣ In ML-gauge, UV-divergent part of TMD PDF (and also the finite one) do not contain any extra terms of the form $\ln p^+$ — related to a cusped contour.

♣ No imaginary term as well (see below).

## RG analysis of TMD PDFs in ML-gauge. Transverse gauge link I

Composite transverse gauge link at light-cone infinity reads

$$
\begin{aligned}
[\mathbf{l}_\perp \tau + \boldsymbol{\xi}_\perp, \mathbf{l}_\perp \tau] \qquad &= \mathcal{P} \exp \left[ +ig \int_0^\infty d\tau \mathbf{l}^\perp \cdot \mathbf{A}^\perp (\infty^-, 0^+; \mathbf{l}_\perp \tau + \boldsymbol{\xi}_\perp) \right] \\
&\times \mathcal{P} \exp \left[ -ig \int_0^\infty d\tau \mathbf{l}^\perp \cdot \mathbf{A}^\perp (\infty^-, 0^+; \mathbf{l}_\perp \tau) \right]
\end{aligned}
$$

and corresponding graph (b) yields

$$
\Sigma^{(b)}_{\mathrm{ML}}(p, \mu, g; \epsilon) = -g^2 C_{\mathrm{F}} \mu^{2\epsilon} \int \frac{d^\omega q'}{(2\pi)^\omega} \int \frac{dq^+}{2\pi} \mathrm{e}^{-iq^+ \infty^-} \int \frac{d^2 q_\perp}{(2\pi)^2} \, \mathbf{l}^\perp \cdot \langle 0 | A^\mu(q) \mathbf{A}^\perp(q') | 0 \rangle
\times \frac{i}{(\mathbf{q}^\perp \cdot \mathbf{l}^\perp) + i0} \, \frac{\gamma^+ (\hat{p} - \hat{q})}{(p - q)^2} \, .
$$

Calculate correlator between longitudinal and transverse gluon fields:

$$\langle 0|A^{\mu}(q)\mathbf{A}^{\perp}(q')|0\rangle = -\frac{\mathbf{q}^{\perp}n^{-\mu}}{(q^2+i0)[q^+]_{\rm ML}}(-i)(2\pi)^4\delta^{(4)}(q+q')$$

using

$$\frac{1}{[q^+]_{\rm ML}} = \frac{q^-}{q^+q^-+i0} = q^-\left[\mathcal{P}\frac{1}{q^+q^-} - i\pi\delta(q^+q^-)\right].$$

Taking the sum of UV-divergent $(a)$ and $(b)$ contributions, we find

$$\Sigma_{\rm ML}^{(a+b){\rm UV}}(p,\mu,\alpha_s;\epsilon) = -\frac{\alpha_s}{\pi}\; C_{\rm F}\left\{\frac{1}{\epsilon}\left[\frac{1}{4} - \frac{\gamma^+\hat{p}}{2p^+}\left(1-\frac{i\pi}{2}\right)\right] - \gamma_E + 4\pi\right\}$$

which yields $(\gamma^+\hat{p}\gamma^+/2p^+ = \gamma^+)$

$$\Sigma_{\rm ML}^{(a+b){\rm UV}}(p,\mu,\alpha_s;\epsilon) = \frac{\alpha_s}{\pi}\; C_{\rm F}\;\left[\frac{1}{\epsilon}\left(\frac{3}{4}+\frac{i\pi}{2}\right) - \gamma_E + 4\pi\right]\;.$$

♣ **Imaginary term killed by mirror contribution of graph (b)**

## Evaluation of soft factor in ML gauge

Verify that proposed modified TMD PDF definition remains valid in ML gauge.

♣ Recall that soft factor introduced in order to remove extra UV divergences and associated anomalous dimension originating from cusped contour.

♣ In leading order, UV singularities of soft factor generated by self-energy of light-like gauge link and one-gluon exchanges between light-like and transverse gauge link (diagrams (c) and (d)):

$$\Phi_{\rm soft}^{\rm LO} = \Phi_{\rm soft}^{(0)} + \Phi_{\rm soft}^{(1)} + O(\alpha_s^2) \quad ,$$

$$\Phi_{\rm soft}^{(0)} = 1 \quad , \quad \Phi_{\rm soft}^{(1)} = \Phi_{\rm soft-virt}^{(1)} + \Phi_{\rm soft-real}^{(1)}$$

$$\Phi_{\rm soft-virt}^{(1)} = \Phi_{\rm soft-virt}^{(c)} + \Phi_{\rm soft-virt}^{(d)}$$

$$\Phi_{\rm soft-virt}^{(c)} = 2ig^2\mu^{2\epsilon} C_{\rm F} \int_0^\infty d\sigma \int_0^\sigma d\tau \int \frac{d^\omega q}{(2\pi)^\omega} \frac{{\rm e}^{-iq^-(\sigma-\tau)}}{2q^+q^- - \mathbf{q}_\perp^2 + i0} \frac{q^-}{q^+ + i0q^-} = 0$$

$$\Phi_{\rm soft-virt}^{(d){\rm ML}} = 0 \qquad [u_\mu = (p^+, 0^-, \mathbf{0}_\perp)]$$

Modified TMD PDF $f_{q/q}^{\rm mod}(x, \mathbf{k}_\perp; \mu, \eta)$ depends on two arbitrary mass-scale parameters:
(i) UV scale $\mu$ (physical resolving power) and
(ii) auxiliary regulator $\eta$ (of light cone singularities)

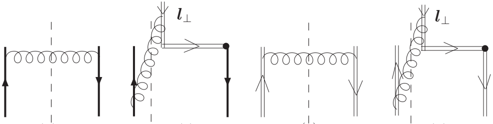
- $\mu$-dependence controlled by standard RG evolution equation driven by the anomalous dimension ($\gamma = \frac{\mu}{2}\frac{1}{Z}\frac{dZ}{d\mu}$)

$$\gamma_{f_{q/q}} = \gamma_{2q} + \sum_{i=1}^{4} \gamma^{i}_{\rm gauge\ link} + \gamma_{\rm R} = \frac{3}{4}\frac{\alpha_s}{\pi} C_{\rm F} + O(\alpha_s^2)$$

♣ $\gamma_{f_{q/q}}$ coincides with anomalous dimension of conventional quark propagator in light-cone gauge, but with opposite sign (different Dirac structure)
♣ Result also coincides with anomalous dimension of gauge-invariant quark propagator in covariant gauge (Craigie & Dorn; Stefanis).

## Evolution behavior with respect to scale $\eta$ — I

To derive the evolution equation with respect to $\eta$, we have to calculate the real-gluon contributions depicted below



(a) (b) (c) (d)

- Diagrams (b) and (d) do not contribute to TMD PDF in light cone gauge
- All diagrams UV finite, but $\eta$-dependent

Remark: Calculation performed in small-$\eta$ limit, which corresponds to large-rapidity $\zeta \to \infty$ limit in Collins-Soper approach $\left(\zeta = \frac{(2P\cdot n)^2}{|n^2|}\right)$

## Evolution behavior with respect to scale $\eta$ — II

**Diagram (a)**

"Feynman" ($\eta$-independent) term reads

$$\Sigma^{(a)\mathrm{real}}_{\mathrm{Feynman}} = \frac{\alpha_s}{2\pi^2}\, C_{\mathrm{F}}\, \frac{|1-x|}{p^+}\; \frac{\mathbf{k}_\perp^2 + x\lambda^2 + x(x-3)p^2}{\left[\mathbf{k}_\perp^2 + x\lambda^2 - x(1-x)p^2\right]^2}$$

$\eta$-dependence appears through pole-contributions:

$$\Sigma^{(a)\mathrm{real}}_{\mathrm{pole}} = \frac{\alpha_s}{\pi^2}\, C_{\mathrm{F}} \left\{ \left[ \frac{x}{(1-x)_+} - \delta(1-x)\ln\frac{\eta}{p^+} \right] \frac{1}{\mathbf{k}_\perp^2 + x\lambda^2 - x(1-x)p^2} \right\}$$

**Diagram (c)**

In small-$\eta$ limit, diagram $(c)$ yields

$$\Sigma^{(c)}_{\mathrm{real}} = \frac{\alpha_s}{2\pi^2}\, C_{\mathrm{F}}\, \delta(1-x) \frac{1}{\mathbf{k}_\perp^2 + \lambda^2} \left( 1 - \ln\frac{\eta}{p^+} \right)$$

## Evolution equation in ML gauge

RG properties of modified TMD PDF in ML gauge controlled by

$$\frac{1}{2}\mu\frac{d}{d\mu}\ f_{i/q}^{\rm mod}(x,\mathbf{k}_\perp;\mu)\ =\int d^2\mathbf{q}_\perp\ \int_x^1\frac{dz}{z}\ P_\perp\left(\frac{x}{z},\mathbf{q}_\perp,\alpha_s\right)\ f_{i/q}^{\rm mod}(z,\mathbf{q}_\perp,\mu)\ ,$$

$$P_\perp\left(y,\mathbf{q}_\perp,\alpha_s\right)\ =\gamma_{\rm ML}\ \delta(1-y)\ \delta^{(2)}(\mathbf{k}_\perp-\mathbf{q}_\perp)+O(\alpha_s^2)\ ,$$

♣ Anomalous dimension of modified TMD PDF coincides with anomalous dimension of standard TMD PDF, i.e.,

$$\gamma_{\rm ML}=-\frac{1}{2}\mu\frac{d}{d\mu}\ln\Sigma_{\rm ML}(\alpha_s,\epsilon)=\frac{3}{4}\frac{\alpha_s}{\pi}\ C_{\rm F}+O(\alpha_s^2)\ ,$$

$$\Sigma_{\rm ML}(\alpha_s;\epsilon)=\frac{\alpha_s}{\pi}\ C_{\rm F}\ \left[\frac{1}{\epsilon}\frac{3}{4}-\gamma_E+4\pi\right]$$

## Connection to Collins-Soper evolution equation

Starting with

$$\eta \frac{d}{d\eta} f_{q/q}^{\mathrm{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = \frac{\alpha_s}{\pi} C_{\mathrm{F}} \, \delta(1-x) \left[ \delta^{(2)}(\mathbf{k}_\perp) \left( \ln \frac{\mu^2}{p^2} - \ln \frac{\mu^2}{\lambda^2} \right) - \frac{1}{\pi} \left( \frac{1}{\mathbf{k}_\perp^2 + x\lambda^2 - x(1-x)p^2} + \frac{1}{\mathbf{k}_\perp^2 + \lambda^2} \right) \right] \phi_0(p)$$

we recast it in a form resembling the Collins-Soper (CS) evolution equation

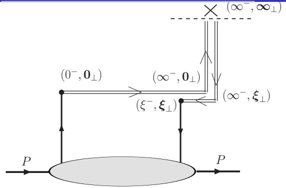
$$\eta \frac{d}{d\eta} f_{q/q}^{\mathrm{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = [\mathcal{K}(\mu) + \mathcal{G}(\mu, \eta)] \otimes f_{q/q}^{\mathrm{mod}}(x, \mathbf{k}_\perp; \mu, \eta)$$

♣ RG behavior of $\mathcal{K}$ and $\mathcal{G}$ determined by universal cups anomalous dimension:

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{K}(\mu) = -\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{G}(\mu, \eta) = \gamma_{\mathrm{cusp}} = \frac{\alpha_s}{\pi} C_{\mathrm{F}} + O(\alpha_s^2)$$

♣ Consistency condition $\mu \frac{d}{d\mu} \left[ \eta \frac{d}{d\eta} f_{q/q}^{\mathrm{mod}}(x, \mathbf{k}_\perp; \mu, \eta) \right] = 0$ ✓

- It was found that composite gauge contour through light cone infinity has a cusp-like junction point, revealed through the entailed anomalous dimension owing to the renormalization effect of gluon radiative corrections.




- Modified definition of TMD PDFs proposed containing soft counter term to dispense with junction-point anomalous-dimensions artifacts.
- Integrated PDF bears no artifacts of contour cusp obstruction and scale $\eta$.
- No contour artifacts appear in light-cone gauge with ML prescription.
- It was pointed out that cusp, lurking at light cone infinity, is origin of intrinsic Coulomb-like phase, as that created through primordial separation of electric charges in QED (Jakob & Stefanis, 1990).
- We found that for unpolarized TMD PDFs $\eta_{\rm SIDIS} \longrightarrow -\eta_{\rm DY}$, but $\gamma_{f_{q/q}}^{\rm SIDIS} = \gamma_{f_{q/q}}^{\rm DY}$, i.e., no sign change.