Lorentz invariance relations and Wandzura-Wilczek approximation

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Outline of the talk

Introduction

- Parton distribution functions (PDFs)
- Transverse momentum dependent PDFs (TMDs)

Unintegrated quark-quark correlator

- Definition and $\mathbf{p}_T$-dependent correlator
- Wilson line in the correlator
- Decomposition of correlator
- TMDs and unintegrated quark-quark correlator

Lorentz invariance relations and Wandzura-Wilczek approximation

- Lorentz invariance relations (LIRs)
- Wandzura-Wilczek (WW) approximation
- LIRs in generalized WW approximation
- Experimental test

Summary and outlook

References:

Metz, Schweitzer, Teckentrup, subm. to Phys. Lett. B, arXiv:0810.5340 [hep-ph]
Avakian, Efremov, Goeke, Metz, Schweitzer, Teckentrup, PRD **77** (2008) 014023

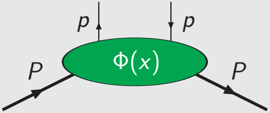
Introduction

- Parton distribution functions (PDFs)
- Transverse momentum dependent PDFs (TMDs)

Parton distribution functions (PDFs)

Definition of PDFs

PDFs appear in QCD description of hard inclusive reactions (e.g. DIS) and are defined by the **integrated correlator**:



longitudinal momentum fraction $x = \frac{p^+}{P^+}$

$$\Phi^{[\Gamma]}(x,S) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi} \langle P,S|\overline{\psi}_j(0)\,\Gamma\,\mathcal{W}_{\mathrm{PDF}}\,\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=\xi_T=0}$$

Correlator for **quarks** can be **parameterized** by **3 PDFs** in twist-2:

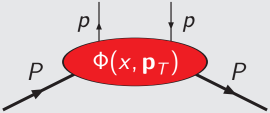
$$\Phi^{[\gamma^+]}(x,S) = f_1(x) \qquad \text{(unpolarized PDF)}$$
$$\Phi^{[\gamma^+\gamma_5]}(x,S) = \lambda\, g_1(x) \qquad \text{(helicity PDF)}$$
$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,S) = S_T^i\, h_1(x) \qquad \text{(transversity PDF)}$$

- **3 PDFs** in twist-3: e, g_T, h_L

Transverse momentum dependent PDFs (TMDs)

Definition of TMDs

TMDs appear in QCD description of hard semi-inclusive reactions (e.g. SIDIS, Drell-Yan) and are defined by the **unintegrated correlator**:



longitudinal momentum fraction $x = \frac{p^+}{P^+}$
transverse parton momentum $\mathbf{p}_T$

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_T}{(2\pi)^2} \, e^{ip\cdot\xi} \langle P, S | \overline{\psi}_j(0) \, \Gamma \, \mathcal{W}_{\text{TMD}} \, \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0}$$

Correlator for **unpol. quarks** can be **parameterized** by **2 TMDs** in twist-2:

$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\varepsilon_T^{ij} p_{Ti} S_{Tj}}{M} \, f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

$$f_1(x) = \int d^2\mathbf{p}_T \, f_1(x, \mathbf{p}_T^2)$$

- **8 TMDs** in twist-2: f_1, $f_{1T}^{\perp}$, g_{1L}, g_{1T}, h_{1T}, $h_{1L}^{\perp}$, $h_{1T}^{\perp}$, $h_1^{\perp}$
- **16 TMDs** in twist-3: e, g_T, h_L, . . .

Features of PDFs and TMDs

Higher twist PDFs and TMDs

- important **information** on **partonic structure** of nucleon
- **additional** to information encoded in **twist-2 PDFs**

Collinear twist-3 PDFs

accessible through spin asymmetries $\propto \frac{1}{Q}$ in

- polarized inclusive deep inelastic scattering (DIS) ($\longrightarrow g_T$)
- polarized Drell-Yan process ($\longrightarrow g_T$, h_L)

TMDs

accessible through spin and azimuthal asymmetries in

- semi-inclusive DIS (SIDIS)
- Drell-Yan process

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Unintegrated quark-quark correlator

- Definition and $\mathbf{p}_T$-dependent correlator
- Wilson line in the correlator
- Decomposition of correlator
- TMDs and unintegrated quark-quark correlator

Unintegrated quark-quark correlator

Definition

Fully **unintegrated quark-quark correlator** for spin-$\frac{1}{2}$ hadron:

$$\Phi_{ij}(P,p,S|n_-) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi}\, \langle P,S\,|\,\overline{\psi}_j(0)\,\mathcal{W}(0,\xi|n_-)\,\psi_i(\xi)\,|\,P,S\rangle$$

- target state:
  - four-momentum P: $P = P^+ n_+ + (M^2/2P^+)n_-,\;\; P^2 = M^2$
    (two light-like vectors n_+, n_-: $n_+^2 = n_-^2 = 0,\; n_+\cdot n_- = 1$)
  - covariant spin vector S: $S^2 = -1,\;\; P\cdot S = 0$
- quark momentum p

$\mathbf{p}_T$-dependent correlator

$\Rightarrow$ appears in QCD description of hard semi-inclusive reactions

$\Phi(x, \mathbf{p}_T, S) = \int dp^-\, \Phi(P,p,S|n_-)$

with longitudinal momentum fraction $x = \frac{p^+}{P^+}$

and transverse parton momentum $\mathbf{p}_T$

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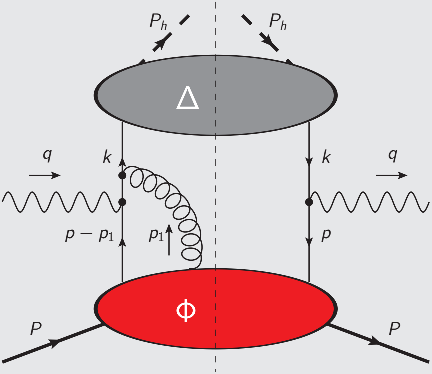
Wilson line in the correlator

Source of Wilson line $\mathcal{W}(0,\xi|n_-)$

Diagram for gluon exchange in SIDIS:



- take into account **gluon exchange** between hard and soft part $\Rightarrow$ **Wilson line**

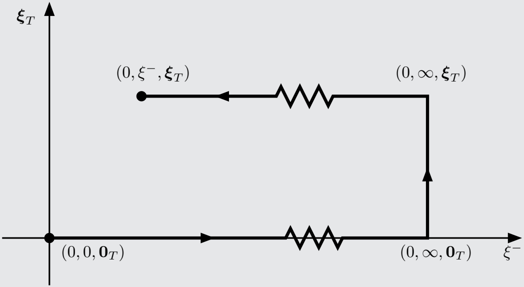
Wilson line in the correlator

Fixing and remarks

$\Rightarrow$ Wilson line $\mathcal{W}(0,\xi|n_-)$ ensures color gauge invariance

$$\mathcal{W}(0,\xi|n_-) = [0,0,\mathbf{0}_T;0,\infty,\mathbf{0}_T]\times[0,\infty,\mathbf{0}_T;\xi^+,\infty,\boldsymbol{\xi}_T]\times[\xi^+,\infty,\boldsymbol{\xi}_T;\xi^+,\xi^-,\boldsymbol{\xi}_T]$$

- $[a^+,a^-,\mathbf{a}_T;b^+,b^-,\mathbf{b}_T]$ denotes gauge link connecting points $a^\mu=(a^+,a^-,\mathbf{a}_T)$, $b^\mu=(b^+,b^-,\mathbf{b}_T)$ along a straight line



Wilson line in the correlator

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$$\mathcal{W}(0,\xi|n_-) =$$
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- $[a^+,a^-,\mathbf{a}_T;b^+,b^-,\mathbf{b}_T]$ denotes gauge link connecting points $a^\mu = (a^+,a^-,\mathbf{a}_T)$, $b^\mu = (b^+,b^-,\mathbf{b}_T)$ along a straight line
- contour not only depends on coordinates of initial and final points but also on light-cone direction n_-
- path chosen such that upon integration over p^- it leads to proper definition of $\mathbf{p}_T$-dependent correlator
- contour depends on process under consideration (here: SIDIS; but all arguments also valid for Drell-Yan)
- Wilson lines near the light-cone more appropriate in connection with unintegrated PDFs
  $\Rightarrow$ reasoning valid if one uses near light-cone direction

Decomposition of correlator

Constraints

- **hermicity**: $\Phi^\dagger(P,p,S|n_-) \;=\; \gamma_0\,\Phi(P,p,S|n_-)\,\gamma_0$
- **parity**: $\Phi(P,p,S|n_-) \;=\; \gamma_0\,\Phi(\bar{P},\bar{p},-\bar{S}|\bar{n}_-)\,\gamma_0$
- 16 possible 4×4-matrices: I, γ^μ, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$, $\gamma^\mu\gamma_5$, γ_5
- possible Lorentz structures: P^μ, p^μ, S^μ, n_-^μ
- $P^2 = M^2$, $P \cdot S = 0$, only linear in S

$\Rightarrow$ correlator parameterized in terms of A_i and B_i

- $A_i = A_i(p \cdot P, p^2)$, $B_i = B_i(p \cdot P, p^2)$
- B_i correspond to structures with n_-
- A_i and B_i are real

Decomposition of correlator

Decomposition

$$\Phi(P,p,S|n_-) \;=$$

$$MA_1 + \not{P}A_2 + \not{p}A_3 + \frac{i}{2M}\,[\not{P},\not{p}]\,A_4 + i(p\cdot S)\gamma_5\,A_5 + M\not{S}\gamma_5\,A_6 + \frac{p\cdot S}{M}\not{P}\gamma_5\,A_7 + \frac{p\cdot S}{M}\not{p}\gamma_5\,A_8$$

$$+\frac{[\not{P},\not{S}]}{2}\gamma_5\,A_9 + \frac{[\not{p},\not{S}]}{2}\gamma_5\,A_{10} + \frac{p\cdot S}{2M^2}[\not{P},\not{p}]\gamma_5\,A_{11} + \frac{1}{M}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho S_\sigma\,A_{12}$$

$$+\frac{M^2}{P\cdot n_-}\not{n}_-\,B_1 + \frac{iM}{2P\cdot n_-}[\not{P},\not{n}_-]\,B_2 + \frac{iM}{2P\cdot n_-}[\not{p},\not{n}_-]\,B_3 + \frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu p_\rho n_{-\sigma}\,B_4$$

$$+\frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}P_\mu p_\nu n_{-\rho}S_\sigma\,B_5 + \frac{iM^2}{P\cdot n_-}(n_-\cdot S)\gamma_5\,B_6 + \frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu n_{-\rho}S_\sigma\,B_7$$

$$+\frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu p_\nu n_{-\rho}S_\sigma\,B_8 + \frac{p\cdot S}{M(P\cdot n_-)}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma}\,B_9 + \frac{M(n_-\cdot S)}{(P\cdot n_-)^2}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu p_\rho n_{-\sigma}\,B_{10}$$

$$+\frac{M}{P\cdot n_-}(n_-\cdot S)\not{P}\gamma_5\,B_{11} + \frac{M}{P\cdot n_-}(n_-\cdot S)\not{p}\gamma_5\,B_{12} + \frac{M}{P\cdot n_-}(p\cdot S)\not{n}_-\gamma_5\,B_{13}$$

$$+\frac{M^3}{(P\cdot n_-)^2}(n_-\cdot S)\not{n}_-\gamma_5\,B_{14} + \frac{M^2}{2P\cdot n_-}[\not{n}_-,\not{S}]\gamma_5\,B_{15} + \frac{p\cdot S}{2P\cdot n_-}[\not{P},\not{n}_-]\gamma_5\,B_{16} + \frac{p\cdot S}{2P\cdot n_-}[\not{p},\not{n}_-]\gamma_5\,B_{17}$$

$$+\frac{n_-\cdot S}{2P\cdot n_-}[\not{P},\not{p}]\gamma_5 B_{18} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{P},\not{n}_-]\gamma_5 B_{19} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{p},\not{n}_-]\gamma_5 B_{20}$$

(Goeke, Metz, Pobylitsa, Polyakov, Phys. Lett. B **567** (2003) 27,
Bacchetta, Mulders, Pijlman, Phys. Lett. B **595** (2004) 309,
Goeke, Metz, Schlegel, Phys. Lett. B **618** (2005) 90)

Decomposition of correlator

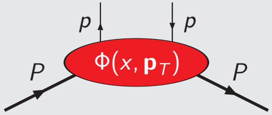
Structure

- **32 matrix structures** multiplied by scalar functions A_i and B_i
- **structures multiplied by A_i**:
  complete decomposition when the Wilson line is neglected, which is not allowed
  ($\Rightarrow$ sufficient parameterization for models without gluons)
- **B_i's** are associated with matrix structures containing light-like **vector n_-** (obviously quark-gluon-quark terms)
- introduction of various powers of target mass M
  $\Rightarrow$ same mass dimension for all scalar functions

TMDs and unintegrated quark-quark correlator

Definition of TMDs

TMDs appear in QCD description of hard semi-inclusive reactions (e.g. SIDIS, Drell-Yan) and are defined by the **unintegrated correlator**:



longitudinal momentum fraction $x = \frac{p^+}{P^+}$
transverse parton momentum $\mathbf{p}_T$

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) \equiv \frac{1}{2}\,\mathrm{Tr}\Big(\Phi(x, \mathbf{p}_T, S)\,\Gamma\Big)$$

$$= \frac{1}{2}\int \frac{d\xi^-}{2\pi}\frac{d^2\xi_T}{(2\pi)^2}\, e^{ip\cdot\xi}\langle P, S|\overline{\psi}_j(0)\,\Gamma\,\mathcal{W}_{\mathrm{TMD}}\,\psi_i(\xi)|P, S\rangle\Big|_{\xi^+=0}$$

with $\mathcal{W}_{\mathrm{TMD}} = \mathcal{W}(0, \xi|n_-)\Big|_{\xi^+=0}$

$\Rightarrow$ **TMDs** defined through **Dirac traces** of $\mathbf{p}_T$-dependent correlator

TMDs and unintegrated quark-quark correlator

Dirac traces

$$\Phi^{[\gamma^+]}(x,\mathbf{p}_T,S) = f_1(x,\mathbf{p}_T^2) - \frac{\epsilon_T^{ij}\, p_{Ti}\, S_{Tj}}{M}\, f_{1T}^{\perp}(x,\mathbf{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(x,\mathbf{p}_T,S) = \lambda\, g_{1L}(x,\mathbf{p}_T^2) + \frac{\mathbf{p}_T\cdot\mathbf{S}_T}{M}\, g_{1T}(x,\mathbf{p}_T^2)$$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,\mathbf{p}_T,S) = S_T^i\, h_1(x,\mathbf{p}_T^2) + \lambda\,\frac{p_T^i}{M}\, h_{1L}^{\perp}(x,\mathbf{p}_T^2) - \frac{p_T^i\, p_T^j + \frac{1}{2}\,\mathbf{p}_T^2\, g_T^{ij}}{M^2}\, S_{Tj}\, h_{1T}^{\perp}(x,\mathbf{p}_T^2) - \frac{\epsilon_T^{ij}\, p_{Tj}}{M}\, h_1^{\perp}(x,\mathbf{p}_T^2)$$

$$\Phi^{[1]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[e(x,\mathbf{p}_T^2) - \frac{\epsilon_T^{ij}\, p_{Ti}\, S_{Tj}}{M}\, e_T^{\perp}(x,\mathbf{p}_T^2)\right]$$

$$\Phi^{[i\gamma_5]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[\lambda\, e_L(x,\mathbf{p}_T^2) + \frac{\mathbf{p}_T\cdot\mathbf{S}_T}{M}\, e_T(x,\mathbf{p}_T^2)\right]$$

$$\Phi^{[\gamma^i]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[-\epsilon_T^{ij}\, S_{Tj}\, f_T(x,\mathbf{p}_T^2) - \lambda\,\frac{\epsilon_T^{ij}\, p_{Tj}}{M}\, f_L^{\perp}(x,\mathbf{p}_T^2) - \frac{p_T^i\, p_T^j + \frac{1}{2}\,\mathbf{p}_T^2\, g_T^{ij}}{M^2}\,\epsilon_{Tjk}\, S_T^k\, f_T^{\perp}(x,\mathbf{p}_T^2) + \frac{p_T^i}{M}\, f^{\perp}(x,\mathbf{p}_T^2)\right]$$

$$\Phi^{[\gamma^i\gamma_5]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[S_T^i\, g_T(x,\mathbf{p}_T^2) + \lambda\,\frac{p_T^i}{M}\, g_L^{\perp}(x,\mathbf{p}_T^2) - \frac{p_T^i\, p_T^j + \frac{1}{2}\,\mathbf{p}_T^2\, g_T^{ij}}{M^2}\, S_{Tj}\, g_T^{\perp}(x,\mathbf{p}_T^2) - \frac{\epsilon_T^{ij}\, p_{Tj}}{M}\, g^{\perp}(x,\mathbf{p}_T^2)\right]$$

$$\Phi^{[i\sigma^{ij}\gamma_5]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[-\epsilon_T^{ij}\, h(x,\mathbf{p}_T^2) + \frac{S_T^i\, p_T^j - p_T^i\, S_T^j}{M}\, h_T^{\perp}(x,\mathbf{p}_T^2)\right]$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}(x,\mathbf{p}_T,S) = \frac{M}{P^+}\left[\lambda\, h_L(x,\mathbf{p}_T^2) + \frac{\mathbf{p}_T\cdot\mathbf{S}_T}{M}\, h_T(x,\mathbf{p}_T^2)\right]$$

TMDs and unintegrated quark-quark correlator

TMDs expressed through A_i and B_i

8 TMDs in twist-2:

$$
\begin{aligned}
f_1(x,\mathbf{p}_T^2) &= 2P^+\int dp^-(A_2 + xA_3)\\
f_{1T}^{\perp}(x,\mathbf{p}_T^2) &= 2P^+\int dp^- A_{12}\\
g_{1L}(x,\mathbf{p}_T^2) &= 2P^+\int dp^-\Big(-A_6 - B_{11} - xB_{12}\\
&\qquad -\tfrac{P\cdot p - M^2 x}{M^2}(A_7 + xA_8)\Big)\\
g_{1T}(x,\mathbf{p}_T^2) &= 2P^+\int dp^-(A_7 + xA_8)\\
h_1(x,\mathbf{p}_T^2) &= 2P^+\int dp^-\Big(-A_9 - xA_{10} + \tfrac{\mathbf{p}_T^2}{2M^2}A_{11}\Big)\\
h_{1L}^{\perp}(x,\mathbf{p}_T^2) &= 2P^+\int dp^-\Big(A_{10} - \tfrac{P\cdot p - M^2 x}{M^2}A_{11} - B_{18}\Big)\\
h_{1T}^{\perp}(x,\mathbf{p}_T^2) &= 2P^+\int dp^- A_{11}\\
h_1^{\perp}(x,\mathbf{p}_T^2) &= 2P^+\int dp^-(-A_4)
\end{aligned}
$$

16 TMDs in twist-3: e, g_T, h_L, ...

Lorentz invariance relations and Wandzura-Wilczek approximation

- Lorentz invariance relations (LIRs)
- Wandzura-Wilczek (WW) approximation
- LIRs in generalized WW approximation
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Lorentz invariance relations (LIRs)

Consequences from unintegrated quark-quark correlator

- **8 TMDs** in twist-2 + **16 TMDs** in twist-3 + **8 TMDs** in twist-4 = **32 TMDs** in total for quarks
- **32** independent **amplitudes** A_i and B_i

$\Rightarrow$ numbers of TMDs and amplitudes **agree**

- all TMDs **linearly independent** structures

$\Rightarrow$ **no exact relations** among TMDs

Historic error

ignore of dependence on light-cone vector n_- (induced by Wilson line):

- **12** independent **amplitudes** $(A_1, \ldots, A_{12})$

$\Rightarrow$ number of TMDs **larger** than number of amplitudes A_i
$\Rightarrow$ rise to **LIRs**

(Mulders, Tangerman, Nucl. Phys. B **461** (1996) 197 [Erratum-ibid. B **484** (1997) 538])

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Lorentz invariance relations (LIRs)

LIRs

LIRs are wrong!

T-even:

$$g_T(x) = g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{(1)}(x)$$

$$h_L(x) = h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x)$$

T-odd:

$$f_T(x) = -\frac{\mathrm{d}}{\mathrm{d}x} f_{1T}^{\perp(1)}(x)$$

$$h(x) = -\frac{\mathrm{d}}{\mathrm{d}x} h_1^{\perp(1)}(x)$$

with $g_{1T}^{(1)}(x) = \int \mathrm{d}^2\mathbf{p}_T \, \frac{\mathbf{p}_T^2}{2M^2} \, g_{1T}(x, \mathbf{p}_T^2)$, etc.

Features

- relations: collinear twist-3 PDFs $\Leftrightarrow$ moments of TMDs
- PDFs on *lhs* of LIRs are twist-3
- PDFs and moments of TMDs on *rhs* of LIRs not suppressed in observables

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Lorentz invariance relations (LIRs)

LIRs are wrong

- first indication from a **model**
  (dressed quark target, light-front perturbative QCD):
  LIRs for T-even PDFs wrong
  (Kundu, Metz, Phys. Rev. D **65** (2002) 014009)
- **model independent** proof:
  LIRs are wrong in context of gauge theory for which a Wilson line is mandatory
  $\Rightarrow$ in presence of Wilson line (n_--dependence) LIRs are no longer fulfilled, they are spoiled by B_i
  (Goeke, Metz, Pobylitsa, Polyakov, Phys. Lett. B **567** (2003) 27)

Lorentz invariance relations (LIRs)

Example for Derivation of LIRs

$$h(x) = -\frac{\mathrm{d}}{\mathrm{d}x} h_1^{\perp(1)}(x)$$

$$\begin{aligned} h(x) &= 2P^+ \int \mathrm{d}p^- \mathrm{d}^2\mathbf{p}_T \left(\tfrac{P\cdot p - M^2 x}{M^2} A_4 + B_2 + xB_3\right) \\ &= \int \mathrm{d}\sigma \mathrm{d}\tau \mathrm{d}^2\mathbf{p}_T\, \delta(\tau - x\sigma + M^2x^2 + \mathbf{p}_T^2) \left(\tfrac{\sigma - 2M^2 x}{2M^2} A_4 + B_2 + xB_3\right) \\ &= \pi \int \mathrm{d}\sigma \mathrm{d}\tau\, \Theta(\tau - x\sigma + M^2x^2) \left(\tfrac{\sigma - 2M^2 x}{2M^2} A_4 + B_2 + xB_3\right) \end{aligned}$$

with $\sigma = 2p\cdot P$, $\tau = p^2$, $A_i = A_i(\sigma,\tau)$, $B_i = B_i(\sigma,\tau)$

$$\begin{aligned} h_1^{\perp(1)}(x) &= 2P^+ \int \mathrm{d}p^- \mathrm{d}^2\mathbf{p}_T\, \tfrac{\mathbf{p}_T^2}{2M^2} (-A_4) \\ &= \int \mathrm{d}\sigma \mathrm{d}\tau \mathrm{d}^2\mathbf{p}_T\, \tfrac{\mathbf{p}_T^2}{2M^2}\, \delta(\tau - x\sigma + M^2x^2 + \mathbf{p}_T^2)\, (-A_4) \end{aligned}$$

Lorentz invariance relations (LIRs)

Example for Derivation of LIRs

relation for solving integral:

$$\int \mathrm{d}\sigma \mathrm{d}\tau \mathrm{d}^2\mathbf{p}_T \, \frac{\mathbf{p}_T^2}{2M^2} \, \delta(\tau - x\sigma + M^2x^2 + \mathbf{p}_T^2) \, \mathcal{F}(x,\sigma,\tau)$$

$$= \quad -\pi \int_x^1 \mathrm{d}y \int \mathrm{d}\sigma \mathrm{d}\tau \, \Theta(\tau - y\sigma + M^2y^2) \left(\frac{\sigma - 2M^2y}{2M^2} \, \mathcal{F}(y,\sigma,\tau) - \frac{\tau - y\sigma + M^2y^2}{2M^2} \, \frac{\partial \mathcal{F}}{\partial y}(y,\sigma,\tau) \right)$$

$$h_1^{\perp(1)}(x) \quad = \quad -\pi \int_x^1 \mathrm{d}y \int \mathrm{d}\sigma \mathrm{d}\tau \, \Theta(\tau - y\sigma + M^2y^2) \left(\frac{\sigma - 2M^2y}{2M^2} \, (-A_4) \right)$$

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comparing with:

$$h(x) \; = \; \pi \int \mathrm{d}\sigma \mathrm{d}\tau \, \Theta(\tau - x\sigma + M^2x^2) \left(\frac{\sigma - 2M^2x}{2M^2} \, A_4 + B_2 + xB_3 \right)$$

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$$= \quad -\pi \int_x^1 \mathrm{d}y \int \mathrm{d}\sigma \mathrm{d}\tau \, \Theta(\tau - y\sigma + M^2 y^2)$$

$$\left(\frac{\sigma - 2M^2 y}{2M^2} \, \mathcal{F}(y,\sigma,\tau) - \frac{\tau - y\sigma + M^2y^2}{2M^2} \, \frac{\partial \mathcal{F}}{\partial y}(y,\sigma,\tau)\right)$$

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$\Rightarrow$ **LIR fulfilled if B_i's are absent**

Wandzura-Wilczek (WW) approximation

Relations between T-even PDFs

$$g_T(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + \tilde{g}'_T(x)$$
$$h_L(x) = 2x \int_x^1 \frac{dy}{y^2}\, h_1(y) + \tilde{h}'_L(x)$$

(Wandzura, Wilczek, Phys. Lett. B **72** (1977) 195;
Jaffe, Ji, Phys. Rev. Lett. **67** (1991) 552, Nucl. Phys. B **375** (1992) 527)

- $\tilde{g}'_T$ and $\tilde{h}'_L$: twist-3 quark-gluon-quark correlations and terms proportional to current quark masses
- isolation of "pure twist-3 terms" in PDFs g_T and h_L
- "working definition" of twist differs from the strict one

Twist

Strict definition:
twist of operator = mass dimension of operator - spin of operator

Working definition:
a PDF is of "twist t" if its contribution to the cross section is suppressed, in addition to kinematic factors, by $1/Q^{t-2}$

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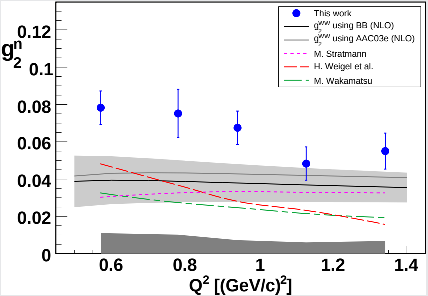
WW approximation for g_T

experimental observation:

$\tilde{g}'_T$ consistent with zero $\quad \Rightarrow \quad g_T(x) \overset{\text{WW}}{\approx} \int_x^1 \frac{dy}{y}\, g_1(y)$

- Jefferson Lab Hall A (Kramer *et al.*, Phys. Rev. Lett. **95** (2005) 142002) neutron spin structure function: $g_2^n(x) \equiv \sum_a e_a^2 (g_T^a(x) - g_1^a(x))$



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- SLAC E155 (Anthony *et al.*, Phys. Lett. B **553** (2003) 18)
  proton/deuteron spin structure function:
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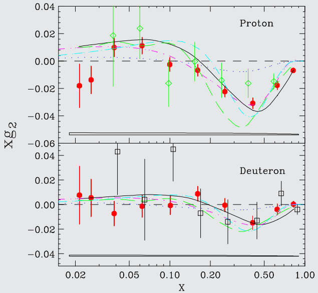

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- Jefferson Lab Hall A (Zheng *et al.* Phys. Rev. C **70** (2004) 065207)
- Jefferson Lab E94-010 (Amarian *et al.* Phys. Rev. Lett. **92** (2004) 022301)
- SLAC E155 (Anthony *et al.*, Phys. Lett. B **553** (2003) 18)
- SLAC E143 (Abe *et al.* , Phys. Rev. D **58** (1998) 112003)
- Spin Muon Collaboration (Adams *et al.*, Phys. Lett. B **336** (1994) 125)

numerical analysis:

numerical analysis of present data for DIS structure function g_2
$\Rightarrow$ violation of WW approximation of order 15 - 40%

(Accardi, Bacchetta, Melnitchouk, Schlegel, arXiv:0907.2942 [hep-ph])

Wandzura-Wilczek (WW) approximation

WW approximation for g_T

further support besides experiment:

- lattice QCD
  (Goeckler *et al.*, Phys. Rev. D **63** (2001) 074506; Phys. Rev. D **71** (2005) 054507)
- instanton model of QCD vacuum
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WW approximation for h_L

instanton model of QCD vacuum:

$$\tilde{h}'_L \text{ small} \quad \Rightarrow \quad h_L(x) \overset{\text{WW}}{\approx} 2x \int_x^1 \frac{dy}{y^2}\, h_1(y)$$

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$\Rightarrow$ no experimental test for this relation exists up to now

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WW-type approximation

QCD equations of motion (EOM)

$$\left.\begin{array}{lcl} g_{1T}^{(1)}(x) & \stackrel{\text{EOM}}{=} & x(g_T(x) - \tilde{g}_T(x)) - \frac{m}{M}\, h_1(x) \\ h_{1L}^{\perp(1)}(x) & \stackrel{\text{EOM}}{=} & -\frac{x}{2}(h_L(x) - \tilde{h}_L(x)) + \frac{m}{2M}\, g_{1L}(x) \end{array}\right\} \Rightarrow \textbf{exact}$$

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- in lightcone gauge:
  $\tilde{g}_T$ and $\tilde{h}_L$, like $\tilde{g}'_T$ and $\tilde{h}'_L$, represent matrix elements of the type $\langle|\bar{\Psi} A_T \Psi|\rangle$

WW-type approximation

- assumption: $\tilde{g}_T$ and $\tilde{h}_L$ are also small
- explicit form of $\tilde{g}_T$ ($\tilde{h}_L$) differs from the one of $\tilde{g}'_T$ ($\tilde{h}'_L$)

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neglect of tilde-functions and quark mass terms in EOM

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LIRs for T-even PDFs in generalized WW approx.

Measure for violation of LIRs

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) + \Delta_g(x)$$
$$h_L(x) = h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x) + \Delta_h(x)$$

LIRs in generalized WW approximation

use WW and WW-type approximation:

$$\Delta_g(x) \overset{\text{WW,WW-type}}{\approx} -g_1(x) - x\,\frac{d}{dx}\int_x^1 \frac{dy}{y}\, g_1(y) = 0$$
$$\Delta_h(x) \overset{\text{WW,WW-type}}{\approx} -h_1(x) - x^2\,\frac{d}{dx}\int_x^1 \frac{dy}{y^2}\, h_1(y) = 0$$

- LIRs for T-even PDFs **work** in generalized WW approximation
- not entirely surprising because violation of LIRs originates from B_i's which are related to quark-gluon-quark correlations

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Experimental test

Generalized WW approximation

instead of 3 functions g_1, g_T, $g_{1T}^{(1)}$ only one independent PDF:

$$g_{1T}^{(1)}(x) \overset{\text{WW,WW}-\text{type}}{\approx} x \int_x^1 \frac{dy}{y}\, g_1(y)$$

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Experimental test of generalized WW approximation

generalized WW approximations can be tested by **single/double spin asymmetries** (SSA/DSA) in SIDIS:

DSA: $A_{LT}^{\cos(\phi-\phi_S)} \propto g_{1T}^{(1)}\ D_1 \Rightarrow$ prel. COMPASS data (Kotzinian *et al.*, PRD **73** (2006) 114017)

SSA: $A_{UL}^{\sin 2\phi} \propto h_{1L}^{\perp(1)}\ H_1^{\perp} \Rightarrow$ HERMES, prel. CLAS

D_1 and $H_1^{\perp}$: fragmentation functions (FF)

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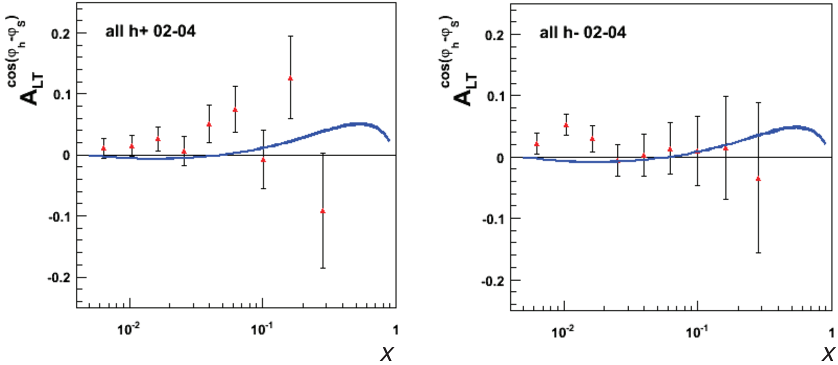
Experimental test with $A_{LT}^{\cos(\phi-\phi_S)}$ at COMPASS

$\mu d \to \mu h^+ X$ $\mu d \to \mu h^- X$



(prel. COMPASS data: Kotzinian, arXiv:0705.2402 [hep-ex]
predictions: Kotzinian, Parsamyan, Prokudin, Phys. Rev. D **73** (2006) 114017)

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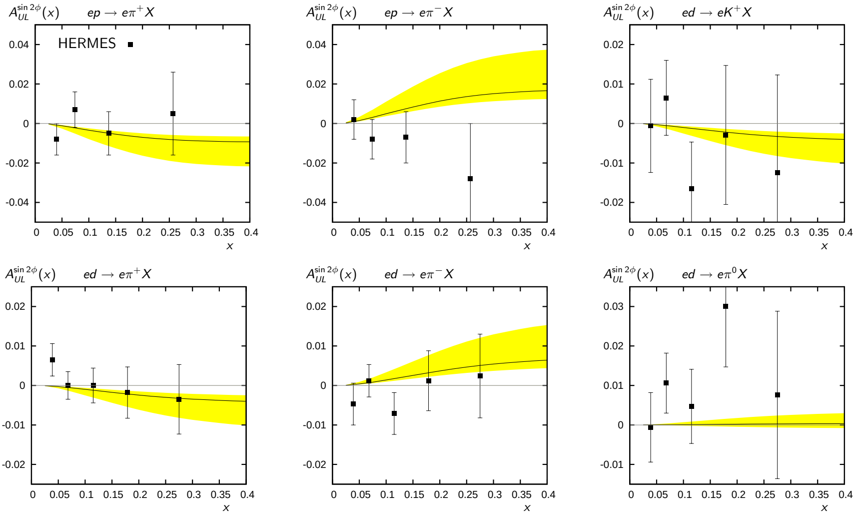
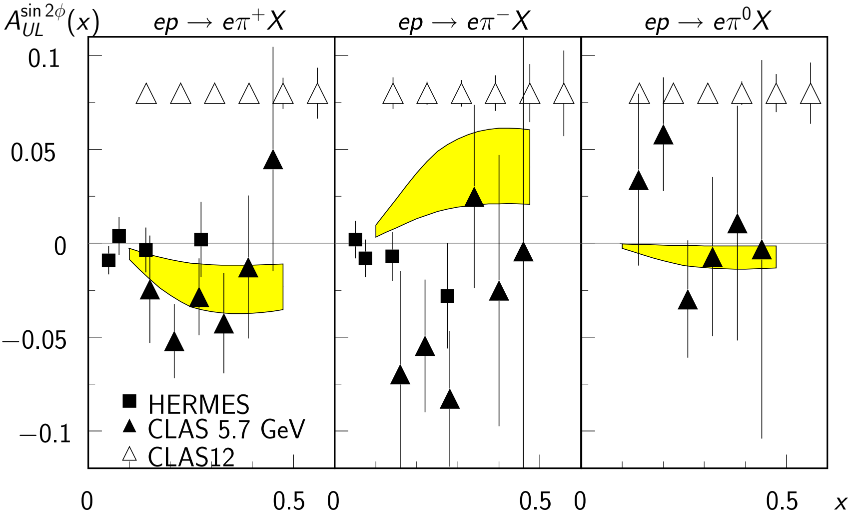


(HERMES data: Airapetian *et al.*, Phys. Rev. Lett. **84** (2000) 4047; Phys. Lett. B **562** (2003) 182)

Experimental test with $A_{UL}^{\sin 2\phi}$ at CLAS



(prel. CLAS 5.7 GeV data: Avakian, Bosted, Burkert, Elouadrhiri, AIP Conf. Proc. **792** (2005) 945)

LIRs for T-odd PDFs in WW-type approximation

Time-reversal invariance

$$f_T(x) = \int d^2\mathbf{p}_T\, f_T(x,\mathbf{p}_T^2) = 0$$

$$h(x) = \int d^2\mathbf{p}_T\, h(x,\mathbf{p}_T^2) = 0$$

$\Rightarrow$ integrated T-odd PDFs f_T and h **vanish**

LIRs for T-odd PDFs

$$\frac{d}{dx} f_{1T}^{\perp(1)}(x) \overset{\text{LIR}}{=} 0 \quad \Rightarrow \quad f_{1T}^{\perp(1)}(x) \overset{\text{LIR}}{=} 0$$

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- $\tilde{f}_T$ and $\tilde{h}$: quark-gluon-quark correlations

WW-type approximation

neglect of $\tilde{f}_T$ and $\tilde{h}$

$$\overset{\text{EOM}}{\Rightarrow} \quad f_{1T}^{\perp(1)}(x) \overset{\text{WW-type}}{\approx} 0, \quad h_1^{\perp(1)}(x) \overset{\text{WW-type}}{\approx} 0$$

LIRs in WW-type approximation

results from WW-type approximation consistent with that from LIRs
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LIRs in WW-type approximation

results from WW-type approximation consistent with that from LIRs

⇒ LIRs for T-odd PDFs **work** in WW-type approximation

Experimental test

Conflict

(1)-moment of Sivers function **vanishes** in **WW-type approx.**
$\Rightarrow$ conflict with **experimental results** because Sivers effect seems **non zero**

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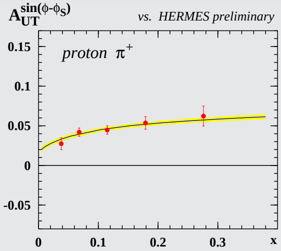
HERMES data (prel.)



(data: Diefenthaler, arXiv:0706.2242 [hep-ex]
fit: Arnold *et al.*, arXiv:0805.2137 [hep-ph])
⇒ Sivers effect small

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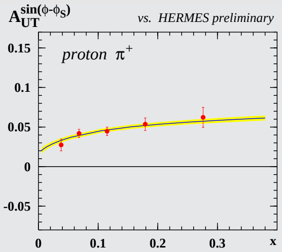

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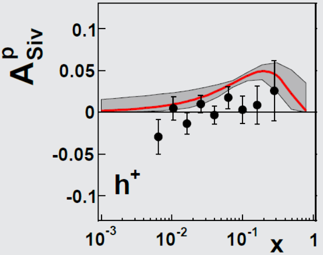
COMPASS data (prel.)



(data: Wollny, arXiv:0902.0519 [hep-ex]
fit: M. Anselmino *et al.*, Eur. Phys. J. A **39** (2009) 89)

$\Rightarrow$ Sivers effect compatible with zero

Summary and outlook

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- LIRs are **wrong** in general
- **Generalized WW approximation**: neglect of quark-gluon-quark correlations and current quark masses
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- Approximation **beyond** standard WW approximation
- Experimental evidence: generalized WW approx. works **well**
- LIRs **helpful tool** to study unknown PDFs

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- Find out **how good** generalized WW approx. and LIRs work
- Further **experimental test** at HERMES, COMPASS and Jefferson Lab

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- Stephan Meißner (Ruhr-Universität Bochum, Germany)
- Andreas Metz (Temple University, Philadelphia, USA)
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Thank you for your attention!