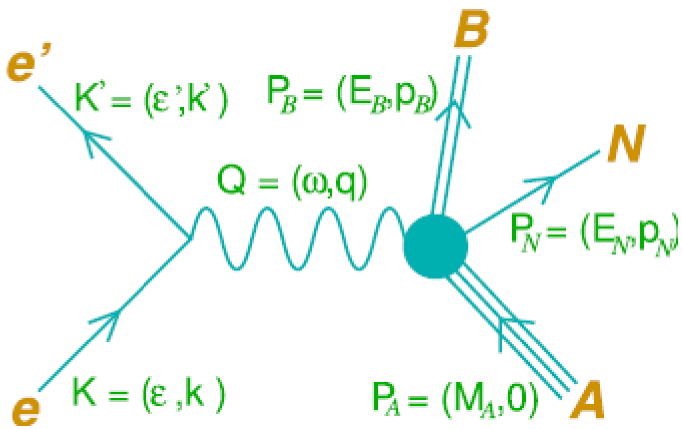
A Model for BCS-type Correlations in Superscaling

R. Cenni, T.W. Donnelly, A. Molinari, MBB Phys. Rev. C 78, 024602 (2008)

- Motivations
  - NN correlations in a relativistic framework
  - Role of correlations in breaking superscaling

- Outline
  - Longitudinal response function in the RFG model
  - The “BCS-inspired” model: Spectral Function
  - The BCS superscaling function
  - Scaling of first and second kinds
  - Possible improvements of the model

The Longitudinal Response Function



- In PWIA

$$R_L(q,\omega) = \frac{2\pi m^2}{q} \iint_{\Sigma} dp\, d\mathcal{E}\, \frac{p}{E_p} S(p,\mathcal{E}) \mathcal{R}_L(q,\omega,p,\mathcal{E})$$

- Scaling function

$$F(q,\omega) = \frac{2\pi m^2}{q} \iint_{\Sigma(q,\omega)} dp\, d\mathcal{E}\, \frac{p}{E_p} S(p,\mathcal{E})$$

- Spectral Function (quantum probability that the residual nucleus is left in a state of given momentum and excitation energy)

$$S(p,\mathcal{E}) = < \Psi_0 | a^{\dagger}_{\mathbf{p}\uparrow} \delta(\mathcal{E} - (\hat{H} - E_0^{(A-1)})) a_{\mathbf{p}\uparrow} | \Psi_0 > \frac{V_A}{A(2\pi)^3} \qquad \int d\mathbf{p}\, d\mathcal{E}\, S(p,\mathcal{E}) = 1$$

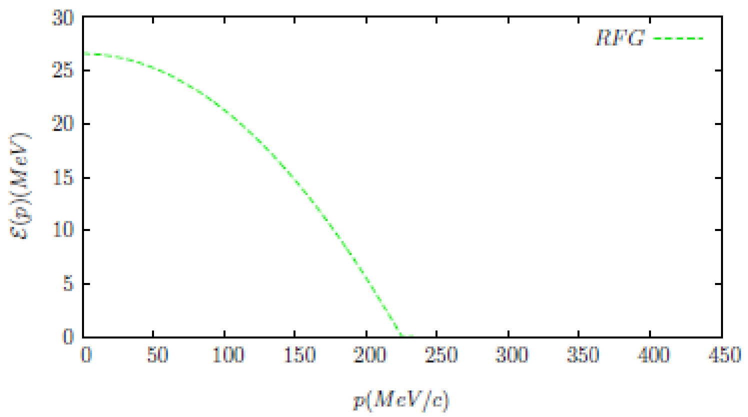
- Single nucleon response

$$\mathcal{R}_L = \frac{q^2}{|Q^2|} \left\{ G_E^2(\tau) + W_2(\tau) \frac{p^2}{m^2} \sin^2\theta \right\}$$

Relativistic Fermi Gas

- Free nucleons descriped by Dirac spinors
- Lorentz covariant and gauge invariant
- Spectral function:

$$S^{RFG}(p,\mathcal{E}) = \theta(k_F - p)\delta(\mathcal{E} - T_F + T_{\rm p})\frac{V_A}{A(2\pi)^3}$$



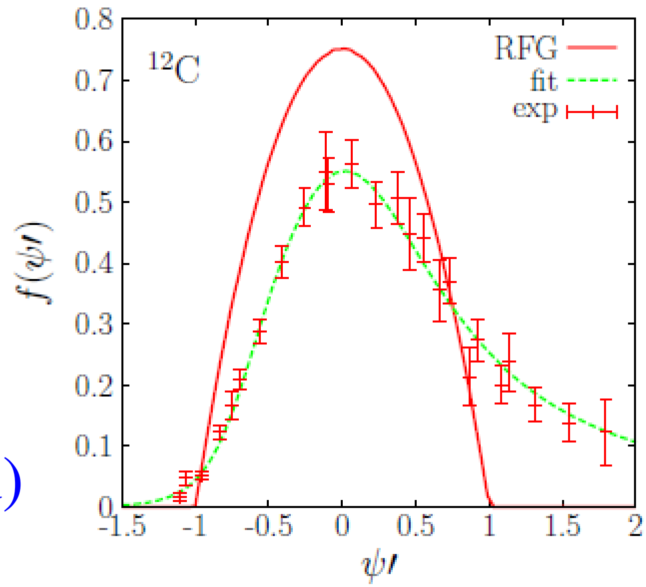
- Scaling function:

$$F(q,\omega) = \underbrace{\left(\frac{2mT_F}{k_F^2}\right)}_{\text{small}}\left(\frac{m}{q}\right)\left(\frac{1}{k_F}\right)\underbrace{\frac{3}{4}\left(1-\psi^2\right)}_{f(\psi)}$$

- The superscaling function $f(\psi) = \frac{3}{4}(1-\psi^2)$ is symmetric in the scaling variable

$$\psi = \frac{1}{\sqrt{\xi_F}}\frac{\lambda-\tau}{\sqrt{(1+\lambda)\tau+\kappa\sqrt{\tau(1+\tau)}}}$$

- q-independent (I kind)
- k_F-independent (II kind)



The BCS-inspired model: ground state

- How can the RFG be extended to include NN correlations?
- Ground state: BCS correlated nuclear wave function

$$|\Phi>=\prod_k (u_k + v_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow})|0> \quad \text{with} \quad |u_k|^2+|v_k|^2=1 \Rightarrow <\Phi|\Phi>=1$$

- The number of particles is not fixed, since the g.s. is not an eigenstate of

$$\hat{n}(k)=\sum_s a^\dagger_{ks} a_{ks}$$

- However we can require that the average number is fixed:

$$\sum_k <\Phi|\hat{n}(k)|\Phi>=\sum_k |v_k|^2 = A$$
$$\stackrel{T.L.}{\Rightarrow} \int \frac{d\mathbf{k}}{(2\pi)^3}|v(k)|^2=\frac{A}{V_A}=\rho_A$$

BCS-inspired model: residual nucleus

- Daughter nucleus: $|D(p)>=\frac{1}{|v'_p(p)|}a_{p\uparrow}\prod_k[u'_k(p)+v'_k(p)a^\dagger_{k\uparrow}a^\dagger_{-k\downarrow}]|0>$

  where u', and v' differ in general from u and v

- Momentum distribution: $n_{D(p)}(k)=<D(p)|\sum_s a^\dagger_{ks}a_{ks}|D(p)>=|v'_k(p)|^2(1-\delta_{kp})$

- Averaged particle number constraint:

$$\sum_k n_{D(p)}(k)=\sum_{k\neq q}|v'_k(p)|^2=A-1$$
$$\stackrel{T.L.}{\Rightarrow}\int\frac{d\mathbf{k}}{(2\pi)^3}|v'(k;p)|^2=\frac{A-1}{V_{A-1}}=\rho_{A-1}$$

- After taking the thermodynamic limit:

$$\rho_A=\rho_{A-1}\Rightarrow v'(k;p)=v(k)$$

The wave-function's coefficients

- We assume the following parameterization:

$$v^2(k) = \frac{c}{e^{\beta(k-\tilde{k})}+1}$$

- The parameter c is fixed by normalization:

$$\int \frac{\mathrm{d}k}{(2\pi)^3}|v(k)|^2 = \frac{A}{V_A} = \rho_A \Rightarrow c(\beta,\tilde{k}) = -\frac{\pi^2\beta^3\rho_A}{Li_3\left(-e^{\beta\tilde{k}}\right)}$$

- The parameter $\tilde{k}$ is related to k_F by the stability condition:

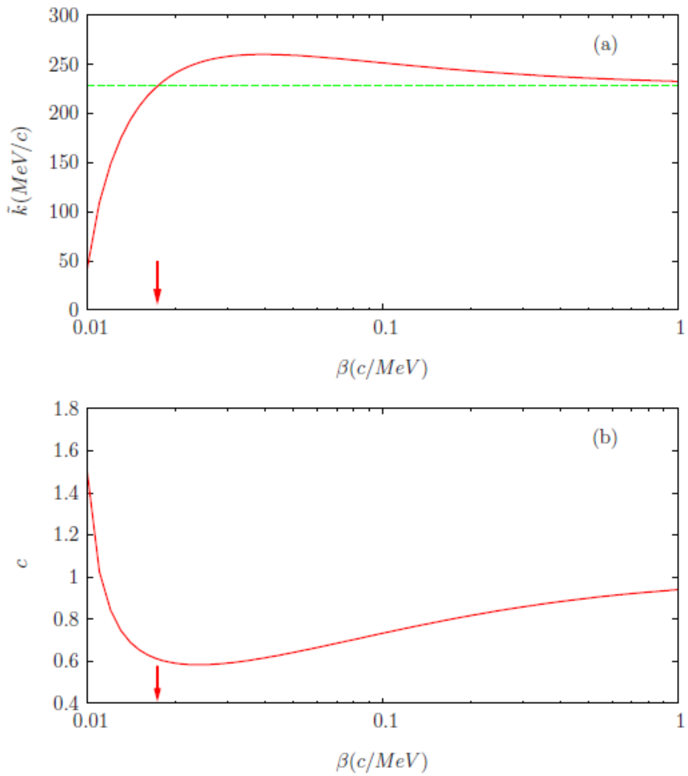
$$\frac{dE_{D(p)}}{dp}\bigg|_{p=k_F} = 0 \Rightarrow \tilde{k} = k_F + \frac{1}{\beta}\log\left[\frac{\beta}{k_F}T_F\left(T_F+m\right)-1\right]$$

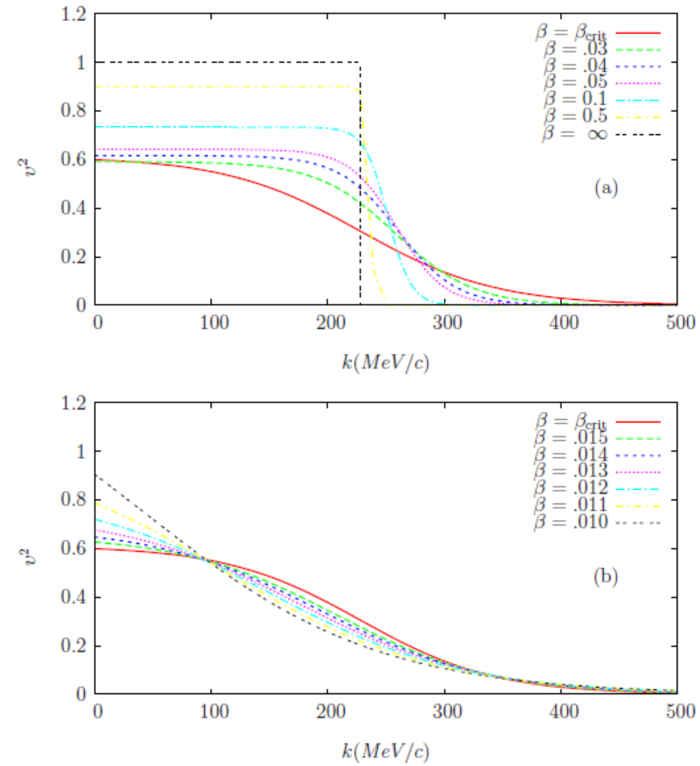
- β , controlling the tail of the Fermi distribution, is the only free parameter of the model. A critical value corresponds to the log change of sign

$$\beta_{\mathrm{crit}} = \frac{2k_F}{T_F(T_F+m)} \simeq 0.0175\, c/MeV$$

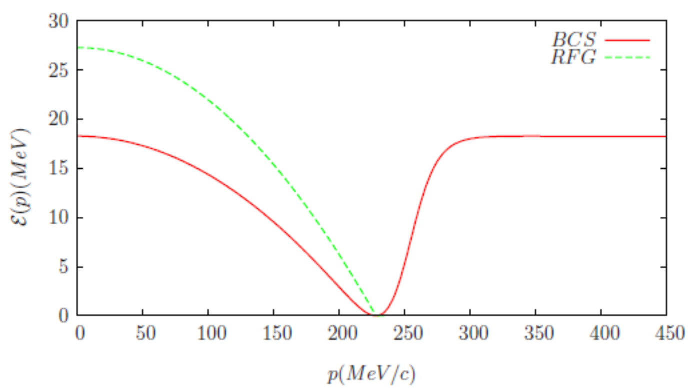
- Below the critical value the system becomes strongly disrupted by correlations

Results





Support of the spectral function:



The BCS-like Superscaling Function

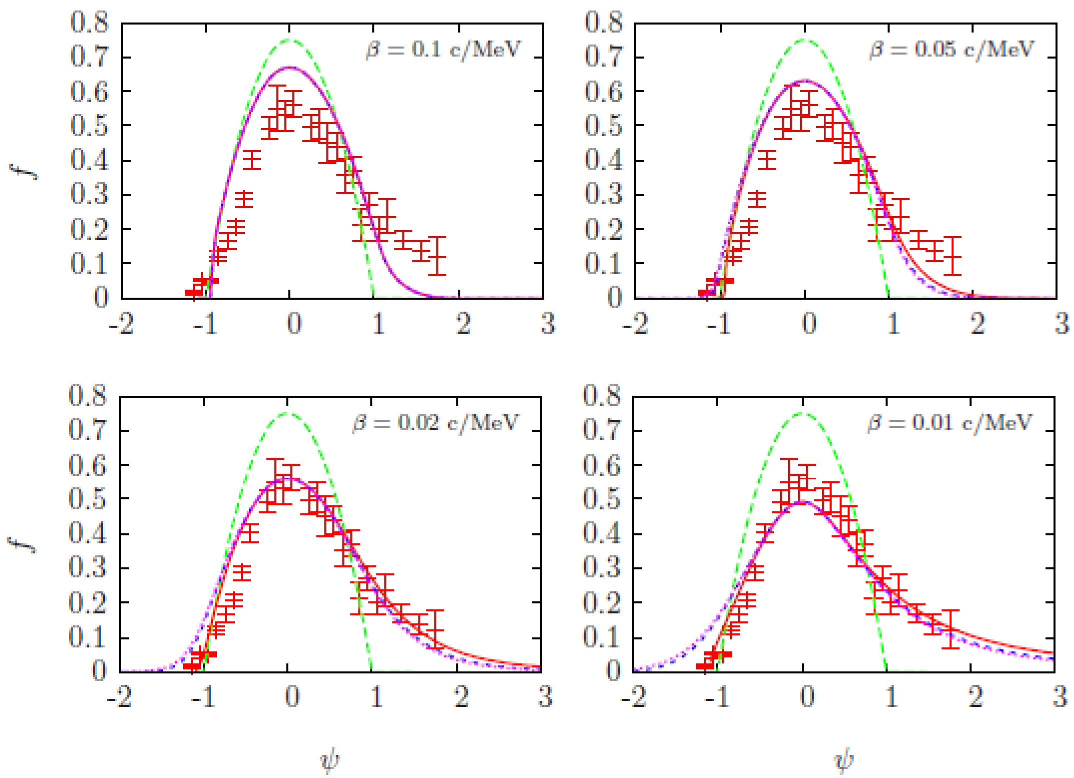
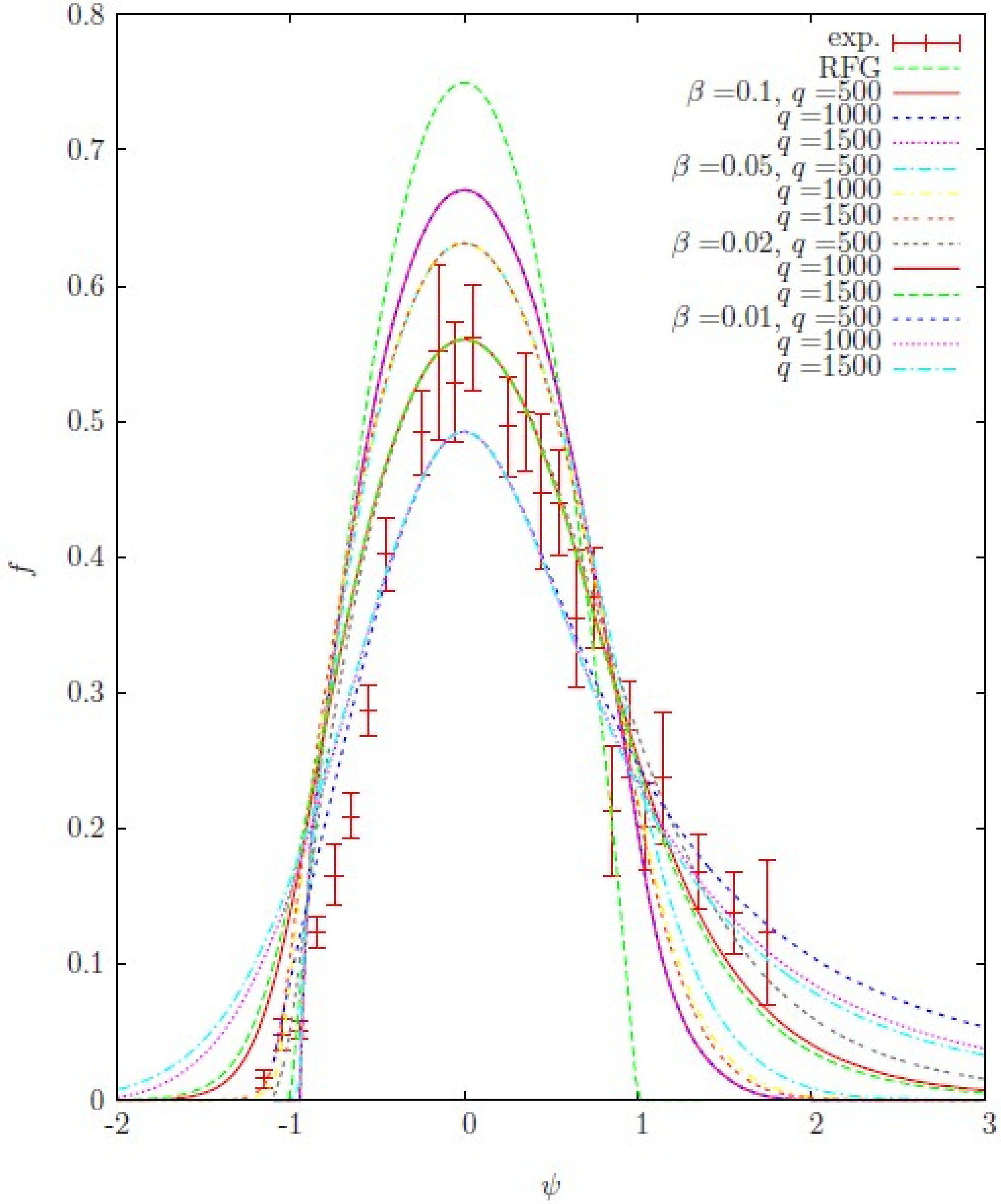


Figure 4. The superscaling function f defined in (5) plotted versus the scaling variable (8) in the RFG model (green) and in the present BCS model for three values of q (red: 500 MeV/c, blue: 1000 MeV/c, magenta: 1500 MeV/c) and different values of β. As usual, k_F =228 MeV/c. Data are taken from [8, 9].

Scaling of I kind



Scaling of I kind

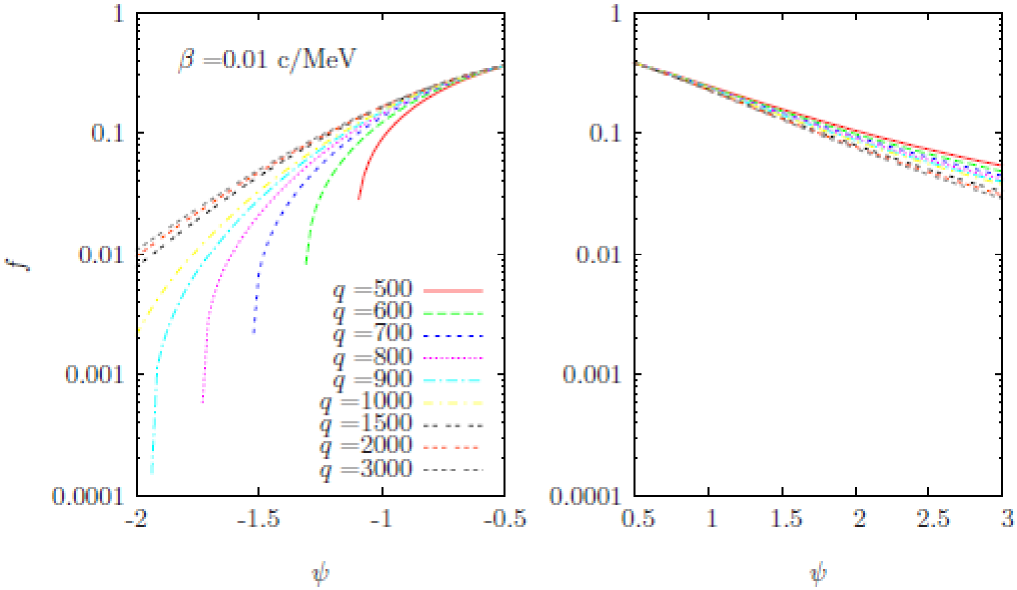


Figure 5. The superscaling function f in the negative ψ region plotted for several values of q (in MeV/c) and $\beta = 0.01$ c/MeV. As usual, k_F =228 MeV/c.

Scaling of II kind

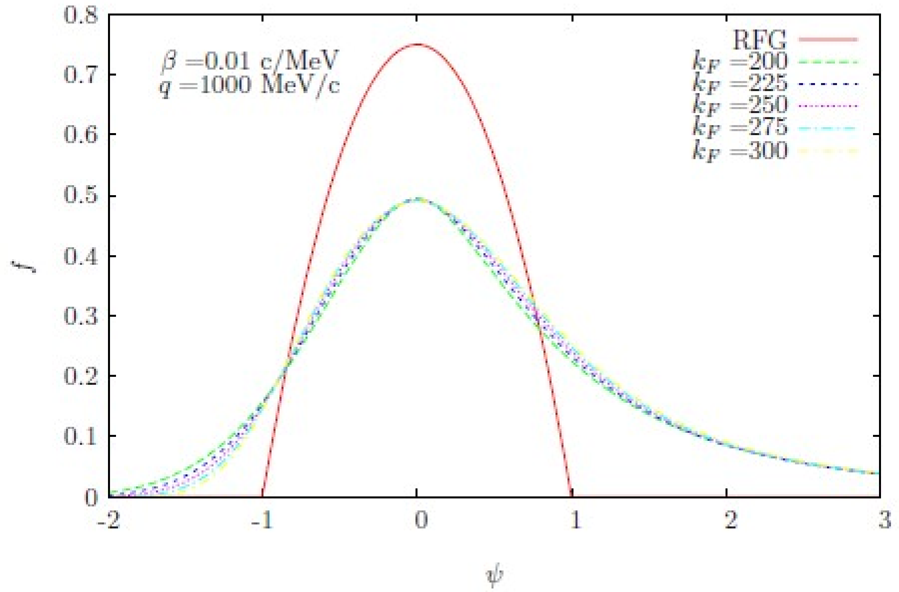


Figure 6. The superscaling function f plotted versus ψ for several values of the Fermi momentum k_F (in MeV/c) and with k_A devised in such a way that the peaks coincide (see text).

Summary and Conclusions

- We have proposed a covariant extension of RFG where pairs of particles
- are promoted from below the Fermi surface to above
- Scaling of I kind occurs not only at the QEP (with onset at q ~ 500 MeV/c) but also at lower and higher energy transfers (with onset at q ~ 2 GeV/c)
- The shape of the superscaling function is non-symmetric around the QEP, in agreement with experiment
- However, scaling of I kind is approached “from below”, in disagreement with experiment
- Scaling of II kind is relatively well satisfied, providing an appropriate momentum scale is chose for each nucleus, with some violations at large negative ψ
- Future developments:
    - Using realistic momentum distributions
    - Using the true BCS theory