In-medium heavy-quark and quarkonium propagators: the static and finite-mass cases

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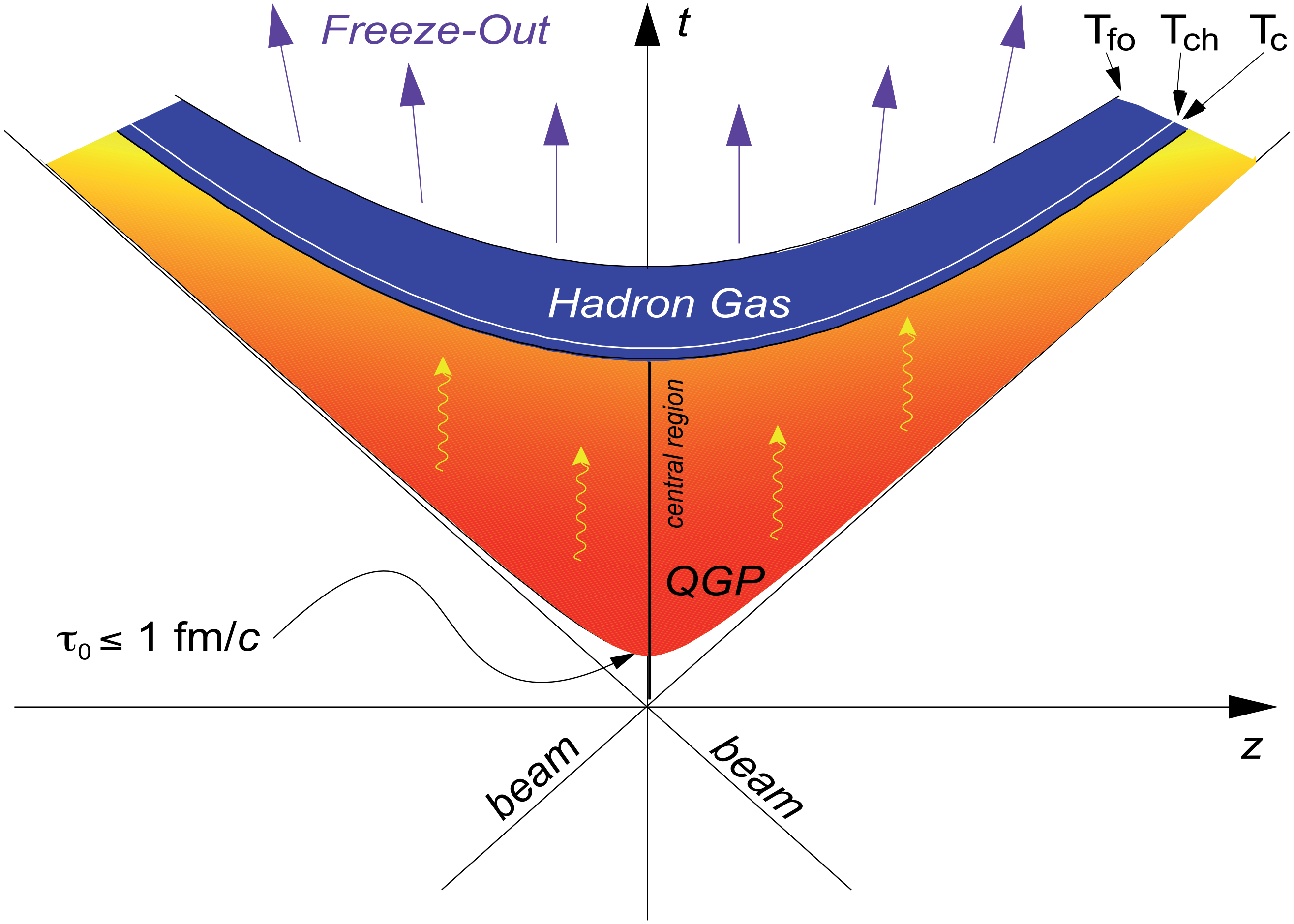
Work in progress in collaboration with J.P. Blaizot (CEA-Saclay),

G. Garberoglio and P. Faccioli (University of Trento)

Space-time evolution of the matter produced in nucleus-nucleus collisions



Particles measured by the detectors

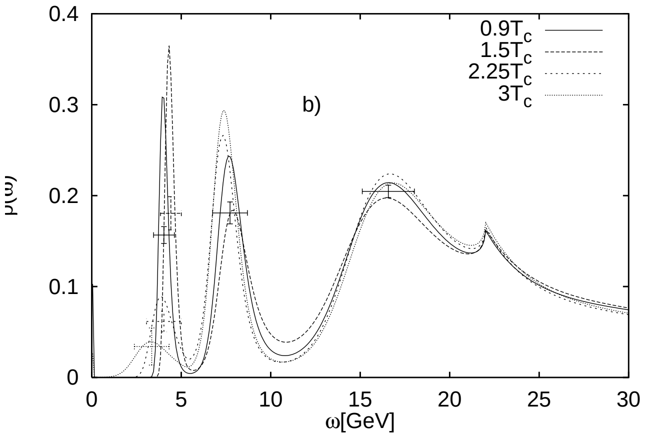
- **Soft hadrons** ($p_T \lesssim 1$GeV): are emitted at the *chemical freeze-out* hyper-surface and represent ~99% of particle multiplicity.
- **Hard probes** (*heavy quarks* and high-p_T particles): produced by pQCD processes in the *very early stages*, they cross the fireball allowing a *tomography* of the matter.
- **Photons and dileptons**: produced during *all the stages* of the fireball evolution.

Outline

- The physical motivations: study of medium effects on the spectral densities of HQ and $Q\overline{Q}$ correlators;
- The static ($M=\infty$) case: how to get a real-time in-medium $Q\overline{Q}$ potential;
- Beyond the static approximation:
  - Easy extension to address the finite-mass case in terms of a QM path-integral;
  - Preliminary numerical results of the MC simulations and of the spectral analysis;
  - Some physical insight: the HQ spectral function from a resummed one-loop calculation;
- Conclusions and future developments.

The physical motivations

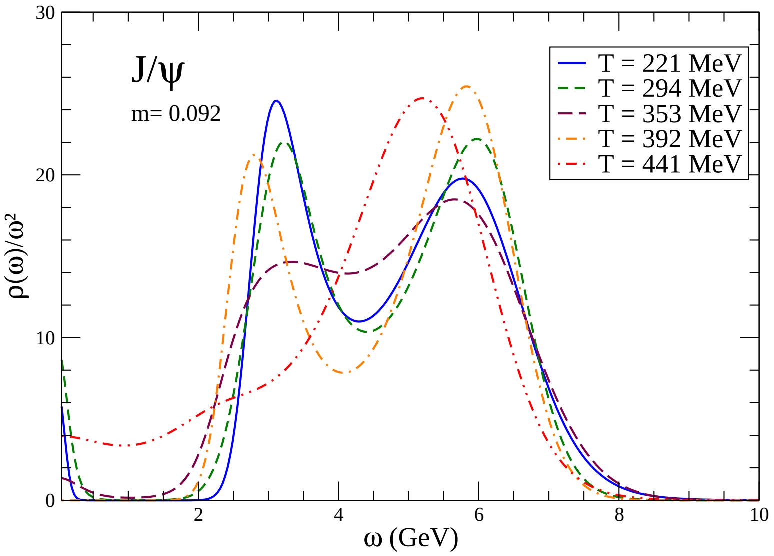
- QGP effects on HQs and quarkonia can be studied through the modifications of the corresponding spectral functions: broadening, shift or disappearance of peaks, development of new peaks and/or non-vanishing strength at low-energy;
- Most of the present spectral studies are based on lQCD calculations of euclidean correlators like $G_M(\tau) \equiv \langle J_M(\tau) J_M^\dagger(0) \rangle$

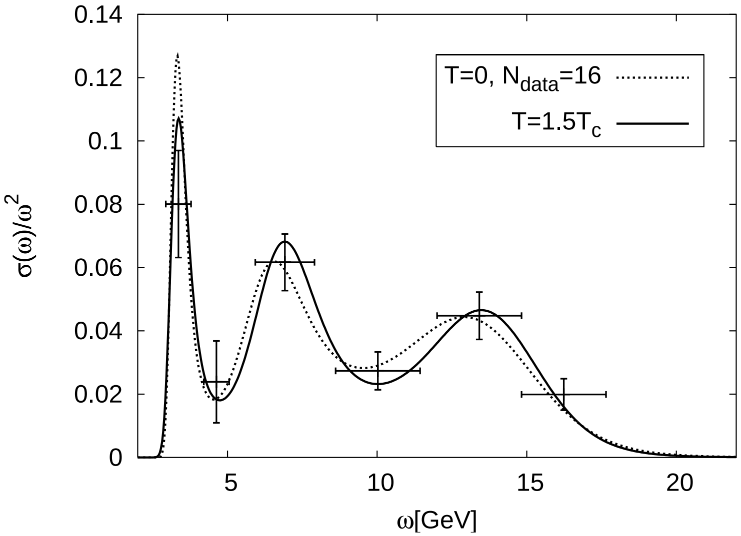


S. Datta, F. Karsch, P. Petreczky and I. Wetzorke,
Phys. Rev. D 69, 094507 (2004)

$$G_M(\tau) = \int_0^\infty d\omega \, \underbrace{\sigma_M(\omega)}_{MSF} \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\beta\omega/2)}$$

The relevance for quarkonium suppression





The vector (left) *and pseudoscalar* (right) *MSFs display well-defined peaks up to temperature* $T\sim 2T_c$[a,b] .

This forced people to reconsider the suppression pattern of J/Ψ as a probe of the deconfined medium.

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[a] G. Aarts *et al.*, arXiv:0705.2198 [hep-lat]

[b] A. Jakovac *et al.*, Phys.Rev. D75 (2007) 014506.

Our goal

We wish to perform a study resulting

- numerically less expensive then lattice calculations (hence allowing a more robust reconstruction of the spectral function);
- capable to get a deeper physical insight on the processes involved.

The basic object of our study

$$G^{>}(t) \equiv \langle \mathcal{O}(t)\, \mathcal{O}^{\dagger}(0) \rangle$$

- $\mathcal{O}^{\dagger}$ creates a Q or a $Q\overline{Q}$ pair;
- Spectral decomposition

$$\begin{aligned} G^{>}(t) &= Z^{-1} \sum_{n} e^{-\beta E_n} \sum_{m} \langle n|\mathcal{O}(t)|m\rangle \langle m|\mathcal{O}^{\dagger}(0)|n\rangle \\ &= Z^{-1} \sum_{n} e^{-\beta E_n} \sum_{m} e^{i(E_n - E_m)t} |\langle m|\mathcal{O}^{\dagger}(0)|n\rangle|^2, \end{aligned}$$

  - $G^{>}(t)$ is an **analytic function** in the strip $-\beta < \mathrm{Im} t < 0 \Longrightarrow$ *unified description of real and imaginary-time propagation*;
  - HQs: *external probe* placed *in a hot/dense medium* of light particles $\Longrightarrow \{|n\rangle\}$ do not contain heavy quarks.

Getting the in-medium spectral function...

- In the general case the spectral density of a correlator would be given by

$$\sigma(\omega) \equiv G^{>}(\omega) \mp G^{<}(\omega);$$

- Dealing with the propagation of an external probe one has $G^{<} \equiv 0$, so that

$$\sigma(\omega) = G^{>}(\omega) \quad \Longrightarrow \quad G^{>}(t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sigma(\omega);$$

- The standard procedure to get $\sigma(\omega)$ is then, exploiting the analyticity of $G^{>}$:

$$\underbrace{G^{>}(t=-i\tau)}_{\text{evaluated}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\; e^{-\omega\tau} \quad \underbrace{\sigma(\omega)}_{\text{reconstructed}} \quad .$$

The static ($M = \infty$) case:

the heavy quarks frozen to their positions

The hot-QED case:

A $Q\overline{Q}$ pair in a plasma of photons, electrons and positrons

$$\mathcal{L}_{\rm QED}^{M=\infty} = \mathcal{L}_{\rm em} + \mathcal{L}_{\rm light} + \underbrace{\psi^\dagger i(\partial_0 - igA_0)\psi}_{\text{heavy } Q} + \underbrace{\chi^\dagger i(\partial_0 + igA_0)\chi}_{\text{heavy } \overline{Q}}$$

It represents a simple abelian model for the QGP, nevertheless *sufficient to study important medium effects*[a].

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[a] A.B., J.P. Blaizot and C. Ratti, Nucl. Phys. A **806**, 312 (2008). ArXiv: 0712.4394

The strategy

- Consider the $Q\overline{Q}$ propagation in a given background configuration of the gauge-field A_μ

$$G_A(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}_1';0,\boldsymbol{r}_2') = \delta(\boldsymbol{r}_1-\boldsymbol{r}_1')\delta(\boldsymbol{r}_2-\boldsymbol{r}_2')\times$$
$$\times \exp\left(ig\int_0^t dt'\, A_0(\boldsymbol{r}_1,t')\right)\exp\left(-ig\int_0^t dt'\, A_0(\boldsymbol{r}_2,t')\right)$$

- Average over the gauge-field configuration with an action accounting for thermal effects

$$G^>(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}_1';0,\boldsymbol{r}_2') = Z^{-1}\int[\mathcal{D}A]G_A(t,\boldsymbol{r}_1;t,\boldsymbol{r}_2|0,\boldsymbol{r}_1';0,\boldsymbol{r}_2')\,e^{iS[A]}$$
$$\equiv \delta(\boldsymbol{r}_1-\boldsymbol{r}_1')\delta(\boldsymbol{r}_2-\boldsymbol{r}_2')\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2)$$

Which is the action to employ to weight the field configurations?

The HTL effective action I

⇒Momentum scales in a relativistic (*weakly coupled*) plasma:

- **Hard** (*plasma particles*):

$$E \sim T^4 \quad N \sim T^3 \quad \Longrightarrow \quad K \sim T;$$

- **Soft** (*collective modes*): $K \sim gT$.

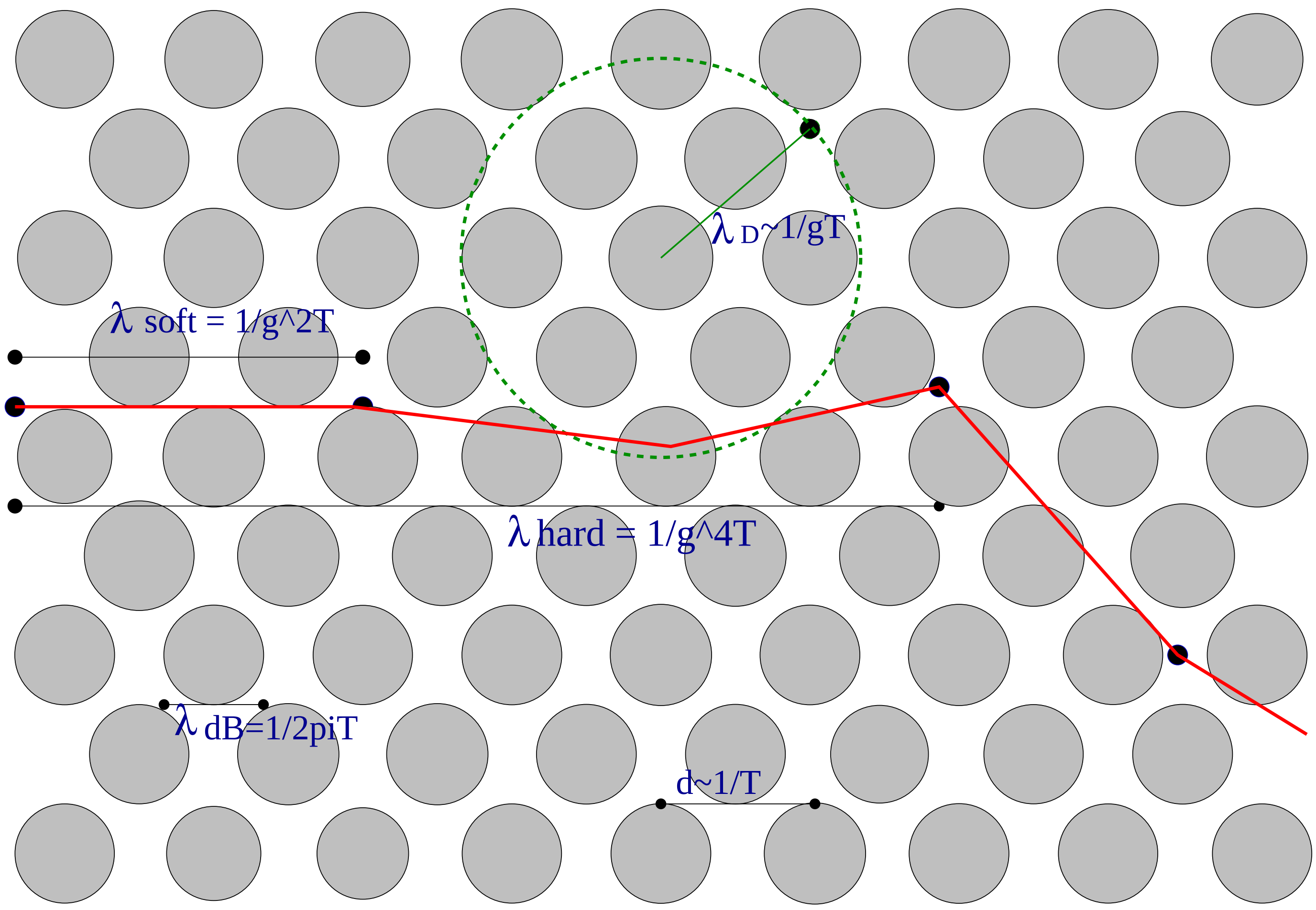
⇒Mean Free Path of a plasma particle:

- For hard momentum exchange: $\lambda_{mfp}^{hard} \sim 1/g^4 T$,
- For soft momentum exchange: $\lambda_{mfp}^{soft} \sim 1/g^2 T$.

For weak coupling one has $\lambda_{mfp}^{soft} \ll \lambda_{mfp}^{hard}$, i.e.

most of the scattering processes involve small momentum transfer.

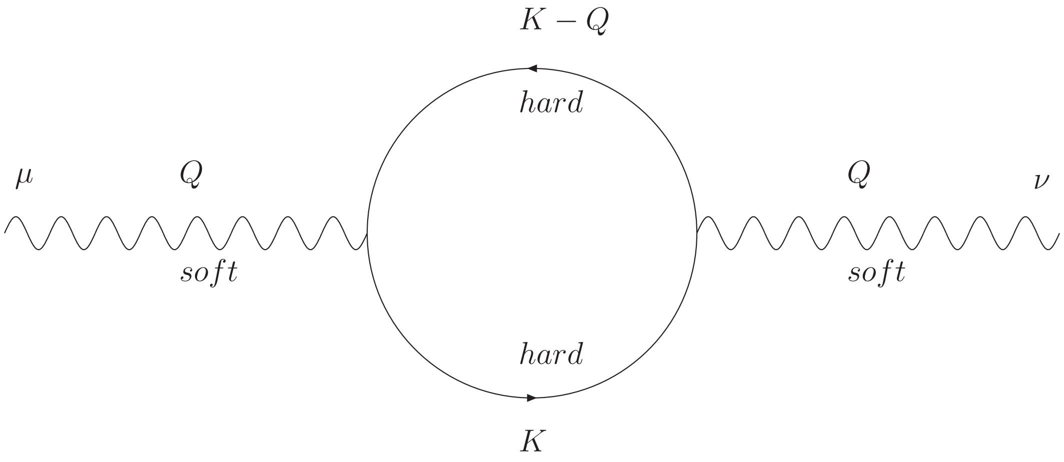
A cartoon...



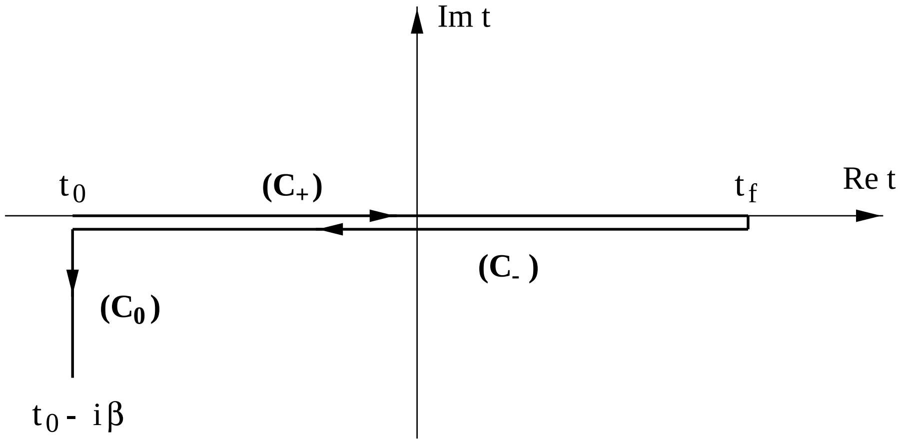
The HTL effective action II

- *Most of the interactions* are mediated by the exchange of **soft gauge-bosons** ($Q \sim gT \ll T$)

- The propagation of soft (long wave-length) gauge-bosons is ***dressed by the interactions with the light plasma-particle*** which are ***hard*** ($K \sim T$)



The HTL effective action III



The HTL effective action will be expressed in terms of the gauge-boson propagator in the *complex-time plane*:

$$iD_{\mu\nu}(x-y) \equiv \theta_C(x^0 - y^0)\langle A_\mu(x)A_\nu(y)\rangle + \theta_C(y^0 - x^0)\langle A_\nu(y)A_\mu(x)\rangle$$

- Along C_+ it coincides with the time-ordered propagator;
- Along C_0 it coincides with the Matsubara propagator.

The HTL effective action IV

- The HTL effective action (*gaussian for a QED plasma!*):

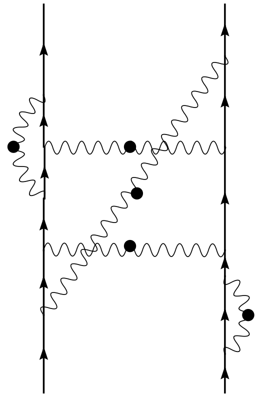
$$S_C^{HTL}[A] = \frac{1}{2}\int_C d^4x \int_C d^4y\, A^\mu(x)\left(D^{-1}\right)^{HTL}_{\mu\nu}(x-y)A^\nu(y).$$

- The functional integral *can be performed exactly*

$$\overline{G}(t, \boldsymbol{r}_1 - \boldsymbol{r}_2) = \exp\left[-\frac{i}{2}\int_C d^4x \int_C d^4y\, J^\mu(x) D^{HTL}_{\mu\nu}(x-y) J^\nu(y)\right]$$

with $J^\mu(x)$ the $Q\overline{Q}$ current.

- In terms of Feynman diagrams:



Real-time $Q\overline{Q}$ propagator

- $Q\overline{Q}$ current non-vanishing along C_+:

$$\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2)=\exp\left[-\frac{i}{2}\int_{C_+}d^4x\int_{C_+}d^4y\,J^\mu(x)D_{\mu\nu}(x-y)J^\nu(y)\right]$$

$$J^\mu(z)=\delta^{\mu 0}\theta(z^0)\theta(t-z^0)[-g\delta(\boldsymbol{z}-\boldsymbol{r}_1)+g\delta(\boldsymbol{z}-\boldsymbol{r}_2)]$$

- Large time behavior: $\overline{G}(t,\boldsymbol{r}_1-\boldsymbol{r}_2)\underset{t\to\infty}{\sim}\exp[-iV_{\text{eff}}(\boldsymbol{r}_1-\boldsymbol{r}_2)t]$, with

$$\underset{\text{effective potential}}{V_{\text{eff}}(\boldsymbol{r}_1-\boldsymbol{r}_2)}\equiv g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)D_{00}(\omega=0,\boldsymbol{q})$$

$$=g^2\int\frac{d\boldsymbol{q}}{(2\pi)^3}\left(1-e^{i\boldsymbol{q}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_2)}\right)\left[\underbrace{\frac{1}{\boldsymbol{q}^2+m_D^2}}_{\text{screening}}-i\underbrace{\frac{\pi m_D^2T}{|\boldsymbol{q}|(\boldsymbol{q}^2+m_D^2)^2}}_{\text{collisions}}\right]$$

$$=-\frac{g^2}{4\pi}\left[m_D+\frac{e^{-m_Dr}}{r}\right]-i\frac{g^2T}{4\pi}\phi(m_Dr)$$

The $Q\overline{Q}$ effective potential: real part

With a *consistent treatment of screened self-energy and interaction*[a]

$$V_{\rm eff}(r) = -\alpha m_D - \frac{\alpha}{r} e^{-m_D r}$$
$$\underset{r\to 0}{\sim} -\alpha m_D - \frac{\alpha}{r} + \alpha m_D = -\frac{\alpha}{r}$$

for bound states of very small size medium effects cancel!

An analogous problem in solid-state physics...

$V_{\rm eff}(r)$ turns out to coincide with the Ecker-Weitzel potential used to study excitons (e-h bound states) in semiconductors.

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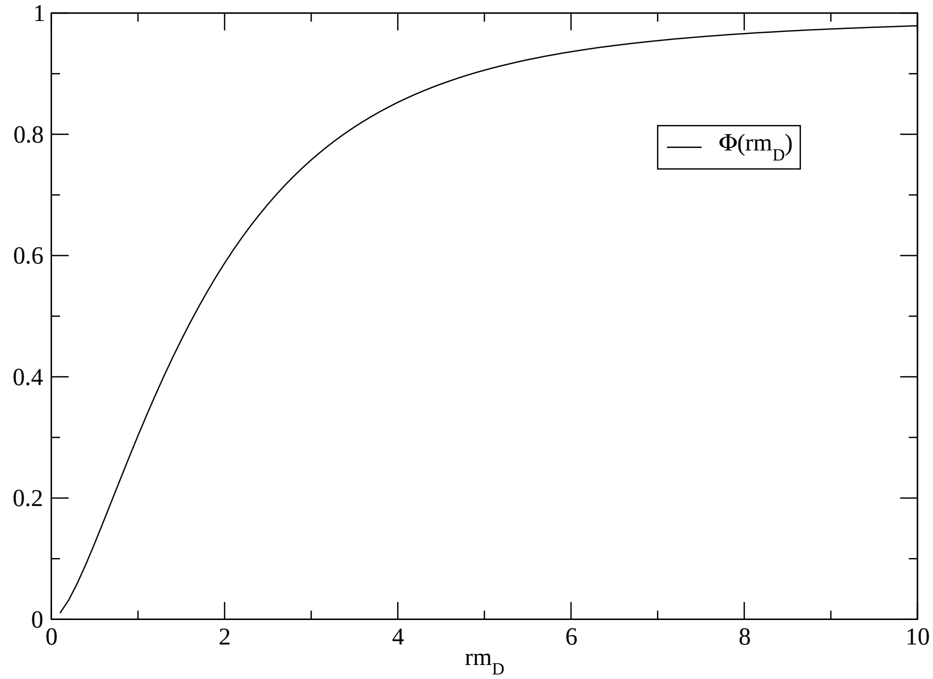
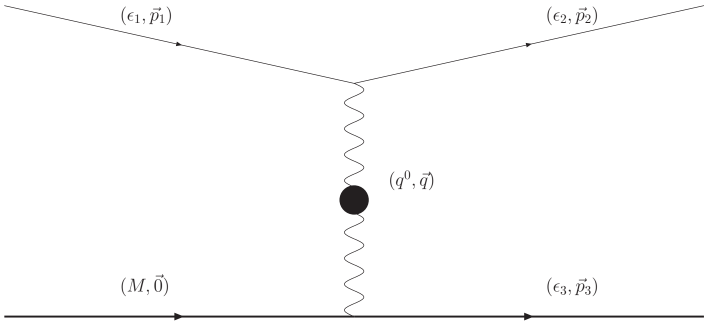
[a] See also R. Rapp, D. Blaschke and P. Crochet, arXiv:0807.2470

The $Q\overline{Q}$ effective potential: imaginary part



- For small separation the $Q\overline{Q}$ pair is seen as a *neutral* object and it does not interact with the particles of the medium;
- For large separation the HQs suffer uncorrelated scatterings with the plasma particles.



Interpretation of the damping as the interaction rate of a heavy fermion in the thermal bath

$$\Gamma(M) = 2\,\frac{1}{2M}\int_{p_1}\int_{p_2}\int_{p_3}(2\pi)^4\delta^{(4)}(P+P_1-P_2-P_3)\times$$
$$\times\left[n_1(1-n_2)(1-n_3)+(1-n_1)n_2n_3\right]\overline{|\mathcal{M}|^2}$$

In the $M\to\infty$ limit:

$$\Gamma(\infty) = g^2T\int\frac{d\boldsymbol{q}}{(2\pi)^3}\,\frac{\pi m_D^2}{(\boldsymbol{q}^2+m_D^2)^2q}$$

NB The resulting width in $G^>(\omega)$ should be interpreted as a *collisional broadening* of the state.

The finite-mass case:

the heavy quarks free to move in the medium

The general idea

Treat the heavy fermion propagating in a thermal bath as a point-like particle in Quantum-Mechanics. Hence:

- Sum over all the possible trajectories in a given background field:

$$\langle \boldsymbol{x}_f \tau_f | \boldsymbol{x}_i \tau_i \rangle = \int_{\boldsymbol{x}(\tau_i)=\boldsymbol{x}_i}^{\boldsymbol{x}(\tau_f)=\boldsymbol{x}_f} [\mathcal{D}\boldsymbol{x}(\tau')] \exp\left[- \int_{\tau_i}^{\tau_f} d\tau' \left(\frac{1}{2} M \dot{\boldsymbol{x}}^2 + V(x) \right) \right],$$

  where $V(x) \equiv g\Phi(x)$ (scalar interaction) and $\dot{\boldsymbol{x}} \equiv d\boldsymbol{x}/d\tau'$.

- Average over all the possible field configurations (the action accounting for medium effects)

$$G^{>}(-i\tau, \boldsymbol{r}_1 | 0, \boldsymbol{r}'_1) = Z^{-1} \int_{\boldsymbol{z}_1(0)=\boldsymbol{r}'_1}^{\boldsymbol{z}_1(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}_1] \int [\mathcal{D}\Phi] \exp\left[- \int_0^{\tau} d\tau' \frac{1}{2} M \dot{\boldsymbol{z}}_1^{\,2} \right] \times$$

$$\times \exp\left[-g \int_0^{\tau} d\tau' \Phi(\tau', \boldsymbol{z}_1(\tau')) \right] e^{-S_E^{\rm eff}[\Phi]}$$

For a gaussian effective action...

Also on the finite-mass case if the action is gaussian

$$S_E^{\rm eff}[\Phi] = \frac{1}{2}\int d^4x_E \int d^4y_E \Phi(x)\Delta^{-1}(x-y)\Phi(y),$$

the integration over the field configurations can be performed exactly:

$$G^>(-i\tau,\boldsymbol{r}_1|0,\boldsymbol{r}_1') = \int_{\boldsymbol{z}(0)=\boldsymbol{r}_1'}^{\boldsymbol{z}(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}]\exp\left[-\int_0^\tau d\tau' \frac{1}{2}M\dot{\boldsymbol{z}}^2\right] \times$$

$$\times \exp\left[\frac{g^2}{2}\int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta(\tau'-\tau'',\boldsymbol{z}(\tau')-\boldsymbol{z}(\tau''))\right],$$

with $\Delta(\tau,\boldsymbol{x})$ the *Matsubara propagator of the exchanged meson.*

A heavy "quark" in hot-QED

We perform Monte Carlo simulations for

$$G^{>}(-i\tau,\boldsymbol{r}_1|0,\boldsymbol{r}_1') = \int_{\boldsymbol{z}(0)=\boldsymbol{r}_1'}^{\boldsymbol{z}(\tau)=\boldsymbol{r}_1} [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \left(M + \frac{1}{2}M\dot{\boldsymbol{z}}^2\right)\right] \times$$

$$\times \exp\left[\frac{g^2}{2}\int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau'-\tau'',\boldsymbol{z}(\tau')-\boldsymbol{z}(\tau''))\right]$$

where

$$\begin{aligned} \Delta_L(\tau,\boldsymbol{q}) &\equiv \Delta_L^{vac}(\tau,\boldsymbol{q}) + \Delta_L^T(\tau,\boldsymbol{q}) \\ &= \frac{-1}{\boldsymbol{q}^2}\delta(\tau) + \int_{-\infty}^{+\infty}\frac{dq_0}{2\pi}e^{-q_0\tau}\rho_L(q_0,\boldsymbol{q})[\theta(\tau)+N(q^0)] \end{aligned}$$

is expressed in terms of the HTL spectral function

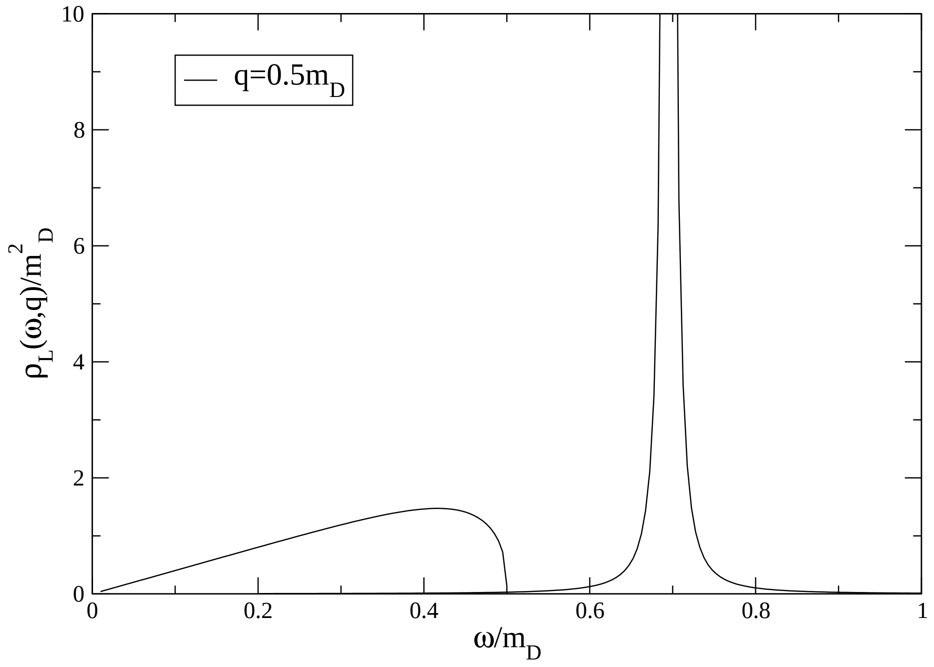
$$\rho_L(\omega>0,q) \equiv 2\pi\left[\underbrace{Z_L(q)\delta(\omega-\omega_L(q))}_{\text{plasmon pole}} + \underbrace{\theta(q^2-\omega^2)\beta_L(\omega,q)}_{\text{Landau damping}}\right]$$

HTL longitudinal spectral function

$$\rho_L(\omega) \equiv 2\,\mathrm{Im} D_L^{\mathrm{ret}}(\omega) = 2\,\mathrm{Im}\Delta_L(\omega + i\eta),$$

where:

$$\Delta_L(q^0, q) = \frac{-1}{q^2 + m_D^2\left(1 - \frac{q^0}{2q}\ln\frac{q^0+q}{q^0-q}\right)}$$



Pole + Continuum. The width is put by hand!

Our long term goal...

...would be to address the $Q\overline{Q}$ case within the same approach:

$$G^{>}(-i\tau;\boldsymbol{r}_1,\boldsymbol{r}_2|0;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = e^{-(M_1+M_2)\tau}\int_{\boldsymbol{r}'_1}^{\boldsymbol{r}_1}[\mathcal{D}\boldsymbol{z}_1]\int_{\boldsymbol{r}'_2}^{\boldsymbol{r}_2}[\mathcal{D}\boldsymbol{z}_2]\times$$

$$\times\exp\left[-\int_0^{\tau}d\tau'\left(\frac{1}{2}M_1\dot{\boldsymbol{z}}_1{}^2-\frac{g^2}{2}\int_0^{\tau}d\tau''\Delta_L^T(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_1(\tau''))\right)\right]\times$$

$$\times\exp\left[-\int_0^{\tau}d\tau'\left(\frac{1}{2}M_2\dot{\boldsymbol{z}}_2{}^2-\frac{g^2}{2}\int_0^{\tau}d\tau''\Delta_L^T(\tau'-\tau'',\boldsymbol{z}_2(\tau')-\boldsymbol{z}_2(\tau''))\right)\right]\times$$

$$\times\exp\left[-g^2\int_0^{\tau}d\tau'\int_0^{\tau}d\tau''\Delta_L(\tau'-\tau'',\boldsymbol{z}_1(\tau')-\boldsymbol{z}_2(\tau''))\right]$$

Numerical results

from the MC simulations for the path-integral

$$\underbrace{G^{>}(t=-i\tau)}_{\text{evaluated}} \equiv G(\tau) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \; e^{-\omega\tau} \qquad \underbrace{\sigma(\omega)}_{\text{reconstructed}} \qquad .$$

- $G(\tau)$ *obtained after averaging over at least* 10^6 *paths!*
- The above data are used the get the HQ spectral density through a MEM analysis.

Maximum Entropy Method

$$G(\tau) = \int_0^\infty d\omega \, K(\omega,\tau)\, \sigma(\omega)$$

- $G(\tau)$: known for $1 \leq \tau_i/a \leq N_\tau (\sim 20)$;
- $\sigma(\omega)$: for it one wants a very fine scan. $\omega_l = l \cdot \Delta\omega$, with $1 \leq l \leq N_\omega (\sim 10^2 - 10^3) \Longrightarrow \chi^2$ method not applicable;
- H: *a priori* information on general properties (e.g. sum rules, positivity...) of the spectral function (key ingredient!)

One looks for the most probable spectral function compatible with the data and the constraints:

$$\frac{\delta P[\sigma|G,H]}{\delta\sigma} = 0,$$

where, from Bayes' theorem

$$P[\sigma|G,H] \quad \sim \quad \underbrace{P[G|\sigma,H]}_{\text{likelihood function}} \quad \times \quad \underbrace{P[\sigma|H]}_{\text{prior probability}}$$

- Likelihood function: $P[G|\sigma, H] \sim e^{-L}$, with

$$L = \frac{1}{2} \sum_{i,j} \left[\underbrace{G(\tau_i)}_{\text{MC data}} - G_\sigma(\tau_i) \right] \underbrace{C^{-1}_{ij}}_{\text{cov matrix}} \left[\underbrace{G(\tau_j)}_{\text{MC data}} - G_\sigma(\tau_j) \right].$$

Maximizing it would correspond to the standard χ^2-fitting.

- Prior probability: $P[\sigma|H] \sim e^{\alpha S}$, with

$$S = \int_0^\infty \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln \frac{\sigma(\omega)}{m(\omega)} \right] d\omega.$$

playing the role an *entropy term*.

  - Default model $m(\omega)$: contains the *a priori* information on the spectral density;
  - *The entropy is maximum* ($S = 0$) *when the spectral function coincides with the default model*. That's what happens in the absence of data!

(Bryan's) Maximum Entropy Method

- *For a given value of* α one looks for the maximum of $P[\sigma|G,H] \sim e^{-L+\alpha S} \equiv e^{Q[\sigma]}$:

$$\left.\frac{\delta Q[\sigma|G,H]}{\delta\sigma}\right|_{\sigma_\alpha(\omega)} = 0,$$

  where α controls the relative weight between

  - L: tends to fit σ to the data;
  - S: tends to fit σ to the default model.

  It's like minimizing the free-energy in statistical mechanics!

- *One finally integrates over different values of* α.

How to choose the default model?

Suppose you know a certain number of moments of the spectral density:

$$\sum_n p_n \omega_n^M = C_M \quad (M = 0, 1, \dots N)$$

The less biased default model is the one which maximizes the (Shannon) entropy respecting the above constraints: $\delta S' / \delta p_n = 0$, with

$$S' = -\sum_n p_n \ln p_n - \sum_{M=0}^{N} \lambda_M \left(\sum_n p_n \omega_n^M - C_M \right) .$$

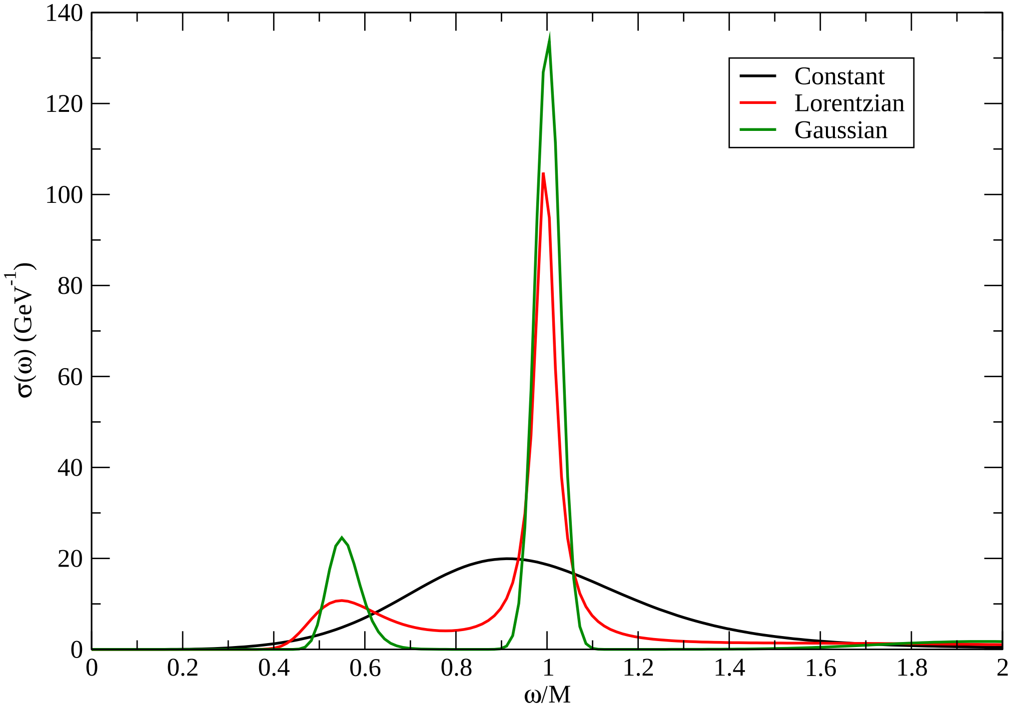
One gets:

$$p_n = \exp\left[-1 - \sum_{M=0}^{N} \lambda_M \omega_n^M \right] .$$

Fixing values of the parameters of possible relevance for the QGP study

$$
\begin{aligned}
\frac{g^2}{4\pi} &\equiv C_F \alpha_s, \quad \textit{with} \quad \alpha_s = 0.3 \\
M &= 1.5 \text{ GeV} \ \ (\text{charm}) \\
T &= 200 - 400 \text{ MeV} \ \ (T/M \ll 1)
\end{aligned}
$$

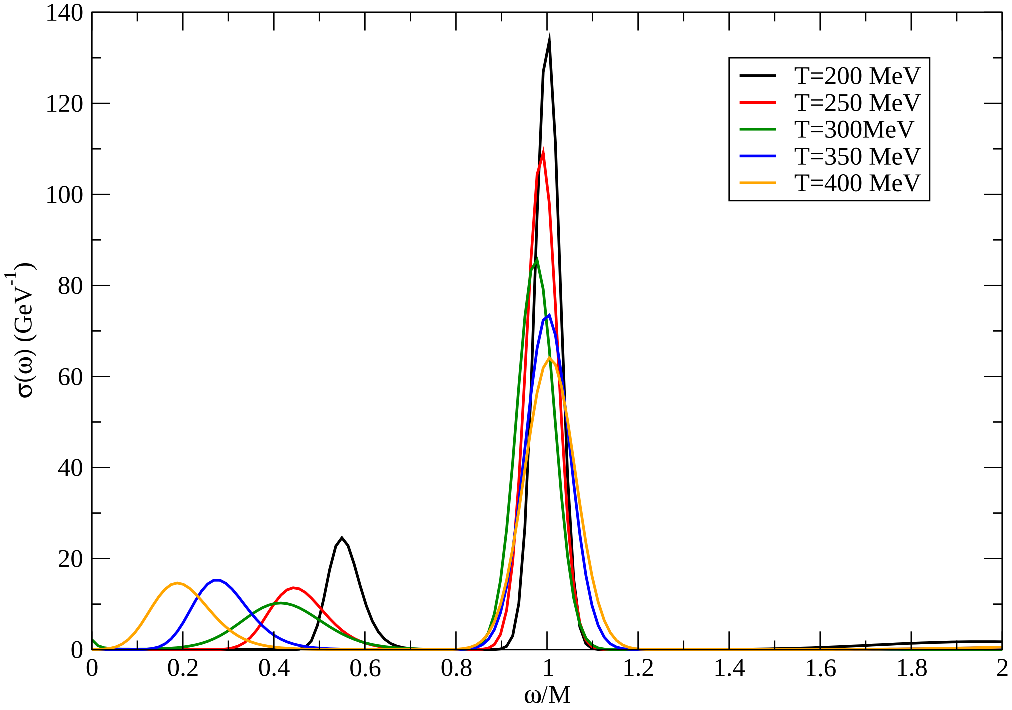
Results for the HQ spectral function I



- We set $T = 200$ MeV;
- We check the sensitivity on the choice of the default model.

The appearance of a *secondary peak* or at least of a *sizable strength at low-energy* seems a robust feature of the spectral density.

Results for the HQ spectral function II



- Using a gaussian default model in the MEM...
- ...we perform a temperature scan.

As the temperature increases
the secondary peak moves toward lower energies.

In order to interpret the numerical outcomes of the simulations....

...some physical insight from (weak-coupling) **thermal field theory calculations**

General setup

- Analytic non-relativistic HQ propagator

$$G(z) = \frac{-1}{z - E_p - \Sigma(z, \boldsymbol{p})},$$

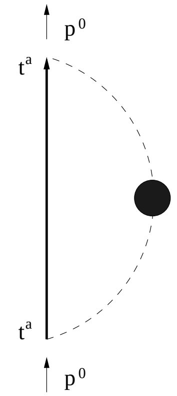
where $E_p = M + p^2/2M$ and setting $z = \omega + i\eta$ corresponds to *retarded boundary conditions*;

- HQ spectral function:

$$\sigma(\omega) \equiv 2\mathrm{Im}\, G^R(\omega) = \frac{\Gamma(\omega)}{[\omega - E_p - \mathrm{Re}\,\Sigma(\omega)]^2 + \Gamma^2(\omega)/4},$$

with $\Gamma(\omega) \equiv -2\mathrm{Im}\,\Sigma^R(\omega) \implies$ *HQ spectral function non-vanishing only for energies for which the self-energy develops an imaginary-part*.

HQ self-energy: resummed one-loop result



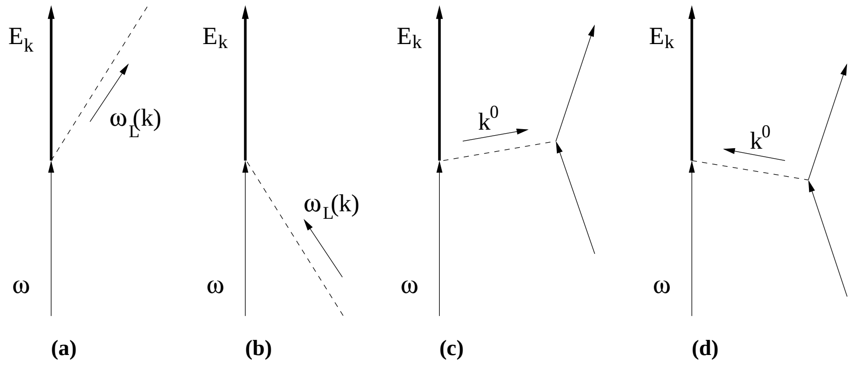
The zero-momentum HQ self-energy reads:

$$\Sigma(p^0) = g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0,k) \frac{1+N(k^0)-n_F(E_k)}{p^0-E_k-k^0}$$

Test-particle limit recovered setting $n_F(E_k)=0$, which arises naturally in the regime $T/M \ll 1$

$$\Sigma^{\rm test}(p^0)=g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3} \int_0^{+\infty} \frac{dk^0}{2\pi} \rho_L(k^0,k) \left[\frac{1+N(k^0)}{p^0-E_k-k^0} + \frac{N(k^0)}{p^0-E_k+k^0}\right]$$

HQ self-energy: imaginary-part



- Plasmon-pole contribution (a and b)

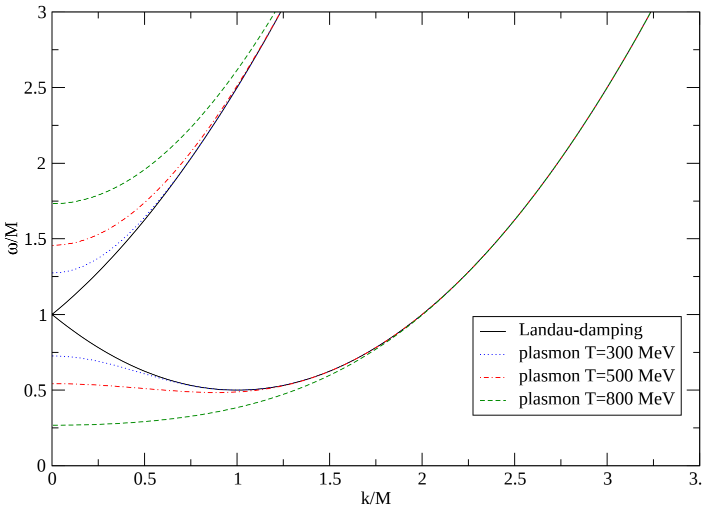
$$\Gamma^{\rm pole}(\omega) = g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3}(2\pi) Z_L(k)\times$$

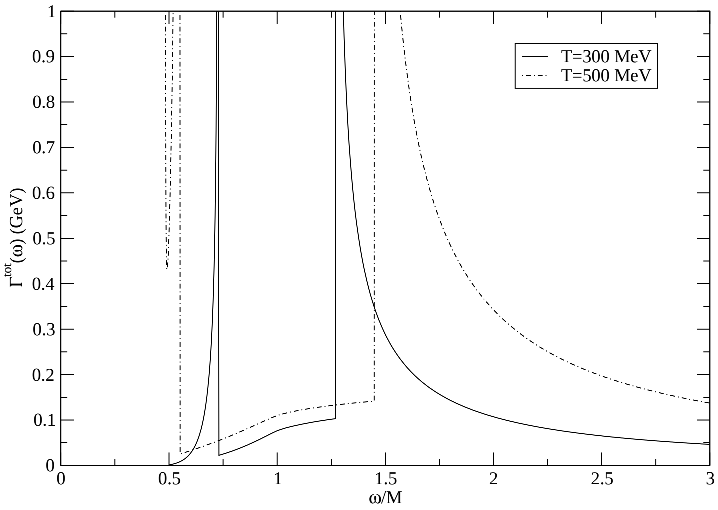
$$\times[(1+N(\omega_L(k)))\,\delta(\omega - E_k - \omega_L(k)) + N(\omega_L(k))\delta(\omega - E_k + \omega_L(k))]$$

- Continuum contribution (c and d)

$$\Gamma^{\rm cont}(\omega) = g^2 C_F \int \frac{d\boldsymbol{k}}{(2\pi)^3}\int_0^k dk^0\,\beta_L(k^0,k)\times$$

$$\times\,(2\pi)\left\{\left[1+N(k^0)\right]\delta(\omega - E_k - k^0) + N(k^0)\delta(\omega - E_k + k^0)\right\}$$



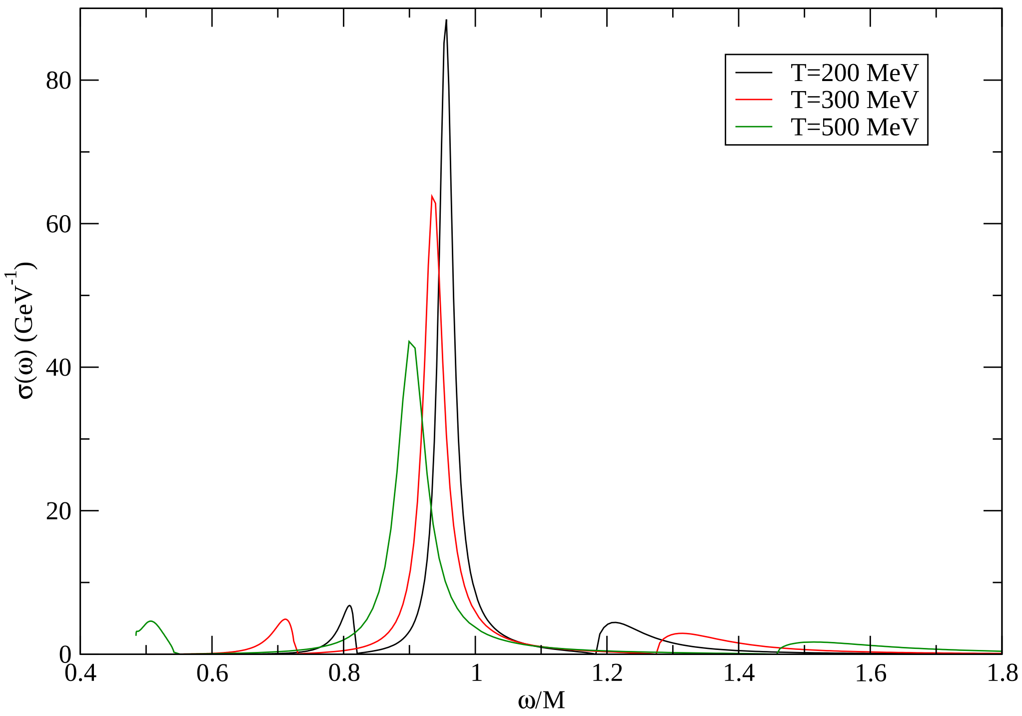


- Spectrum displaying a **threshold close to $M/2$**;
- Very narrow *peaks arising from a divergence in the density of states* (Van-Hove singularities). Defining $\omega \equiv E_{k_{1/2}} \pm \omega_L(k_{1/2})$

$$\Gamma^{\text{pole}}(\omega) = \frac{g^2 C_F}{\pi} \left\{ \frac{k_1^2}{|E'_{k_1} + \omega'_L(k_1)|} Z_L(k_1) \left[1 + N(\omega_L(k_1))\right] + \right.$$

$$\left. + \sum_{k_2} \frac{k_2^2}{|E'_{k_2} - \omega'_L(k_2)|} Z_L(k_2) N(\omega_L(k_2)) \right\}$$

HQ spectral-function



- Negative shift and broadening of the principal peak;
- Appearance of secondary peaks at energies corresponding to a large density of states for *plasmon absorption/emission processes*

Summary

- The effective-action approach, introduced to derive a real-time *static potential*, results very convenient to address also the finite-mass case: *QFT problem reduced to a QM problem!*
- Numerical results for $G(\tau)$ indicates the possible existence of secondary peaks and an important spectral strength at low-energy;
- Resummed one-loop calculation of interest to shed light on possible processes responsible for such a strength.

Future developments

- Systematic study for different values of the HQ mass, the temperature and the coupling;
- *Addressing the $Q\overline{Q}$ case*.

Back-up slides

Evaluation of the path-integral I

We can reduce

$$\begin{aligned} G(\tau, r) &= \int [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{\boldsymbol{z}}^2\right)\right] \times \\ &\quad \times \exp\left[\frac{g^2}{2} \int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau' - \tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau''))\right] \\ &\equiv \int [\mathcal{D}\boldsymbol{z}] \exp[-S[\boldsymbol{z}]] \end{aligned}$$

to the evaluation of an *expectation value*, by *rescaling the coupling* $g^2 \to \alpha g^2$

$$G_\alpha(\tau, r) \equiv \int [\mathcal{D}\boldsymbol{z}] \exp[-S_\alpha[\boldsymbol{z}]],$$

so that

$$\frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \left\langle \frac{g^2}{2} \int d\tau' \int d\tau'' \Delta_L^T(\tau' - \tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau'')) \right\rangle_\alpha$$

Evaluation of the path-integral II

- For a given α the *expectation value* is evaluated by generating paths distributed according to

$$W_\alpha[\boldsymbol{z}] = \frac{1}{G_\alpha} \exp(-S_\alpha[\boldsymbol{z}])$$

- By integrating over the parameter α one gets:

$$\int_0^1 d\alpha \, \frac{\partial \ln G_\alpha(\tau, r)}{\partial \alpha} = \ln\left(\frac{G(\tau, r)}{G_{\text{free}}(\tau, r)}\right) = \int_0^1 d\alpha \, \langle \Delta \rangle_\alpha ,$$

where

$$G_{\text{free}} = [M/(2\pi\tau)]^{3/2} \exp[-Mr^2/(2\tau)].$$

Renormalization of the path-integral correlator

In the path-integral correlator

$$G_{\rm MC}(\tau,\boldsymbol{r}) = \int_{\boldsymbol{z}(0)=\boldsymbol{0}}^{\boldsymbol{z}(\tau)=\boldsymbol{r}} [\mathcal{D}\boldsymbol{z}] \exp\left[-\int_0^\tau d\tau' \left(M + \frac{1}{2} M \dot{\boldsymbol{z}}^2\right)\right] \times$$

$$\times \exp\left[\frac{g^2}{2}\int_0^\tau d\tau' \int_0^\tau d\tau'' \Delta_L^T(\tau'-\tau'', \boldsymbol{z}(\tau') - \boldsymbol{z}(\tau''))\right],$$

the interaction term is evaluated as:

$$\exp\left[\frac{g^2}{2}\sum_{i\neq j=1}^{\tau/a_t} a_t^2 \Delta_L^T(i-j, \boldsymbol{r}_i - \boldsymbol{r}_j)\right]$$

neglecting the $i=j$ contribution:

$$\exp\left[\frac{g^2}{2}\sum_{i=1}^{\tau/a_t} a_t^2 \Delta_L^T(0,\boldsymbol{0})\right] = \exp\left[\frac{g^2}{2} a_t \Delta_L^T(0,\boldsymbol{0})\,\tau\right]$$

- The finite time-step a_t provides a cutoff which insures dealing always with finite quantities in the intermediate steps;
- however we don't want to change the continuum physics.

The link with the continuum renormalized result is:

$$G_{\mathrm{ren}}(\tau, \boldsymbol{r}) = [Z(a_t)]^{\frac{\tau}{a_t}} G_{\mathrm{MC}}(\tau, \boldsymbol{r}|a_t).$$

The renormalization factor $Z(a_t)$ can be determined in the static case:

$$\overline{G}_{\mathrm{ren}}^{M=\infty}(\tau, \boldsymbol{r} = \boldsymbol{0}) = \exp\left\{\frac{g^2}{2}\int \frac{d\boldsymbol{q}}{(2\pi)^3}\left(\frac{1}{\boldsymbol{q}^2} - \frac{1}{\boldsymbol{q}^2 + m_D^2}\right)\tau\right\} \times$$

$$\times\exp\left\{g^2\int \frac{d\boldsymbol{q}}{(2\pi)^3}\int_0^{+\infty}\frac{dq^0}{2\pi}\frac{\rho_L(q^0, \boldsymbol{q})}{(q^0)^2}\left[\frac{\cosh q^0(\tau - \beta/2)}{\sinh(\beta q^0/2)} - \coth(\beta q^0/2)\right]\right\},$$

$$\overline{G}_{\mathrm{MC}}^{M=\infty}(\tau, \boldsymbol{r} = \boldsymbol{0}) = \exp\left[\frac{g^2}{2}\sum_{i\neq j=1}^{\tau/a_t} a_t^2 \Delta_L^T(i - j, \boldsymbol{r} = \boldsymbol{0})\right].$$