How scaling ideas apply to lepton-nucleus scattering:

Nuclear effects, Momentum Distributions, Coulomb Sum Rule, Scaling violations...

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Electroweak Interactions with Nuclei: Superscaling and connections between electron and neutrino scattering

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SCALING in PHYSICS

Phenomenon occurring when interacting probes (leptons, neutrons) are scattered from composite systems (atoms, nuclei, nucleons,...)

Solid State, Atomic, Nuclear, Molecular, Particle Physics,... (wide range in energy)

SCALING *in projectile–composite system scattering reactions: the response of the complex system does not longer depend on two independent variables, ω, q, but only on a particular combination of those, called "scaling" variable.*

- "Scaling-y" (QEP). The (e, e') differential cross section, divided by an appropriate factor, depends only on the variable $y(q, \omega) \Longrightarrow$ incoherent elastic scattering from constituents in the nucleus.
- Extension to other processes: (ν_l, l), $(\nu, N)\nu'$ and other kinematic regions: Δ-excitation and beyond.

"SCALING": spectral function of the constituents in the complex system & physics of final state

Scaling in QE (e, e') processes

Basic mechanism in QE (e, e') process: elastic scattering of nucleons in nuclei with (free?) ejection of a nucleon

$$\left[\frac{d\sigma}{d\epsilon' d\Omega'}\right]_{(e,e')} = \sum_{i=1}^{A} \int_{-y(q,\omega)}^{Y(q,\omega)} p dp \int_{0}^{\varepsilon_M(q,\omega;p)} d\varepsilon \int_{0}^{2\pi} d\phi_N \left(\frac{E_N}{q p_N^2}\right) \left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N}\right]_{(e,e'N)}$$

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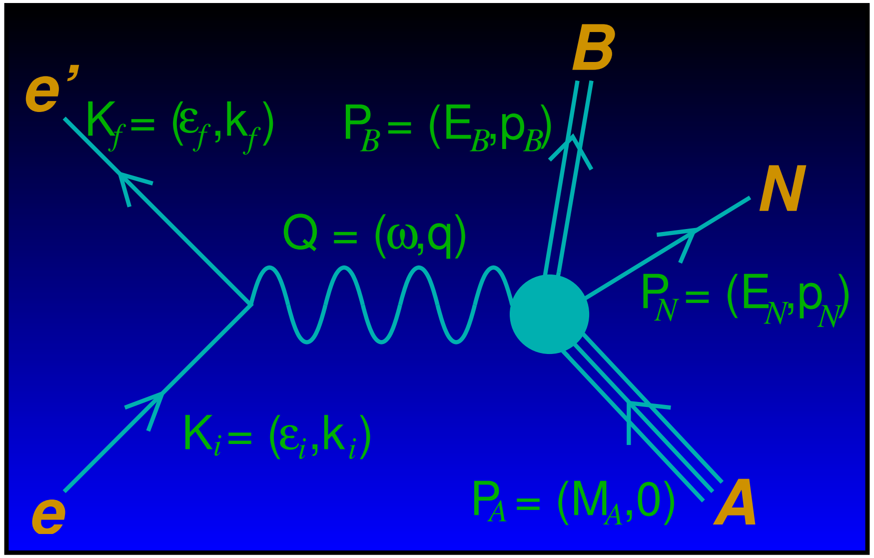

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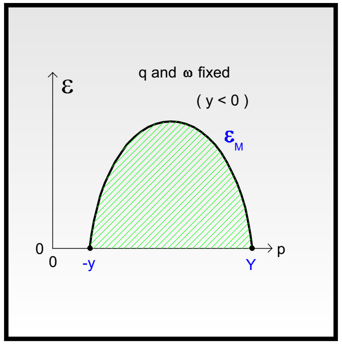
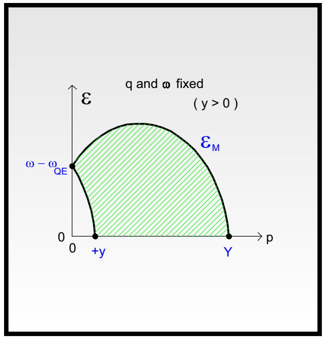
- Kinematics of (e, e') processes

  p: missing momentum

  $\varepsilon(p) = E^2_{A-1} - E^{0^2}_{A-1}$: exc. en.

  $y(q, \omega)$: minimum value of p





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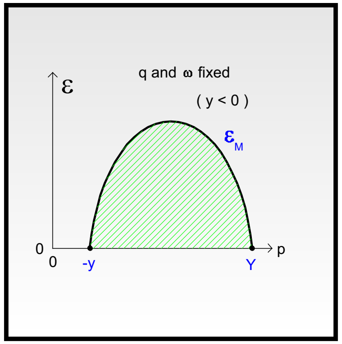
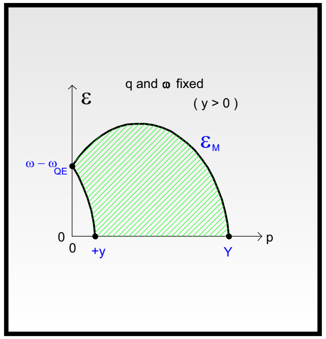
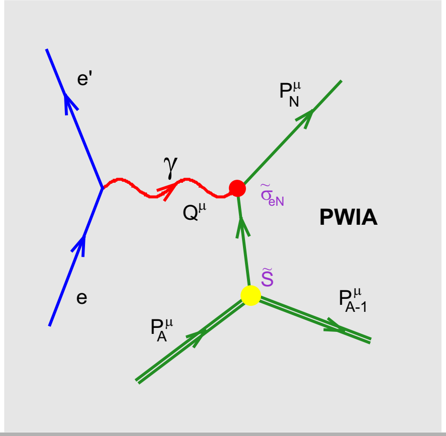




- Plane Wave Impulse Approximation (PWIA). Factorization in $(e, e'N)$:



$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N}\right]_{(e,e'N)} = K\sigma_{eN}(q,\omega;p,\varepsilon,\phi_N)S(p,\varepsilon)$$

$$\left[\frac{d\sigma}{d\Omega' d\omega}\right]_{(e,e')} \approx \overline{\sigma}_{eN}(q,\omega;p=-y,\varepsilon=0) \cdot F(q,y)$$

$$F(q,y) \equiv 2\pi \int_{-y(q,\omega)}^{Y(q,\omega)} pdp \int_0^{\varepsilon_M(q,\omega;p)} d\varepsilon S(p,\varepsilon)$$

WHAT QE (e, e') DATA ARE SHOWING US?

Analysis of experimental cross sections

Experimental scaling function:

$$F(q, y) = \frac{[d\sigma/d\omega d\Omega']_{exp}}{\overline{\sigma}_{eN}(q, \omega; p = -y, \varepsilon = 0)}$$

$$\overline{\sigma}_{eN}(q, \omega; p, \varepsilon) \equiv \frac{1}{2\pi} \int d\phi_N \frac{E_N}{q} \left[Z\sigma_{ep}(q, \omega; p, \varepsilon, \phi_N) + N\sigma_{en}(q, \omega; p, \varepsilon, \phi_N) \right]$$

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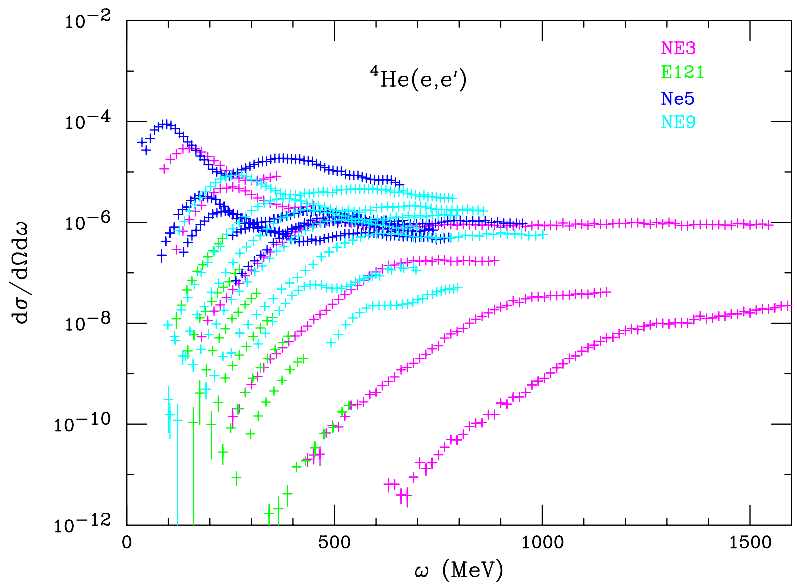

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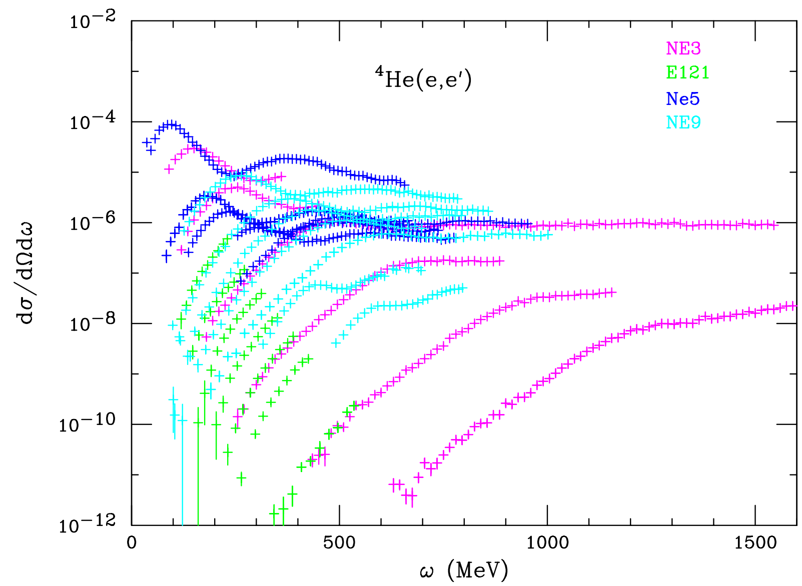
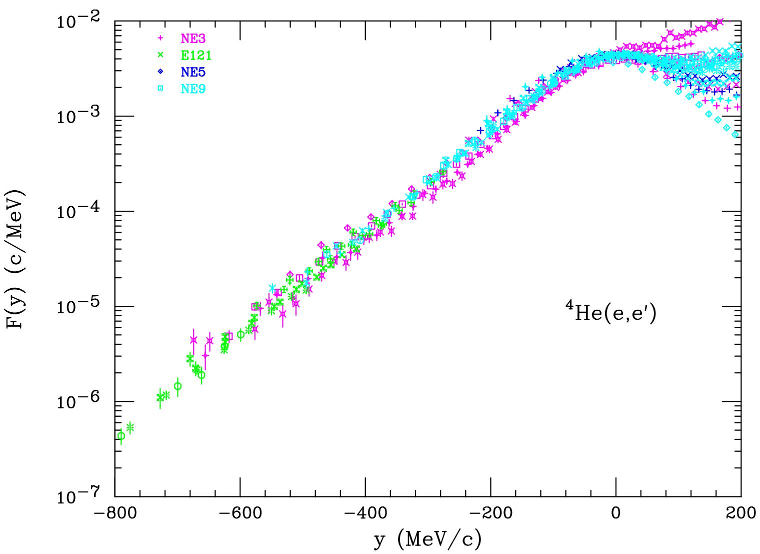
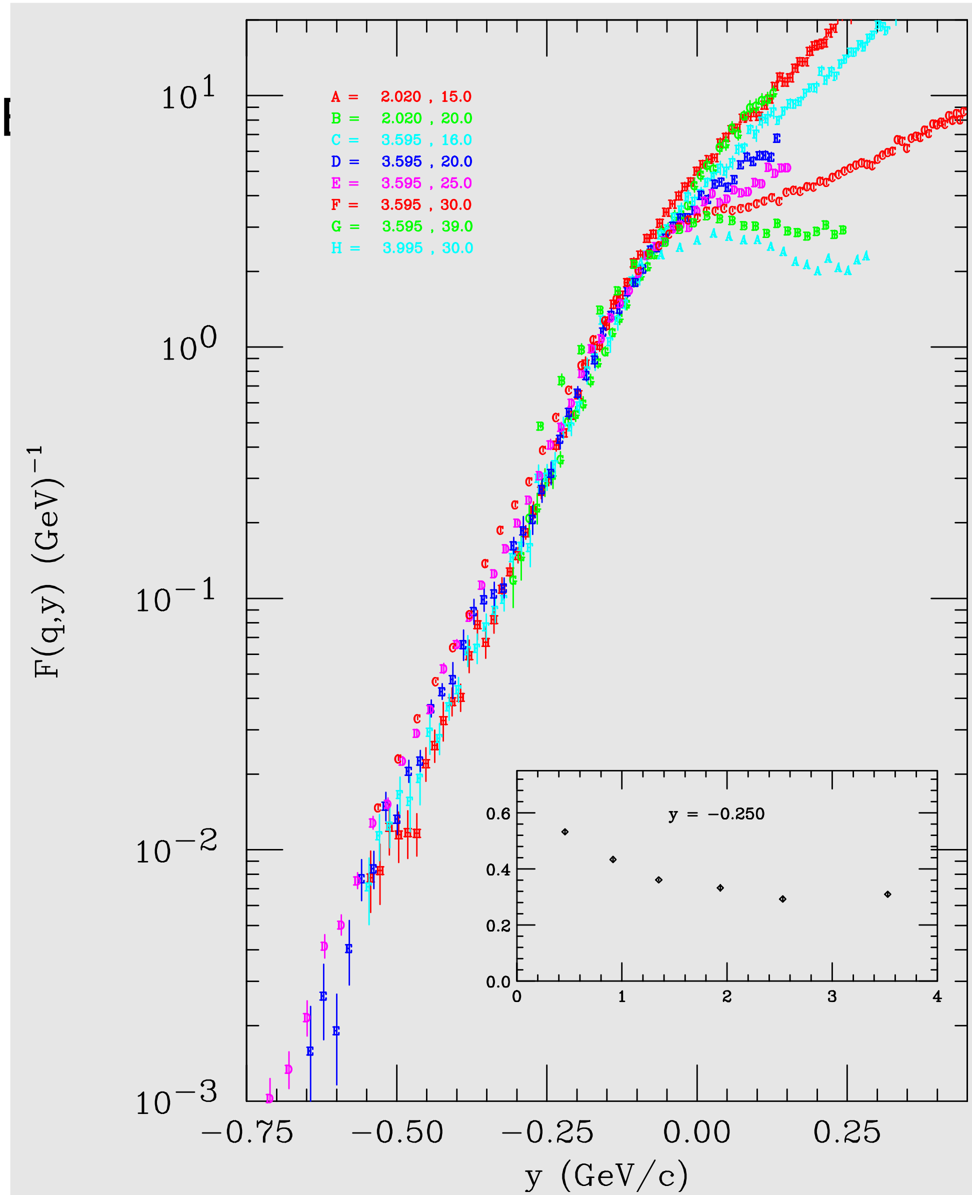




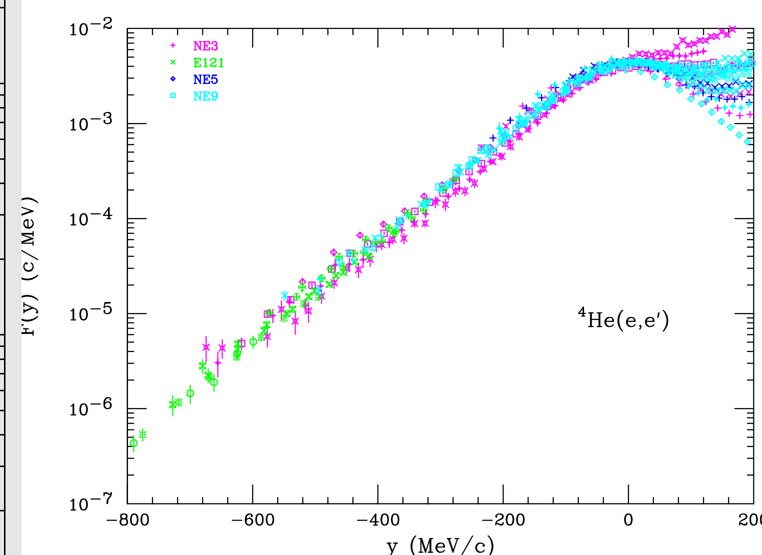
Scaling of the first kind: $q \to \infty \Longrightarrow$ $F(q,y) \longrightarrow F(y) \equiv F(\infty, y)$

Analysis of experimental cross sections



$$F(q,y) = \frac{[d\sigma/d\omega d\Omega']_{exp}}{\overline{\sigma}_{eN}(q,\omega;p=-y,\varepsilon=0)}$$

$$q,\omega;p,\varepsilon,\phi_N) + N\sigma_{en}(q,\omega;p,\varepsilon,\phi_N)]$$



$$\Rightarrow \quad F(q,y) \longrightarrow F(y) \equiv F(\infty,y)$$

SUPERSCALING (*guided by RFG*): analysis of data

$$f(q,\psi) \equiv k_F \frac{[d\sigma/d\omega d\Omega_e]}{\sigma_M\,[v_L G^L + v_T G^T]}, \quad f^L(q,\psi) \equiv k_F \frac{R^L(q,\omega)}{G^L}, \quad f^T(q,\psi) \equiv k_F \frac{R^T(q,\omega)}{G^T}$$

- Scaling of the first kind: $f_{exp}(q,\psi) \stackrel{q\to\infty}{\longrightarrow} f_{exp}(\psi);\ \ \psi \approx y/k_F$ – *superscaling variable*
- Scaling of the second kind: $f_{exp}(\psi)$ – *independence on the nuclear system*

SUPERSCALING

- Scaling of the zeroth kind: $f_{exp}(q,\psi) = f^L_{exp}(q,\psi) = f^T_{exp}(q,\psi)$

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Relativistic Fermi Gas (RFG):

$$f_{RFG}(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2); \quad \psi = \frac{y}{k_F}\left[1 + \mathcal{O}(\eta_F^2)\right]$$

$f_{RFG}(\psi)$: *independent on the transferred momentum q and valid for all nuclear systems*

Universality character of the Scaling Function

SUPERSCALING (*guided by RFG*): analysis of data

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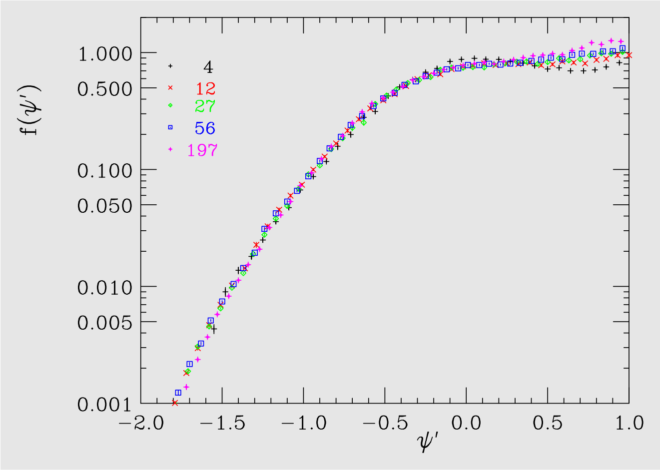

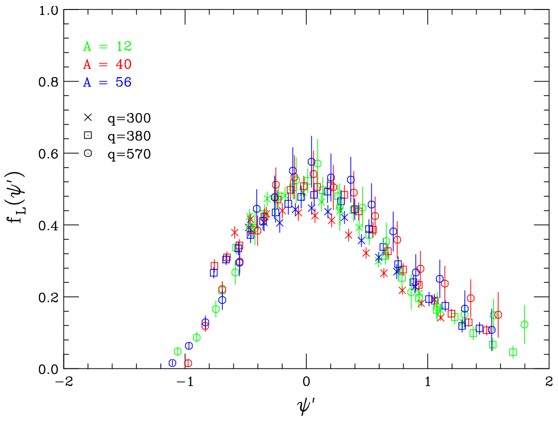

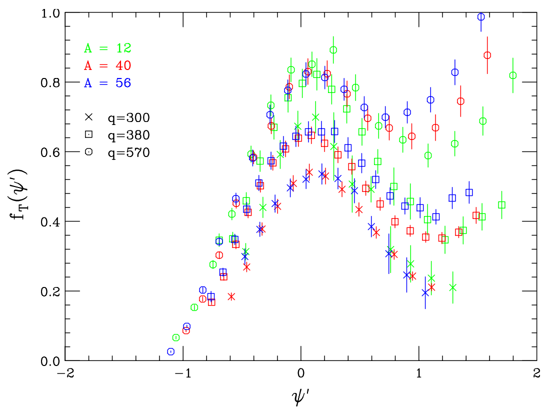

BASIC CONCLUSIONS

- *Reasonable scaling of the first kind below the QE peak* ($\psi \leq 0$)
- *Excellent scaling of the second kind in the same region*
- *Breaking of scaling, particularly of the first kind, above the QE peak* ($\psi > 0$) $\Longrightarrow$ *Effects beyond the IA (mainly in the T channel)*
- *The L response superscales*

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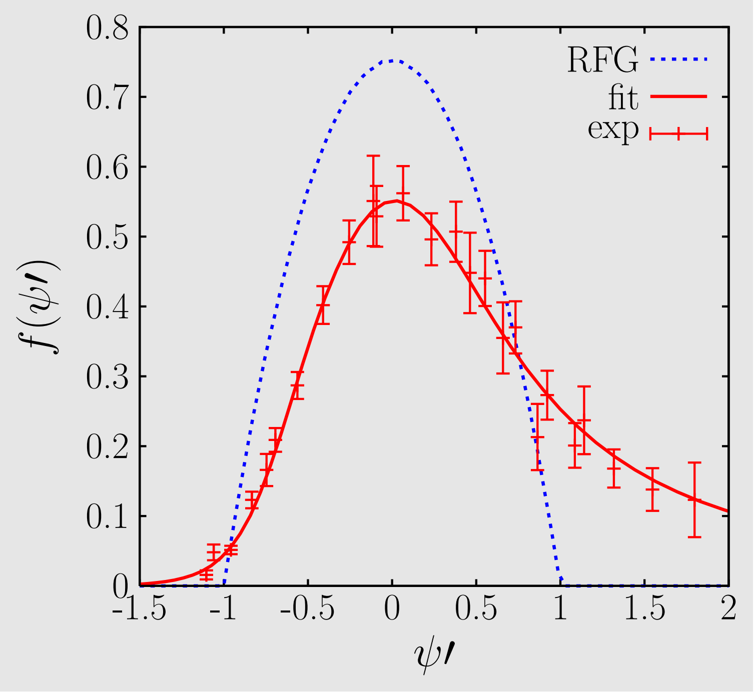
- *The L response superscales*

Experimental superscaling function: asymmetric shape with a long tail extended to positive ψ-values

PRC60 (1999) 065502

PRL82 (1999) 3212

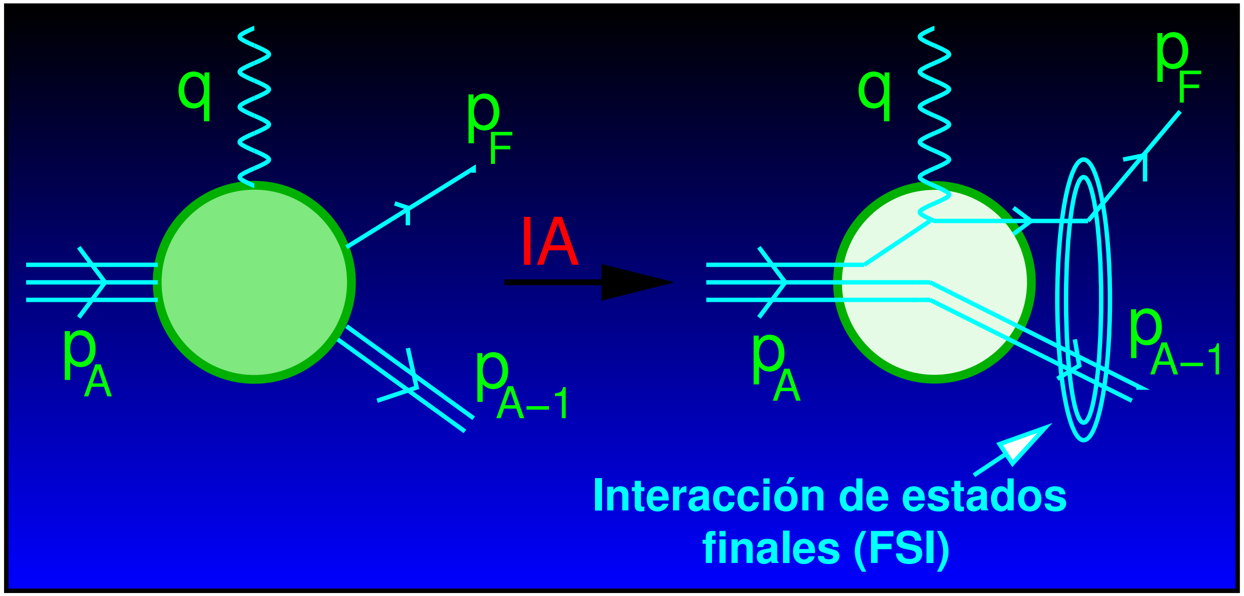
PRC65 (2002) 025502



THE MODEL: RELATIVISTIC IMPULSE APPROXIMATION

APPLICATION TO (e, e') PROCESSES

Relativistic Impulse Approximation (RIA)



Nuclear Current $\Longrightarrow$ One-body operator

$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p}\, \overline{\Psi}_F(\vec{p}+\vec{q}) \hat{J}_N^\mu \Psi_B(\vec{p})$$

Scattering off a nucleus $\Longrightarrow$ incoherent sum of single–nucleon scattering processes

Ingredients in RIA: nucleon w.f. & current operator

Solutions of Dirac equation with phenomenological relativistic potentials

- Ψ_B: **Bound nucleon w.f.** $\Longrightarrow$ **Relativistic Mean Field (RMF)**
- Ψ_F: **Ejected nucleon w.f.** $\Longrightarrow$ **Final State Interactions (FSI)**

RMF $\Leftrightarrow$ rROP $\Leftrightarrow$ RPWIA $\Leftrightarrow$ RGFA

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RMF $\Leftrightarrow$ rROP $\Leftrightarrow$ RPWIA $\Leftrightarrow$ RGFA

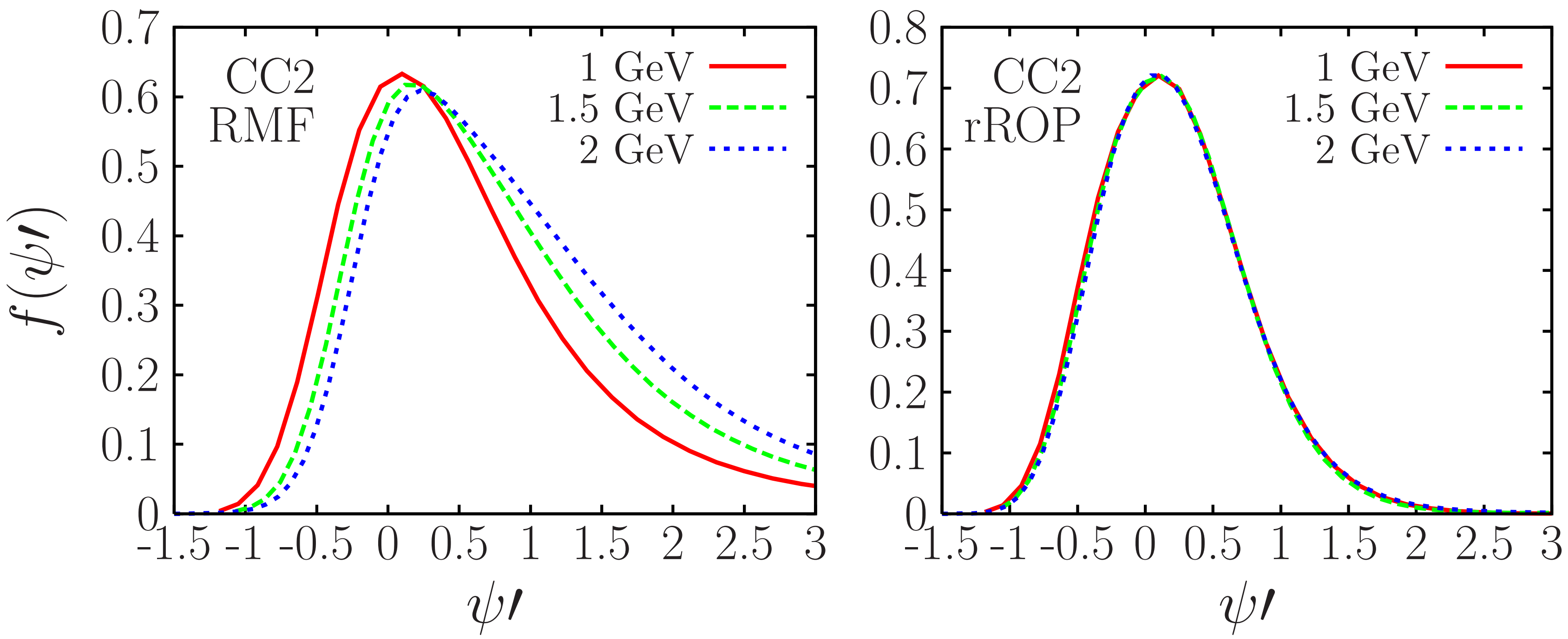
- **Electromagnetic current:** (e, e')

$$\hat{J}^{\mu}_{cc1} = (F_1 + F_2)\gamma^{\mu} - \frac{F_2}{2m_N}(\overline{P} + P_N)^{\mu}$$

$$\hat{J}^{\mu}_{cc2} = F_1\gamma^{\mu} + \frac{iF_2}{2m_N}\sigma^{\mu\nu}Q_{\nu}$$

Off-shell & Gauge ambiguities ($Q_{\mu}J^{\mu} \neq 0$)

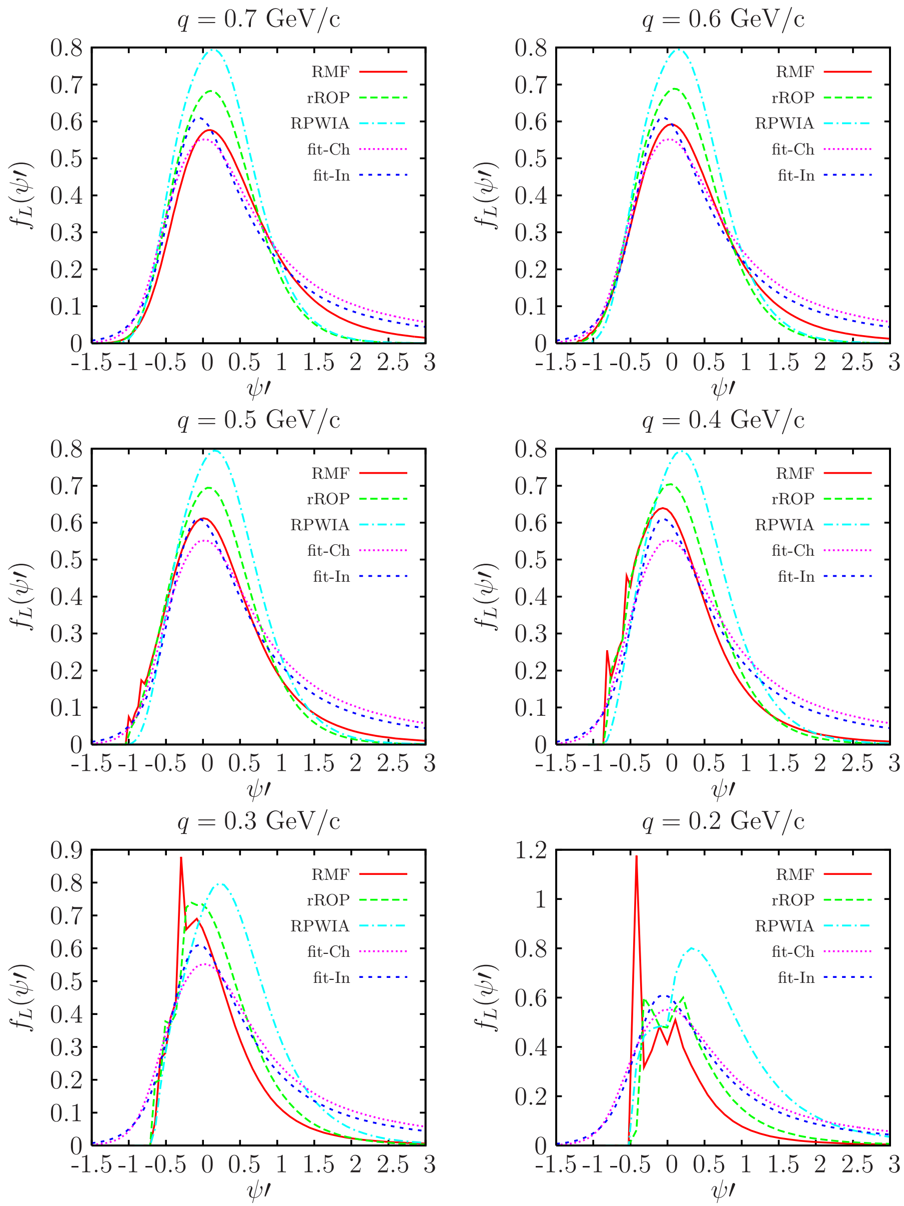
Scaling of the first kind in $^{12}C(e,e')$



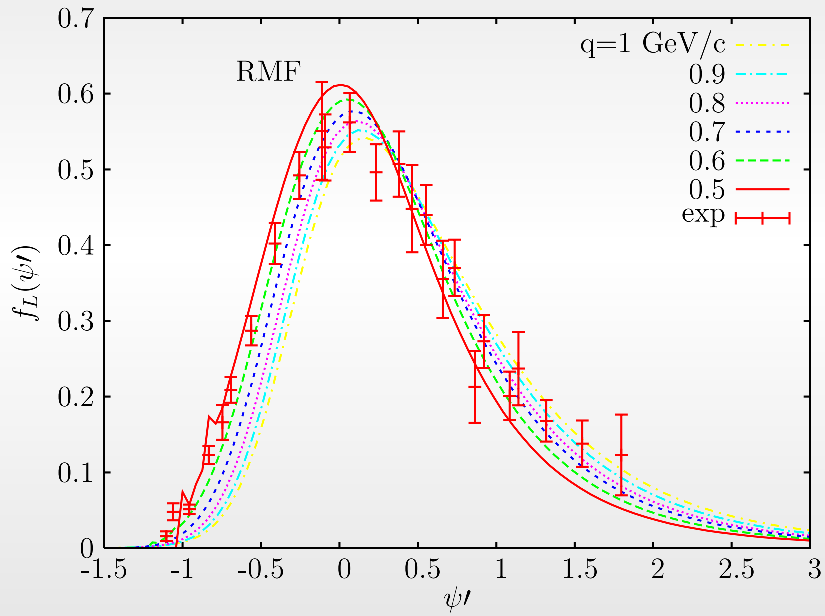
RMF: shift in $\psi' < 0$ and breakdown of scaling at roughly $\sim 25 - 30\%$ for $\psi' > 0$ (compatible with data).

Scaling of the first kind: excelent in rROP approach (and RPWIA)

Breaking of scaling of the 1^{er} kind

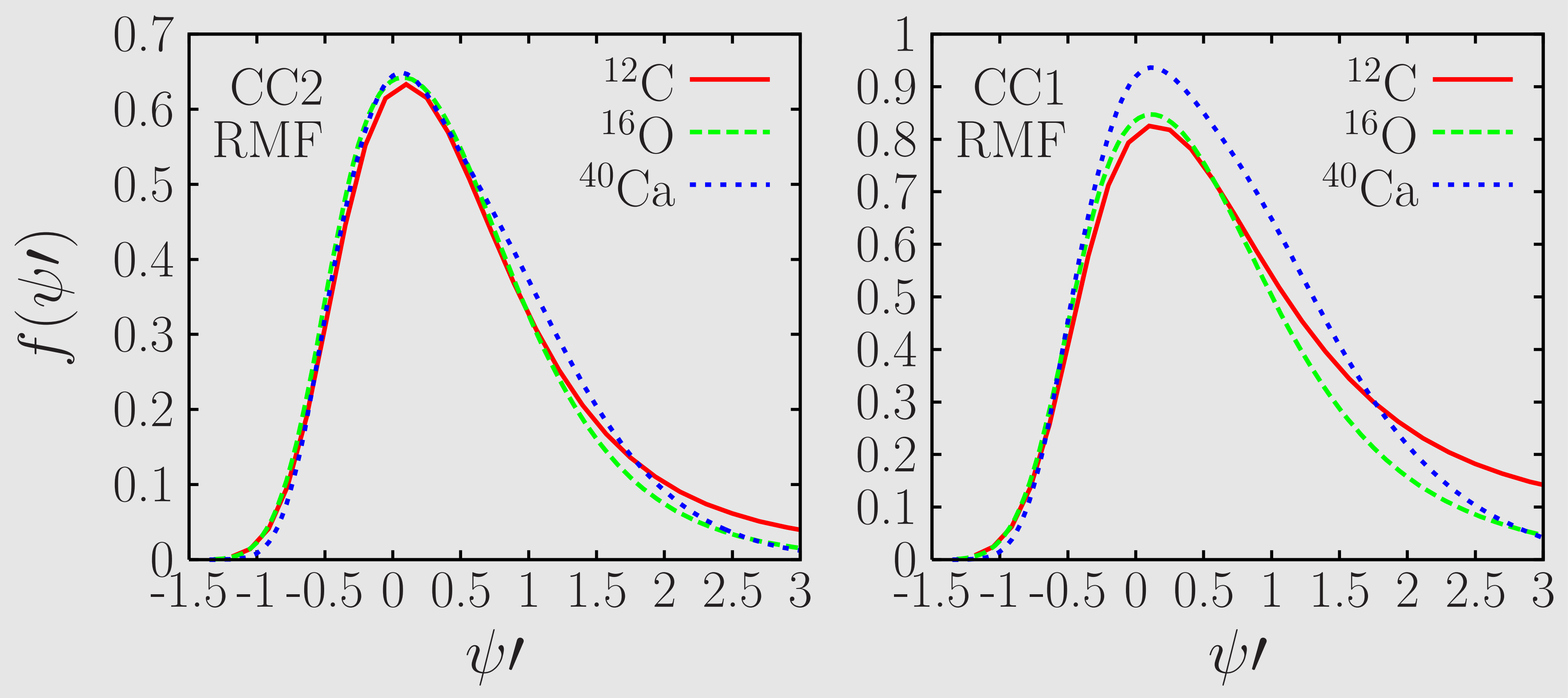


Scaling of first kind. Results for $^{12}C(e,e')$ and $\epsilon_e = 1$ GeV. CC2 current prescription



Scaling phenomenon of 1^{er} kind (independence on the transfer momentum q) is observed for values: $q \geq 0.4 - 0.5$ GeV/c.

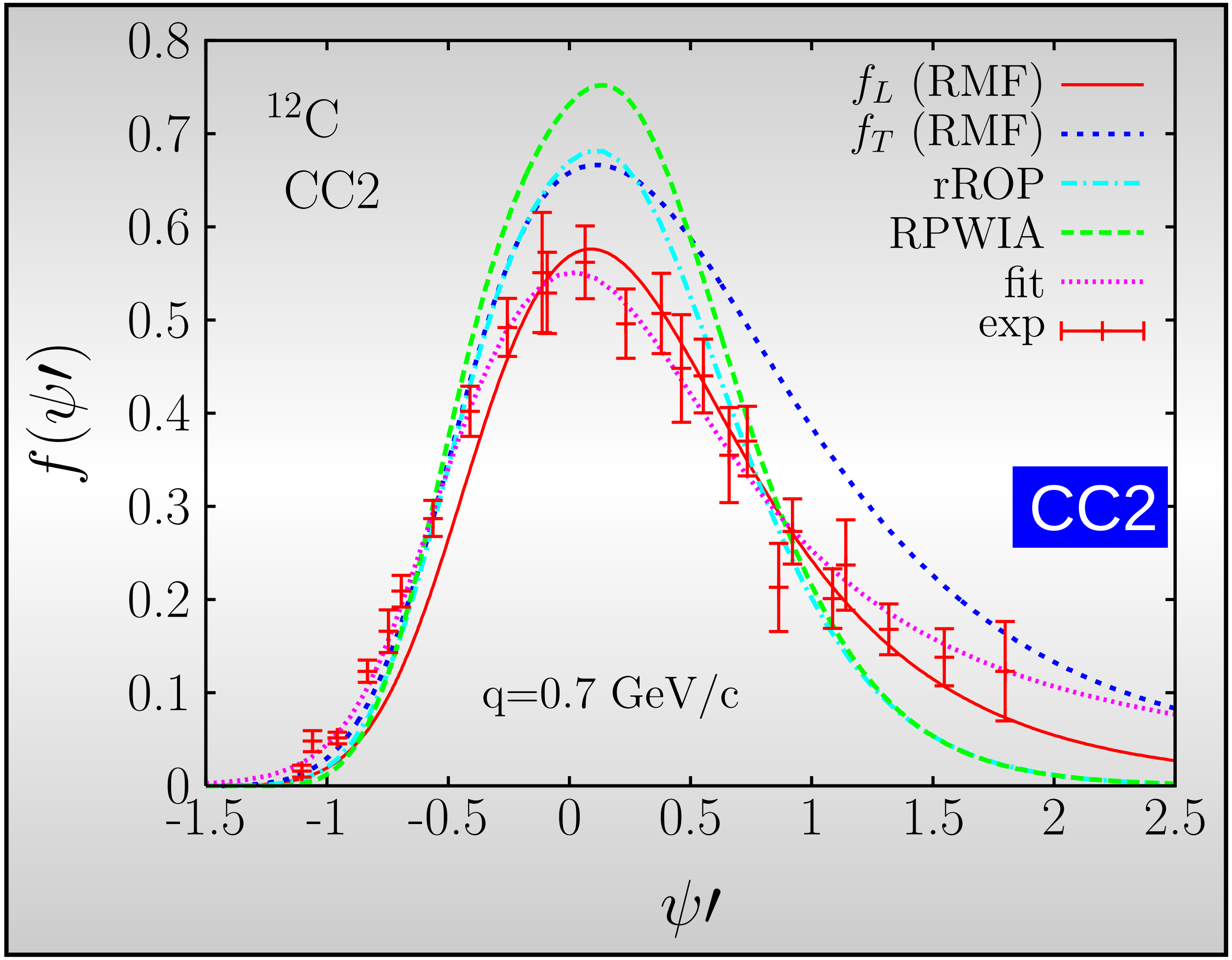
Scaling of the second kind in RIA



Scaling of 2^a kind: excellent with the CC2 current operator

Visible scaling violation for CC1 (due to the T contribution)

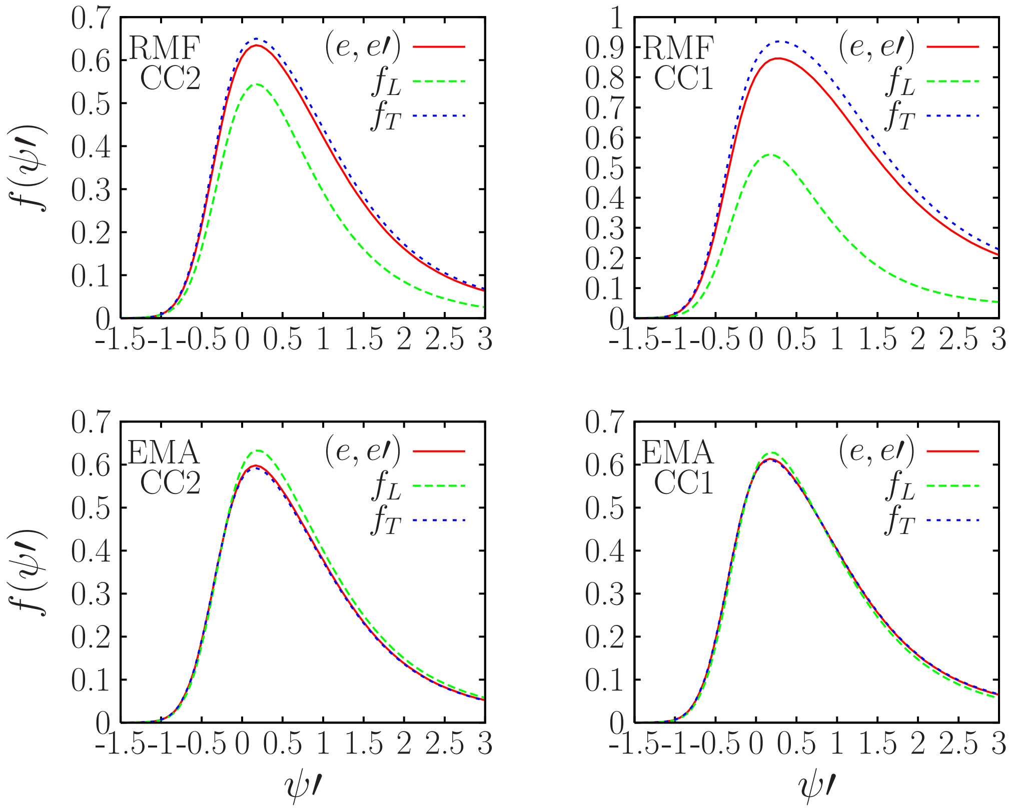
Comparison with (e, e') data



Only the description of FSI provided by RMF leads to an asymmetric function $f(\psi')$ in accordance with the behavior shown by data.

Scaling of the zeroth kind in RIA

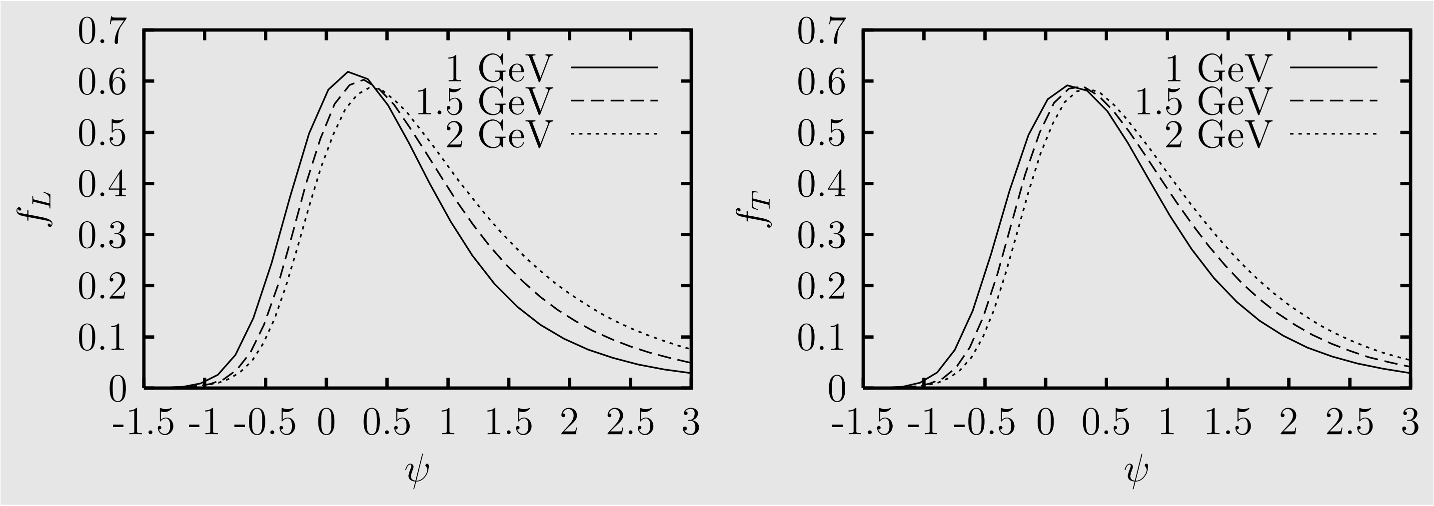
$\varepsilon_e = 1$ GeV
$q = 1$ GeV/c



EMA ("Effective Momentum Approximation"): nucleon (bound and ejected) wave functions are projected over positive-energy states. Results in the figure indicate that relativistic nuclear dynamics in the final channel in presence of strong relativistic potentials (RMF model) leads to very significant effects in $f(\psi)$ concerning zeroth kind scaling. SR models are not able to produce these effects.

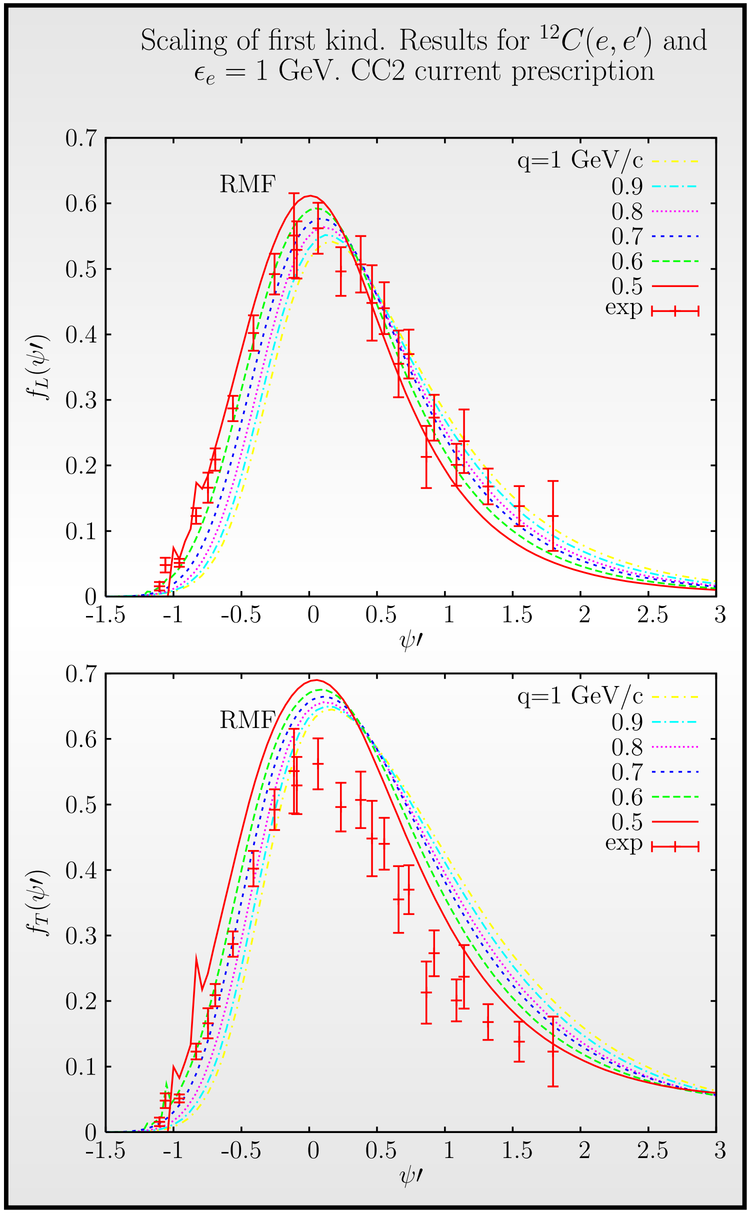
Scaling of the 0^{th} kind (other models)

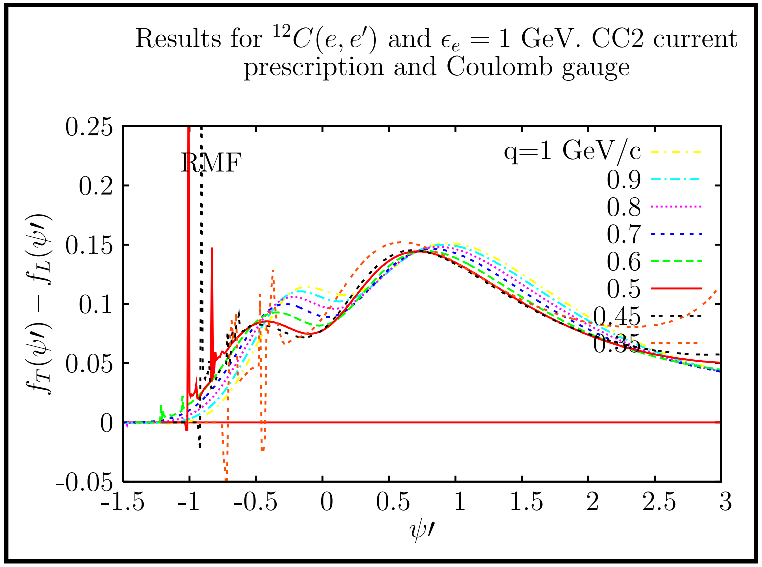
- RFG: by construction $f_L(\psi) = f_T(\psi) = f(\psi)$
- RPWIA: $f_L(\psi) = f_T(\psi) = f(\psi)$–symmetric
- Semi-relativistic (SR) and/or NR approaches with FSI:
  - *Woods-Saxon potential: symmetric scaling functions.*
  - *Dirac Equation-Based (DEB) potential: leads to an asymmetric function $f(\psi)$.*

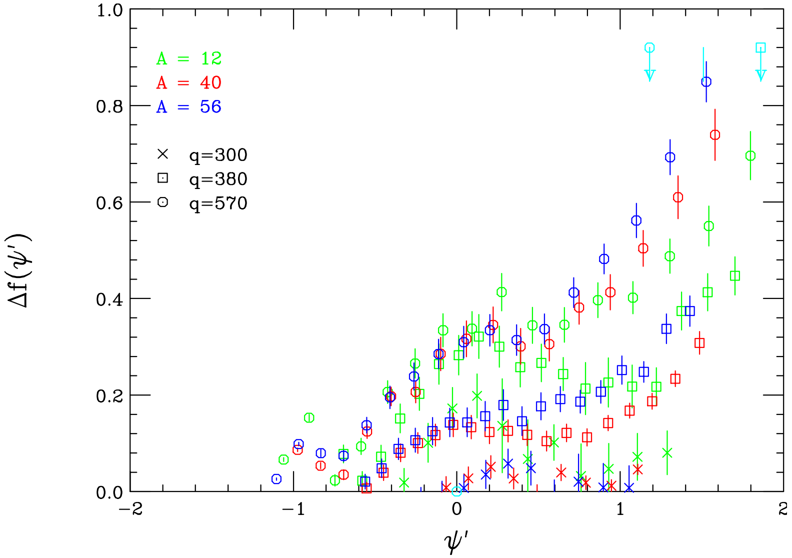


In all cases: $f_L(\psi) = f_T(\psi) = f(\psi)$

Zeroth order scaling violation shown by data?





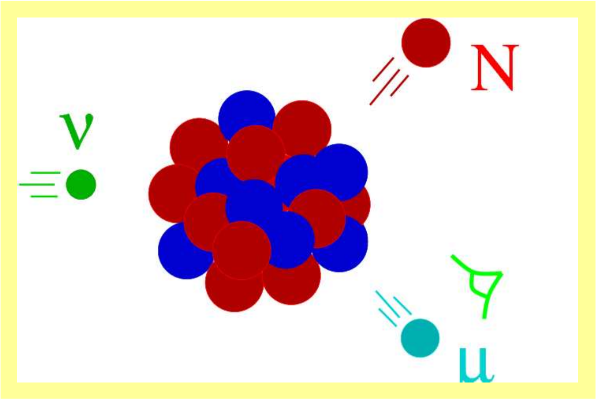


APPLICATION TO (ν_μ, μ) PROCESSES

Quasielastic (e, e') versus (ν, μ) reactions

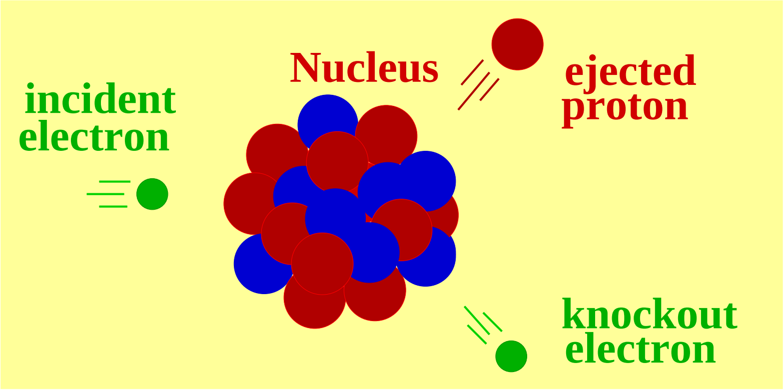
Neutrinos

$$\nu + A \Rightarrow \mu + N + B$$



Electrons

$$e + A \Rightarrow e' + N + B$$



Kinematics in electron and neutrino INCLUSIVE scattering are very similar. One should check models of neutrino scattering against inclusive electron data.

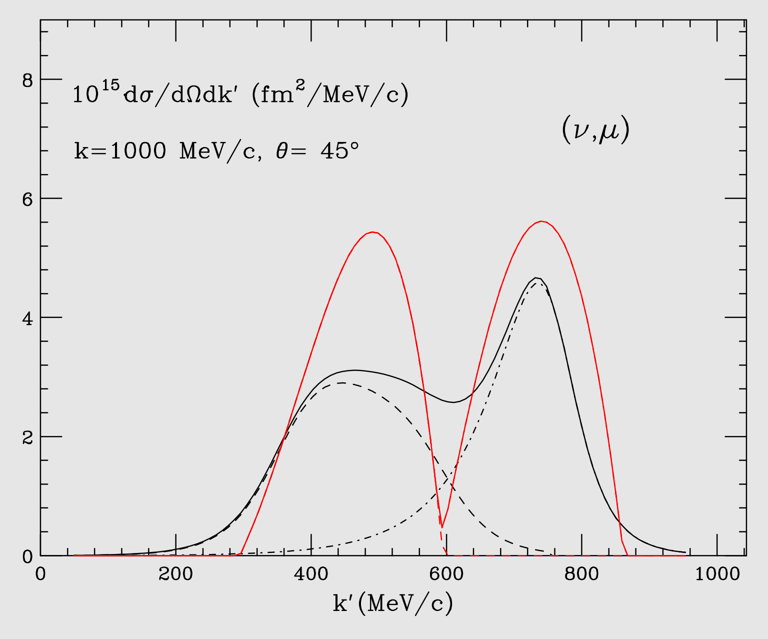
However, caution with models that do not succeed in reproducing the experimental (e, e') scaling function.

Scaling approach to (ν, μ) processes

HOW TO PROCEED?

- *Extract the single-nucleon contribution from the differential cross section corresponding to (ν, μ) reactions (use RFG as a guide)*
- **Hyphotesis:** *the universal character of the function $f_{exp}(\psi')$ extracted from the analysis of (e, e') data $\Longrightarrow$ it results also valid for CC (ν_μ, μ) processes*.
  Prediction of "realistic" (ν, μ) cross sections [MiniBoone]

SuSA *("SuperScaling Analysis")*

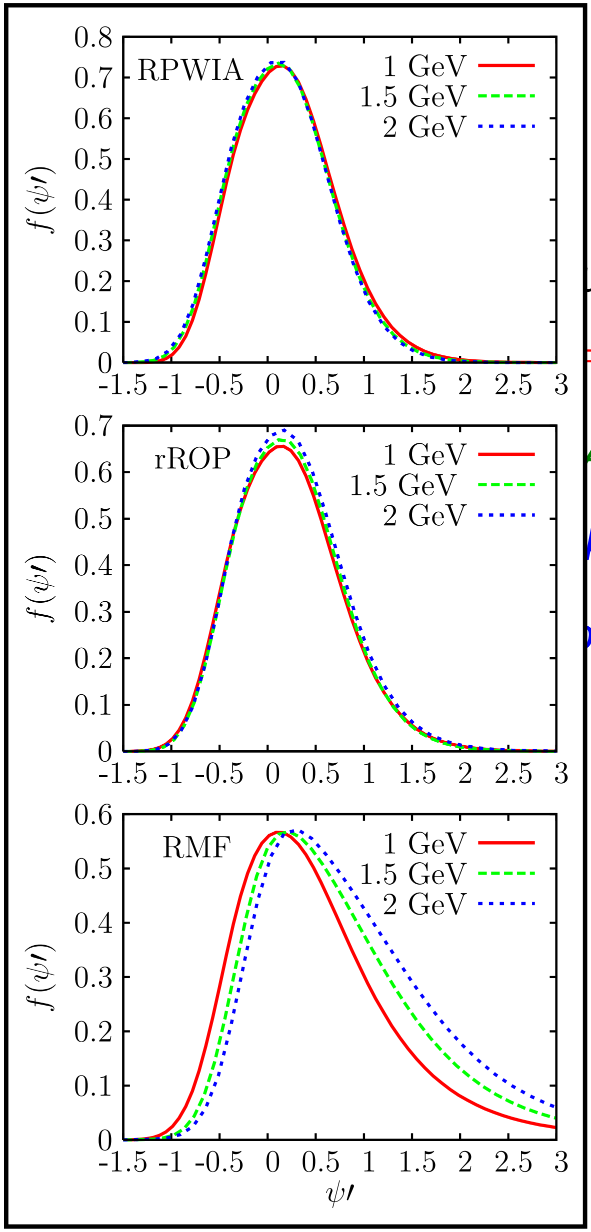


Other scaling approach to (ν, μ): **RIA analysis**

PROCEDURE

- *Evaluate the inclusive (ν, μ) cross section with a specific RIA model and divide it by the corresponding single-nucleon cross section [weighted by the appropriate proton (Z) and neutron (N) numbers]* $\Longrightarrow$ **THEORETICAL SCALING FUNCTION**
- *Does the theoretical RIA scaling function satisfy scaling properties?*
  - *Scaling of the first kind:* $f(q, \psi) \stackrel{q \to \infty}{\longrightarrow} f(\psi)$
  - *Scaling of the second kind:* $f(\psi)$ – *independent on the nucleus*

Other scaling approach to (ν, μ): RIA analysis



PROCEDURE

$\nu, \mu)$ cross section with a specific RIA model and divide it by the

ucleon cross section [weighted by the appropriate proton (Z) and

$\Longrightarrow$ **THEORETICAL SCALING FUNCTION**

A scaling function satisfy scaling properties?

kind: $f(q, \psi) \xrightarrow{q \to \infty} f(\psi)$

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Other scaling approach to (ν, μ): RIA analysis

PROCEDURE

- *Evaluate the inclusive (ν, μ) cross section with a specific ... the corresponding single-nucleon cross section [weighted by ... and neutron (N) numbers]* $\Longrightarrow$ **THEORETICAL SCALING F...**
- *Does the theoretical RIA scaling function satisfy scaling p...*
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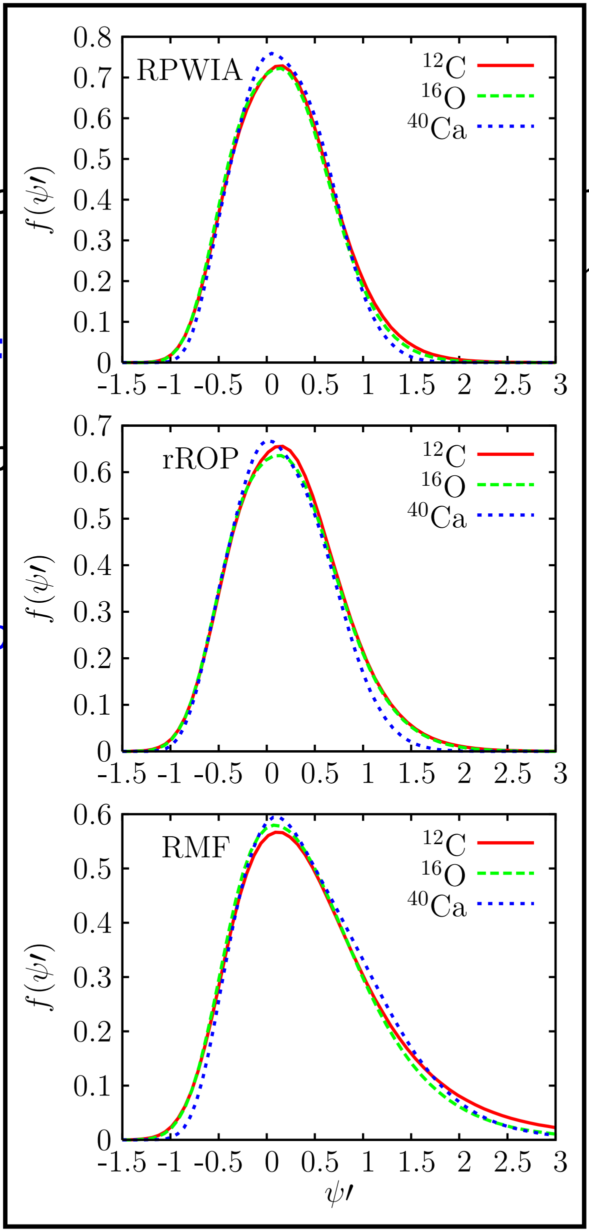

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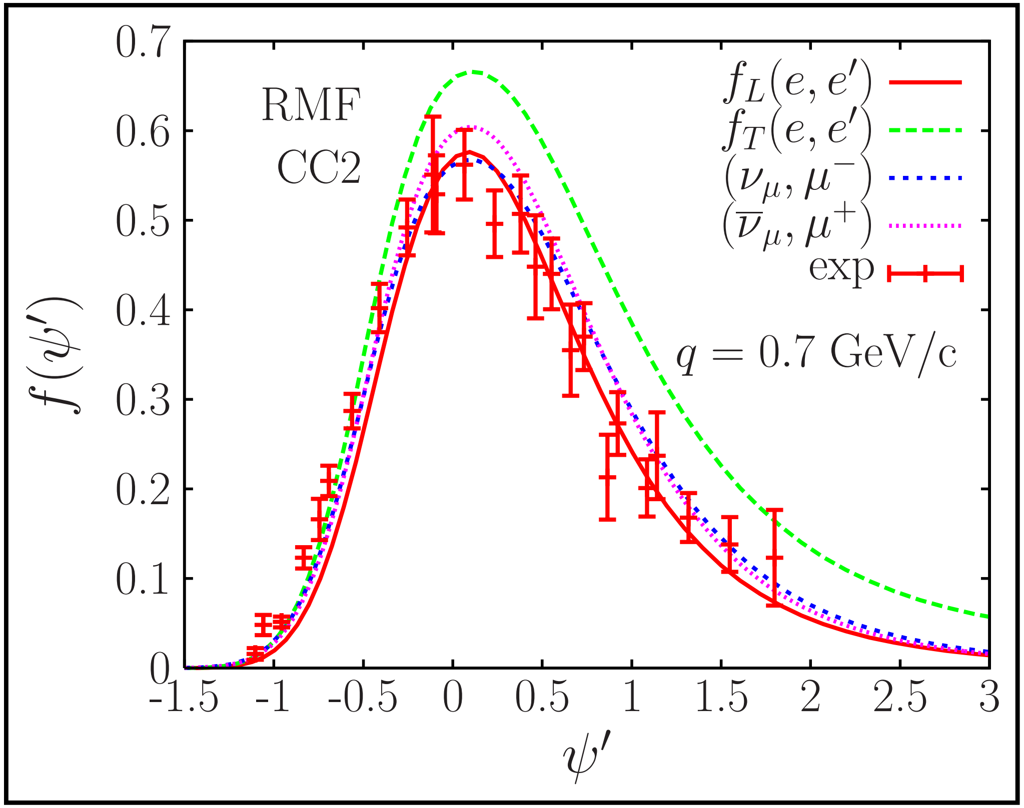
- *Evaluate the inclusive (ν, μ) cross section with a specific RIA model and divide it by the corresponding single-nucleon cross section [weighted by the appropriate proton (Z) and neutron (N) numbers]* $\Longrightarrow$ **THEORETICAL SCALING FUNCTION**
- *Does the theoretical RIA scaling function satisfy scaling properties?*
  - *Scaling of the first kind:* $f(q, \psi) \stackrel{q\to\infty}{\longrightarrow} f(\psi)$
  - *Scaling of the second kind:* $f(\psi)$ – *independent on the nucleus*
- *Is the function $f(\psi)$ obtained from (ν, μ) cross sections evaluated within RIA consistent with the function $f(\psi)$ obtained from (e, e') calculations (with the same model)?, and with $f_{exp}(\psi)$?*

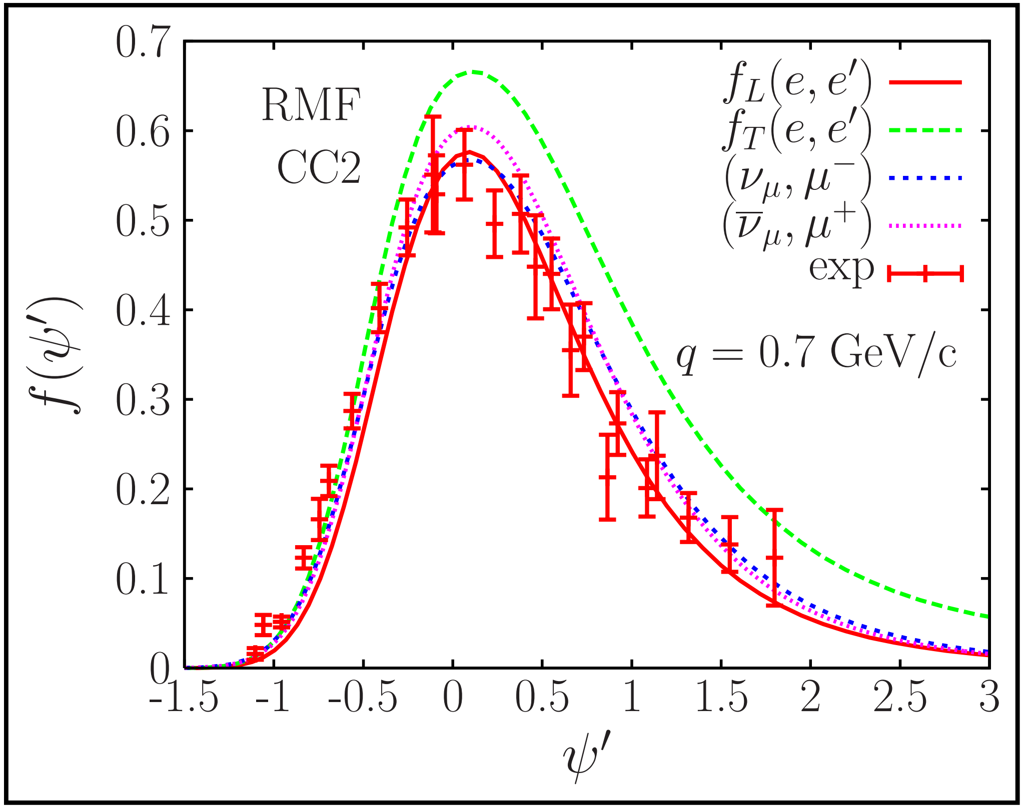
Similar scaling function $f(\psi)$ for (e, e') and (ν, μ) processes?

(e, e') vs (ν, μ) scaling and experiment



Basic result: *the function $f(\psi)$ evaluated for (ν, μ) processes agrees better with the contribution $f_L(\psi)$ [corresponding to (e, e')] than with $f_T(\psi)$.*

(e,e') vs (ν,μ) scaling and experiment



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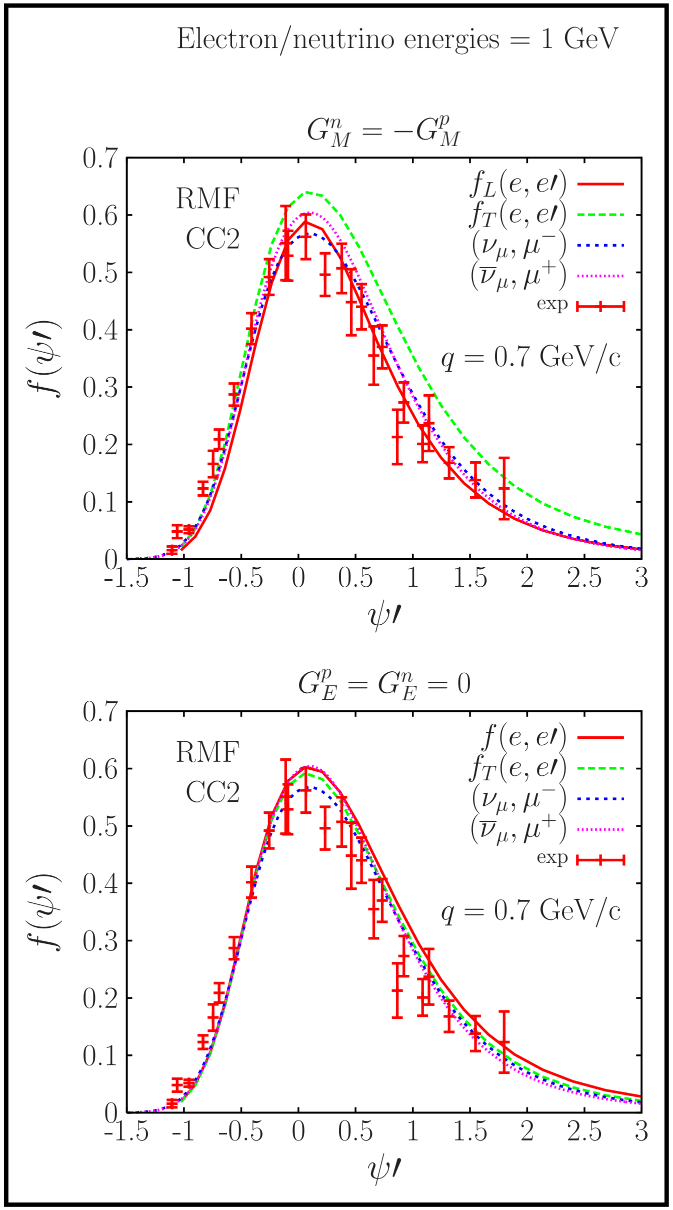
The previous result, that may seem strange because in (ν,μ) processes the channel that contributes the most is pure transverse, can be explained from the isospin contributions in both processes: whereas in (ν,μ) only pure isovector form factors enter, (e,e') contain isoscalar and isovector contributions.

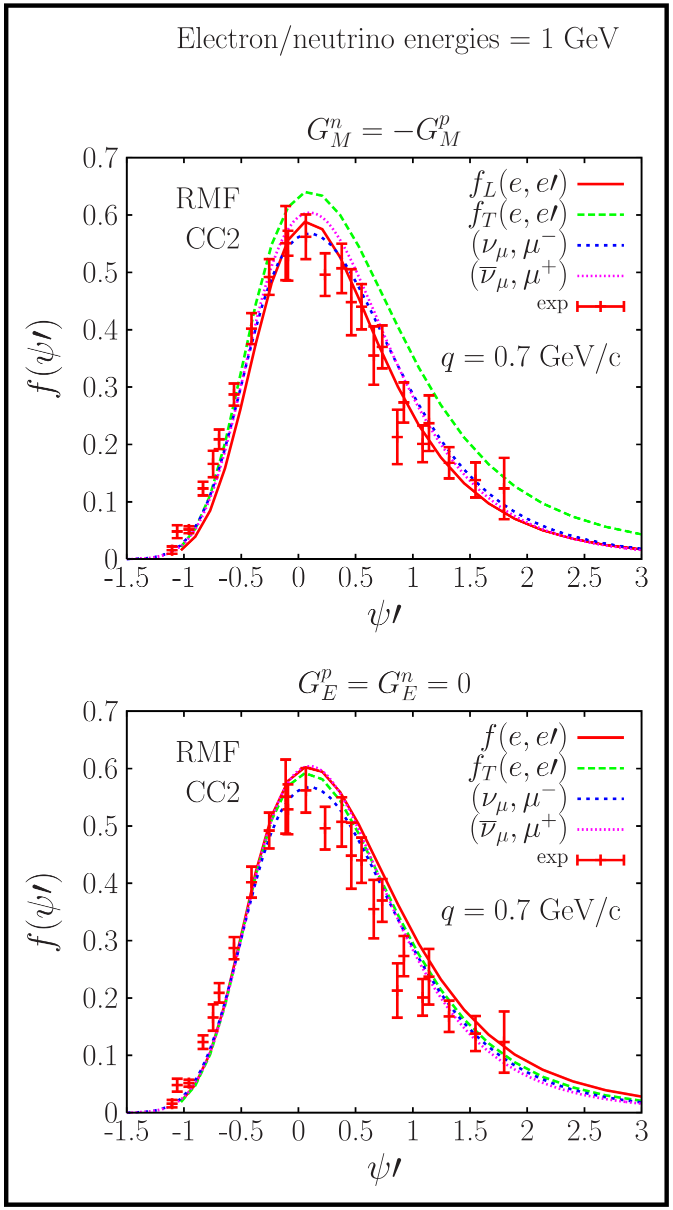
ISOSPIN: isoscalar vs isovector (3^{er} kind scaling)



Neglecting the isoscalar contribution in G_M [$G_M^n = -G_M^p$] leads to a smaller difference between $f_T(\psi)$ and $f_L(\psi)$.

When convective terms are turned off, [$G_E^n = G_E^p = 0$], a unique (universal) scaling function emerges from the analysis of (e,e') calculated cross sections. Moreover, this function (without isoscalar terms) is in accordance with the result evaluated from (ν,μ) processes (pure isovector contribution), and with $f_{exp}(\psi)$ extracted from the analysis of longitudinal (e,e') world data.

ISOSPIN: isoscalar **vs** isovector (3^{er} kind scaling)



Neglect … *tion in* G_M

[G^n_M = … *r difference*

between

When … *turned off,*

[G^n_E = … *(universal)*

scaling … *he analysis*

of $(e,e$ … *ons. More-*

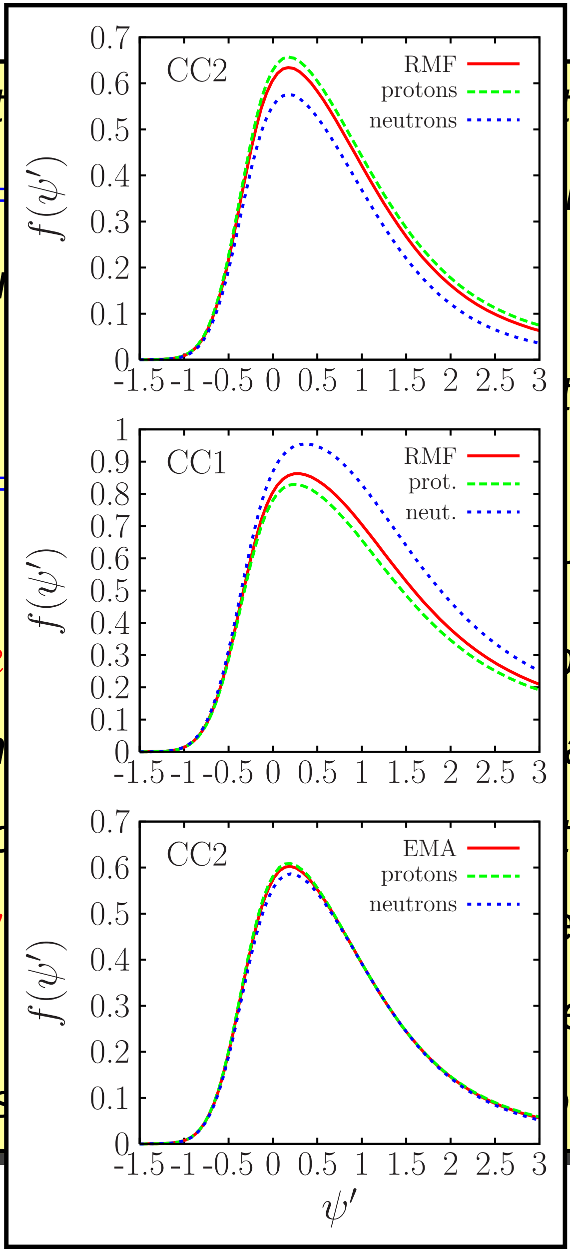
over, th … *alar terms)*

is in ac … *evaluated*

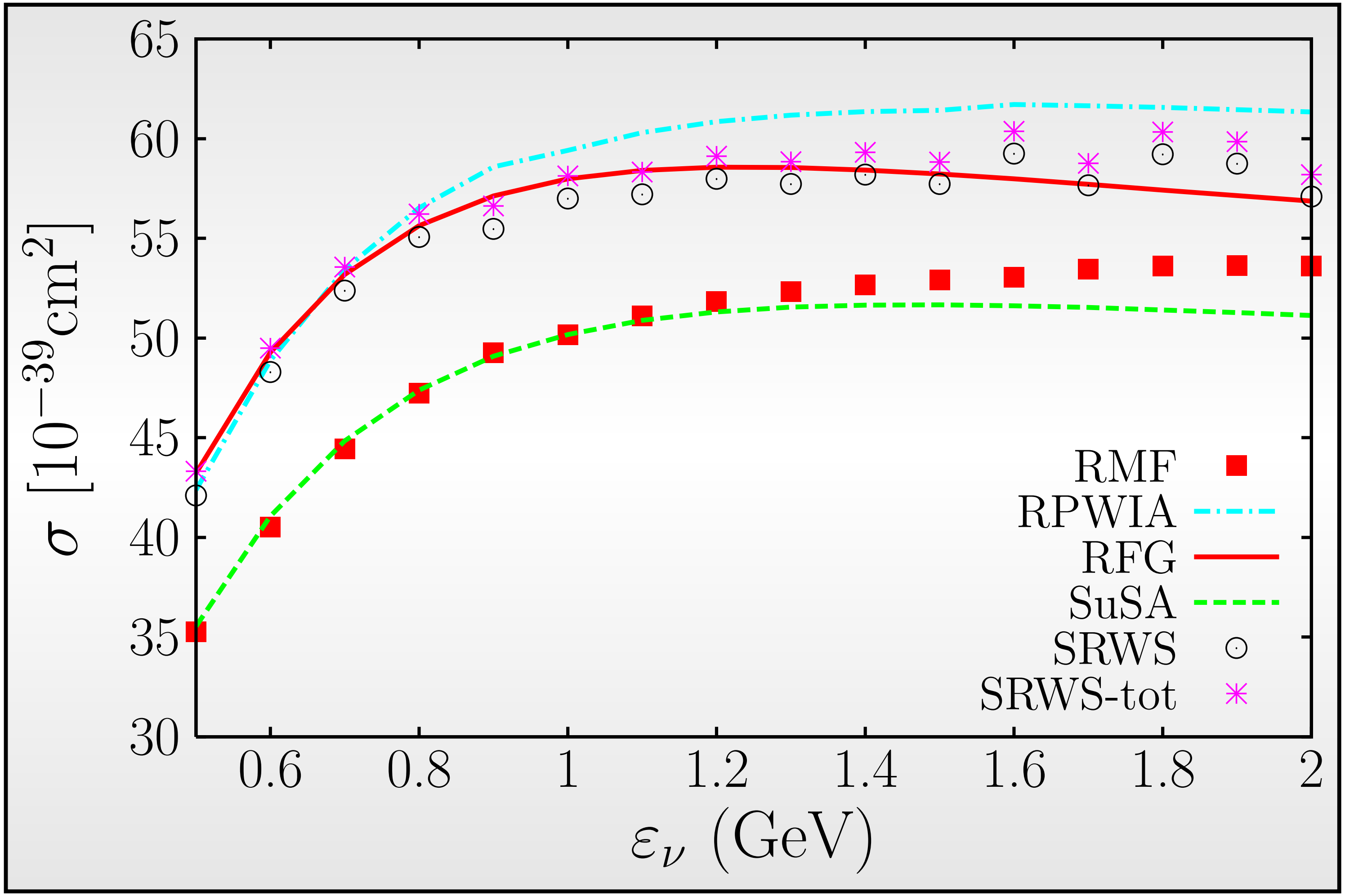
from $(\nu$ … *ector contri-*

bution), … *ed from the*

analysis … *rld data.*



INTEGRATED CROSS SECTION



Physical Review Letters 98, 242501 (2007)

BASIC CONCLUSIONS: (e, e') & (ν_μ, μ)

ELECTRON SCATTERING

- *Superscaling shows up in RIA, even with FSI: rROP & RMF.*
- *RMF-FSI description leads to an asymmetric superscaling function which fits data.*
- *Some first-kind scaling violation with FSI switched on.*

Contrary to most non-relativistic models, $f_L(\psi) \neq f_T(\psi)$ within RMF

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NEUTRINO SCATTERING

- *Superscaling fulfilled by RIA calculations.*
- *Scaling functions from QE (e,e') and (ν,μ) cross sections follow similar trends.*
- *Differences between (e,e') and (ν,μ) RIA+RMF results are consistent with the isoscalar/isovector nucleon f.f. contributions in the two processes.*

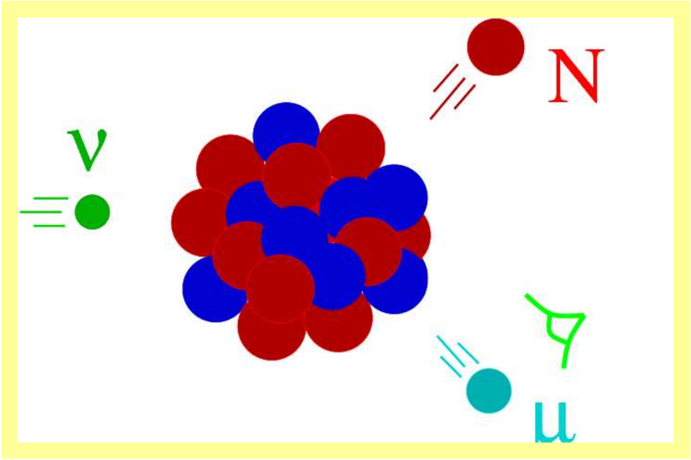
APPLICATION TO NEUTRAL CURRENT NEUTRINO PROCESSES: $(\nu, N)\nu'$

NC vs CC neutrino-nucleus QE scattering

The dominant processes in the QE region are assumed to be:

Charged-Current

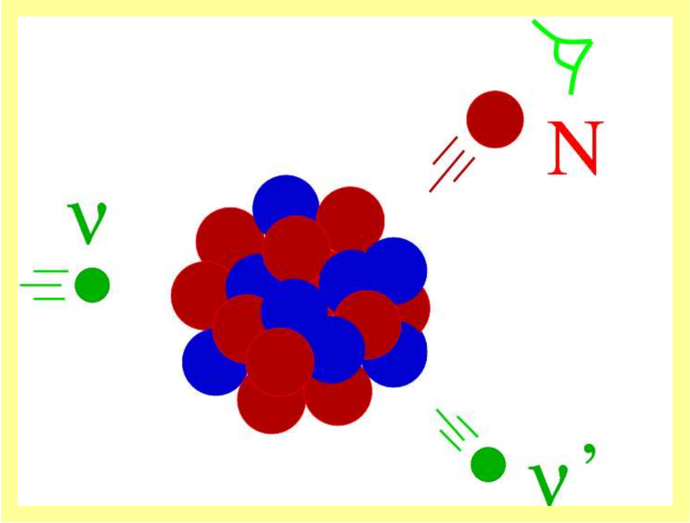
$$\nu + A \Rightarrow \mu + N + B$$



Outgoing lepton detected, fixed Q^2 as in (e, e'): **t-channel kinematics**

Neutral-Current

$$\nu + A \Rightarrow \nu' + N + B$$



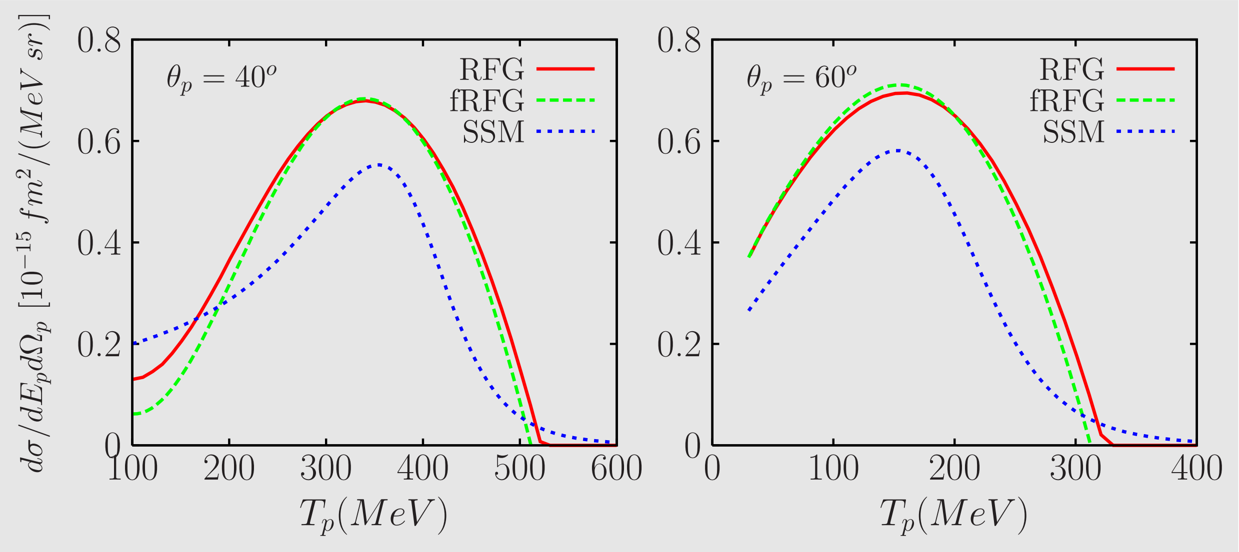
Only the nucleon is detected, Q^2 is not fixed: **u-channel kinematics**

Very different kinematics in both processes. Do they reveal different sensitivity to the nuclear dynamics underlying scaling?

Scaling and factorization for NC in RFG

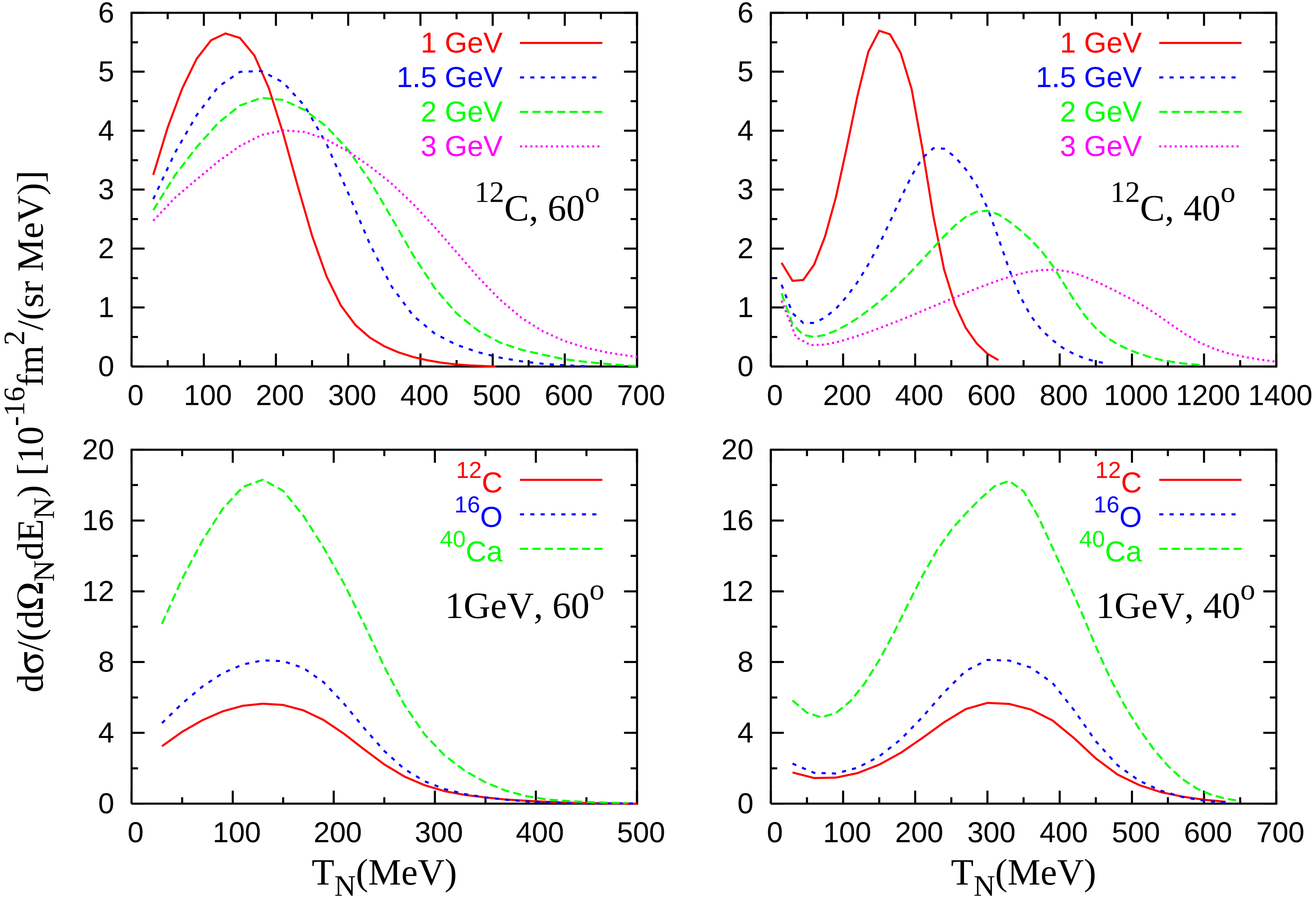
- Validity of scaling analyzed within the RFG model:

$$Factorization: \quad \frac{d\sigma}{d\Omega_N dp_N} \approx \overline{\sigma}_{sn}^{(u)} F(\psi^{(u)}, q') \approx \overline{\sigma}_{sn}^{(u)} F(\psi^{(u)})$$



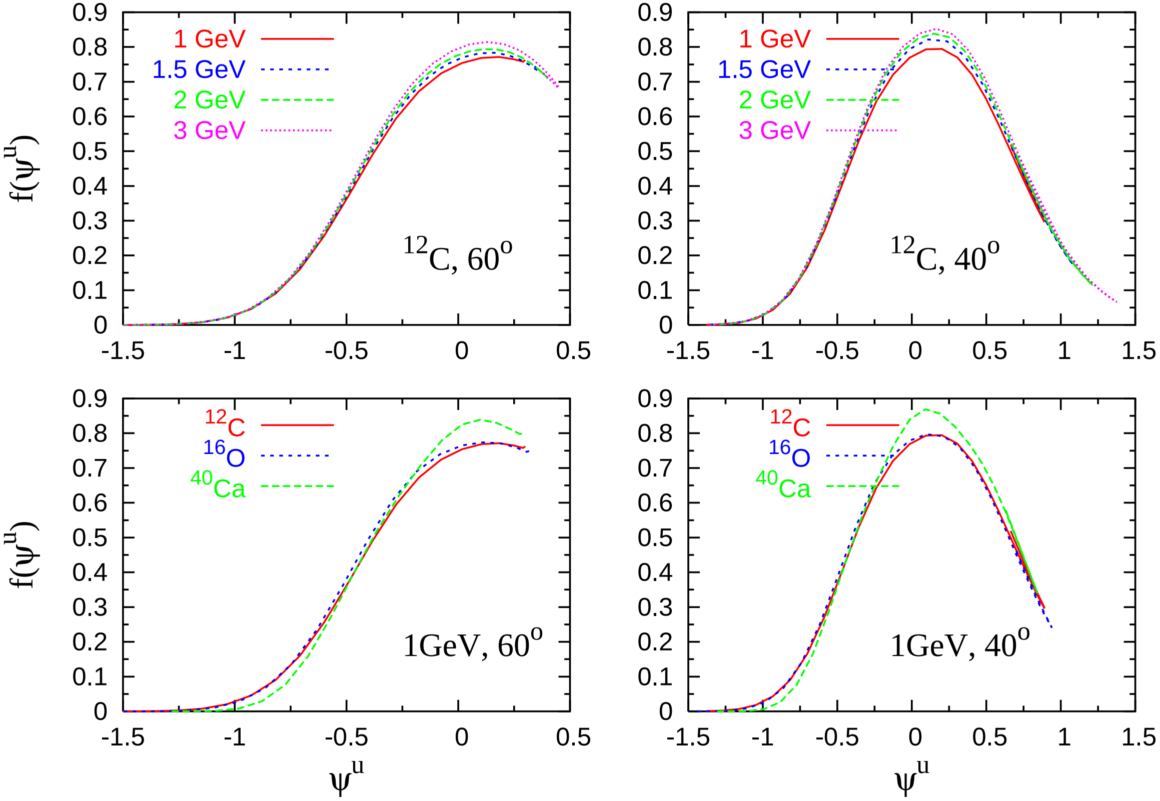
Cross section predictions are provided making use of the phenomenological (e, e') scaling function. However, how to be sure that $f_{exp}(\psi)$ should be also used to predict NC cross sections?

Relativistic Impulse Approximation and cross sections



Superscaling for NC in RPWIA

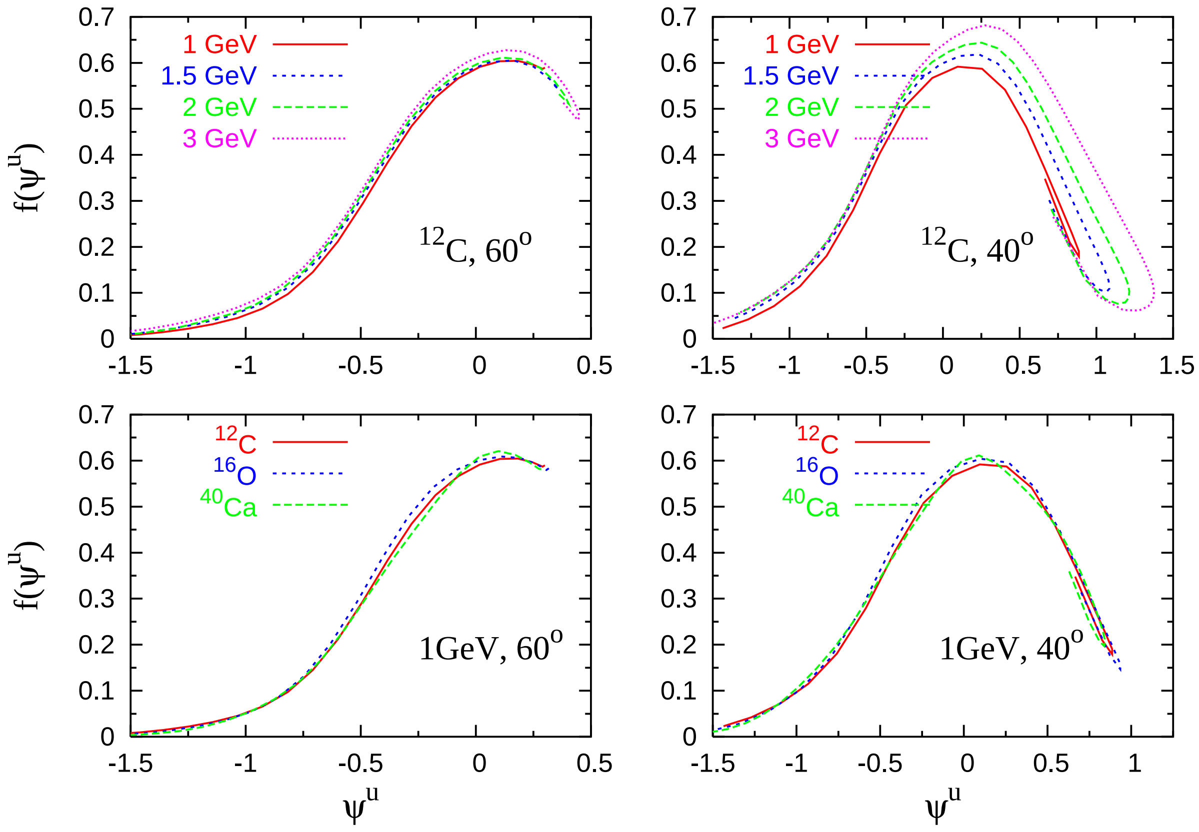
$$f(\psi^{(u)}) = k_F \frac{\left[\frac{d\sigma}{d\Omega_N dp_N}\right]^{RI}_{NC}}{\overline{\sigma}^{(u)}_{sn}}$$



Superscaling also occurs at high degree within RPWIA for NC

What happens when FSI are incorporated?

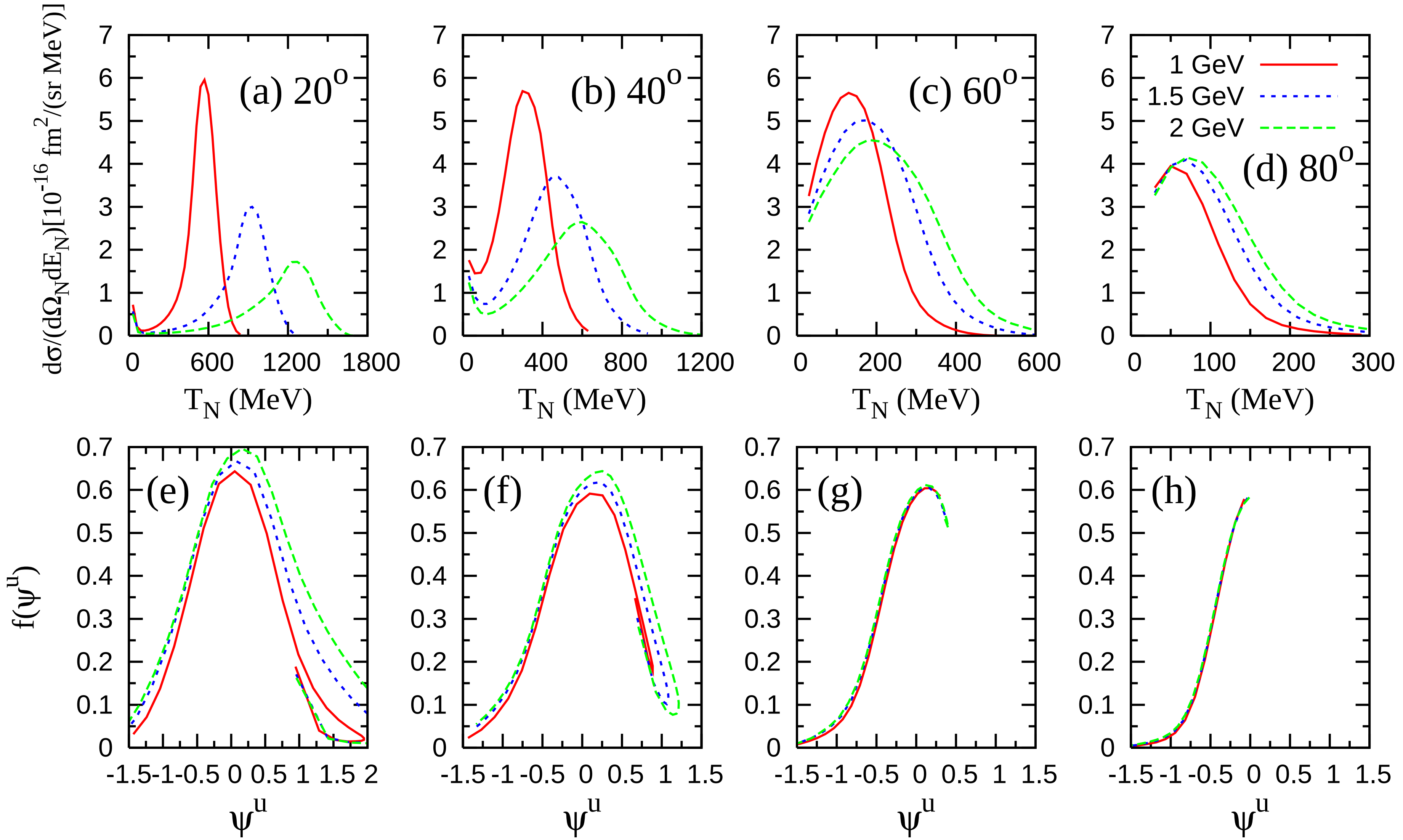
Analysis within RMF (FSI turned on)



- *Within RIA-RMF, first kind scaling (independence on q) is O.K. for 60^0, but not so good for 40^0 in the positive $\psi^{(u)}$-region. Second kind scaling is preserved.*

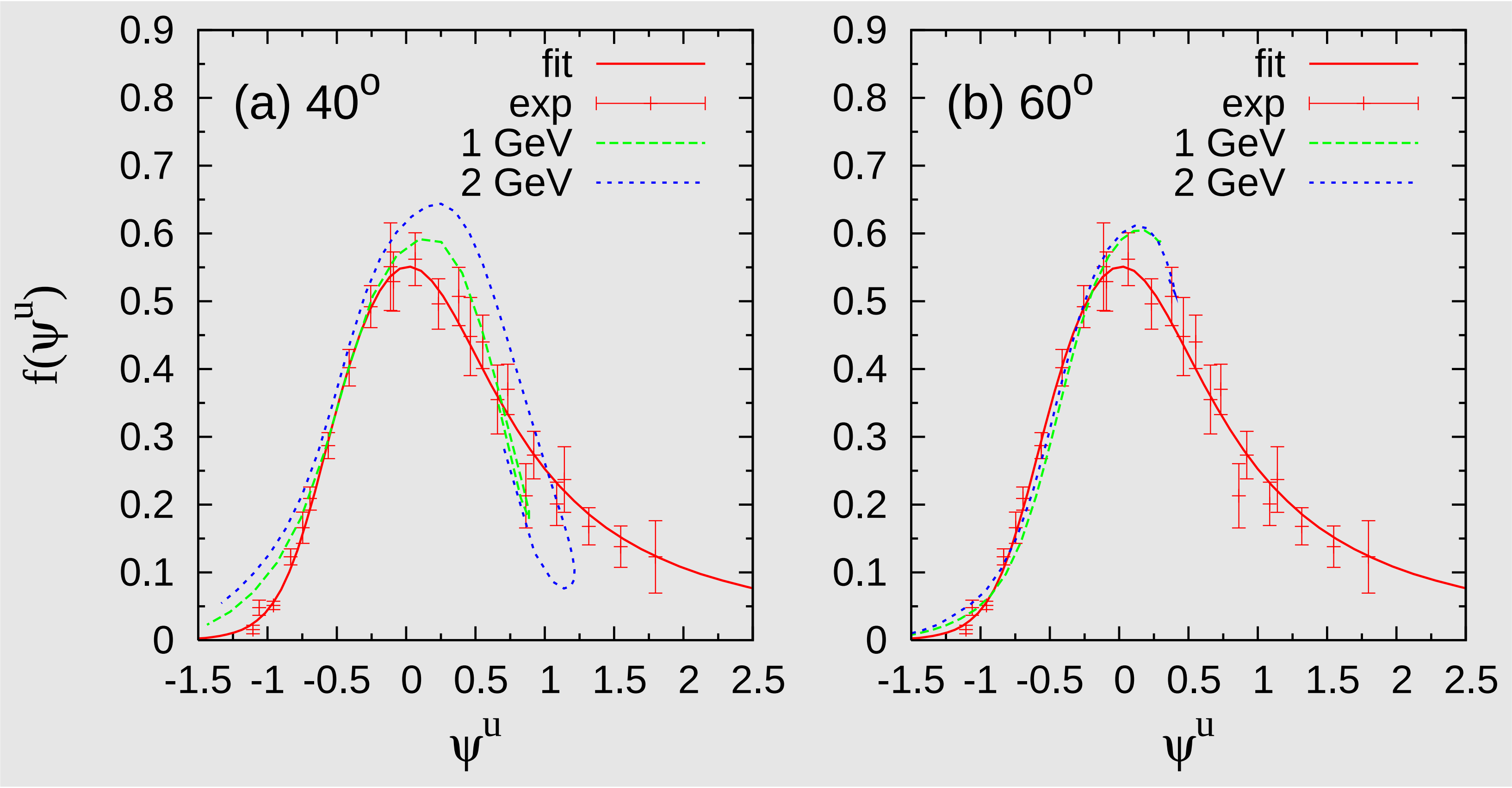
First kind scaling violation emerges due to FSI. Why and under which conditions?

First-kind scaling vs nucleon detection angle



FSI redistributes strength depending on T_N. When the change in T_N is high (small angles), the scaling procedure cannot account for that redistribution...

Comparison with $f_{exp}(\psi)$ [from (e, e')]



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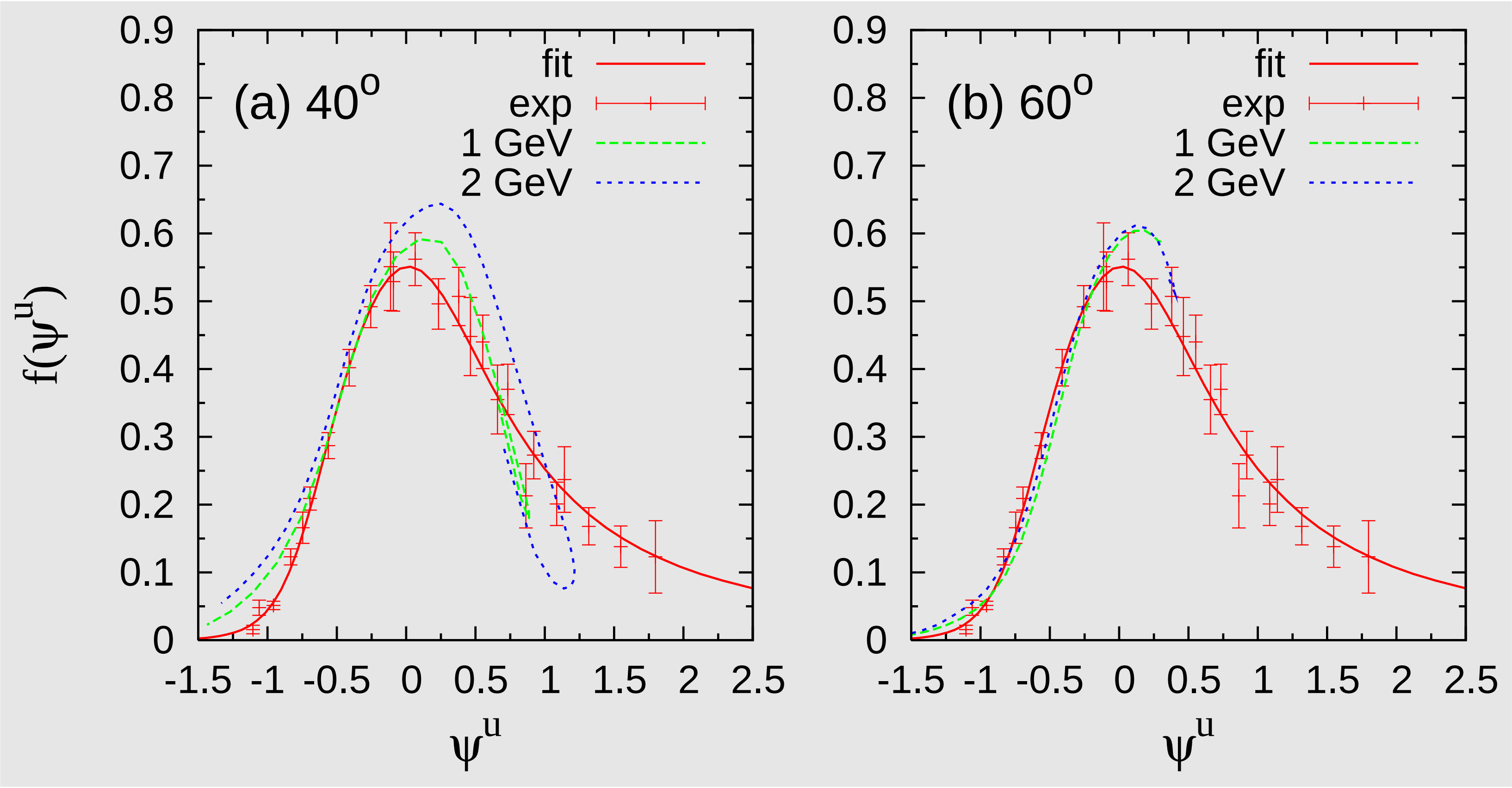


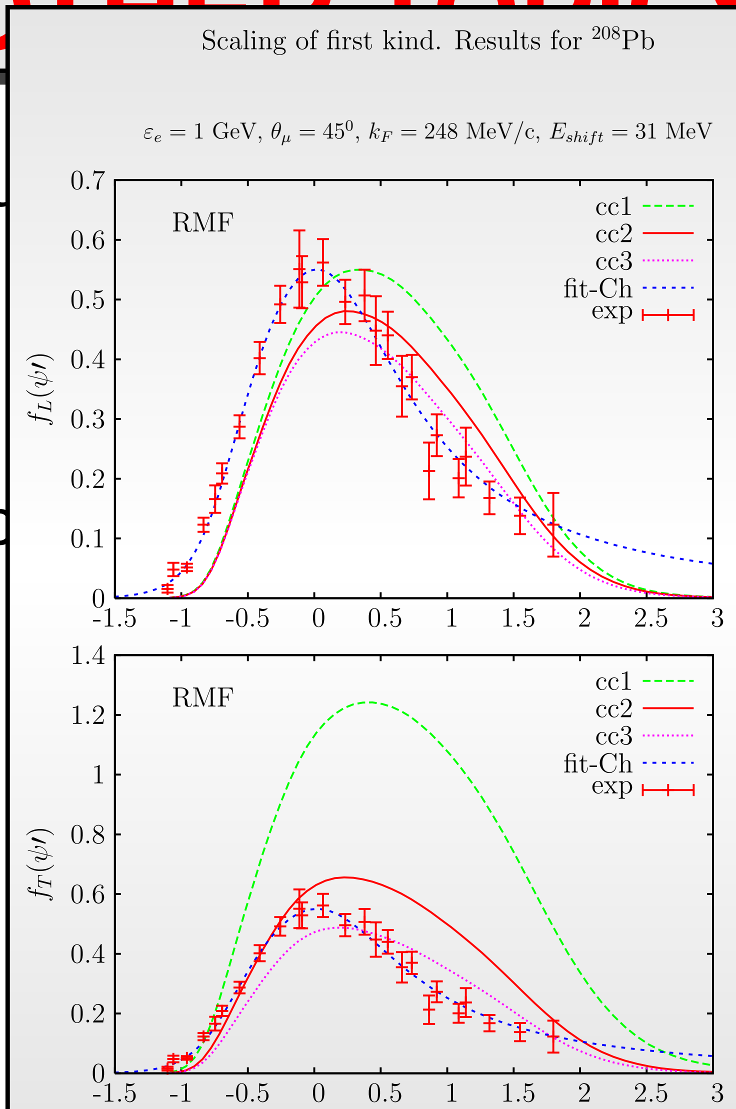
- *FSI break scaling of first kind for small scattering angles. On the contrary, for large values ($\geq 60^0$), which give the largest contribution to the integrated cross section, scaling of first kind is highly fulfilled.*
- *Comparison with $f_{exp}(\psi)$ suggests that a similar scaling function results valid for electrons (longitudinal), CC and NC neutrino reactions.*

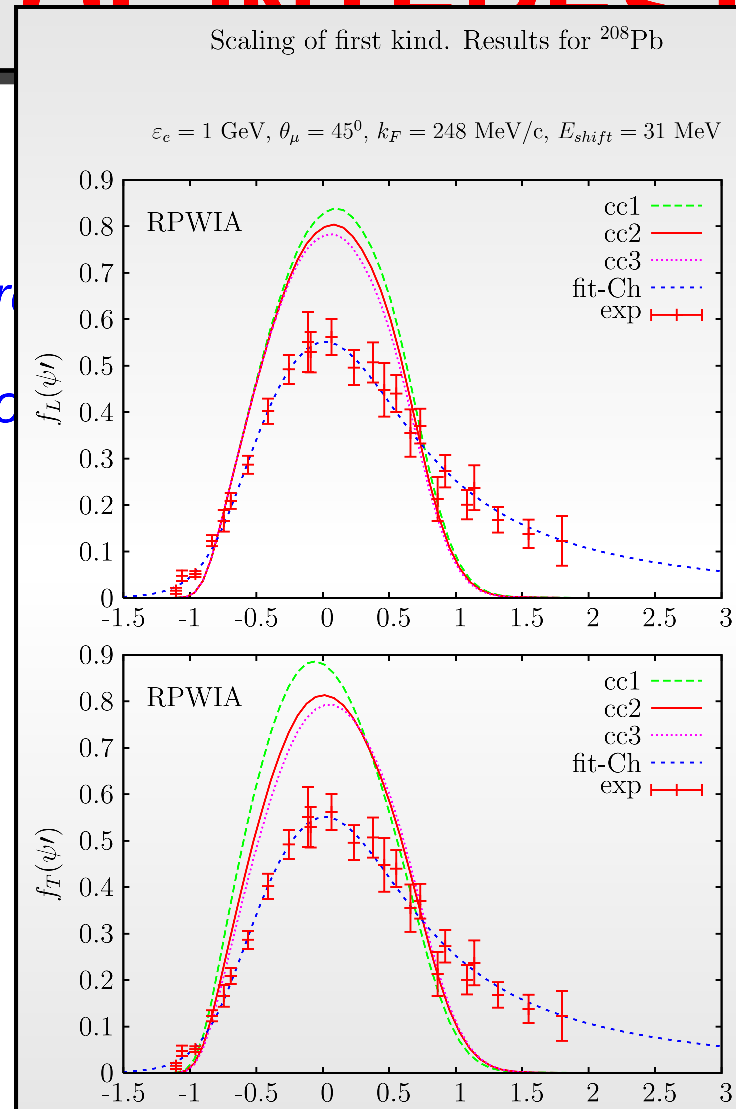
OTHER TOPICS OF INTEREST

- Nuclear Effects & FSI
  - *Relativistic Green Function Approach (RGF): talk by C. Giusti*
  - *Coherent Density Fluctuation Model (CDFM): talk by A. Antonov*
- Scaling & Coulomb Distortion in heavy $N \neq Z$ nuclei

OTHER TOPICS OF INTEREST

- Nu
  - ppr
  - Mo
- Sc on





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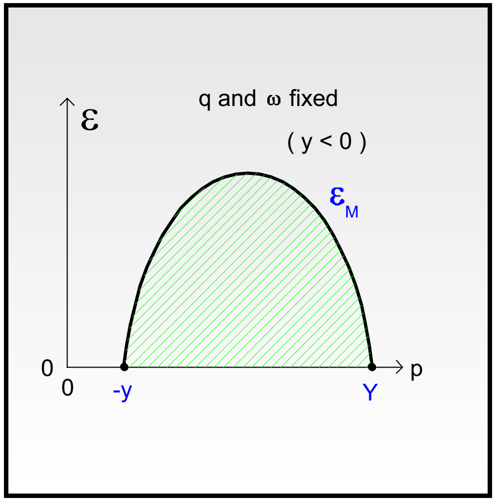
1. Scaling vs Momentum Distributions

2. Coulomb Sum Rule (CSR)

Scaling Function vs Nucleon Momentum Distribution



$$F(q,\xi) \equiv 2\pi \int_{\xi \equiv -y}^{Y(q,\xi)} p\,dp \int_0^{\varepsilon_M(q,\xi;p)} d\varepsilon S(p,\varepsilon)$$

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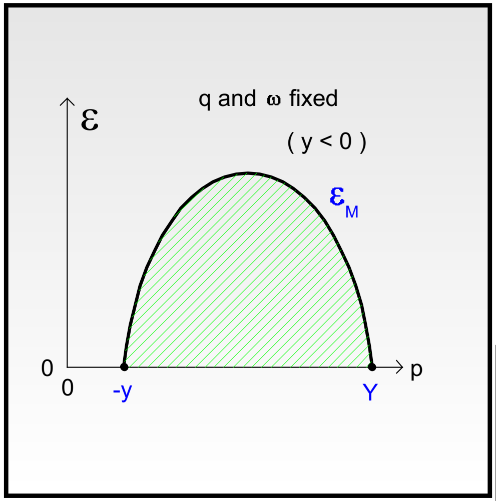

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Introducing: $S(p,\varepsilon) = n_0(p)\delta(\varepsilon) + S_1(p,\varepsilon);\ S_1(p,\varepsilon=0)=0$:

$$F(q,\xi) \equiv 2\pi \int_\xi^{Y(q,\xi)} p\,dp\,n_0(p) + 2\pi \int_\xi^{Y(q,\xi)} p\,dp \int_0^{\varepsilon_M(q,\xi,p)} S_1(\varepsilon,p)$$

$$\frac{\partial F}{\partial \xi} = 2\pi \left\{ Y\,n_0(Y)\left(\frac{\partial Y}{\partial \xi}\right) - \xi\,n_0(\xi) + \left(\frac{\partial \varepsilon_M}{\partial \xi}\right) \int_\xi^Y p\,dp\,S_1(p,\varepsilon_M) \right\}$$

$$\frac{\partial F}{\partial q} = 2\pi \left\{ Y\,n_0(Y)\left(\frac{\partial Y}{\partial q}\right) + \int_\xi^Y p\,dp \left(\frac{\partial \varepsilon_M}{\partial q}\right) S_1(p,\varepsilon_M) \right\}$$

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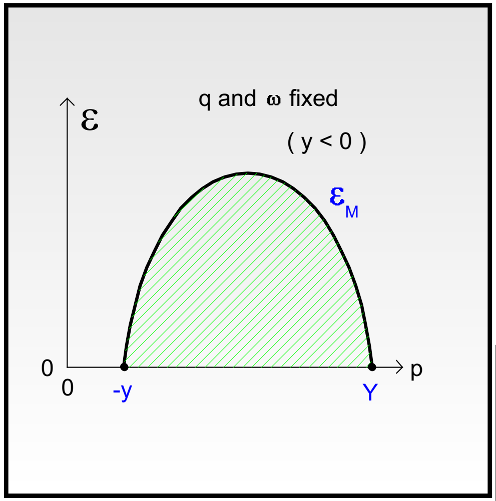

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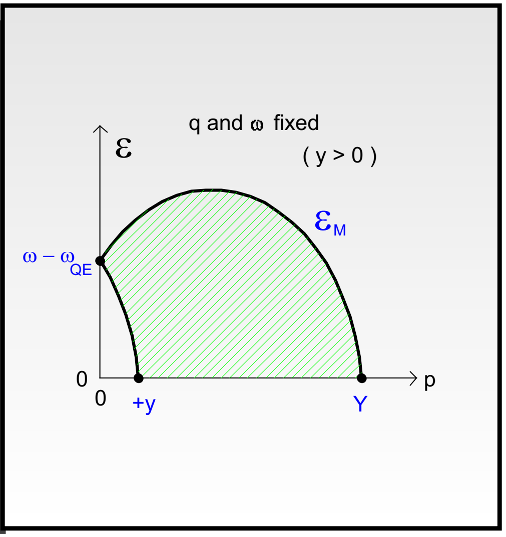
USUAL APPROACH: $n(\xi) = \frac{-1}{2\pi\,\xi} \left(\frac{\partial F}{\partial \xi}\right)$

Physics in $F(q,\xi)$ comes from initial as well as from final states!!

L channel (negative & positive y-regions)

$$F(q,y) \equiv 2\pi \int_0^y p\,dp \int_{\varepsilon_m(q,y;p)}^{\varepsilon_M(q,y;p)} d\varepsilon S(p,\varepsilon) + 2\pi \int_y^{Y(q,y)} p\,dp \int_0^{\varepsilon_M(q,y;p)} d\varepsilon S(p,\varepsilon)$$



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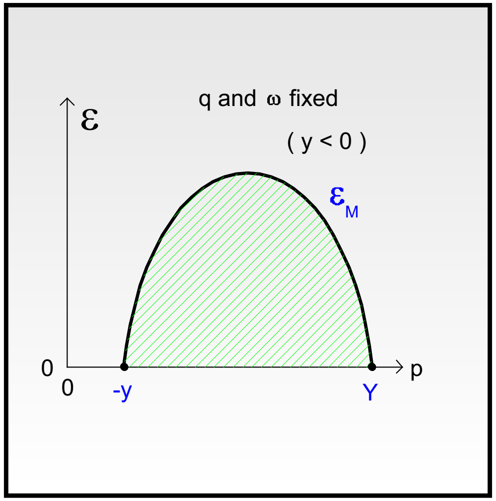

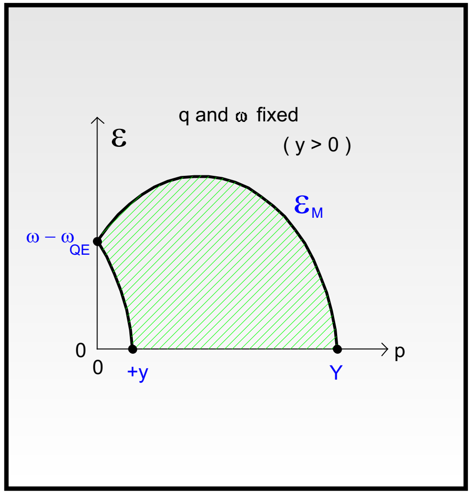

- **Negative $y \equiv -\xi$-region: ($Y \to \infty$)**

$$\frac{1}{2\pi}\left(\frac{\partial F}{\partial \xi}\right) = -\xi\, n_0(\xi) - \frac{q-\xi}{\overline{E}_{(q-\xi)}} \int_{\xi}^{\infty} p\, dp\, S_1(p, \varepsilon_M)$$

- **Positive y-region: ($Y \to \infty$)**

$$\frac{1}{2\pi}\left(\frac{\partial F}{\partial y}\right) = -y\, n_0(y) + \frac{q+y}{\overline{E}_{(q+y)}} \left\{ \int_0^{\infty} p\, dp\, S_1(p, \varepsilon_M) - \int_0^{y} p\, dp\, S_1(p, \varepsilon_m) \right\}$$

Momentum distribution from scaling function

Let us consider: $$\frac{\partial F}{\partial y} = -2\pi\, y\, n(y) \Longrightarrow n(k) = \frac{-1}{2\pi y}\left(\frac{\partial F}{\partial y}\right)\Big|_{|y|=k}$$

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- **Negative values** $y \equiv -\xi < 0$: **(limit** $Y \to \infty$**)**

$$n^-(k) = \frac{-1}{2\pi y}\left(\frac{\partial F}{\partial y}\right)\Big|_{-y=k} = n_0(y) - \frac{q+y}{y\sqrt{(q+y)^2+m_N^2}} \int_{-y}^{\infty} p\, dp\, S_1(p, \varepsilon_M^-)\Big|_{-y=k} > n_0(k)$$

- **Positive values** $y \geq 0$: **(limit** $Y \to \infty$**)**

$$n^+(k) = \frac{-1}{2\pi y}\left(\frac{\partial F}{\partial y}\right)\Big|_{y=k} =$$

$$n_0(y) - \frac{q+y}{y\sqrt{(q+y)^2+m_N^2}} \left\{\int_0^{\infty} p\, dp\, S_1(p, \varepsilon_M^+) - \int_0^{y} p\, dp\, S_1(p, \varepsilon_m^+)\right\}\Big|_{y=k} \lesseqgtr n_0(k)$$

Analysis of $F(q, y)$ in both regions, positive and negative y, may provide important & complementary information about the nucleon momentum distribution

Some preliminary conclusions on $n^{\pm}(k)$

Let us consider a small value of the nucleon momentum: $k \equiv |y| \ll q$

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$$n^{-}(k) \approx n_0(k) + \frac{q}{k\sqrt{q^2+m_N^2}}\int_{k}^{\infty} p\,dp\,S_1(p,\varepsilon_M^-) > n_0(k)$$

Some preliminary conclusions on $n^{\pm}(k)$

Let us consider a small value of the nucleon momentum: $k \equiv |y| \ll q$

- **Positive $y \equiv k$: (limit $Y \to \infty$)**

$$n^{+}(k) = \frac{-1}{2\pi y}\left(\frac{\partial F}{\partial y}\right)\Big|_{y=k} =$$

$$n_0(y) - \frac{q+y}{y\sqrt{(q+y)^2+m_N^2}}\left\{\int_0^{\infty} p\,dp\,S_1(p,\varepsilon_M^+) - \int_0^{y} p\,dp\,S_1(p,\varepsilon_m^+)\right\}\Big|_{y=k} \lesseqgtr n_0(k) \Rightarrow$$

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Hence, $n^{+}(k) \le n_0(k) \le n^{-}(k)$ **for** k **small.**

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Let us consider a small value of the nucleon momentum: $k \equiv |y| \ll q$

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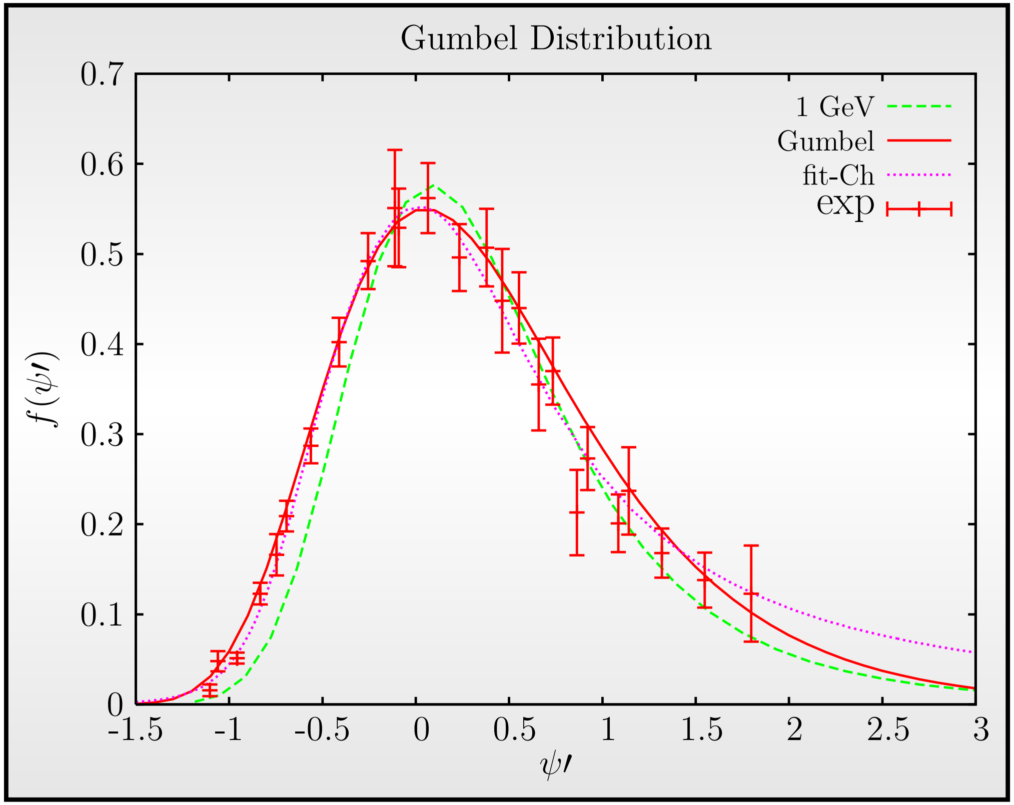
Hence, $n^{+}(k) \leq n_0(k) \leq n^{-}(k)$ **for k small.**

As k grows up, n^{+} and n^{-} cross each other at some specific k, being $n^{+}(k) > n^{-}(k)$ *for larger k. This is so because the main contribution comes from the term:* $\int_0^{y} p\,dp\,S_1(p, \varepsilon_m^{+})$ *in the case of positive-y*

Some examples: Gumbel function and CDFM

$$f_G(\psi') = \frac{1}{\sigma} e^{-\psi'/\sigma} e^{-e^{-\psi'/\sigma}}; \int_{-\infty}^{\infty} d\psi' f_G(\psi') = 1$$



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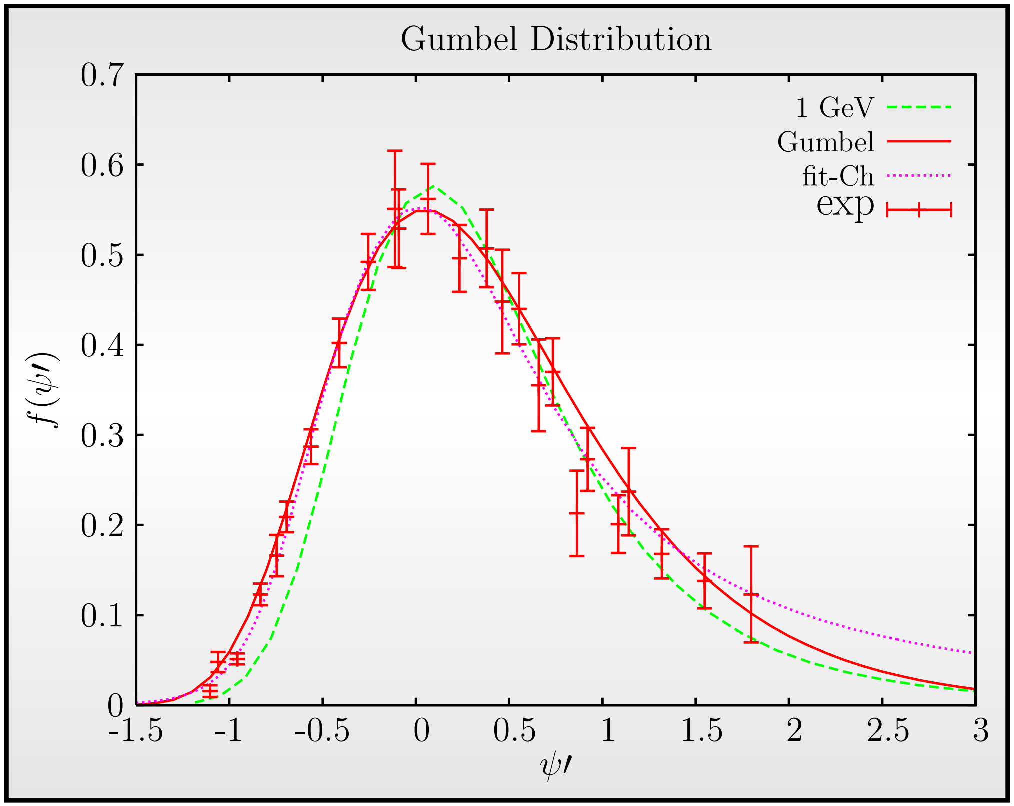
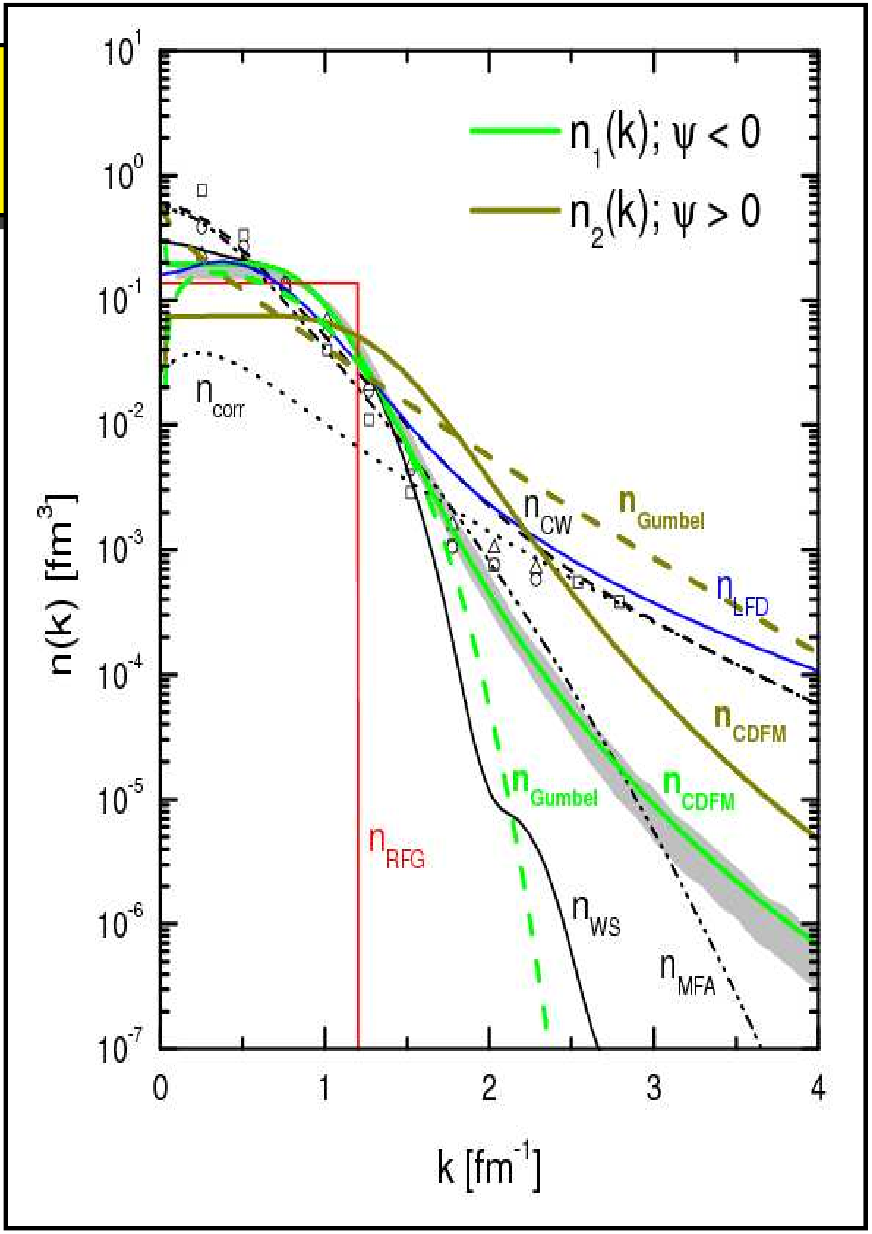
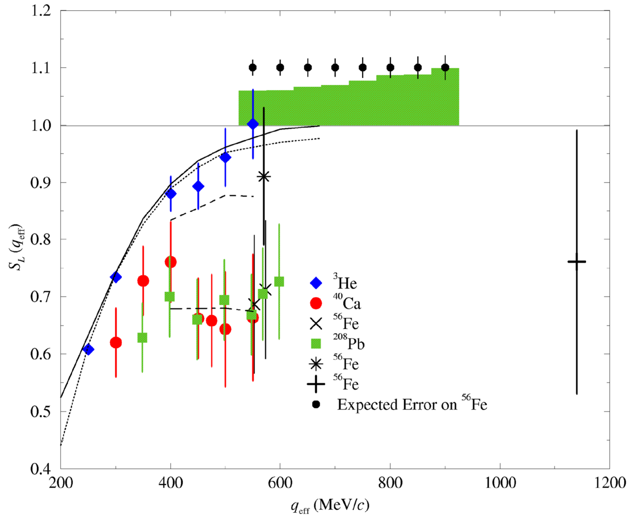
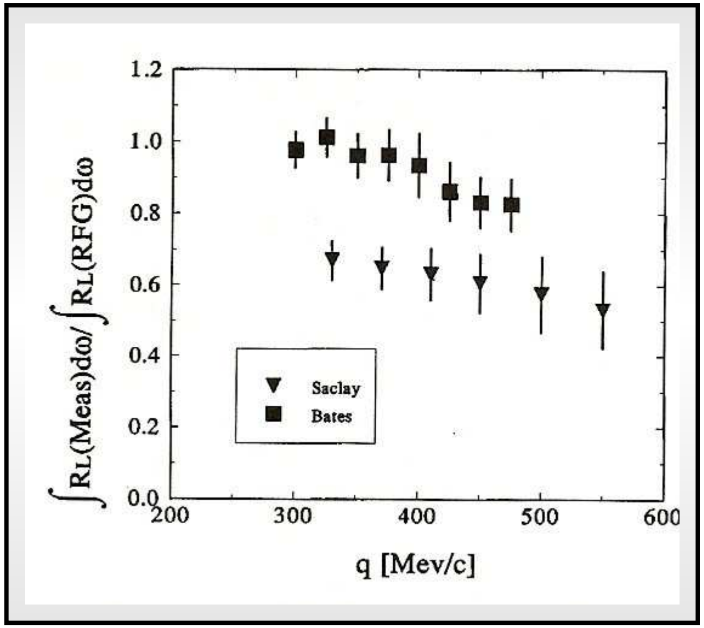




Coulomb Sum Rule (CSR): experimental status

Expected Uncertainty on Coulomb Sum





^{40}**Ca**

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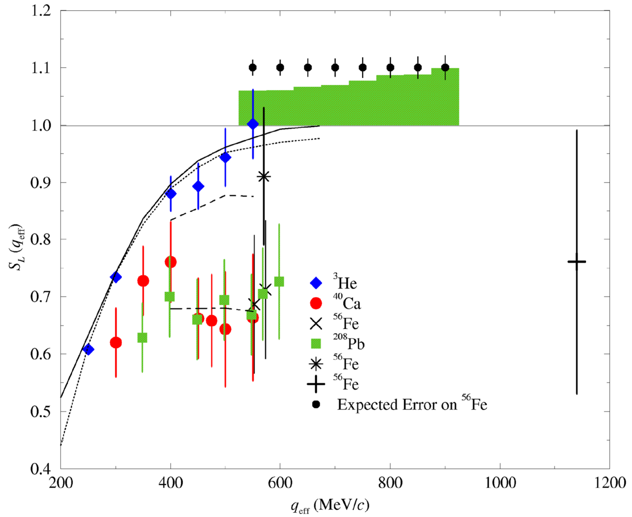

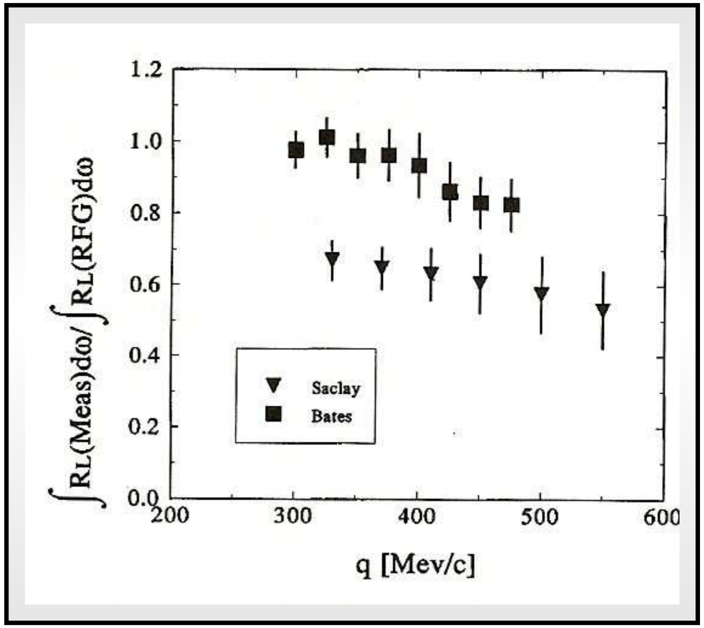

^{40}**Ca**

Long-standing problem with (still) controversial experimental answers: *whereas Jourdan concludes the L response not to be suppressed, Meziani claims a significant quenching in the CSR, up to 40%!!, due to the modification of the nucleon properties inside the nuclear medium.*

Why so large discrepancies in the CSR?

- Limited significance of data (L/T separation?)
- Analysis of Coulomb distortion for heavy nuclei?
- Modification of the nucleon properties inside the nuclear medium?

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- Limited significance of data (L/T separation?)
- Analysis of Coulomb distortion for heavy nuclei?
- Modification of the nucleon properties inside the nuclear medium?

Integration of the total strength in the L channel? $\Longrightarrow$

Behavior of L data at large ω $\Longrightarrow$

Asymmetrical Scaling Function!

CSR and SUPERSCALING

- Expressions for the CSR:

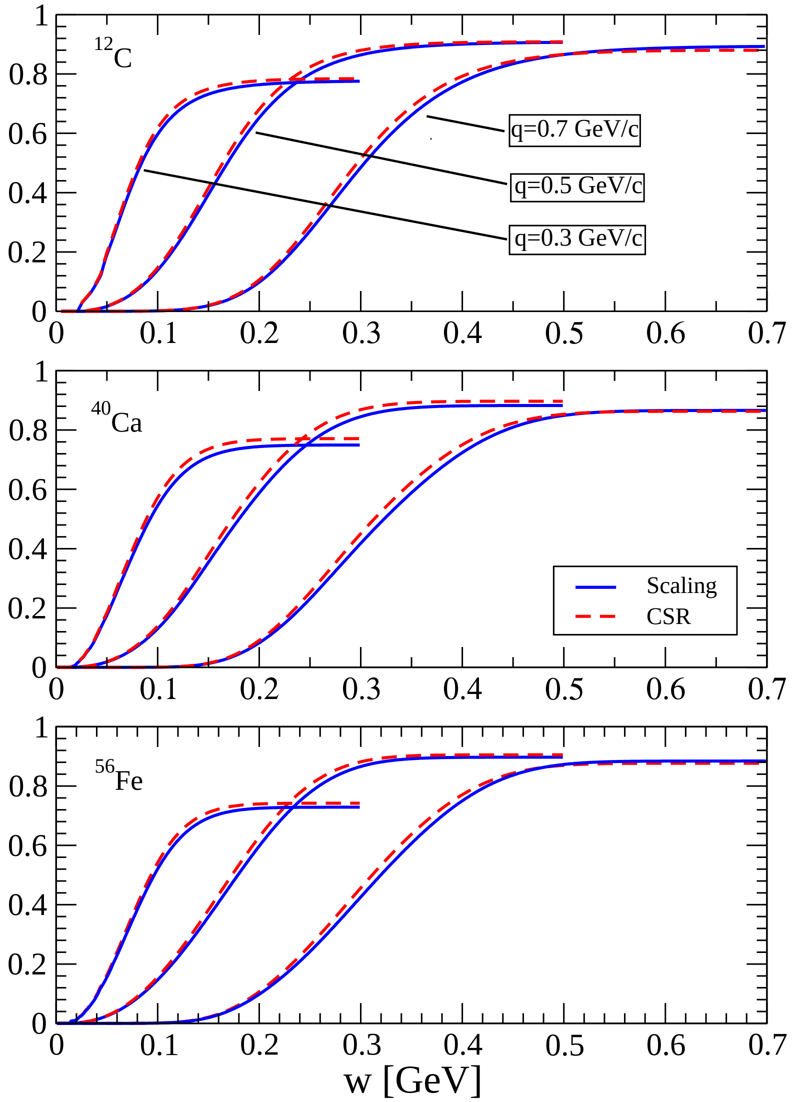
$$CSR(q) = \frac{1}{Z}\int_{\omega^+}^{\infty} \frac{R^L(q,\omega)}{\tilde{G}_E^2(Q^2)} d\omega$$

$$CSR_{scal}(q) = \int_{-\infty}^{\infty} d\psi' \, f_L(\psi')$$

$$\tilde{G}_E^2(Q^2) = \left[G_{Ep}^2(Q^2) + \frac{N}{Z} G_{En}^2(Q^2)\right] \frac{(1+\tau)}{(1+2\tau)}$$

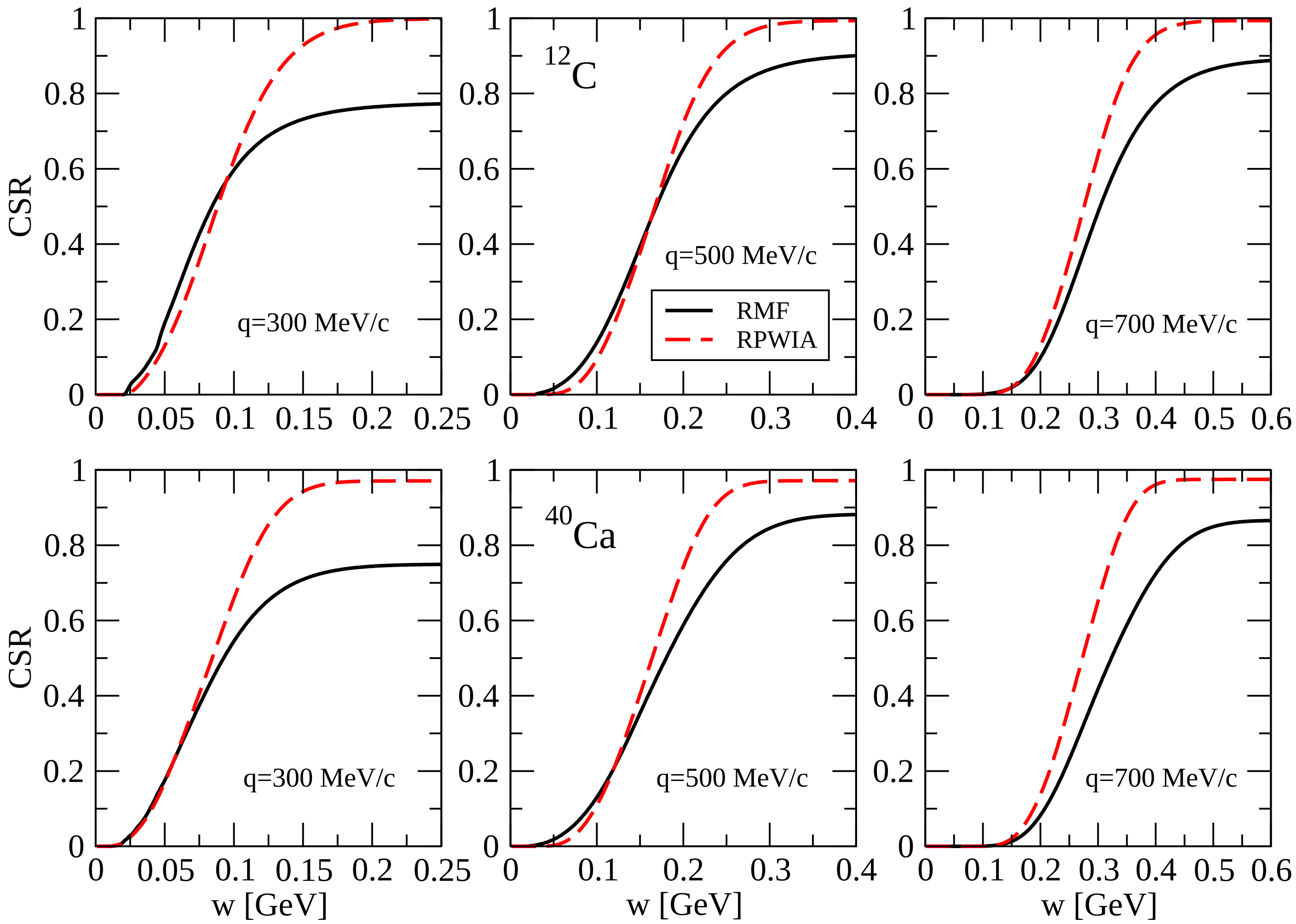
1. CSR: saturates to a unique value for $q \geq 0.5$ GeV/c.

2. CSR: similar results for all nuclei.

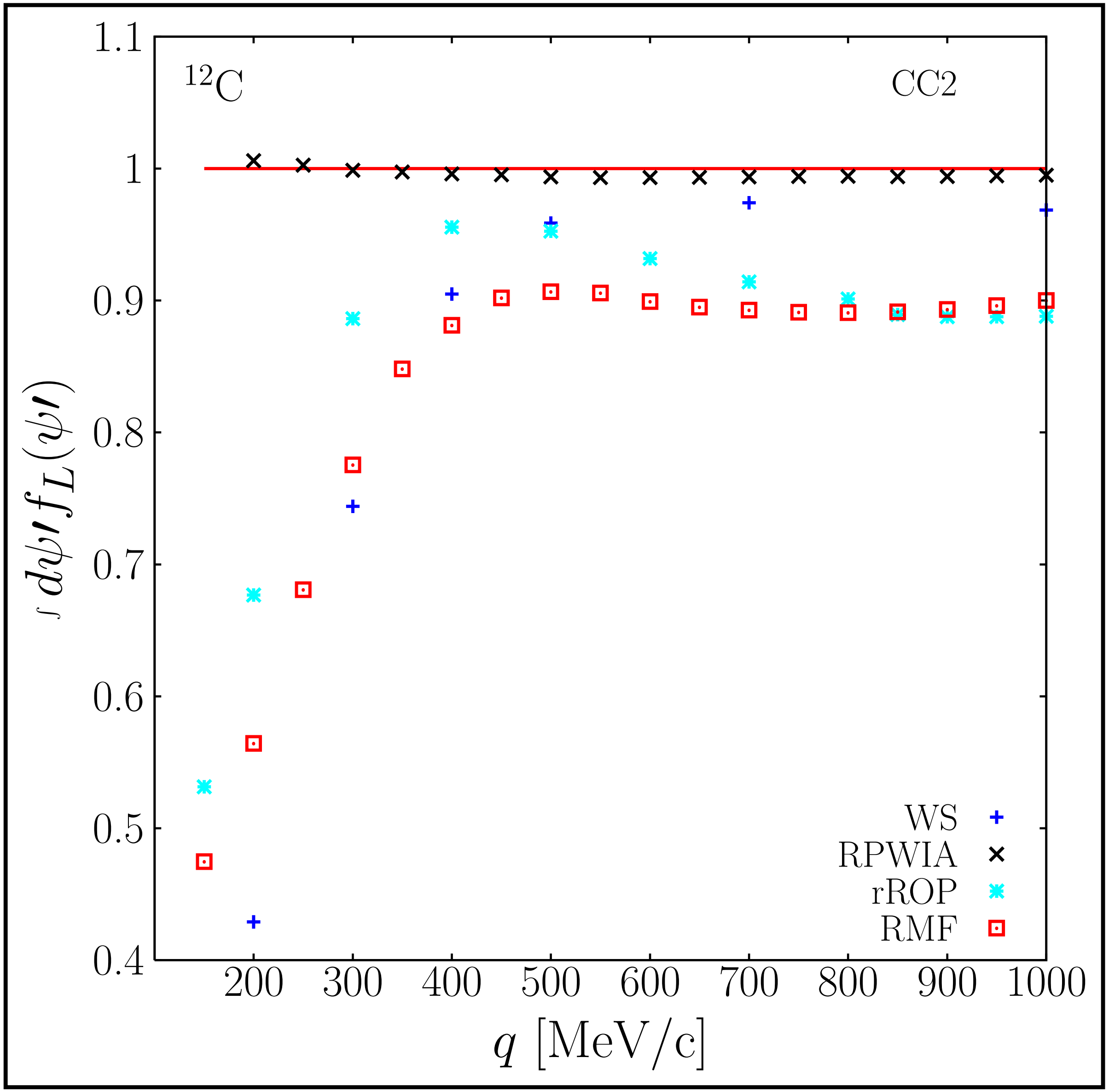


Dependence with the model: RMF vs RPWIA

Presence of a long tail in $f_L(\psi)$ (RMF) $\Longrightarrow$ effects in CSR?



CSR vs the transfer momentum (q)



CSR and experimental energy cut-off

1. How much strength in the L channel is already included in getting CSR from the experiment?

2. How the CSR value depends on the specific cut-off ω values considered?

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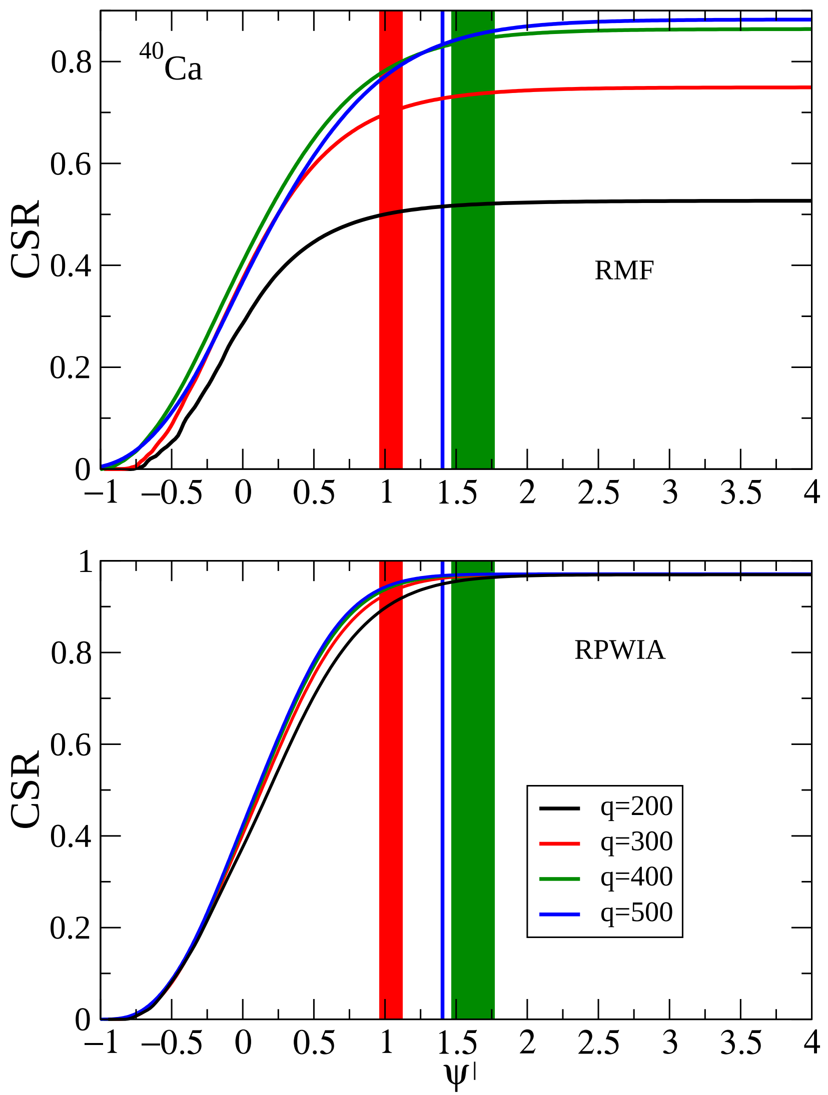


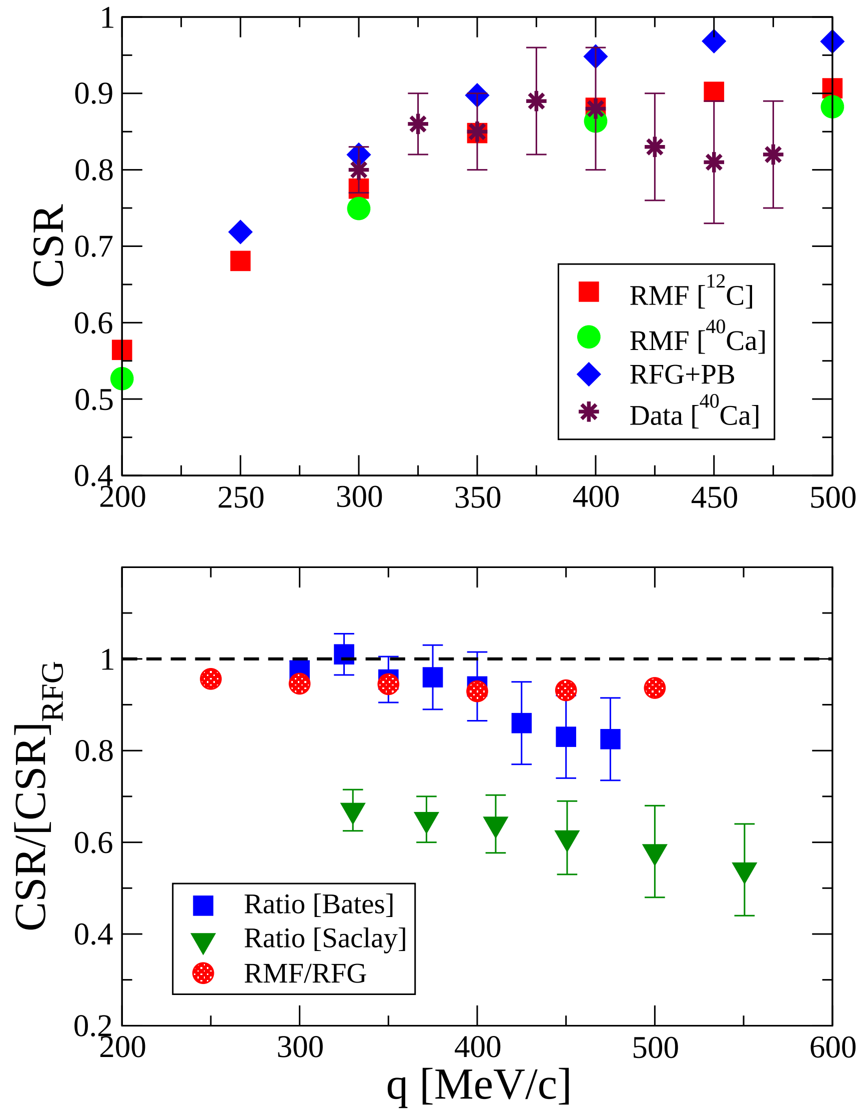
Results indicate that by using the experimental ω cut-off values, the CSR result has already reached $\gtrsim 93-95\%$ of its total strength.

CSR: theory vs experiment

^{40}Ca (Bates): *PRC 56, 3152 (1997)*

RMF results for the CSR are in accordance with Bates data on ^{40}Ca. Similar comments apply to the ratio between the integrated L response and the RFG.

No significative quenching in the L response occurs, at most of the order of $\sim 10 - 15\%$. This is in contrast with Saclay data analyzed by Meziani et al., who state a reduction in the L channel of the order of $\sim 40 - 50\%$



SUMMARY

How scaling & superscaling ideas work in the QE domain

RELATIVISTIC IMPULSE APPROXIMATION

- Electron-nucleus scattering: (e, e').
- CC neutrino-nucleus scattering: (ν_μ, μ).
- NC neutrino-nucleus scattering: $(\nu, N)\nu'$.
- Coulomb Sum Rule & Momentum distribution vs scaling

OTHER TOPICS

- Beyond the IA: correlations, MEC $\Longrightarrow$ Breaking of scaling.
- Other kinematical regions: Δ, high inelasticities...

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