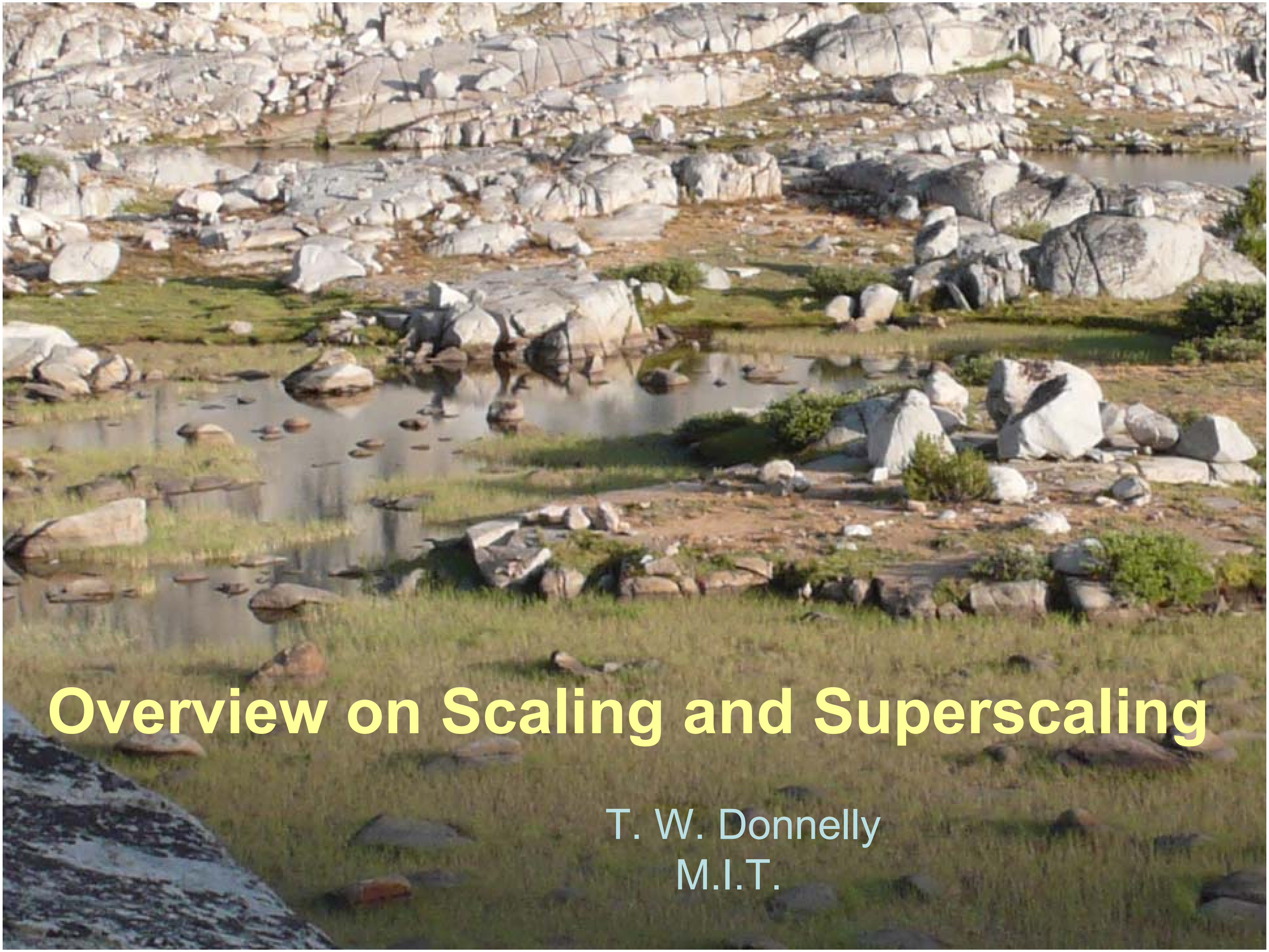

Overview on Scaling and Superscaling

T. W. Donnelly
M.I.T.

Outline:

- Basics of QE electron scattering
- Scaling of the 1st kind (y-scaling)
- Scaling of the 2nd kind
- Scaling of the 0th kind, 1p-1h MEC effects
  and Superscaling
- Non-QE scaling
  - Inelastic scattering
  - 2p-2h MEC effects
- Predicting ν cross sections using scaling
  and scaling of the 3rd kind

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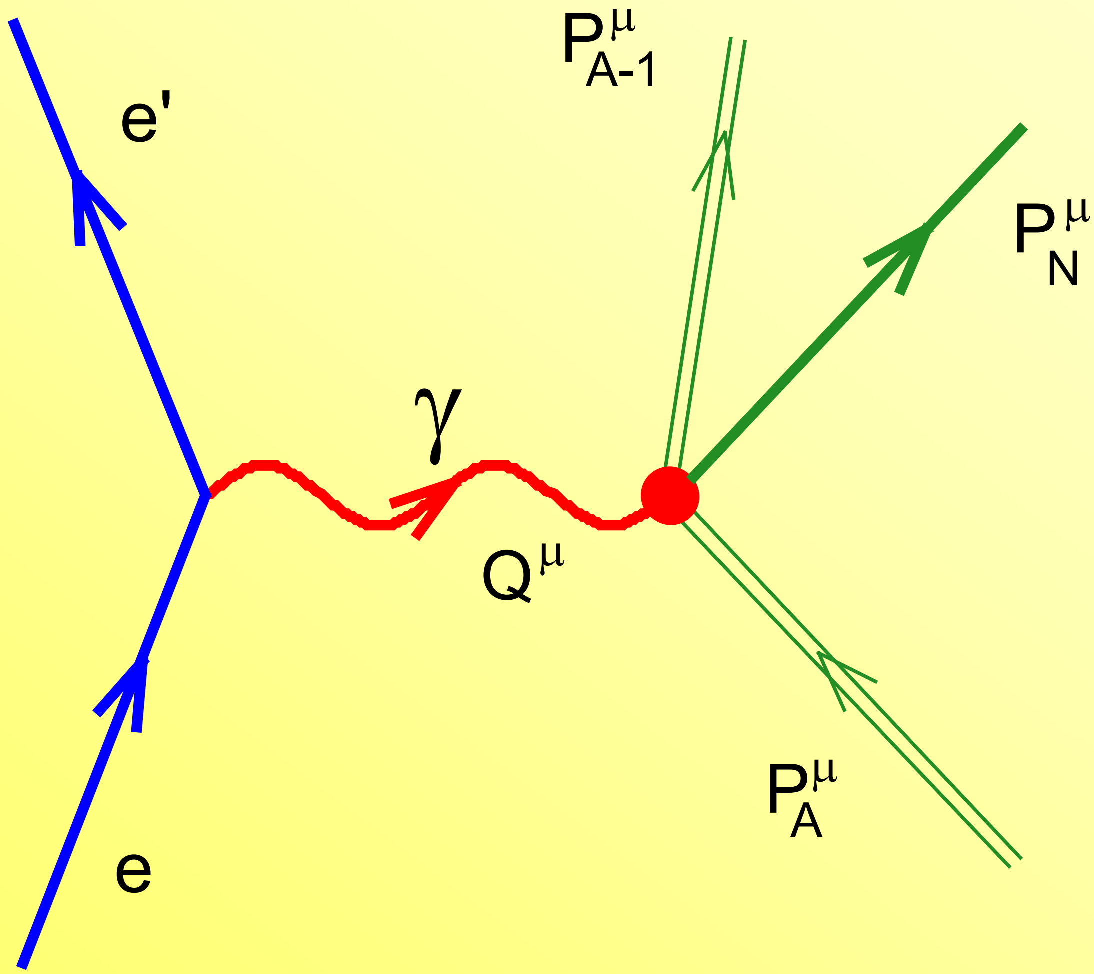
Begin by assuming that QE scattering is dominated by (e,e´N):



The daughter nucleus has 4-momentum

$$P^{\mu}_{A-1} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P^{\mu}_{A} - P^{\mu}_{N}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an "excitation energy" (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M^0_{A-1})^2 + p^2}$$

where

$$M^0_{A-1} = M^0_A - m_N + E_s$$

with E_s the separation energy and M^0_{A-1} the daughter rest mass

Energy conservation gives

$$M_A^0 + \omega = E_N + E_{A-1}$$
$$= \sqrt{m_N{}^2 + p_N{}^2} + E_{A-1}^0 + \mathcal{E}$$
$$= \sqrt{m_N{}^2 + (\mathbf{q} + \mathbf{p})^2} + \sqrt{(M_{A-1}^0)^2 + p^2} + \mathcal{E}$$

which can be turned around to yield an expression for the excitation energy:

$$\mathcal{E} = M_A^0 + \omega - \sqrt{(M_{A-1}^0)^2 + p^2} - \sqrt{m_N{}^2 + q^2 + p^2 + 2pq\cos\theta}$$

One can let the angle θ between p and q vary over all values and impose the constraints

$$p \geq 0$$

$$\mathcal{E} \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane.

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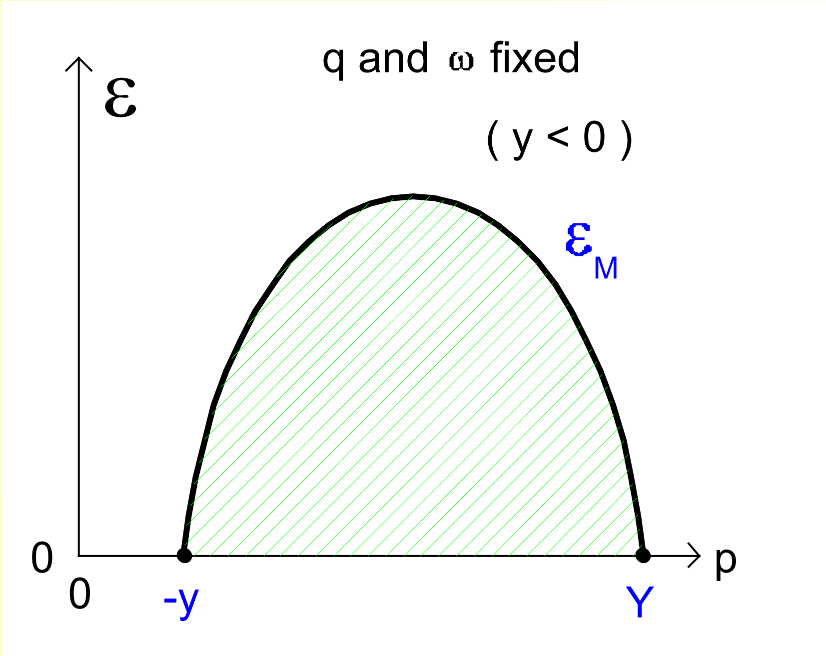
$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When

$$\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$$ one finds



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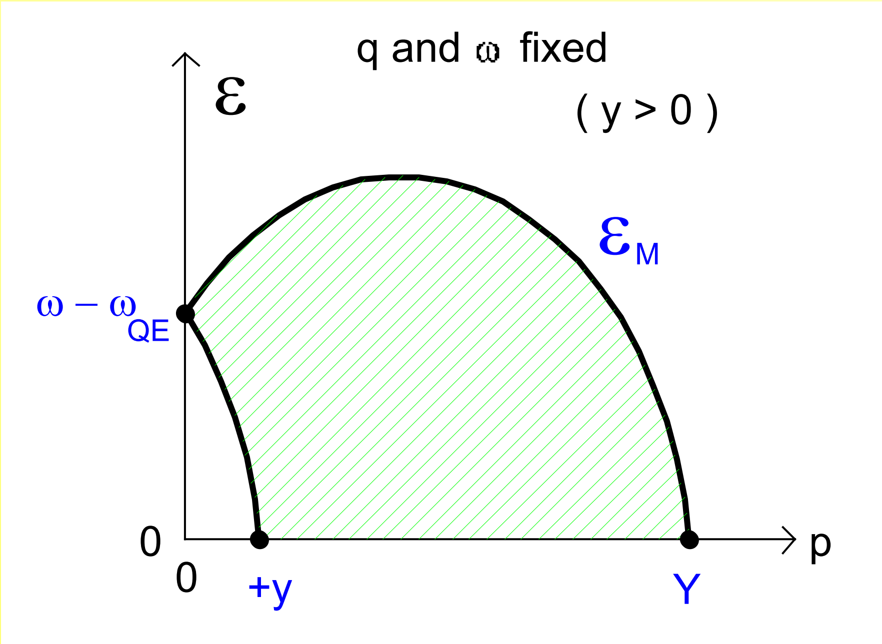


where one has the smallest and largest values of the missing momentum at zero excitation energy occurring at

$$y = \frac{1}{2W^2}[\alpha - \beta]$$

$$Y = \frac{1}{2W^2}[\alpha + \beta]$$

with

$$W = \sqrt{(M_A^0 + \omega)^2 - q^2} \geq W_T = M_{A-1}^0 + m_N$$

$$\alpha = (M_A^0 + \omega)\sqrt{W^2 - W_T^2}\sqrt{W^2 - (W_T - 2m_N)^2}$$

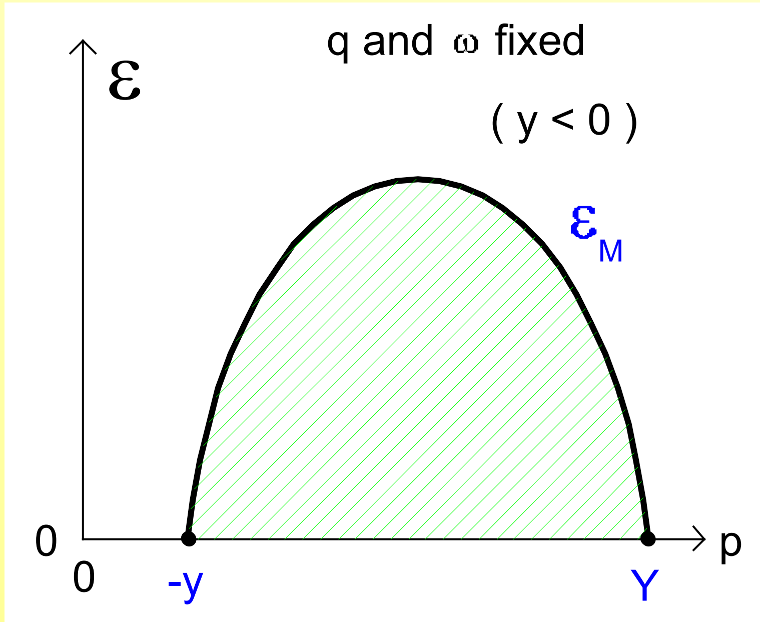
$$\beta = q\left[W^2 + W_T\left(W_T - 2m_N\right)\right]$$

The so-called **y-scaling variable** is approximately given by

$$y \cong \sqrt{\nu(2m_N + \nu)} - q$$

$$\nu \equiv \omega - E_s$$

For the case *y<0*



one has

$$\mathcal{E}_M = \sqrt{m_N{}^2 + (q+y)^2} - \sqrt{m_N{}^2 + (q-p)^2} + \sqrt{(M_{A-1}^0)^2 + y^2} - \sqrt{(M_{A-1}^0)^2 + p^2}$$

or, at high momentum transfers, approximately

$$\mathcal{E}_M \xrightarrow[\substack{q\to\infty \\ M_{A-1}^0\to\infty}]{} (p+y) - (p^2 - y^2)/2M_{A-1}^0$$

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- First, one uses ***(q,y)*** rather than ***(q,ω)*** for the functional dependence of the inclusive cross section. The inclusive cross section is assumed to be the sum of the integrals over the semi-inclusive (e,e'p) and (e,e'n) cross sections, *i.e.,* over the momentum of the ejected nucleon $\mathbf{p}_N$. These can be turned into integrals over p and ε covering the regions discussed above.

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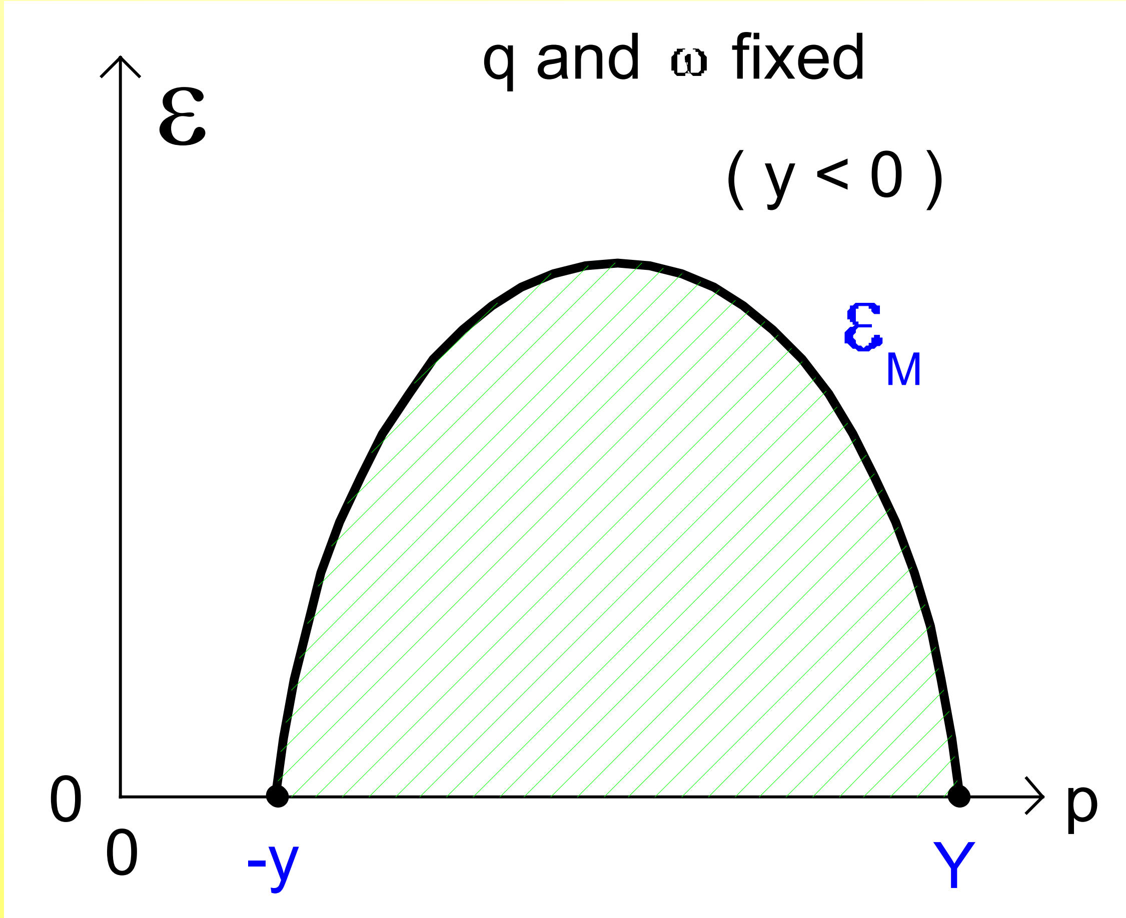
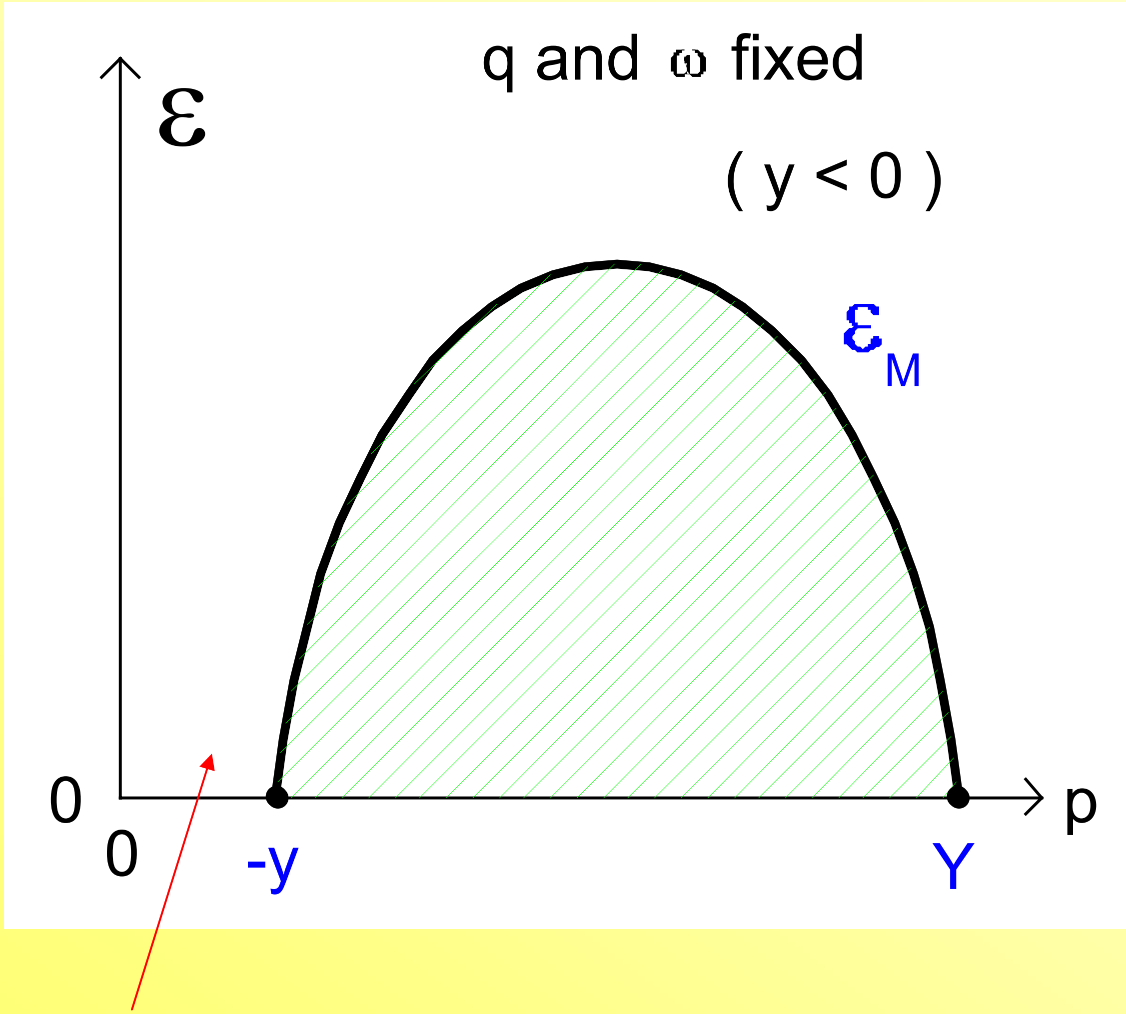
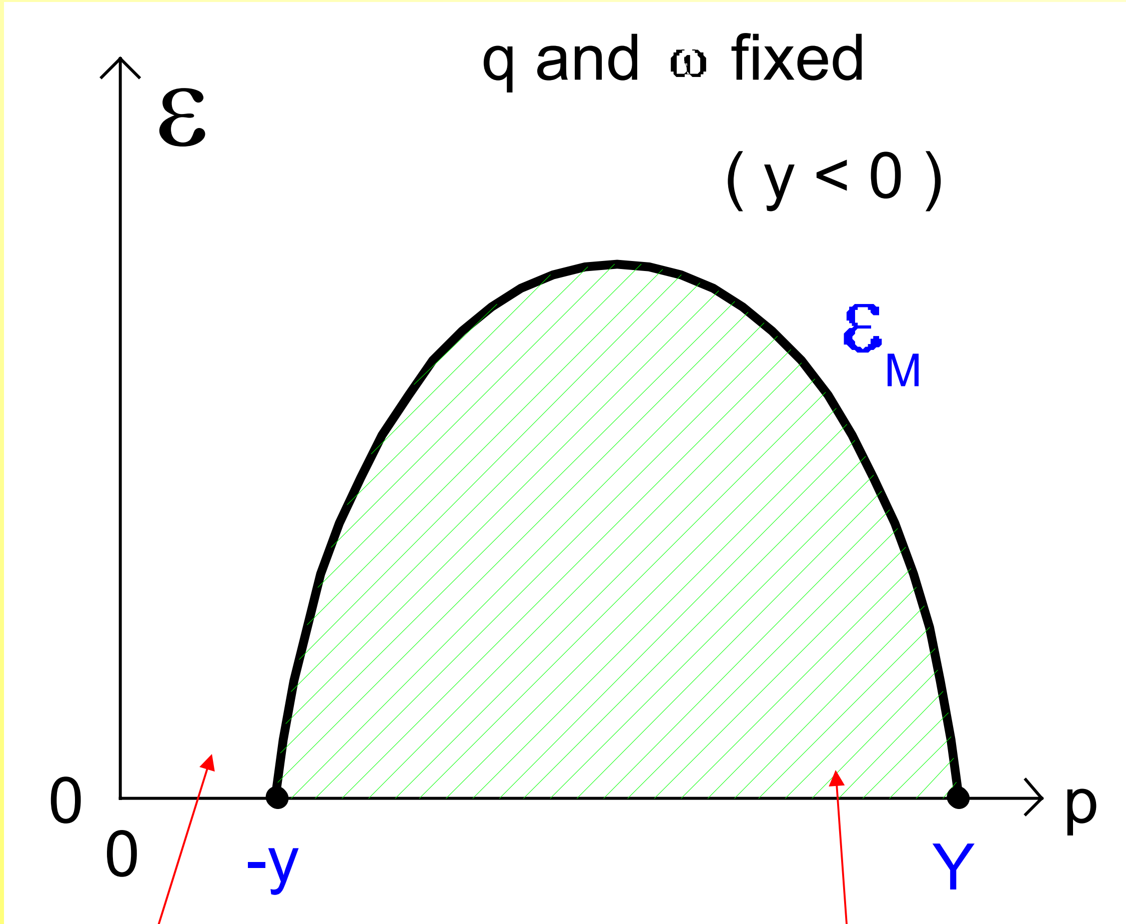
Actually not completely true… double counting in general.


q and ω fixed
(y < 0)
ε
ε_M
0
0
-y
Y
p



The semi-inclusive cross section is

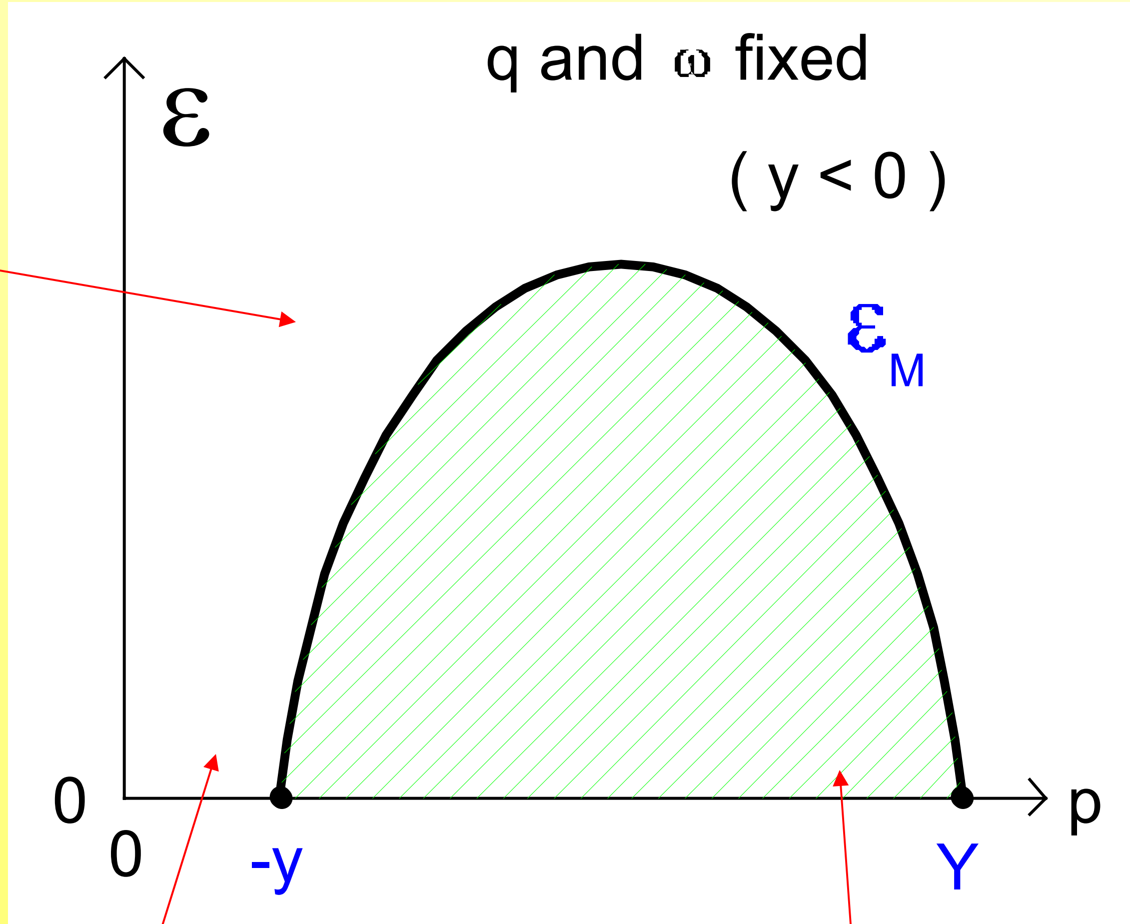
typically largest at small **p** and ε



The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

For given **y<0** the region at small **p**, but high ε is inaccessible



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What is optimal?

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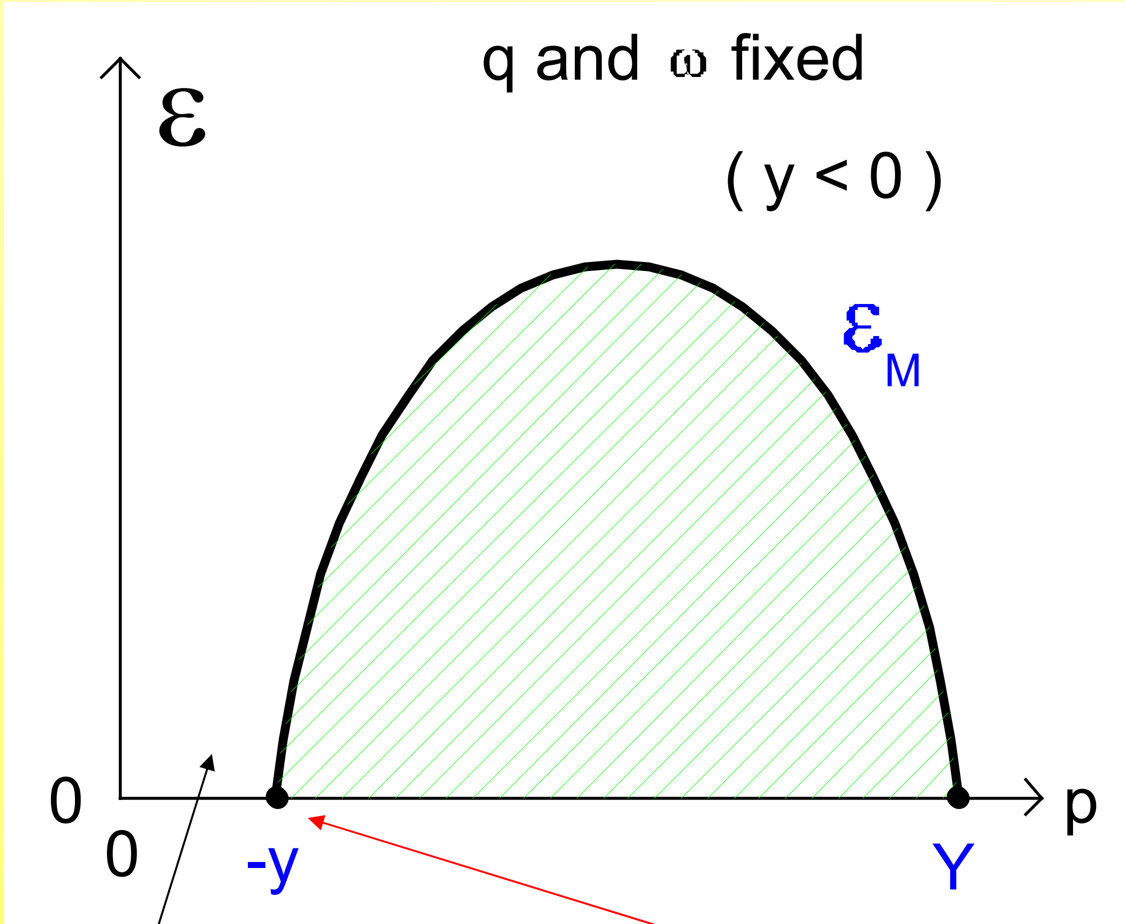
What is optimal?

From the discussions above one is led to a choice such as the one made in many analyses of scaling, namely, set **p to |y|** and ε **to 0**:

$$\Sigma_{eN}^{eff} = \frac{1}{A}\left[Z\overline{\sigma}_{ep}^{elastic} + N\overline{\sigma}_{en}^{elastic} \right]_{p=|y|,\, \varepsilon=0}$$



Evaluate the single-nucleon cross section at this point and remove from integral

The semi-inclusive cross section is typically largest at small **p** and ε

Dividing by this effective single-nucleon cross section leads to
The definition of the scaling function, F(q,y):

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In the Plane-Wave Impulse Approximation (PWIA) this can be written as an integral of the spectral function (here for *y*<*0*):

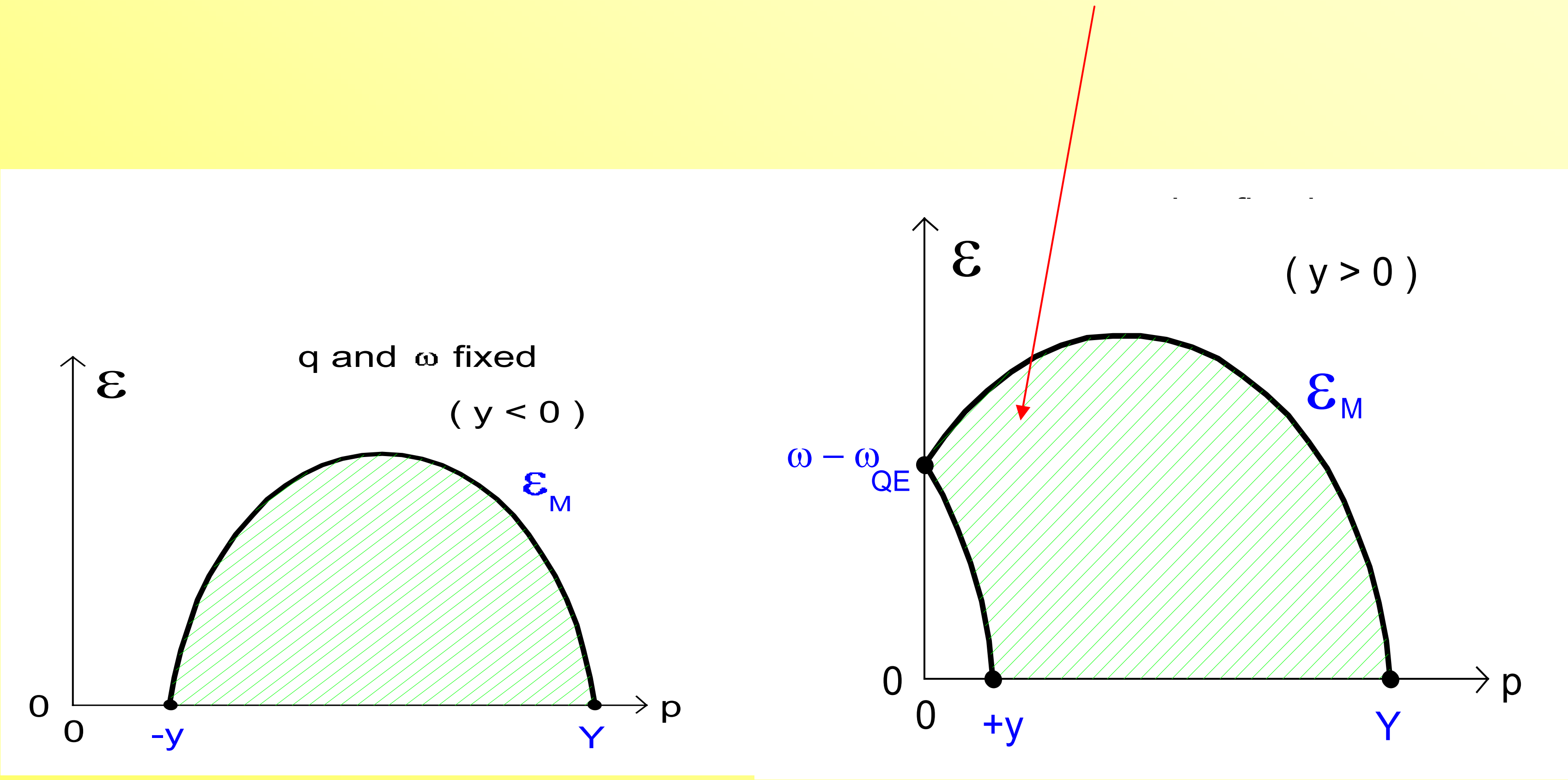
$$F(q,y) = 2\pi \int_{-y}^{Y} p dp\, n(q,y;p)$$

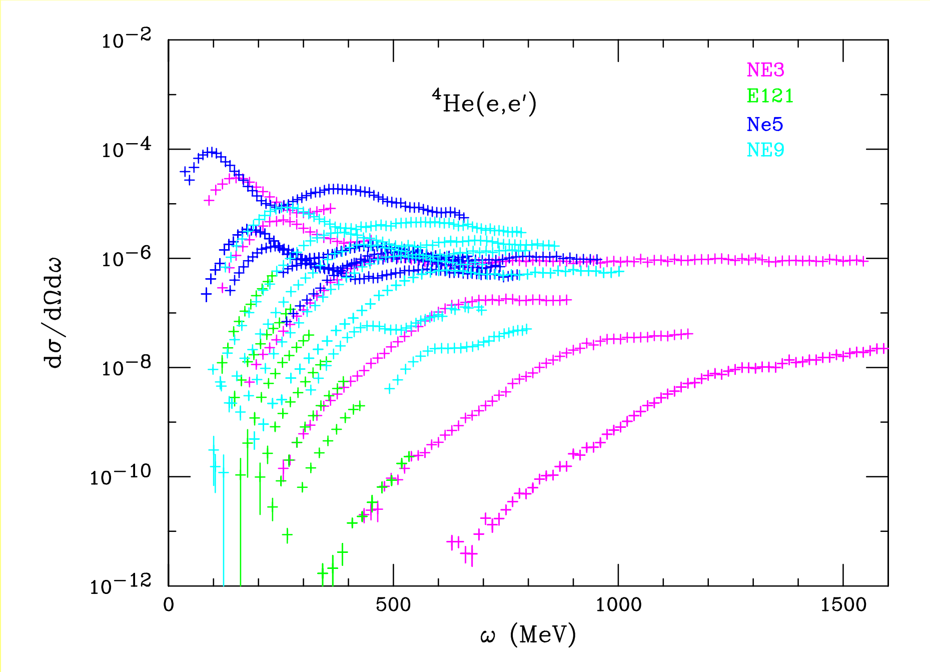
$$n(q,y;p) = \int_{0}^{\varepsilon_M} d\mathcal{E}\, S(p,\mathcal{E})$$

note: $n(q,y;p)$

is not the momentum distribution

Typically the region of integration for y>0 is larger than for y<0. In PWIA this would imply an **asymmetry**, namely, more strength when y>0 than when y<0.





Example using ^{4}He data from SLAC:

when the inclusive cross section for various beam energies and electron scattering angles

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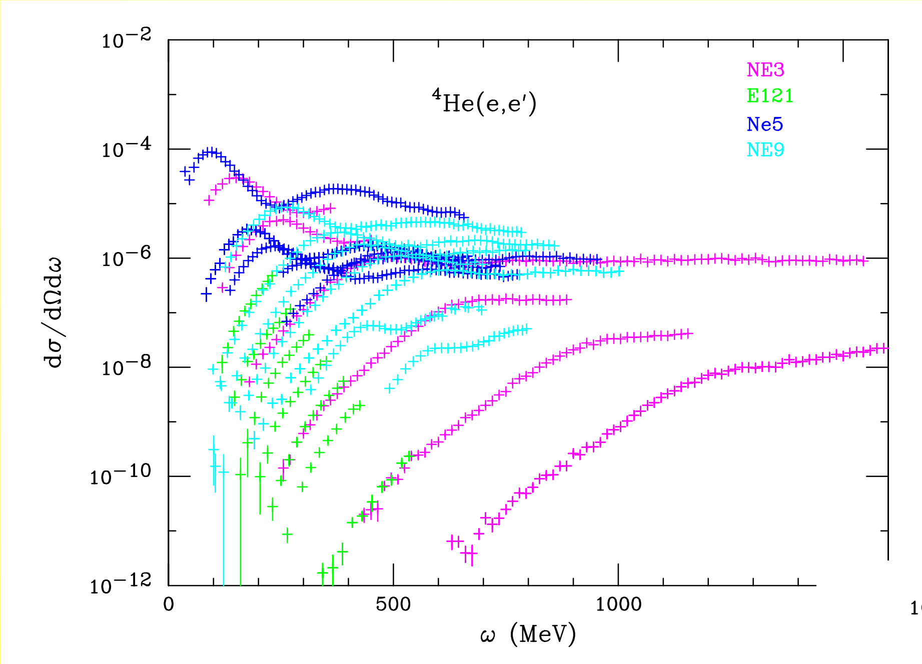
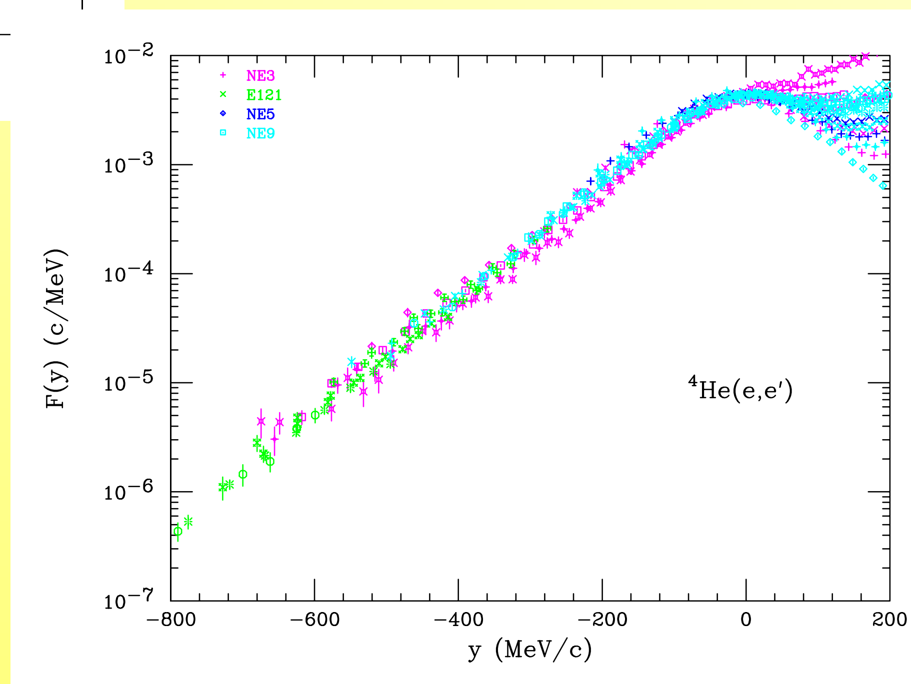
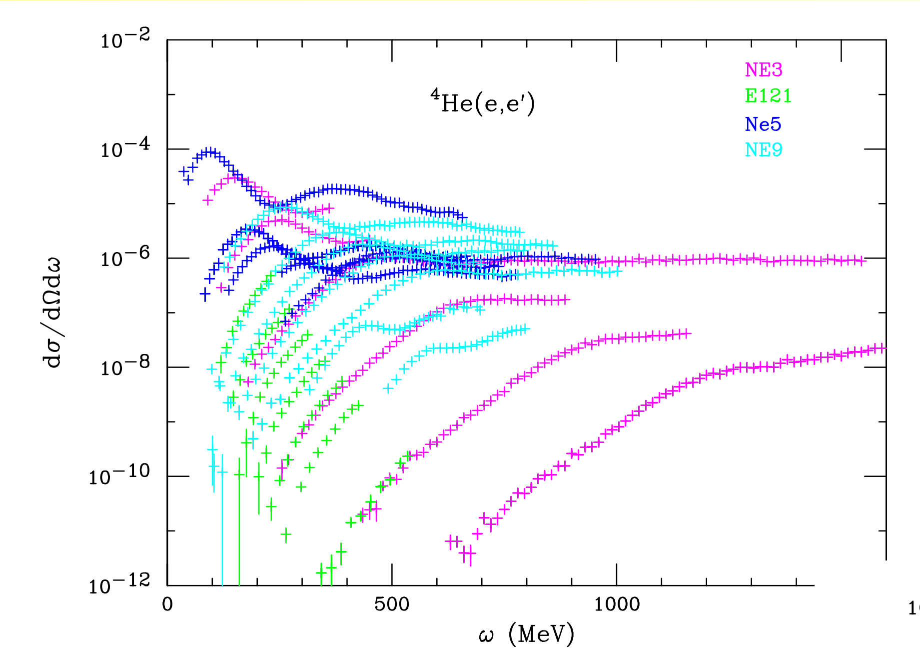


when the inclusive cross section for various beam energies and electron scattering angles

is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds





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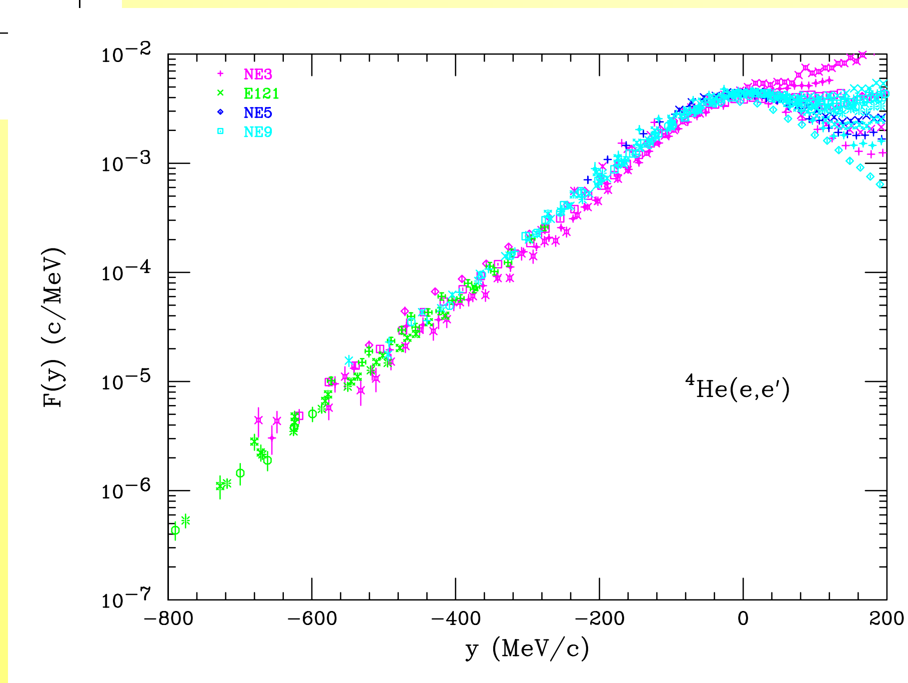


Independence of q

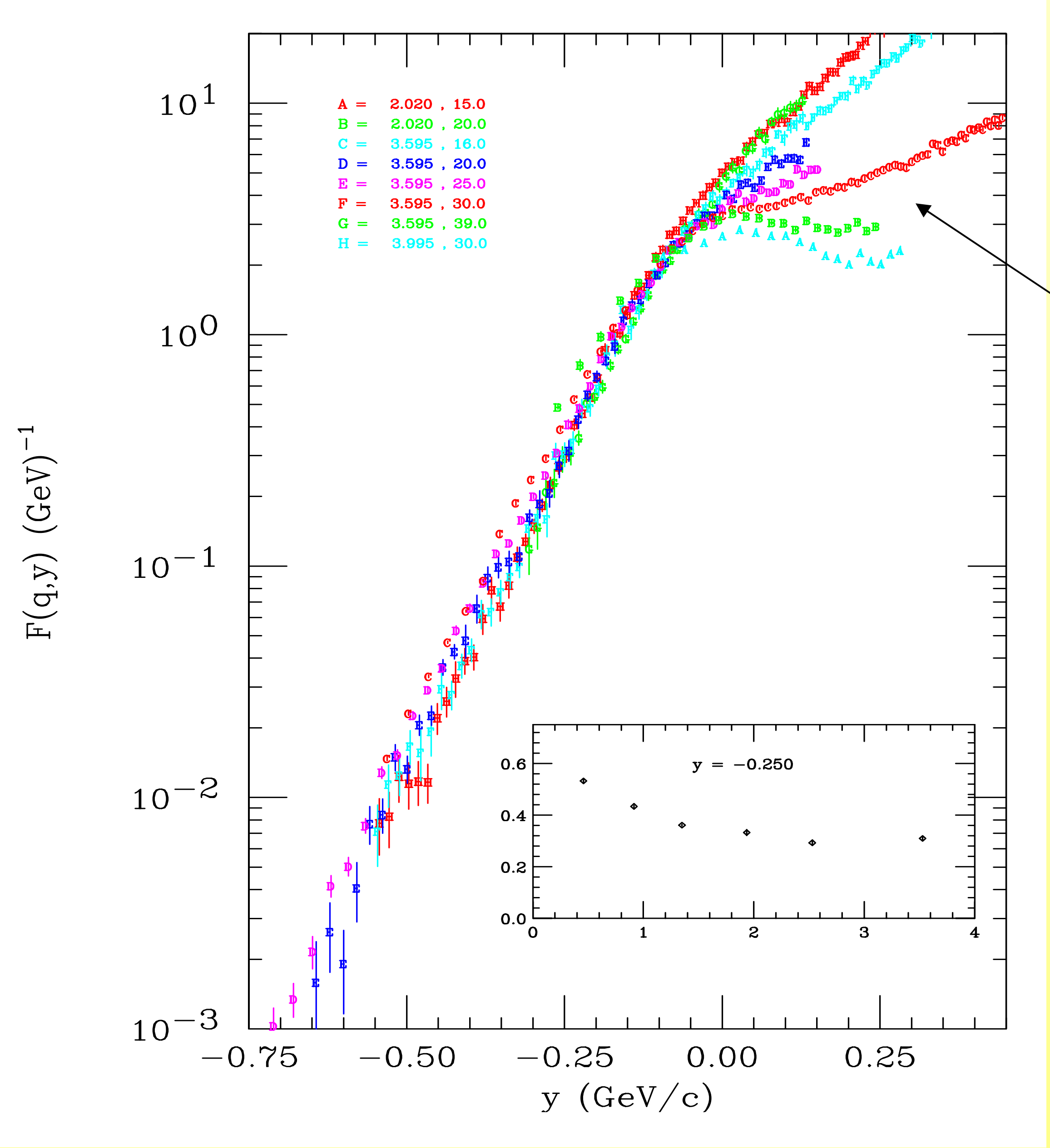
↕

SCALING OF THE 1st KIND

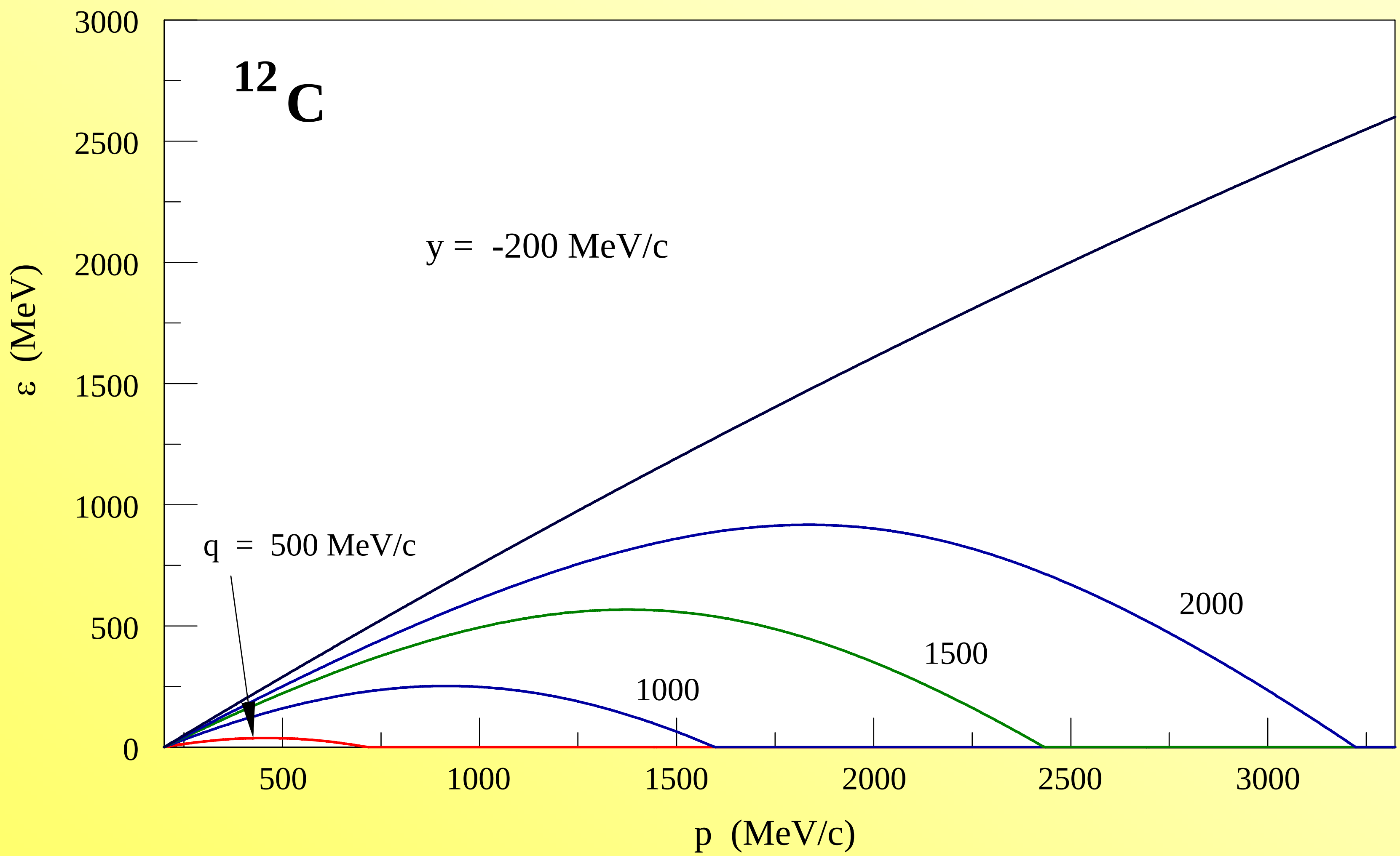
(y-scaling)

$$F(q,y) \xrightarrow[q\to\infty]{} F(y) \equiv F(\infty, y)$$

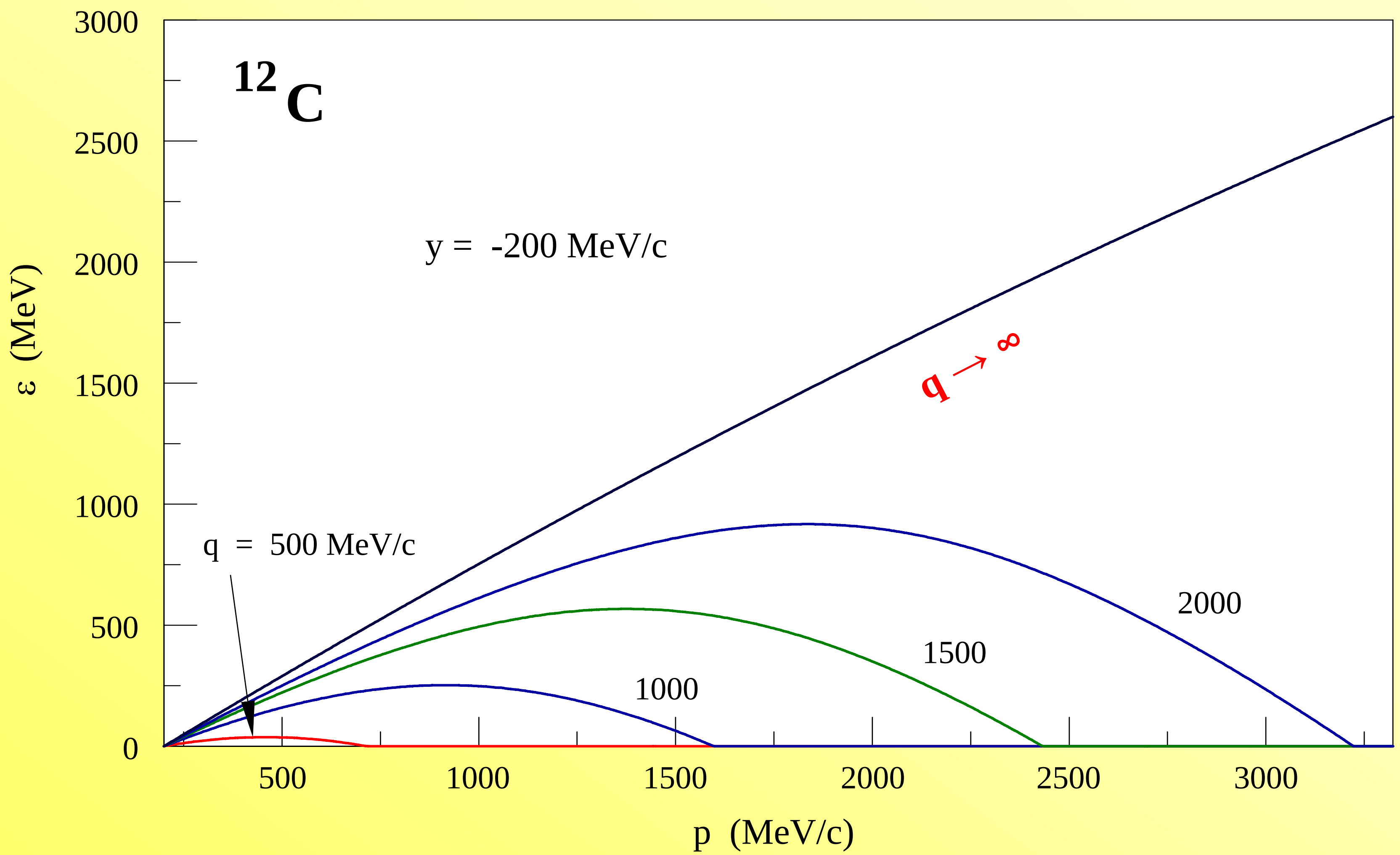
Example of ^{56}Fe



Note that at *y>0* the scaling is not good, due to the presence of resonances, meson production, *etc.* (see later, however)



^{12}C
y = -200 MeV/c
q = 500 MeV/c
1000
1500
2000
ε (MeV)
p (MeV/c)
0
500
1000
1500
2000
2500
3000



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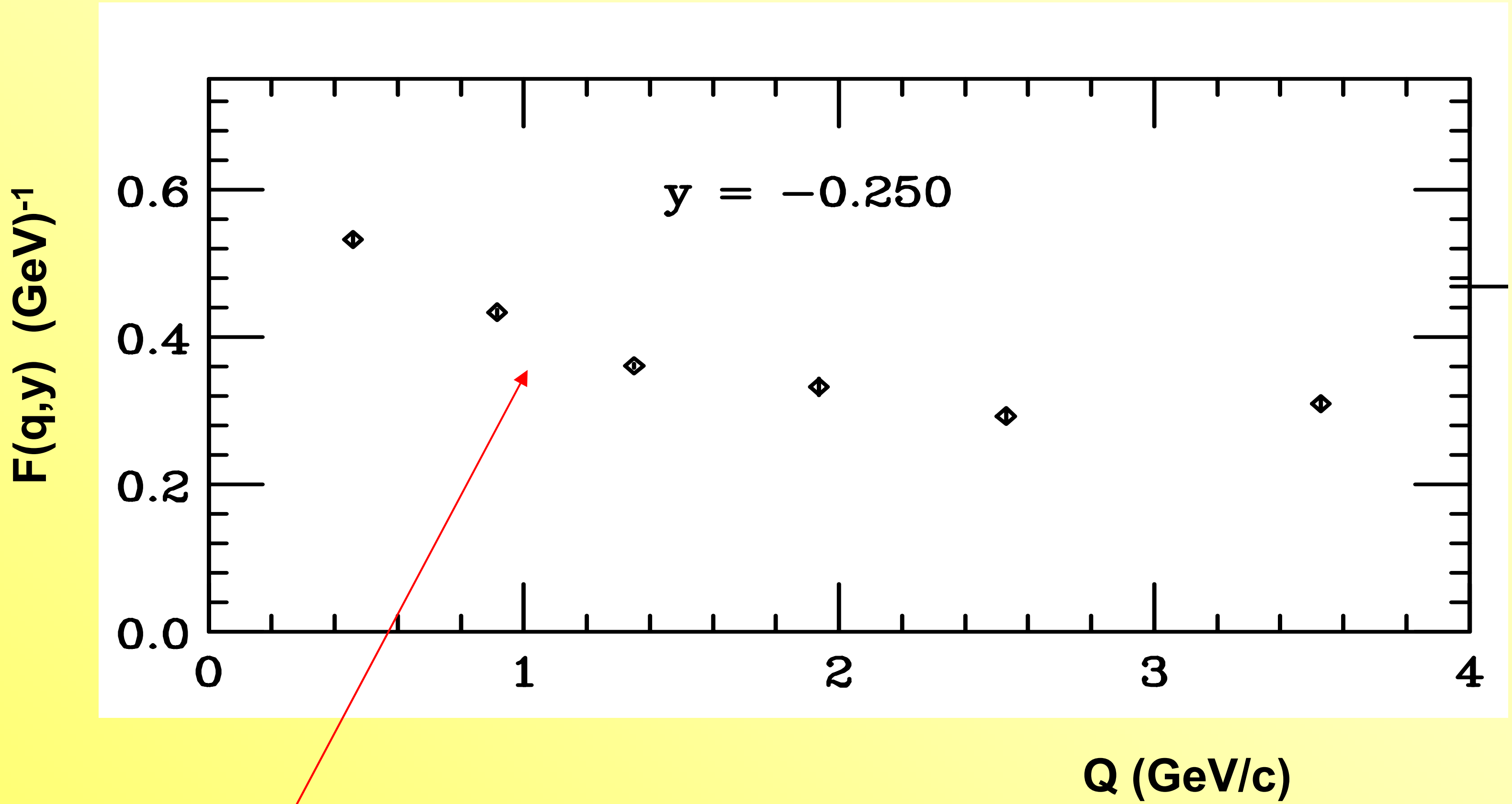

Approach from above

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Next we introduce a characteristic momentum scale for a given nuclear species

$$k_A = \sqrt{\langle k^2 \rangle_A}$$

and use this to define a dimensionless function

$$f(q,y) \equiv k_A \bullet F(q,y)$$

Correspondingly, one wishes to introduce a dimensionless scaling variable ψ and then to plot $f(q,\psi)$ versus ψ for various values of momentum transfer q

The Relativistic Fermi Gas (RFG) model is used to motivate the choice of scaling variable.

In the RFG one has

$$\left[k_A\right]^{RFG} = k_F$$

The RFG (dimensionless) scaling variable is given by

$$\psi = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\left[(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}\right]^{1/2}}$$

$$\cong \frac{1}{\eta_F}\left[\lambda\sqrt{1+1/\tau} - \kappa\right]$$

where the following dimensionless variables are used

$$\kappa \equiv q/2m_N$$

$$\lambda \equiv \omega/2m_N$$

$$\tau \equiv \left|Q^2\right|/4m_N^2 = \kappa^2 - \lambda^2 > 0$$

$$\eta_F \equiv k_F / m_N \sim 0.25$$

$$\xi_F \equiv \sqrt{1+\eta_F^2} - 1 \cong \eta_F^2/2 \sim 0.03$$

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Roughly: $\psi \approx y / k_A$

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$$\eta_F \equiv k_F / m_N \sim 0.25$$

$$\xi_F \equiv \sqrt{1+\eta_F^2} - 1 \cong \eta_F^2 / 2 \sim 0.03$$

To obtain the RFG scaling scaling function one uses as an effective single-nucleon EM cross section the following

$$\left[\Sigma_{eN}^{eff}\right]^{RFG} = \sigma_M \frac{1}{A}\left[\frac{\kappa}{2\tau} v_L \widetilde{G}_E^2 + \frac{\tau}{\kappa} v_T \widetilde{G}_M^2\right] + O(\eta_F^2)$$

$$\widetilde{G}_E^2 = ZG_{Ep}^2 + NG_{En}^2$$

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$$\widetilde{G}_E^2 = Z G_{Ep}^2 + N G_{En}^2$$

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relativistic factors required

small corrections from higher-order terms

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required relativistic factors

small corrections from higher-order terms

The RFG scaling function is found to be

$$f^{RFG} = k_F F^{RFG} = \frac{3}{4}\left(1-\psi^2\right)\theta\left(1-\psi^2\right) + O(\eta_F^2)$$

Note that it is a function only of ψ, but not of q or k_F (to lowest order)

Finally, introducing a small shift in energy to correct a deficiency of the RFG

$$\omega \to \omega' \equiv \omega - E_{shift} \quad , E_{shift} \approx 20\,\mathrm{MeV}$$

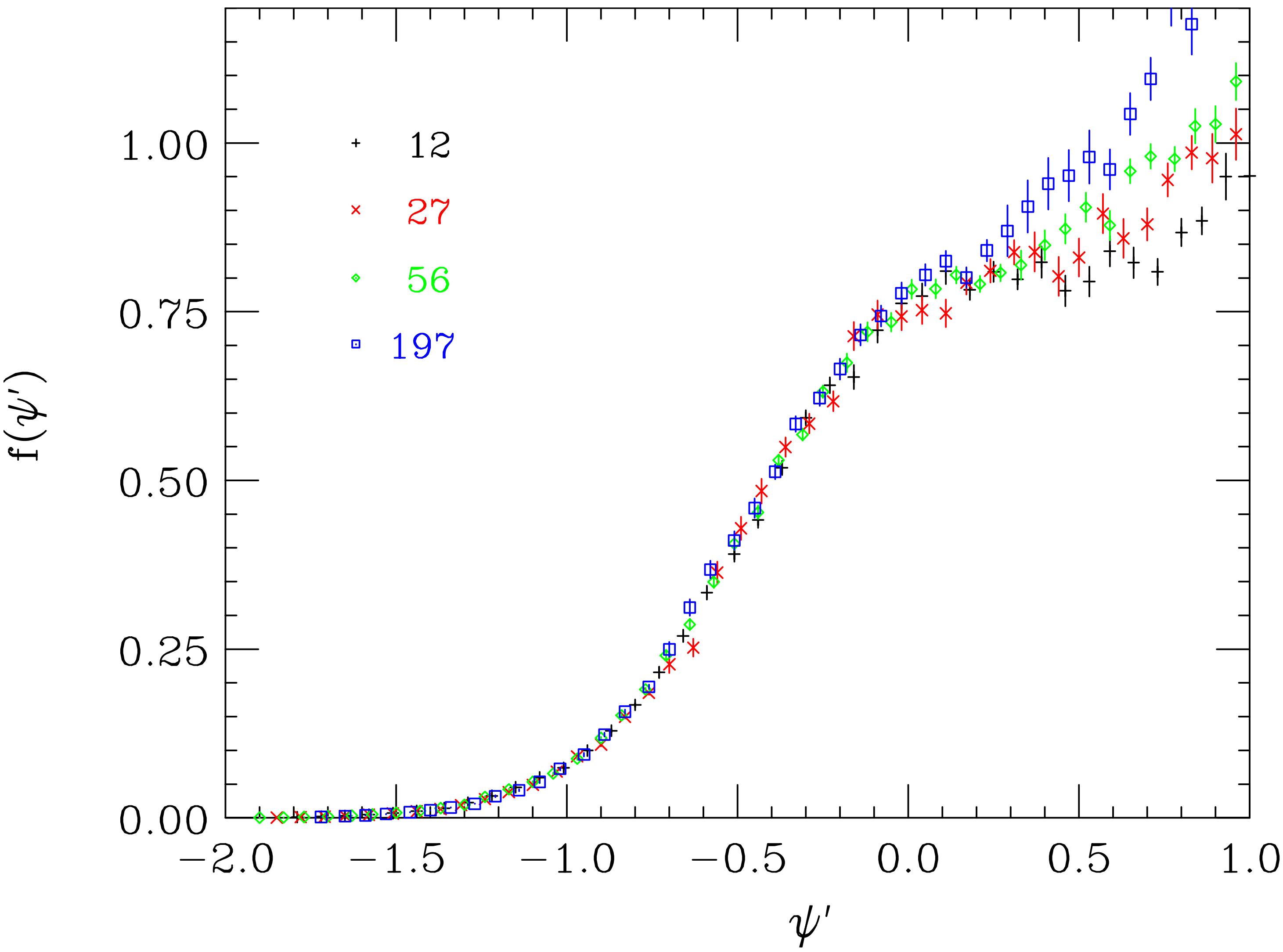
$$\lambda \to \lambda' \equiv \omega' / 2m_N$$

$$\tau \to \tau' \equiv \kappa^2 - \lambda'^2$$

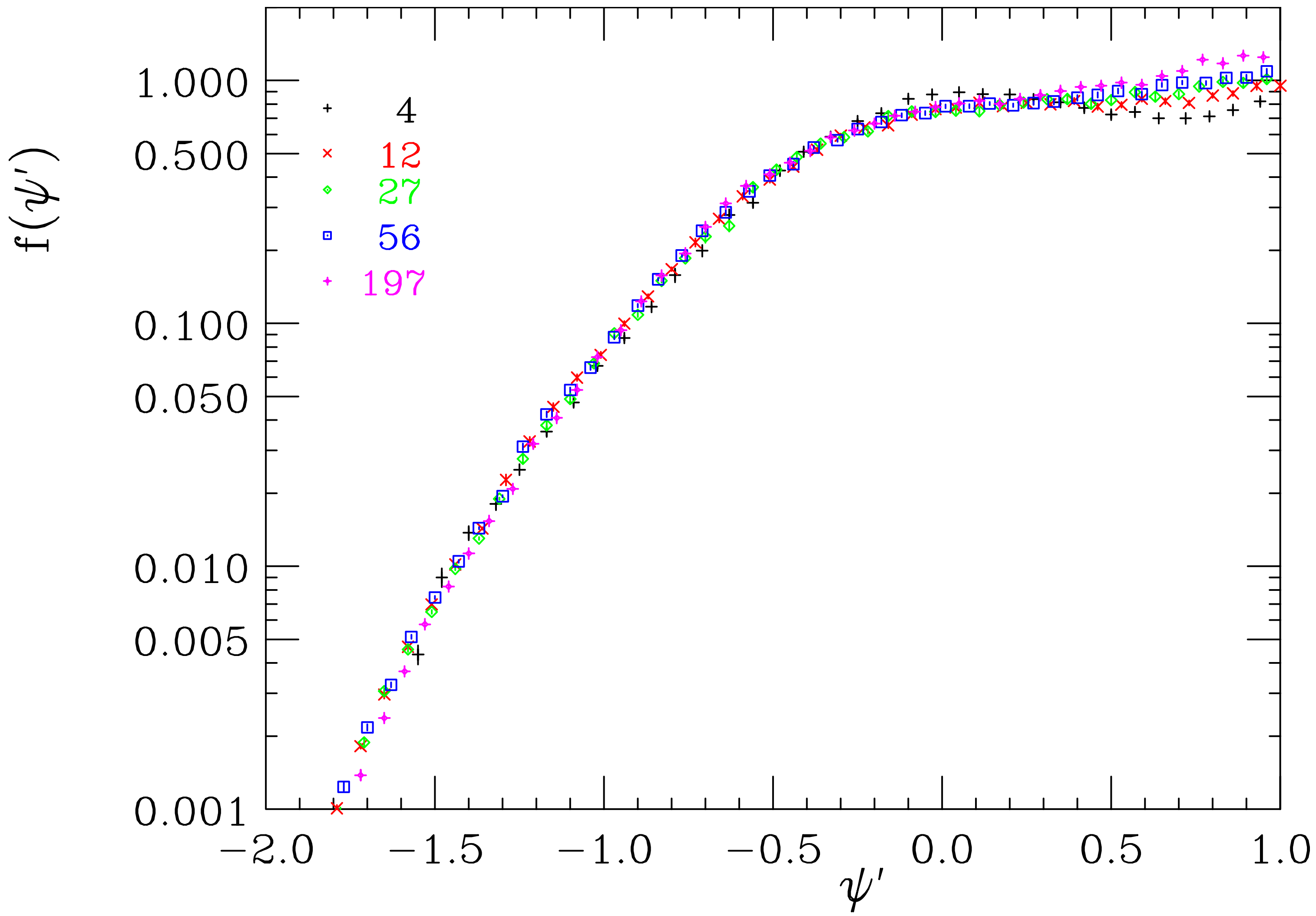
one can plot the dimensionless scaling function versus the dimensionless scaling variable obtained from the RFG result above using the shift:

$$\psi' \equiv \psi(\kappa, \lambda \to \lambda', \tau \to \tau')$$

Doing this using data for electron scattering from a variety of nuclei, all at the same kinematics (ε = 3.6 GeV, θ_e = 16°) yields the following:



f(ψ′)
ψ′
1.00
0.75
0.50
0.25
0.00
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
12
27
56
197



1.000
0.500
0.100
0.050
0.010
0.005
0.001
f(ψ′)
4
12
27
56
197
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
ψ′

In the scaling region ($\psi' < 0$) a universal behavior is seen, with

very little dependence on the nuclear species

↕

SCALING OF THE 2nd KIND

In the region above $\psi' = 0$ where resonances, meson production and the start of DIS enter the 2nd-kind scaling is not as good (see below)

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Although the amount of data separated into longitudinal (L) and transverse (T) responses is small, one can attempt a scaling analysis with what does exist. The inclusive cross section may be written

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left[v_L R_L(q,\omega) + v_T R_T(q,\omega) \right]$$

$$v_L = \left| Q^2 / q^2 \right|^2$$

$$v_T = \tfrac{1}{2}\left| Q^2 / q^2 \right| + \tan^2 \theta_e / 2$$

From which L and T scaling functions can be defined as above

$$F_L(q,y) \equiv \frac{R_L(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_L / \sigma_M v_L}$$

$$F_T(q,y) \equiv \frac{R_T(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_T / \sigma_M v_T}$$

→

as can their dimensionless analogs

$$f_L(q,y) \equiv k_A \bullet F_L(q,y)$$

$$f_T(q,y) \equiv k_A \bullet F_T(q,y)$$

What results is the following:

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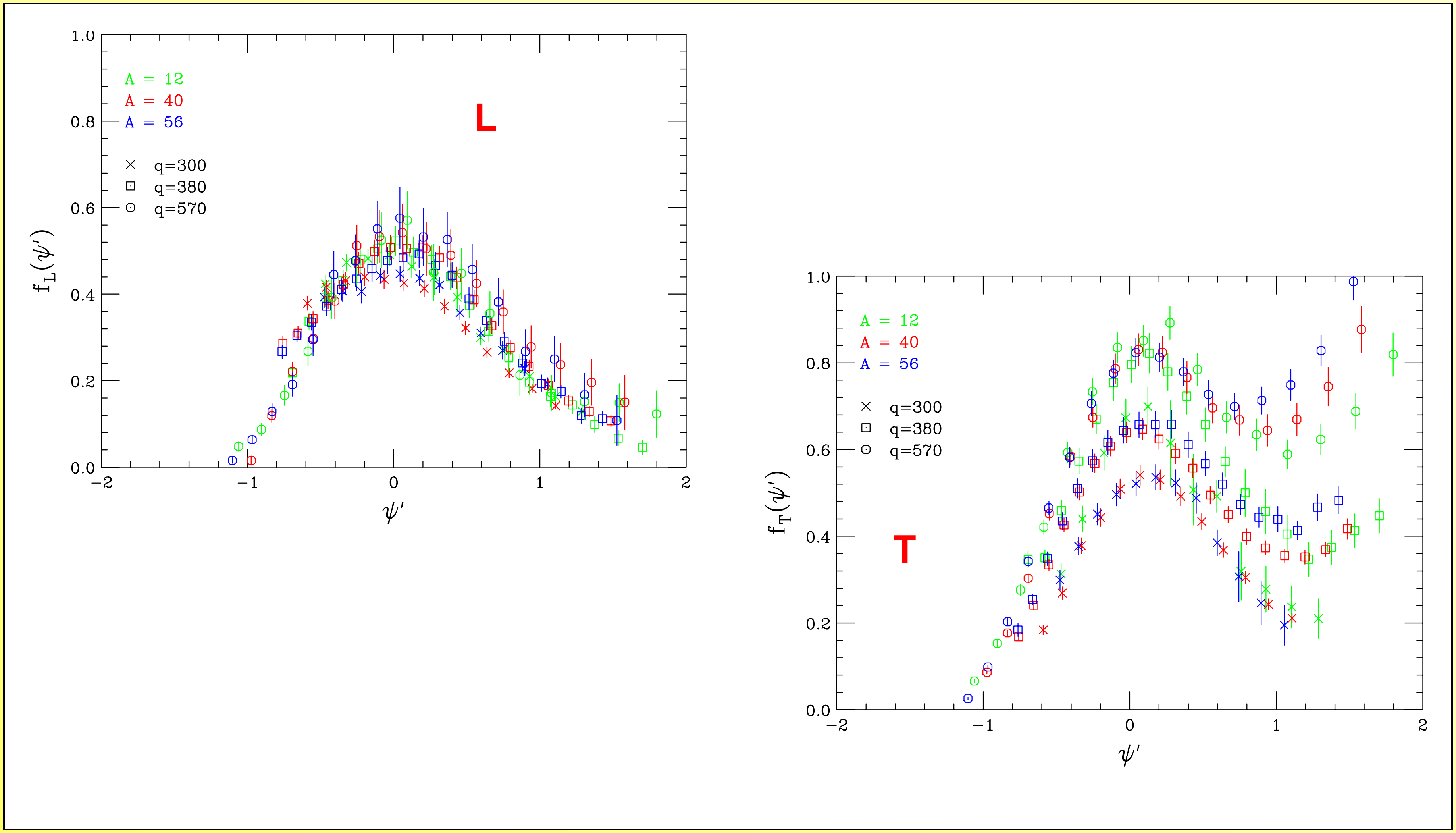

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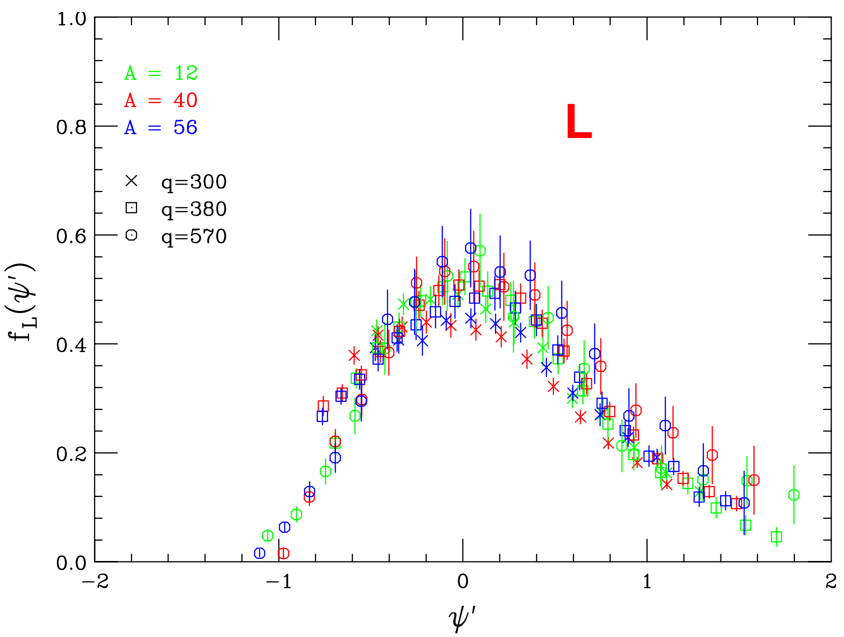
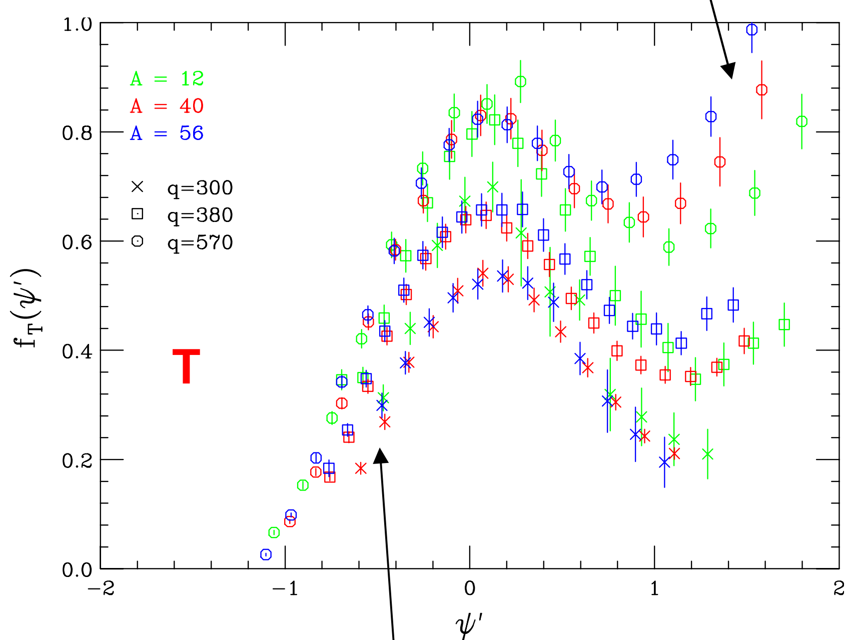
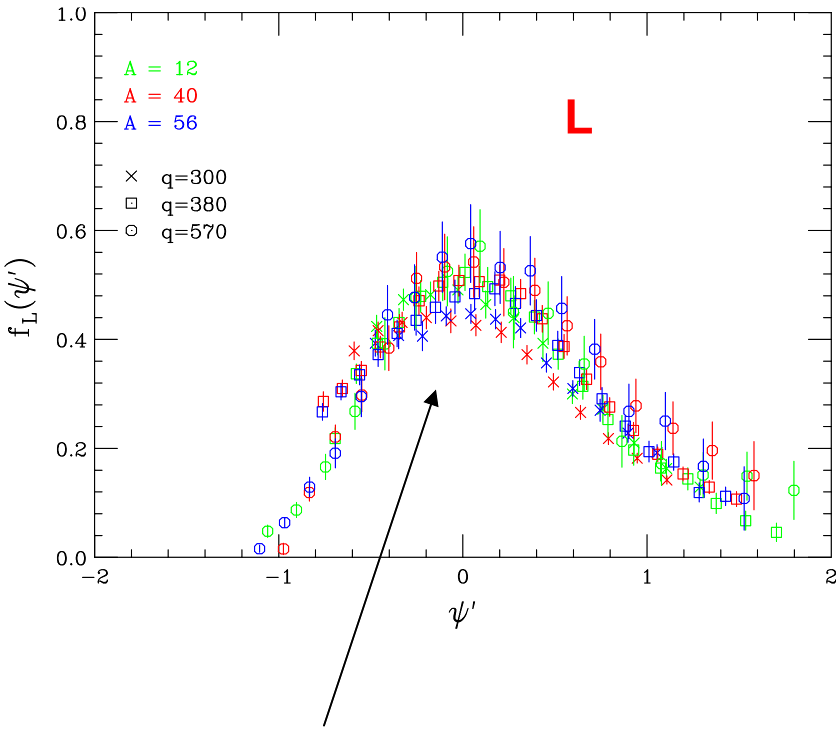


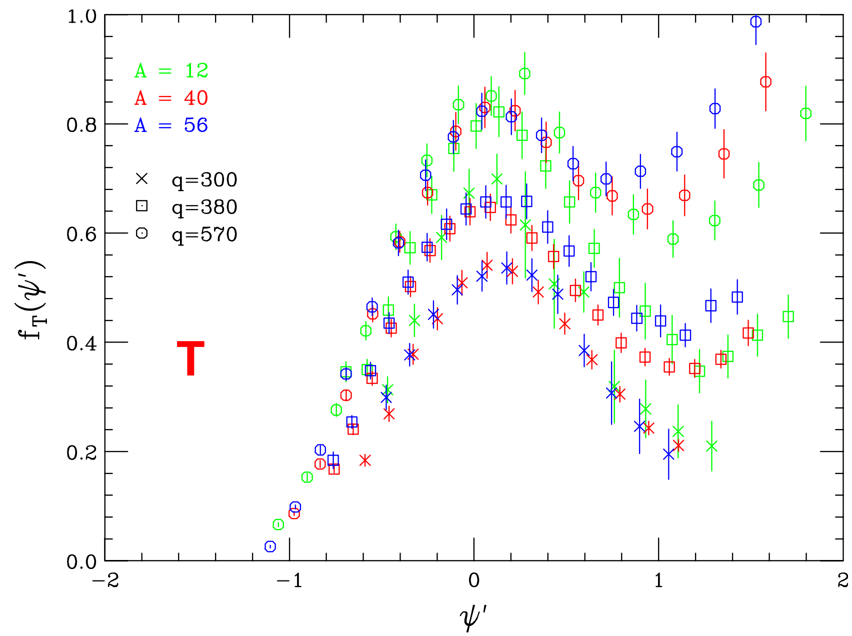
Inelastic contributions (mainly T)



… however, still some residual below the QE peak

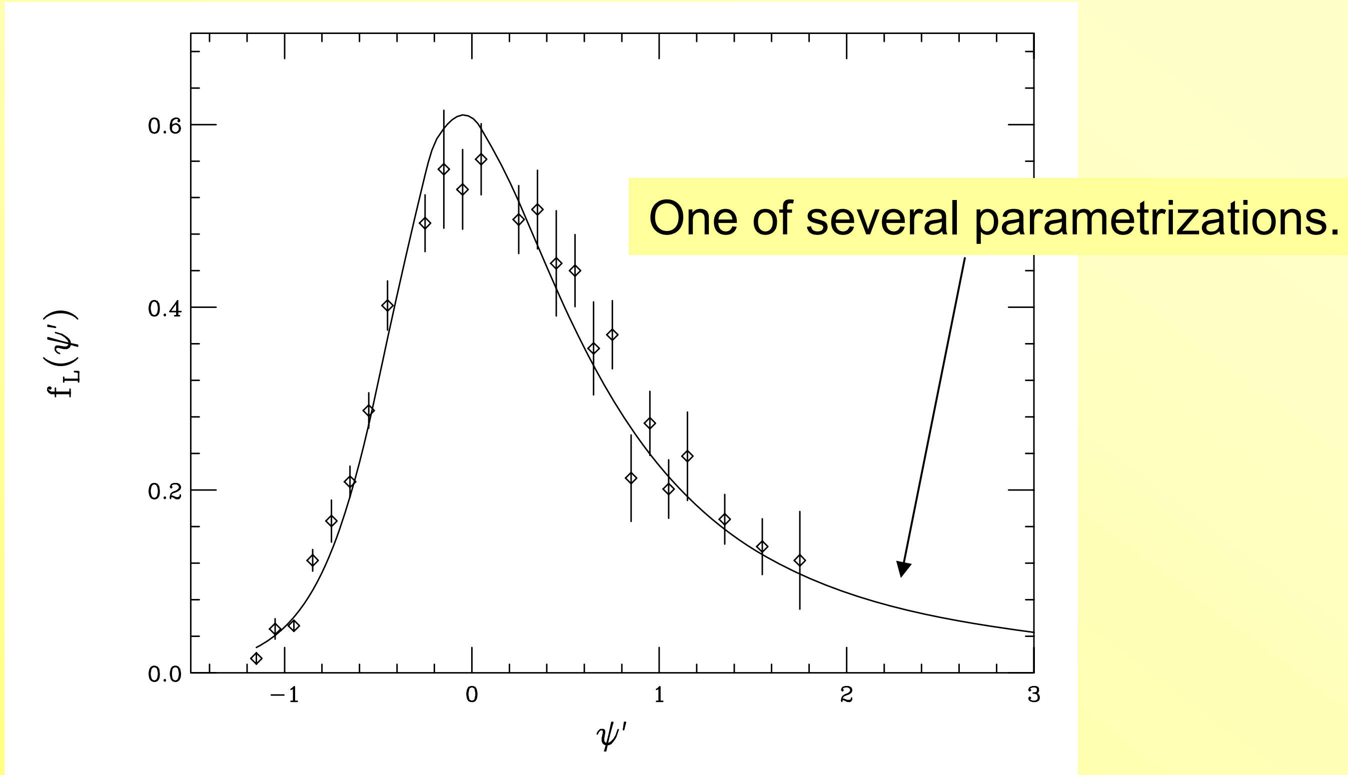
What results is the following:





In contrast, the L results show a **universal behavior**

That is, one has:



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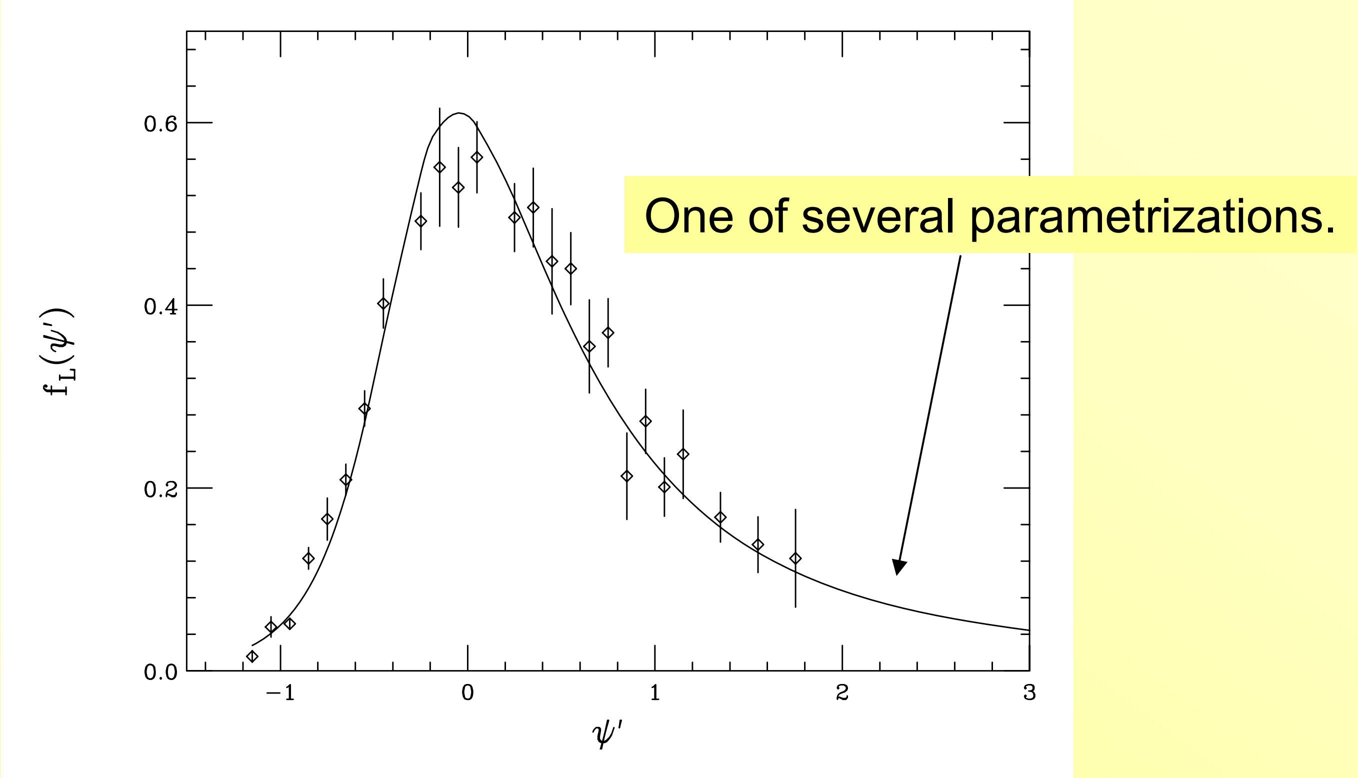

which is seen to be both independent of q (scaling of the 1st kind) and also independent of nuclear species (scaling of the 2nd kind)

↔ SUPERSCALING

Essentially the same asymmetric function is found in what are believed to be the best (relativistic) models:

- Relativistic Mean Field (RMF) approaches
- Semi-relativistic modeling
- BCS-inspired modeling

(see talks to follow)

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… but not in other models:

- Non-relativistic modeling
- PWIA with simple spectral functions
- RFG

Note: in the RFG one has

$$[f_L]^{RFG} = [f_T]^{RFG} = [f]^{RFG}$$

which has been called SCALING OF THE 0^{th} KIND

If it were not for

- contributions from resonances, meson production and DIS (which should not scale, since they involve different elementary cross sections, not elastic eN scattering, and since the scaling variables constructed above are appropriate only for QE scattering; see the discussions to follow), and for

- effects from meson-exchange currents (dominantly in T)

one might expect scaling of the 0^{th} kind to be found.

1p-1h MEC: at the QE peak 1p-1h MEC contributions typically interfere destructively, reducing the cross section from the result obtained in their absence; however, in new work a "bump" is predicted in the region where $\psi > 0$ (see later talk). At the QE peak this is roughly a 10-15% effect (minus).

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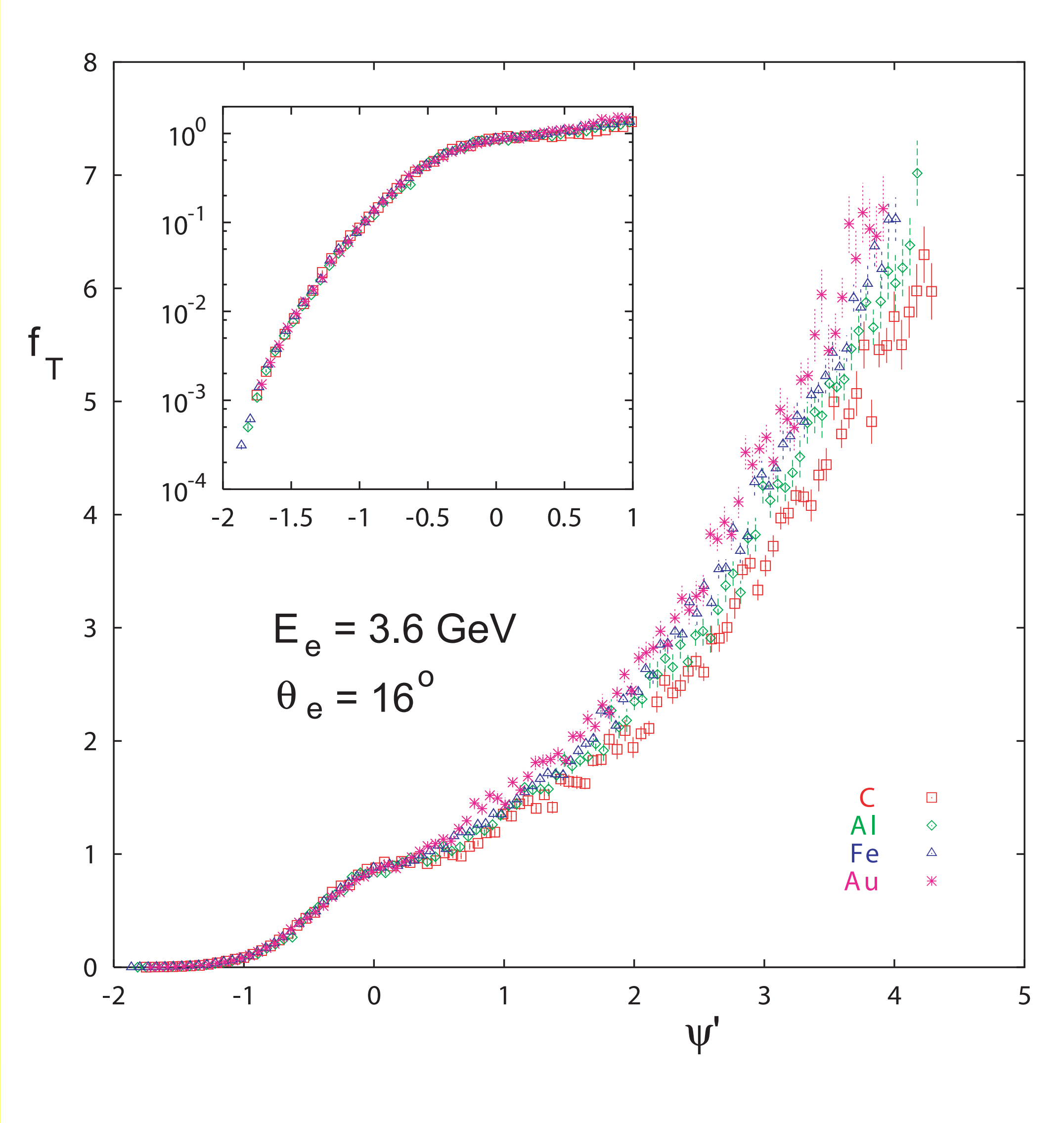
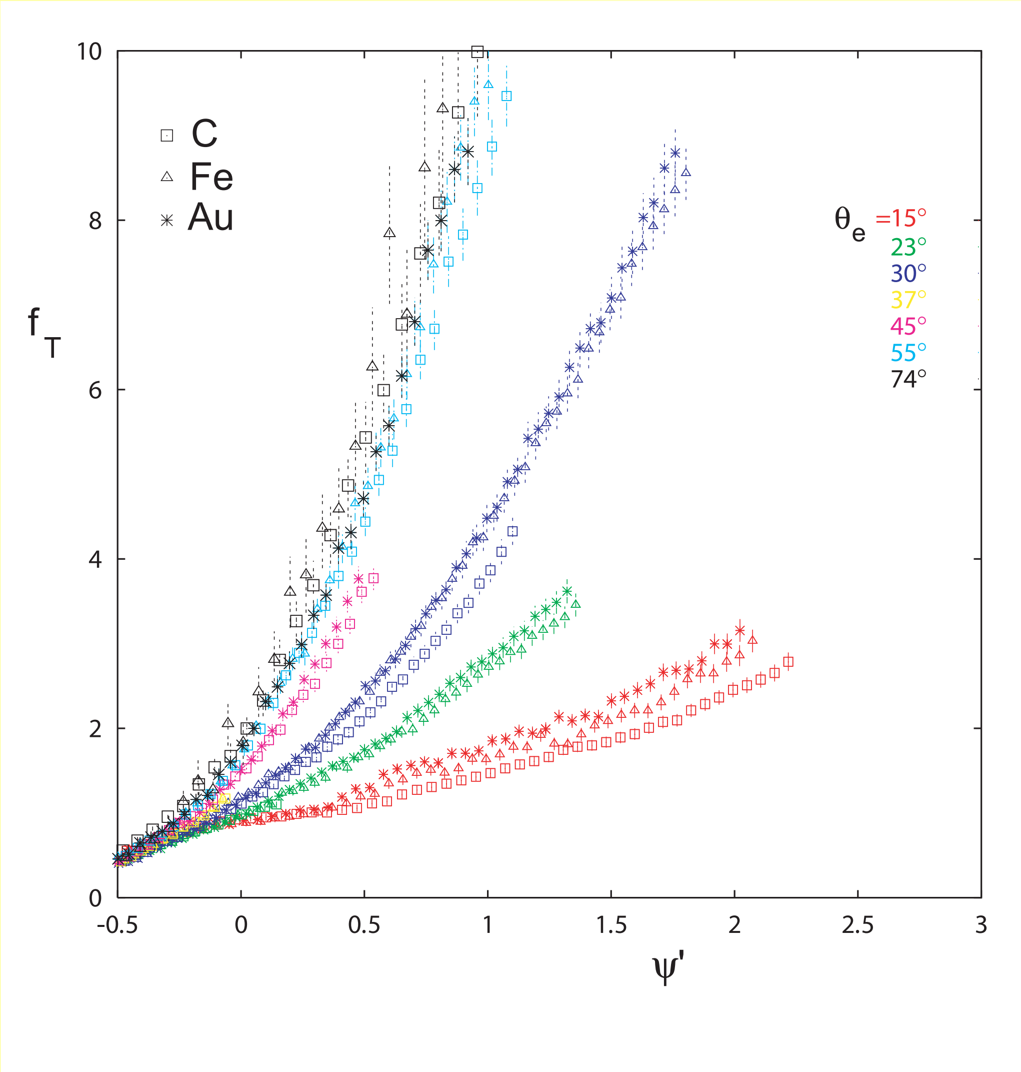
… net result: perhaps roughly compensate?

Outline:

- Basics of QE electron scattering
- Scaling of the 1st kind (y-scaling)
- Scaling of the 2nd kind
- Scaling of the 0th kind, 1p-1h MEC effects and Superscaling
- Non-QE scaling
  - Inelastic scattering
  - 2p-2h MEC effects
- Predicting ν cross sections using scaling and scaling of the 3rd kind



Large ψ' region



Breaking of 1st and 2nd kind scaling at high ψ'

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As will be seen later in the week, the net result of adding together the **universal L scaling function**, the **inelastic contributions** obtained using this as well, and the **2p-2h MEC contributions** is in reasonable agreement with experiment.

Outline:

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  - 2p-2h MEC effects
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  and scaling of the 3rd kind

Just as for the electron scattering reactions in the QE and Δ regions, we **use the scaling functions determined above**, but now multiply by the corresponding **charge-changing neutrino reaction cross sections** for the Z protons and N neutrons in the nucleus.

For the QE region we have the elementary reactions

$$\nu_\mu + n \rightarrow p + \mu^-$$
$$\bar{\nu}_\mu + p \rightarrow n + \mu^+$$

While in the Δ region we have

$$\nu_\mu + p \rightarrow \Delta^{++} + \mu^-$$
$$\nu_\mu + n \rightarrow \Delta^{+} + \mu^-$$
$$\bar{\nu}_\mu + p \rightarrow \Delta^{0} + \mu^+$$
$$\bar{\nu}_\mu + n \rightarrow \Delta^{-} + \mu^+$$

… and so on.

Note that these reactions are **isovector** only, whereas electron scattering contains both isoscalar and isovector contributions (the transverse EM response is, in fact, predominantly isovector at high energy).

Thus, in going from electron scattering where the universal scaling function came from the L response (essentially 50% isoscalar and 50% isovector) to CC neutrino reactions we have had to invoke

Scaling of the 3rd Kind

where the isospin nature of the scaling functions is assumed to be universal.

First, some **basic kinematics**:

incident and final leptons of masses *m* and *m´* with 3-momenta **k** and **k´** energies

$$\varepsilon = \sqrt{m^2 + k^2}$$

$$\varepsilon' = \sqrt{m'^2 + k'^2}$$

Energy and 3-momentum transfer carried by $W^{+/-}$

$$\omega = \varepsilon - \varepsilon'$$

$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$

Define $$\omega_0 \equiv \left(W^2 - (M_A^0)^2\right) / 2M_A^0 \geq 0$$

and then

$$\omega = \omega_0 + \left|Q^2\right| / 2M_A^0$$

Then the final lepton has energy and 3-momentum

$$k' = \left[\varepsilon_2^2 \left(k \cos \theta_{kk'} \right) + \left(M_A^0 + \varepsilon \right) \sqrt{\varepsilon_2^4 - m'^2 \varepsilon_1^2} \right] / \varepsilon_1^2$$

$$\varepsilon' = \left[\varepsilon_2^2 \left(M_A^0 + \varepsilon \right) + \left(k \cos \theta_{kk'} \right) \sqrt{\varepsilon_2^4 - m'^2 \varepsilon_1^2} \right] / \varepsilon_1^2$$

where

$$\varepsilon_1 \equiv \sqrt{(M_A^0)^2 + 2M_A^0 \varepsilon + m^2 + k^2 \sin^2 \theta_{kk'}}$$

$$\varepsilon_2 \equiv \sqrt{M_A^0 (\varepsilon - \omega_0) + (m^2 + m'^2)/2}$$

The charge-changing neutrino cross section may be written

$$\left[\frac{d^2\sigma}{d\Omega_{kk'}dk'}\right]_\chi = \sigma_0 R_\chi$$

nuclear response function

$$\sigma_0 \equiv \frac{\left(G\cos\theta_c\right)^2}{2\pi^2}\left[k'\cos\tilde{\theta}_{kk'}\right]^2$$

elementary cross section

$$\tan^2\tilde{\theta}_{kk'} \equiv \frac{\left|Q^2\right|}{4\varepsilon\varepsilon' - \left|Q^2\right|}$$

effective angle

for neutrinos and anti-neutrinos:

$$\chi = +(-) \longleftrightarrow \nu(\overline{\nu})$$

The nuclear response function may be decomposed into a generalization of the familiar Rosenbluth expression from studies of electron scattering (see above):

$$R_\chi = \left[\widehat{V}_{CC} R_{CC} + 2\widehat{V}_{CL} R_{CL} + \widehat{V}_{LL} R_{LL} + \widehat{V}_T R_T\right] + \chi\left[\widehat{V}_{T'} R_{T'}\right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

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with kinematic factors

$$\widehat{V}_{CC} = 1 - \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2} \cdot \delta^2$$

$$\widehat{V}_{CL} = \nu + \frac{1}{\rho'} \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2} \cdot \delta^2$$

$$\widehat{V}_{LL} = \nu^2 + \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2} \left(1 + \frac{2\nu}{\rho'} + \rho \cdot \delta^2\right) \cdot \delta^2$$

$$\widehat{V}_T = \left[\tfrac{1}{2}\rho + \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2}\right] - \frac{1}{\rho'} \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2} \left(\nu + \tfrac{1}{2}\rho\rho' \cdot \delta^2\right) \cdot \delta^2$$

$$\widehat{V}_{T'} = \left[\frac{1}{\rho'} \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2}\right] \left(1 - \nu\rho' \cdot \delta^2\right)$$

$$m = m_\nu = 0$$

$$m' = m_\mu$$

$$\delta \equiv m' / \sqrt{|Q^2|}$$

$$\nu \equiv \omega / q = \lambda / \kappa$$

$$\rho \equiv |Q^2| / q^2 = \tau / \kappa^2 = 1 - \nu^2$$

$$\rho' \equiv \tan \tfrac{\tilde{\theta}_{kk'}}{2} / \sqrt{\rho + \tan^2 \tfrac{\tilde{\theta}_{kk'}}{2}}$$

The cross section is typically **transverse** (T, T')

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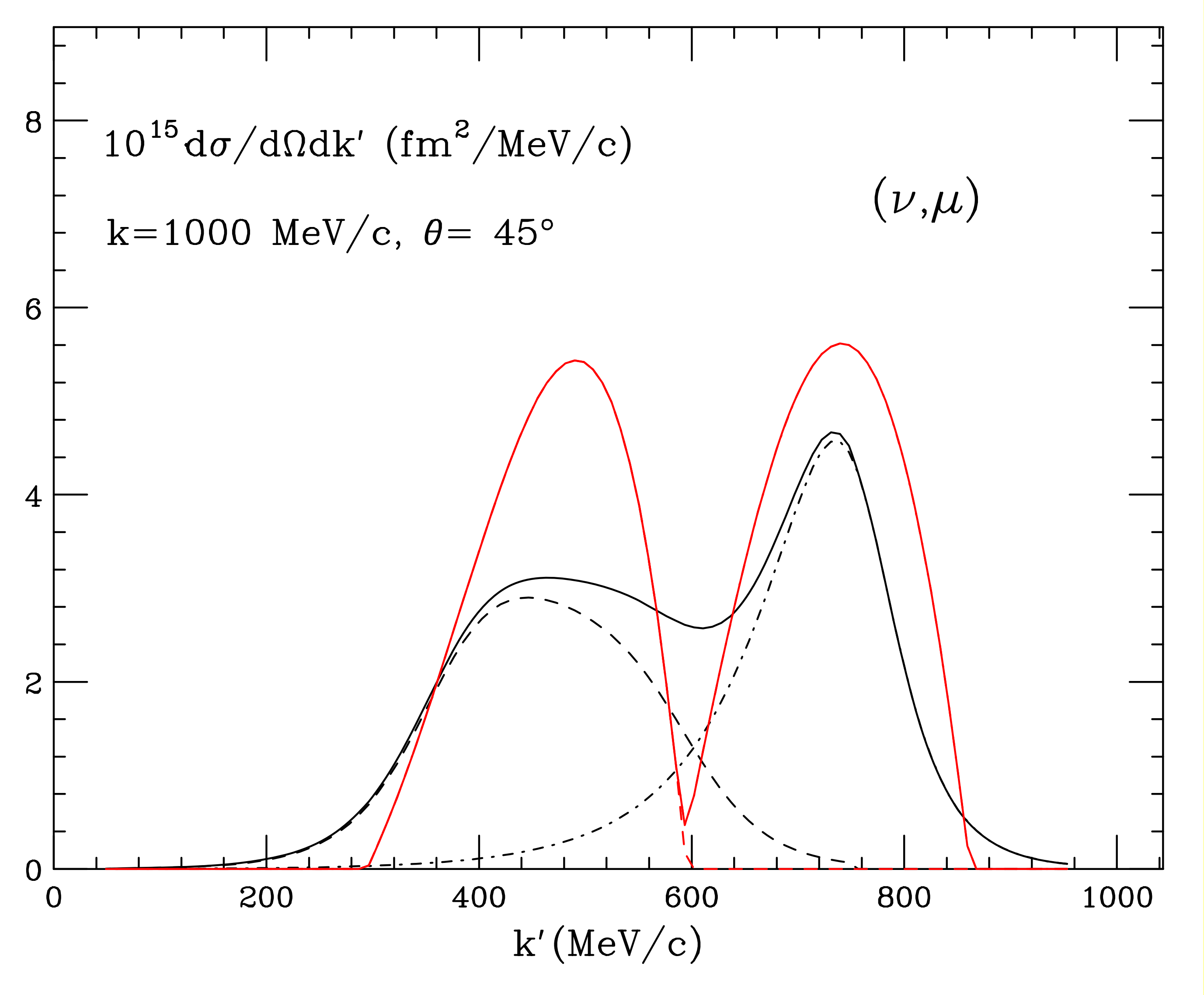
… and has VV, **AA** and **VA** contributions

The VV response has the same (isovector) contributions as occur for electron scattering, including the 2p-2h MEC contributions; however, the **transverse axial-vector matrix elements have no MEC pieces** in leading order and thus the **AA** and **VA** contributions do not contain the scaling violations from MEC


$10^{15} d\sigma/d\Omega dk'$ (fm^2/MeV/c)
k=1000 MeV/c, θ= 45°
(ν,μ)
0
2
4
6
8
0
200
400
600
800
1000
k' (MeV/c)