

Soft-gluon resummation: precision physics for hadron colliders

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- 1 Introduction
 - Analytical resummation
- 2 Higgs production
 - Total cross section
 - Differential distributions
- 3 Vector boson production
 - Predictions for the LHC
 - Comparison with Tevatron data
- 4 Summary

Outline

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The need for resummation

Partonic cross section as a perturbative series

$$\begin{aligned} \sigma_{ab}^{part}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) &= \alpha_s^k(\mu_R) [\sigma_{LO}(p_1, p_2, Q, Q_i) \\ &+ \alpha_s(\mu_R) \sigma_{NLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) \\ &+ \alpha_s^2(\mu_R) \sigma_{NNLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) + \dots] \end{aligned}$$

- The fixed-order result gives reliable result only when all the scales are of the same order of magnitude
- If $Q_i \gg Q$ or $Q_i \ll Q$, the appearance of $\alpha_s \log(Q_i/Q)$ terms could spoil the perturbative result: **they need to be resummed!**

Well-known examples

- $\log(Q/Q_0)$
 - evolution of pdfs from input scale Q_0 to hard scale Q
 - collinear radiation from colliding partons: single logs
 - systematically resummed by **DGLAP equation**
- $\log(Q/\sqrt{S})$
 - hadronic c.m. energy \sqrt{S} much larger than hard scale Q
 - multiple radiation over wide rapidity range: single logs
 - systematically resummed by **BFKL equation**
- $\log(Q^2/q_T^2)$
 - systems with invariant-mass $Q \gg q_T$
 - soft and collinear gluon emission: single and double logs
 - treated by means of **soft-gluon resummation**
- $\log(1 - Q^2/S)$
 - hadronic c.m. energy \sqrt{S} comparable to hard scale Q
 - soft and collinear gluon emission: single and double logs
 - treated by means of **soft-gluon resummation**

An example: the Drell-Yan cross section

$$\sigma(\tau) = \sigma_0 \int_{\tau}^1 \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \sum_{ij} f_i(x_1) f_j(x_2) (C_{ij}^{(0)}(z) + \frac{\alpha_s}{\pi} C_{ij}^{(1)}(z) + \dots)$$

$$\tau = \frac{Q^2}{S} \quad z = \frac{Q^2}{x_1 x_2 S} = \frac{Q^2}{s}$$

$$C_{q\bar{q}}^{(0)} = \delta(1 - z)$$

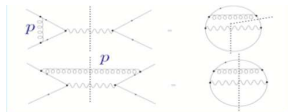
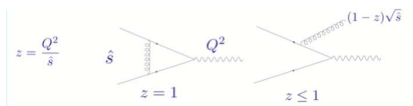
$$C_{q\bar{q}}^{(1)} = C_F (2(1 + z^2) \left(\frac{\log(1 - z)}{1 - z}\right)_+ - (1 + z^2) \frac{\log z}{1 - z} + \left(\frac{\pi^2}{3} - 4\right) \delta(1 - z))$$

- The plus distribution is regular but large if $z \rightarrow 1$
- Dominant contribution if $\tau \rightarrow 1$
- Convolution with pdfs: selecting small/intermediate x

→ Partonic threshold usually reached

→ $z \rightarrow 1$ contributions relevant even far from hadronic threshold

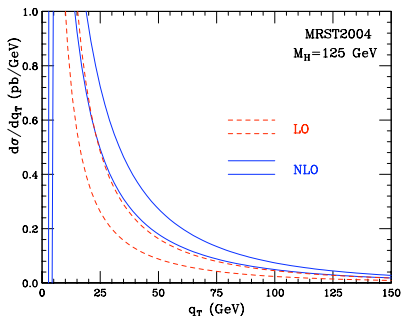
The origin of logs



- Higher orders made up of real ($z < 1$) and virtual ($z = 1$) contributions (both IR divergent)
- Different cuts of the same diagram: cancellation of IR divergences (KLN theorem)
- Near "critical" regions of phase space we have kinematical unbalance (full virtual - partial real) \rightarrow incomplete cancellation \rightarrow large logs! $\alpha_S L^2$ (soft and collinear emission) $\alpha_S L$ (only soft emission)

Another example: the small- q_T region ($q_T \ll Q$)

- Bulk of the events in the region $q_T \ll Q$
 - Kinematical unbalance between real and virtual contributions
- perturbative coefficients enhanced by $\alpha_S^n \log^m\left(\frac{Q^2}{q_T^2}\right)$
- convergence of perturbative result completely spoiled



→ **need for resummation!** [Collins, Soper, Sterman (1985)]

Resummation: the main idea

$\alpha_s L^2$	$\alpha_s L$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
...
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$...	$\mathcal{O}(\alpha_s^n)$	(N^n LO)
LL	NLL	NNLL	

- Ratio of two successive rows: $\mathcal{O}(\alpha_s L^2)$
- improved expansion
 - *reorganization* of the terms into *towers of logs*
 - *all-order summation* of the terms in each class
- key-point: *exponentiation*

$$\sigma^{res} \sim \exp [Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots]$$

- Ratio of two successive columns: $\mathcal{O}(1/L)$

Exponentiation 1

The observable must fulfill factorization properties both for

- dynamics (matrix element)

→ in the soft limit, multigluon amplitudes fulfill *generalized factorization formulae* given in terms of *single gluon emission probability*

$$\frac{1}{n!} \left[\underbrace{J^{\mu a}(q) J_{\mu}^a(q)} \right]^n$$

$$g^2 \left[\sum_a T_i^a T_i^a \right] \left(\frac{-2 p_1 \cdot p_2}{p_1 \cdot k p_2 \cdot k} \right)$$

- kinematics (phase space)

→ usually factorizable working in *conjugate space*

$$\delta^{(2)}(q_T - q_{T1} - \dots - q_{Tn}) = \int d^2 b e^{ib \cdot q_T} \prod_i e^{ib \cdot q_{Ti}}$$

$$\log(Q^2/q_T^2) \rightarrow \log(Q^2 b^2)$$

→ generalized exponentiation of single gluon emission

Exponentiation 2

$$\sigma(z) \sim \sigma_0 \left[1 + \sum_{n=1}^{\infty} \int_0^1 dz_1 \dots dz_n \frac{d\omega_n(z_1 \dots dz_n)}{dz_1 \dots dz_n} \Theta_{PS}^{(n)}(z_1 \dots dz_n) \right]$$

Dynamical factorization

[Bassetto, Ciafaloni, Marchesini (83)]

$$\frac{d\omega_n(z_1 \dots dz_n)}{dz_1 \dots dz_n} = \frac{1}{n!} \prod_{i=1}^n \frac{d\omega(z_i)}{dz_i}$$

Kinematical factorization

$$\Theta_{PS}^{(n)}(z, z_1 \dots dz_n) = \prod_{i=1}^n \Theta_{PS}(z, z_i)$$

Exponentiation

[Collins, Soper, Sterman (85); Parisi, Petronzio (79); Catani, Trentadue (89)]

$$\begin{aligned} \sigma(z) &\sim \sigma_0 \left[1 + \sum_{n=1}^{\infty} \frac{1}{n!} \left[\int_0^1 dz_i \frac{d\omega(z_i)}{dz_i} \Theta_{PS}(z, z_i) \right]^n \right] \\ &\sim \sigma_0 \exp \left[\int_z^1 dz' \frac{d\omega(z')}{dz'} \Theta_{PS}(z, z') \right] \sim \sigma_0 \exp[\alpha_s L^2 + \dots] \end{aligned}$$

Matching with fixed-order

The resummed result has to be properly matched with the fixed-order calculation to avoid double counting

$$\sigma = \sigma^{res} + \sigma^{fix} - \sigma^{asym}$$

where σ^{asym} = expansion of resummed result to same order

- $q_T \ll Q$: $\sigma^{fix} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{res}$
- $q_T > Q$: $\sigma^{res} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{fix}$
- intermediate q_T : matching $\rightarrow \sigma$

Problem

Resummation involves integration over b from 0 to ∞ :

$$\alpha_s(1/b) \text{ large when } b \rightarrow 1/\Lambda_{QCD}$$

Going back to the physical space

- Proposed solutions

- return to p_T space (expansion of the exponent + inverse transformation performed analytically)

[Ellis, Veseli(97); Frixione, Nason, Ridolfi(99); Kulesza, Stirling(99-03)]

- integration over a complex b -plane to avoid singularities

[Laenen, Sterman, Vogelsang(00); Kulesza, Sterman, Vogelsang(02) Bozzi, Catani, DeFlorian, Grazzini(05-09)]

- extrapolation of perturbative results into large- b region [Qiu, Zhang(01)]

- using Borel resummation [Bonvini, Forte, Ridolfi(08)]

- Improved matching [Bozzi, Catani, DeFlorian, Grazzini(05-09)]

$$\tilde{L} = \log\left(\frac{bQ}{b_0} + 1\right) \rightarrow \int dp_T \frac{d\sigma_{NLO}}{dp_T} = \sigma_{NNLO}$$

- Other approaches

- joint resummation: resum both threshold and recoil logs

[Laenen, Sterman, Vogelsang(00)]

- resummation for double differential (p_T, y) distributions

[Bozzi, Catani, DeFlorian, Grazzini(08)]

Non-perturbative effects

- Important non-perturbative (NP) effects for q_T -distributions (large- b region).
 \equiv intrinsic q_T of the partons, inside the hadrons.

- Resummation formula $\rightarrow \exp(S + F_{NP})$

$$F_{NP}(b, Q, x_a, x_b) = \exp\left[\left(-g_1 - g_2 \log\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \log(100x_a x_b)\right)b^2\right]$$

- NP form factor (g_1, g_2, g_3) obtained from experimental data:
 - Ladinsky, Yuan(94)
 - Brock-Landry-Nadolsky-Yuan(03)
 - Konyshov-Nadolsky (06)

The all-orders crew

- Drell-Yan lepton pairs at low Q

→ [Balazs, Yuan(97);Fai,Qiu,Zhang(03)]

- W and Z boson production

→ [Balazs, Yuan, Ladinsky, Qiu, Landry, Brock, Nadolsky, Berge, Olness, Konychev(97-05); Ellis, Ross, Veseli(97-98); Kulesza, Stirling(99-01); Laenen, Sterman, Vogelsang(00); Qiu, Zhang(01); Kulesza, Sterman, Vogelsang(02); Bozzi, Catani, Ferrera, de Florian, Grazzini(09)]

- SM Higgs boson production

→ [Bozzi, Catani, de Florian, Grazzini(03-05); Berger, Qiu(03); Balazs, Yuan(00); Cao, Chen(07); Kulesza, Stirling(03); Kulesza, Sterman, Vogelsang(04)]

- Di-photon production

→ [Balazs, Berger, Mrenna, Yuan, Nadolsky, Schmidt(98-07)]

- W-pair production

→ [Grazzini(06)]

- Z-pair production

→ [Balazs, Yuan(00);Frederix, Grazzini(08)]

The all-orders crew (continued)

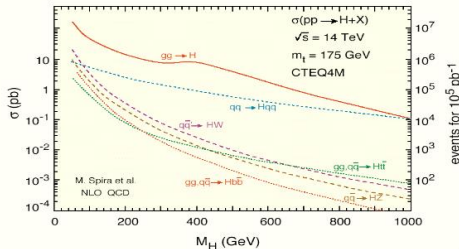
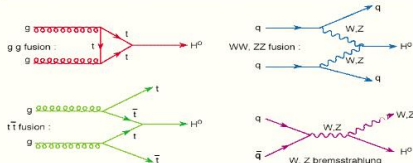
- SUSY Higgs production
 - [Field(04);Belyaev,Nadolsky,Yuan(06)]
- Slepton pair production
 - [Bozzi,Fuks,Klasen(06-08)]
- Z' production
 - [Fuks,Klasen,Ledroit,Li,Morel(08)]
- Upsilon production
 - [Berger,Qiu,Wang(05)]
- Polarized vector boson production (RHIC)
 - [Nadolsky,Yuan(03)]
- Transversely polarized Drell-Yan
 - [Kawamura,Kodaira,Shimizu,Tanaka(06-08)]

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Higgs production

H^0 production at hadron colliders:



- **Gluon fusion:** dominant production channel over entire mass range (large gluon luminosity)
- **Vector boson fusion:** very clean experimental signature
- **Associated production:** maybe important in the low mass region (provided a good b-tagging!)
- **Higgs-strahlung:** relevant at Tevatron for $M_H \leq 130$ GeV, very difficult at the LHC

The $gg \rightarrow H$ channel

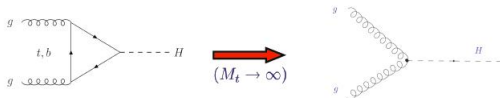
- Coupling mediated by triangular heavy quark loops
- Higgs coupling \propto fermion mass \rightarrow top loops dominate
- LO = $\mathcal{O}(\alpha_S^2)$ computed a long time ago...

[Georgi, Glashow, Machacek, Nanopoulos (1978)]

- NLO QCD corrections very large (**K-factor $\sim 80-100\%$!**)

[Spira, Djouadi, Graudenz, Zerwas (1991, 1995)]

- Higher-order calculations extremely difficult: considerable simplifications arise when $m_H \leq 2m_t$



- $\mathcal{L}_{eff} = -\frac{1}{4} \left[1 - \frac{\alpha_S}{3\pi} \frac{H}{V} (1 + \Delta) \right] \text{Tr} G_{\mu\nu} G^{\mu\nu}$

[Ellis, Gaillard, Nanopoulos (1976)]

[Δ known to $\mathcal{O}(\alpha_S^3)$ Chetyrkin, Kniehl, Steinhauser (1997)]

- Good agreement (up to 4% for $M_H < 200$ GeV) with full result

State of the art: total cross section

- **NNLO** ($\mathcal{O}(\alpha_s^4)$): another 15-20% enhancement ($m_t \rightarrow \infty$)

[Harlander (2000); Harlander, Kilgore (2001, 2002); Catani, deFlorian, Grazzini (2001, 2002);]

[Anastasiou, Melnikov (2002); Ravindran, Smith, vanNeerven (2003)]

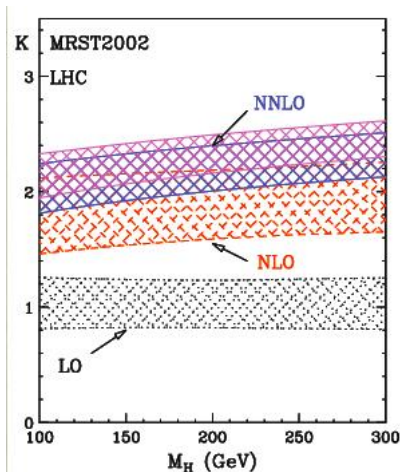
- Bulk of radiative corrections due to virtual and soft-gluon contributions \rightarrow (*insensitive to top quark loop*)
- Higher-order perturbative contributions reliably estimated by resumming multiple soft-gluon emissions
- **NNLL+NNLO**: perturbative uncertainty reduced to $\pm 10\%$

[Catani, deFlorian, Grazzini, Nason (2003)]

- Soft-gluon terms at **NNNLO**: effects consistent with NNLL+NNLO uncertainty

[Moch, Vogt (2005); Laenen, Magnea (2006); Idilbi, Ji, Ma, Yuan (2006)]

Higgs total cross section



- NNLO: 10-20% increase wrt NLO
- Threshold resummation further improves stability (6% wrt NNLO)
- 10% uncertainty due to scale variation
- 2-loop EW also available: 5-8% effect below WW threshold

[Aglietti, Bonciani, Degrassi, Vicini (2004)]

[Catani, deFlorian, Grazzini, Nason (2003)]

State of the art: differential distributions

- Transverse-momentum distribution

[Hj:deFlorian,Grazzini,Kunszt (1999)]: NLO

[Hj:Ravindran,Smith,vanNeerven (2002);Glosser,Schmidt (2002)]: NLO

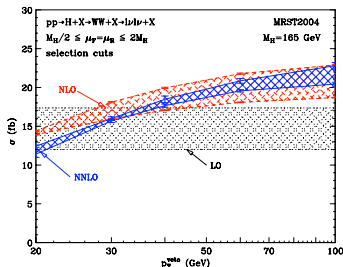
[Hjj:Campbell,Ellis,Zanderighi (2006)]: NLO

- Fully exclusive σ with arbitrary cuts

[FEHIP: Anastasiou,Melnikov,Petriello (2004,2005)]: NNLO

- Fully exclusive parton level event generator including
 $H \rightarrow \gamma\gamma, H \rightarrow WW, H \rightarrow ZZ$ decays

[HNNLO: Catani,Grazzini (2007)]: NNLO



Our work

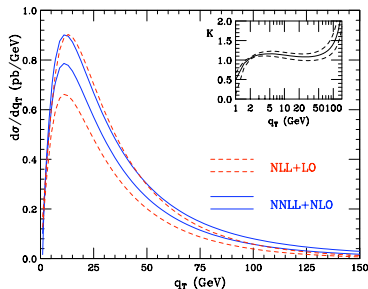
[Bozzi, Catani, deFlorian, Grazzini (2003, 2005)]

- Resummation at **NNLL** at small q_T
- Perturbative calculation at **NLO** at large q_T
- Matching at $O(\alpha_s^4)$ in the intermediate region
- Code **HqT** available at <http://theory.fi.infn.it/grazzini/codes.html>

[Bozzi, Catani, deFlorian, Grazzini (2007)]

- Extension including Higgs rapidity
- Impact parameter and double Mellin moments used
- NNLL+NLO accuracy for full-differential (q_T, y) cross section
- New version of HqT to appear

The q_T spectrum [BCdFG (2003, 2005)]

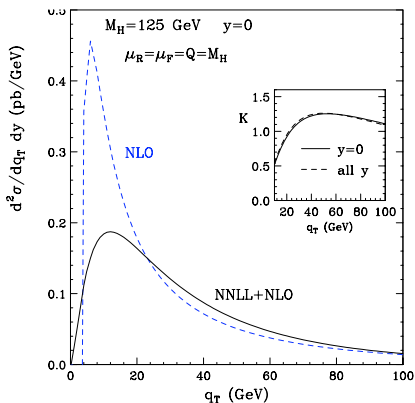


- NNLL+NLO uncertainty band overlaps with NLL+LO one
 - very good convergence of the resummed perturbative result
- q_T -dependent K-factor

$$K(q_T) = \frac{d\sigma_{\text{NNLL+NLO}}(\mu_F, \mu_R)}{d\sigma_{\text{NLL+LO}}(\mu_F = \mu_R = M_H)}$$

- ~ 1.1 - 1.2 in the central region
- increase (decrease) drastically for $q_T > 50$ ($q_T < 2$)
 - no simple rescaling of NLL+LO
- similar features when including rapidity dependence

Fixed rapidity [BCdFG (2007)]



- NLO

- divergent
- unphysical peak

- NNLL+NLO

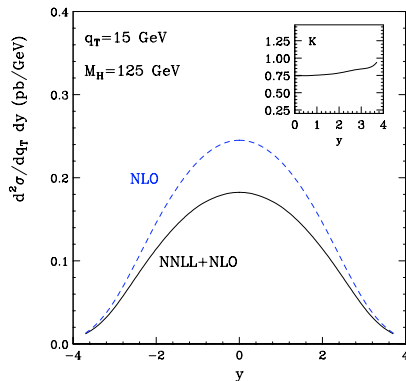
- well-behaved
- physical peak
- converges to NLO at high q_T

- q_T -dependent K-factor

$$K(q_T, y) = \frac{d\sigma_{\text{NNLL+NLO}}/(dq_T dy)}{d\sigma_{\text{NLO}}/(dq_T dy)}$$

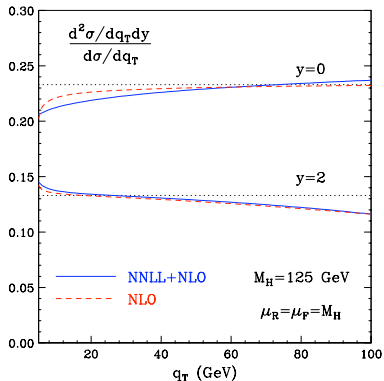
- mild rapidity dependence
- resummation relevant both at small and intermediate q_T

Fixed transverse-momentum [BCdFG (2007)]



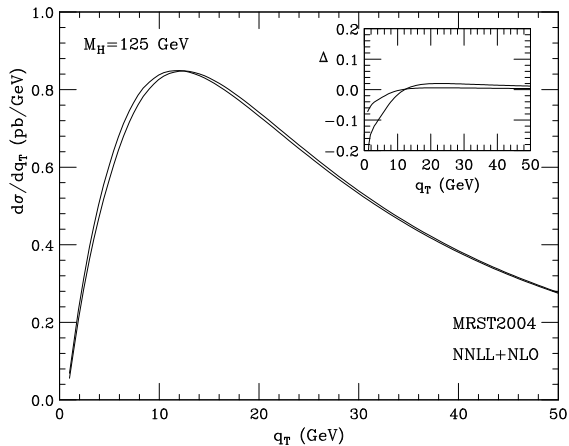
- NNLL+NLO reduces the cross section
- $y=0 \rightarrow 25\%$ suppression
- mild dependence on y in the central region
- more important in forward and backward regions (where σ is rather small)

Normalized results [BCdFG (2007)]



- $y=0$ lines above $y=2$ lines
 - expected, since σ decrease with y
- q_T slope decreases with increasing rapidity
 - q_T spectrum slightly softer at higher rapidity
- overall decrease going from $y=0$ to $y=2$: $\sim 40\%$
 - going from central to off-central rapidity regions, cross sections vary more in absolute value than in q_T shape

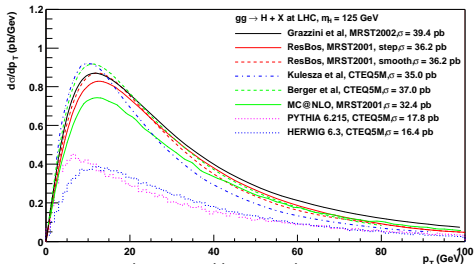
Non-perturbative effects



Important NP effect!

Higgs production via gluon fusion at the LHC

[Balazs, Grazzini, Huston, AK, Puljak'04]



NNLL+NLO

“Sudakov” NNLL + LO

“Sudakov” NNLL + LO

“Sudakov” NNLL + (N)LO

MC@NLO

PYTHIA

HERWIG

b-space with constraint:

$$\int dp_T \frac{d\sigma^{\text{NLO}}}{dp_T} = \sigma^{\text{NNLO}}$$

b-space

joint

b-space

LO p_T -distribution + parton shower

with hard matrix el. corrections

without hard matrix el. corrections

[Bozzi et al.'03'05]

[Berger, Qiu'02]

[A.K., Sterman, Vogelsang'03]

[Balazs, Yuan'00]

[Frixione, Webber'02]

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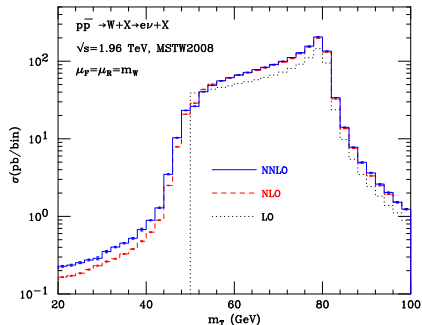
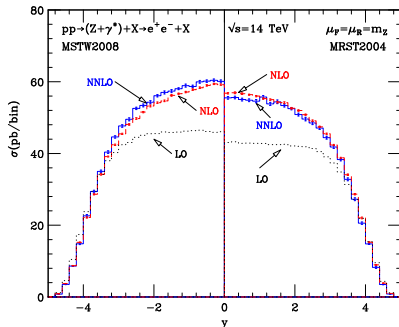
Drell-Yan at hadron colliders

NNLO QCD results now available also for W, Z production at the LHC

Anastasiou, Dixon, Melnikov, Petriello [03]

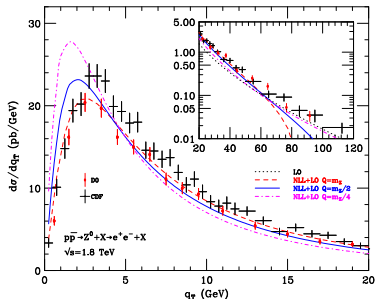
Melnikov, Petriello [06]

Catani, Cieri, DeFlorian, Ferrera, Grazzini [09]



- Z: result changes with different sets of pdfs
- W: large NNLO effects at low m_T , instabilities at $m_T \sim 50$ GeV

Drell-Yan at NLL+LO [Bozzi, Catani, deFlorian, Ferrera, Grazzini (08)]

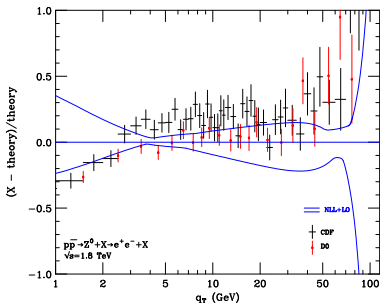


- Dependence on the resummation scale Q
- Q controls size of q_T -region influenced by soft gluon effects
- $q_T \leq 50$ GeV:
good agreement for large Q
- $q_T \geq 50$ GeV:
lower Q values preferred
- **NLL+NLO is mandatory**
(work in progress)

Drell-Yan at NLL+LO [Bozzi, Catani, deFlorian, Ferrera, Grazzini (08)]

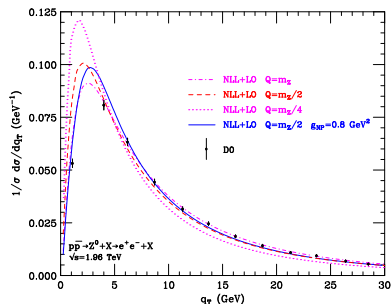
$$\frac{(d\sigma/dq_T)_X - (d\sigma/dq_T)_{NLL+LO}(\mu_F=\mu_R=2Q=m_Z)}{(d\sigma/dq_T)_{NLL+LO}(\mu_F=\mu_R=2Q=m_Z)}$$

(where X = data or different scales μ_R, μ_F, Q)



- Large q_T :
NLL+LO not accurate
- Low and intermediate q_T :
good agreement with data
(mostly D0)

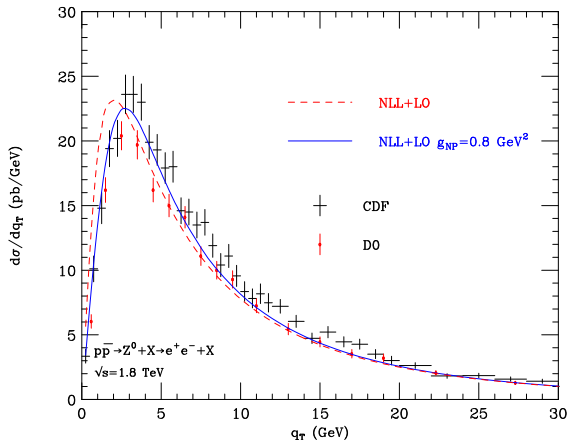
Drell-Yan at NLL+LO [Bozzi, Catani, deFlorian, Ferrera, Grazzini (08)]



- Normalized q_T distribution
- Scales fixed to Z mass
- Uncertainty dominated by Q variation
- Good agreement with Run II D0 data

Experimental errors are smaller than theoretical uncertainty
 → more accurate perturbative predictions (NNLL+NLO)

Non-perturbative effects



NP effect inside the NLL+LO uncertainty band

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Summary

- Precise knowledge of Higgs and vector boson differential distributions is very important at hadron colliders
 - Big theoretical effort in the last years
 - New contributions: $d\sigma/(dq_T dy)$ at (N)NLL+(N)LO for (H)W,Z
- importance of resummation at low and intermediate q_T
- stability of the main features with respect to perturbative uncertainties, NP dependence still deserves better investigation

Thanks for your attention!