Overview of TMD theory

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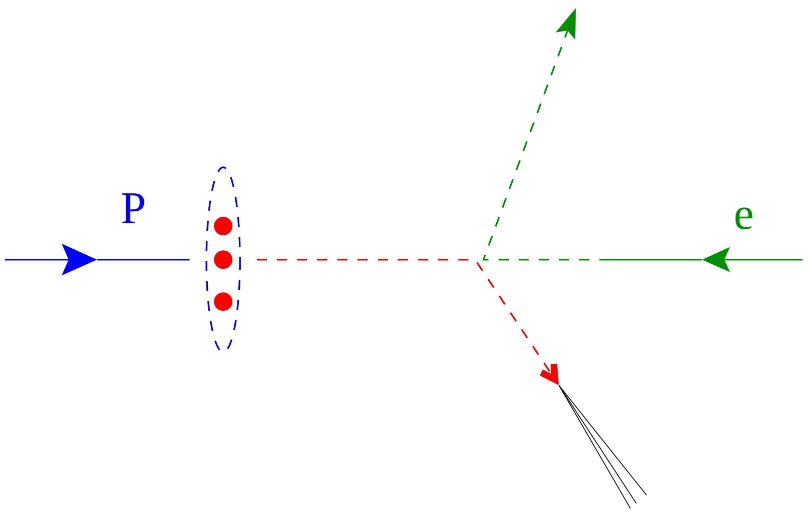
- Motivation
- TMD factorization
- Definitions of TMD functions
- Evolution, etc
- Universality
- Outlook

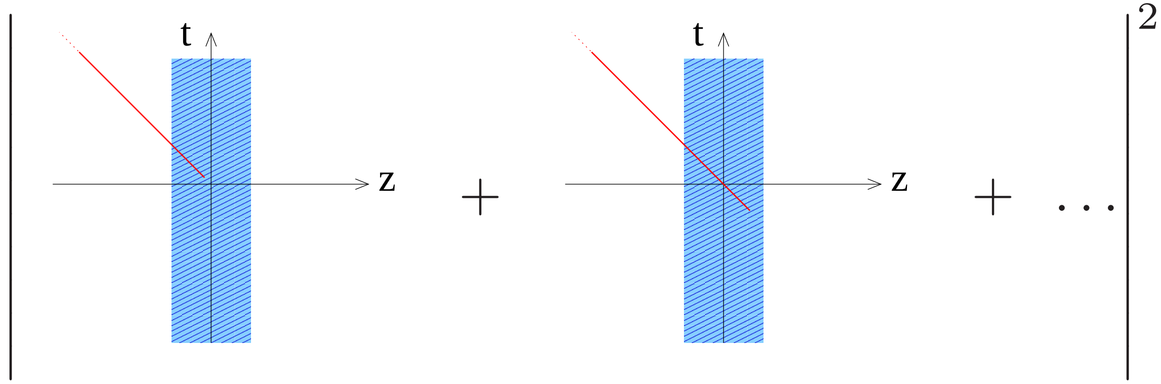
Motivating intuition, from DIS



- Short-distance hard scattering
- Density of partons $f_{j/H}(x)$
- SIDIS (semi-inclusive DIS, $eP \longrightarrow e + h + X$):
  - Fragmentation function $d_{\pi/j}(z)$
  - Deviation from basic parton model configuration
    $\Longrightarrow$ need parton k_T (in pdf $f_{j/H}(x, \boldsymbol{k}_\mathsf{T})$ and fragmentation function $d_{\pi/j}(z, \boldsymbol{k}'_\mathsf{T})$)

Light-front quantization

Source of scattered parton on light-like plane



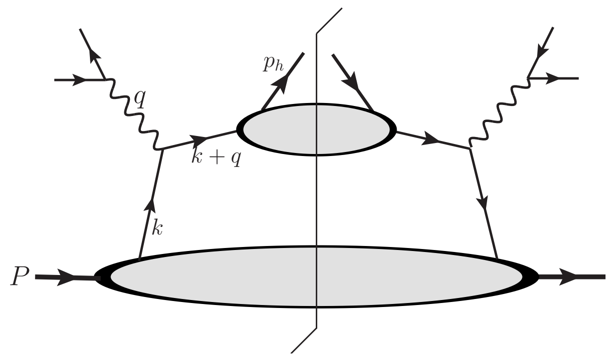
Integrated pdf:

$$f(x) \overset{?}{=} \frac{1}{2x(2\pi)^3} \int \mathrm{d}^2\boldsymbol{k}_\mathsf{T} \, \frac{\langle P|\; b^\dagger(xP^+, \boldsymbol{k}_\mathsf{T}) \; b(xP^+, \boldsymbol{k}_\mathsf{T}) \; |P\rangle}{\langle P|P\rangle}$$

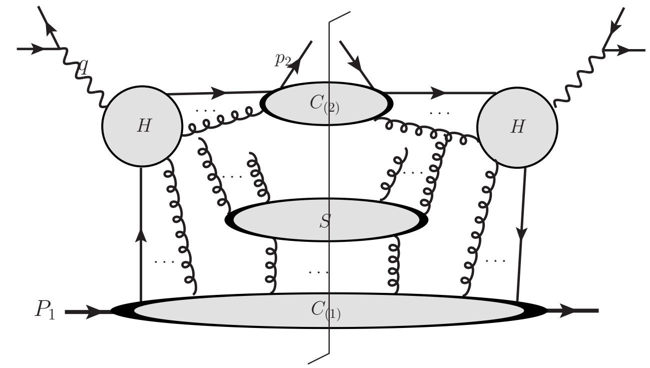
TMD pdf:

$$f(x, \boldsymbol{k}_\mathsf{T}) \overset{?}{=} \frac{1}{2x(2\pi)^3} \frac{\langle P|\; b^\dagger(xP^+, \boldsymbol{k}_\mathsf{T}) \; b(xP^+, \boldsymbol{k}_\mathsf{T}) \; |P\rangle}{\langle P|P\rangle}$$

QCD complications

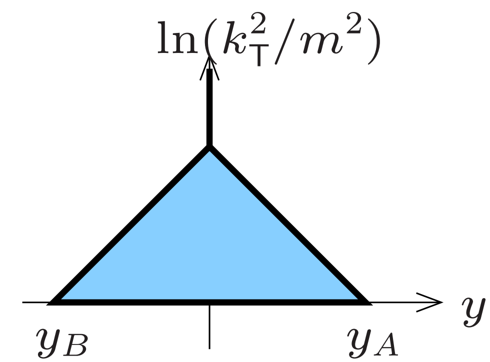


parton model



QCD

- Multiple gauge-dependent, gluon exchanges between subgraphs with very different kinematics
- Logarithmic integrals over rapidity and k_T. E.g., over



TMD factorization for SIDIS at small q_{T}

$$W^{\mu\nu}(\gamma^*(q) + P \to h(p_B) + X)$$
$$= \text{kin.} \sum_j H^{\mu\nu}(\ldots) \int \mathrm{d}^2\boldsymbol{k}_{\mathsf{T}}\, f_{j/P}(x, \boldsymbol{k}_{\mathsf{T}}; \zeta_A, \mu) D_{h/j}\big(z, -z(\boldsymbol{q}_{\mathsf{T}} + \boldsymbol{k}_{\mathsf{T}}); \zeta_B, \mu\big)$$

- Why useful:
  - Separation of scales. Parameters ζ and μ: . . .
  - Evolution equations
  - Single-scale perturbative quantities
  - Universal non-perturbative single-particle densities, etc
- Soft factors absorbed into new defns. of pdfs and frag. fns.

Which processes for TMD functions/TMD factorization?

Classic cases:

Process	frag. fn.	pdf
$e^+e^- \to h_A + h_B + X$	✓	
SIDIS: $eP \to e + h + X$	✓	✓
DY: $P_A + P_B \to (\gamma^*, W, Z) + X$		✓

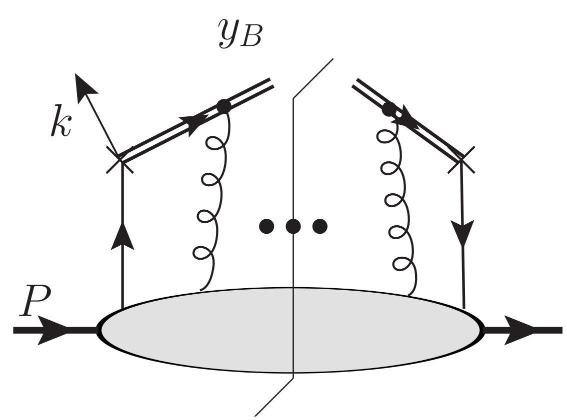
N.B. Mulders and Rogers talks about $H_1 + H_2 \to \ldots$

Definitions of TMD functions

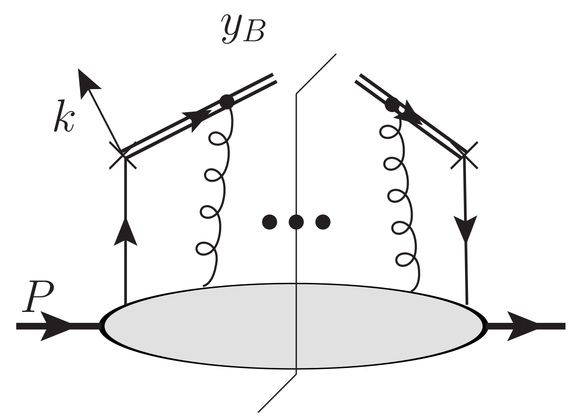
- Operator matrix element needed. E.g., to relate to lattice QCD calculations (Euclidean space)
- Definition must be compatible with proof of factorization
- My current best idea. (Fourier tfmn. $\boldsymbol{k}_\mathsf{T}$-space to $\boldsymbol{b}_\mathsf{T}$-space)

$$\tilde{f}_{f/P}(x,\boldsymbol{b}_\mathsf{T};\zeta_A;\mu) \stackrel{\text{def}}{=} \lim_{\substack{y_A\to+\infty \\ y_B\to-\infty}} \tilde{f}^{\text{unsub}}_{f/P}(x,\boldsymbol{b}_\mathsf{T};y_P-y_B)\sqrt{\frac{\tilde{S}_{(0)}(b_\mathsf{T},y_A,y_n)}{\tilde{S}_{(0)}(b_\mathsf{T},y_A,y_B)\ \tilde{S}_{(0)}(b_\mathsf{T},y_n,y_B)}}$$

$$\times \text{ UV renormalization factor}$$

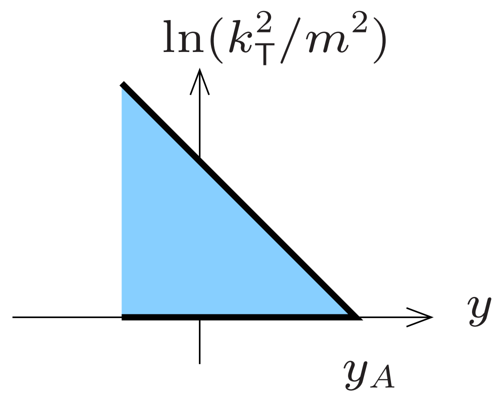
$=$  $\times$ Soft factors

$$f_{f/P}^{\mathsf{unsub}}(x, \boldsymbol{k}_{\mathsf{T}}; y_P - y_B) =$$



$$= \operatorname*{Tr}_{\text{color}} \int \frac{\mathrm{d}w^- \, \mathrm{d}^2\boldsymbol{w}_{\mathsf{T}}}{2\pi} e^{-ixP^+w^- + i\boldsymbol{k}_{\mathsf{T}}\cdot\boldsymbol{w}_{\mathsf{T}}} \left\langle P|\bar{\psi}_f(w) W(w; n_B)^\dagger \frac{\gamma^+}{2} W(0; n_B) \psi_f(0)|P\right\rangle_c$$

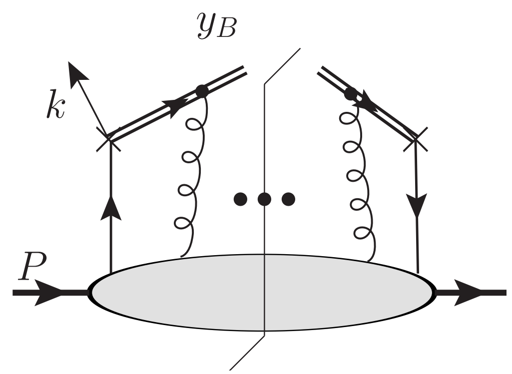
- Gluon kinematics roughly:



$\Longrightarrow$ Rapidity divergences if $y_B \to -\infty$: . . .

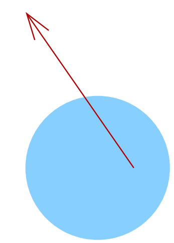
$\Longrightarrow$ UV divergences in virtual corrections: renormalization

Meaning of Wilson line

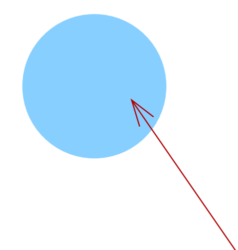


Represents recoilless approximation to struck quark

(SI)DIS: WL to future



DY: WL from past



Relation by TP transformation

Meaning of parameters ζ and μ

- Valid definition must be compatible with proof of factorization
- Must implement cutoffs on y and k_T of gluons
- $\zeta_A = M^2 x^2 e^{2(y_P - y_n)}$
- μ: renormalization, like k_T cutoff
- Need evolution equations

$\longrightarrow$ Contrary to basic definition using light-front quantization:

$$\int \mathrm{d}^2\boldsymbol{k}_\mathsf{T}\, f_{j/H}(x, \boldsymbol{k}_\mathsf{T}) \quad \neq \quad f_{j/H}(x)$$

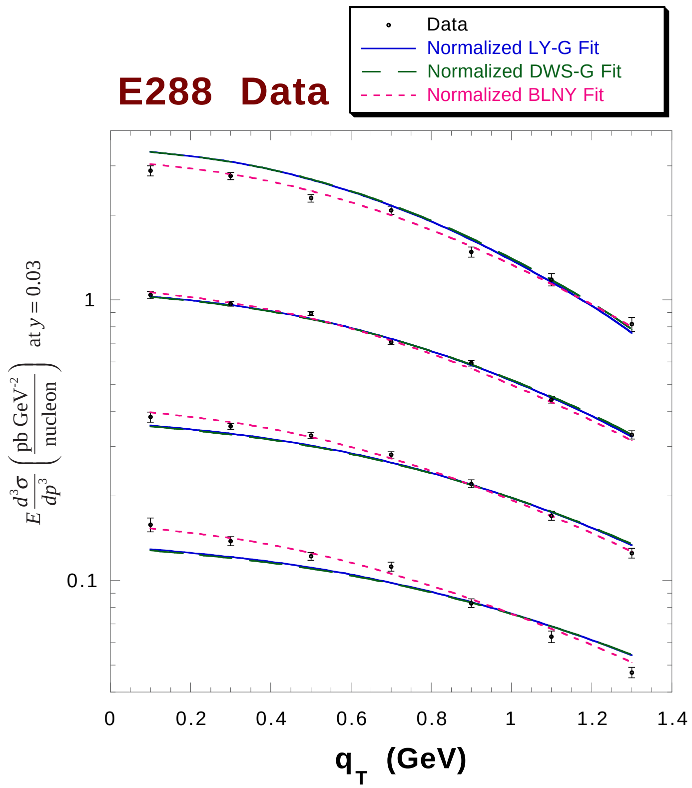
Comparison with other definitions

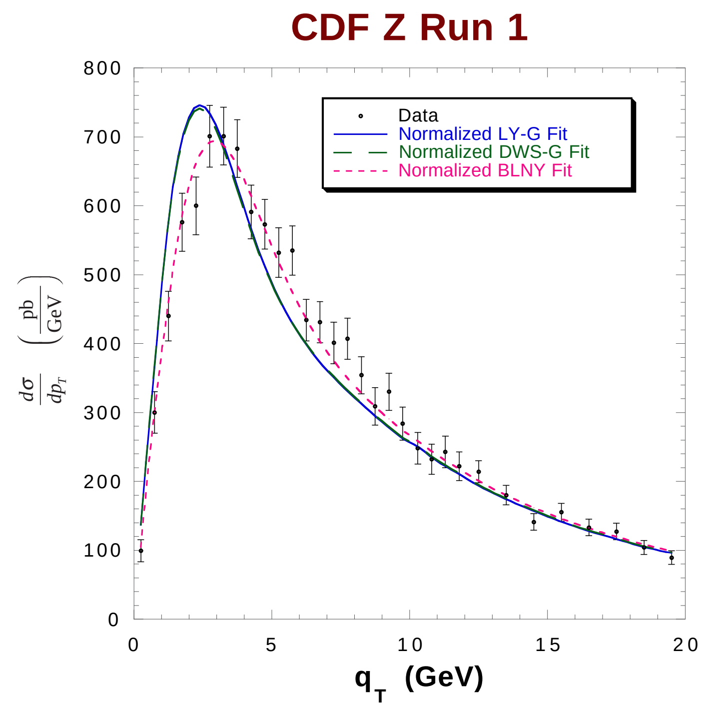
- CS(S)
- Ji, Ma Yuan
- CCFM
- . . .
- Cherednikov & Stefanis

Evolution

- CS: $\dfrac{\partial \tilde{f}_{f/P}(x, \boldsymbol{b}_\mathrm{T}; \zeta_A; \mu)}{\partial \ln \zeta_A}$, RG: $\dfrac{\partial \tilde{f}_{f/P}(x, \boldsymbol{b}_\mathrm{T}; \zeta_A; \mu)}{\partial \ln \mu}$

$\Longrightarrow$ Energy-dependent k_T distribution (DY):



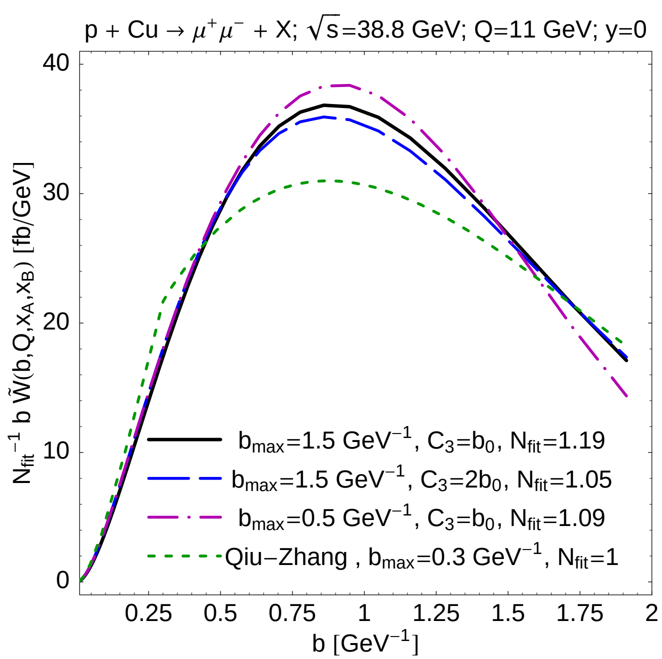


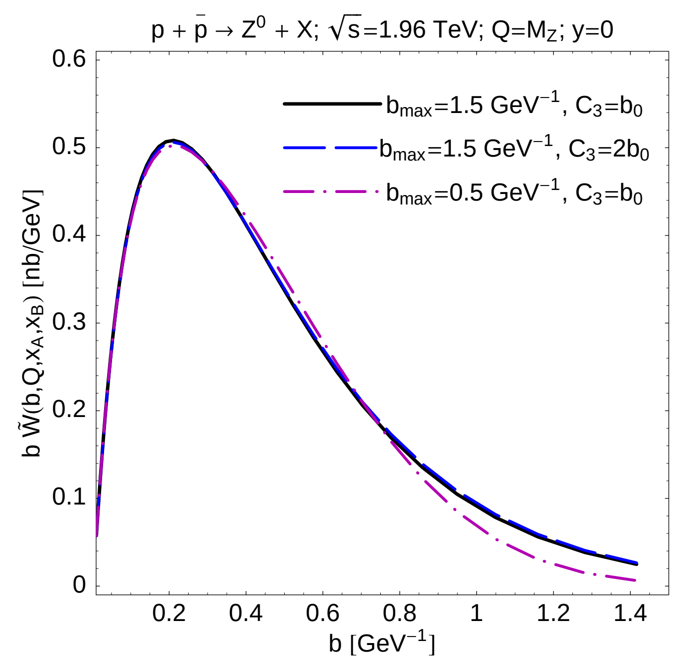
$\sqrt{s} = 27.4\,\mathrm{GeV}$ $\sqrt{s} = 1800\,\mathrm{GeV}$

(Landry, Brock, Nadolsky, & Yuan, 2003)

Non-perturbative information

- Formulate in transverse-coordinate space
- "Intrinsic transverse momentum": large b_T, energy-independent
- Gluon-radiation function in CS(S) evolution at large b_T
- Results of fits





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(Konychev & Nadolsky, 2006)

Polarization: terms in angular momentum decomposition

- Azimuth in $\boldsymbol{k}_\mathrm{T}$. Polarization of hadron. Polarization of parton.
- Parton densities:

		Azimuth	Hadron	Parton
f_1	Density	Uniform	Unpol.	Unpol.
h_1	Tr. dens.	Uniform	Tr. pol.	Tr. pol.
$h_1^\perp$	Boer-Mulders	$\sin\Delta\theta$	Unpol.	Tr. pol.
$f_{1T}^\perp$	Sivers	$\sin\Delta\theta$	Tr. pol.	Unpol.
$h_{1T}^\perp$	Pretzelocity	$\cos 2\Delta\theta$	Tr. pol.	Tr. pol.
g_1	Hel. dens.	Uniform	Hel.	Hel.
g_{1T}		$\sin\Delta\theta$	Hel.	Tr. pol.
$h_{1L}^\perp$		$\sin\Delta\theta$	Tr. pol.	Hel.

- Fragmentation functions: similarly

Universality

- Predictive power from factorization:

  Purely perturbative calculations (no logarithms)
  $+$ Universality of parton densities and fragmentation functions

- Modified by
  - Evolution
  - "T-odd" parton densities (Sivers, Boer-Mulders):
    - $*$ Vanish without Wilson lines
    - $*$ Reverse sign between (SI)DIS and Drell-Yan

$\Longrightarrow$ More predictions

$\Longrightarrow$ Including solution by Boer-Mulders of long-standing puzzle in DY:

Coeff. of $\sin^2\theta\cos 2\phi$ is $O(q_\mathsf{T}^2/M^2)$, not $O(q_\mathsf{T}^2/Q^2)$

Outlook on TMDs

- TMD functions give more information:
  - Detailed final-state distribution
  - More information on hadron wave function
  - Stress test of QCD dynamics
- Their use should be more widespread
- Important technicalities:
  - Must include evolution
  - Definition issues
- Non-factorization in hadron production in hadron-hadron collisions: interesting non-trivial dynamics