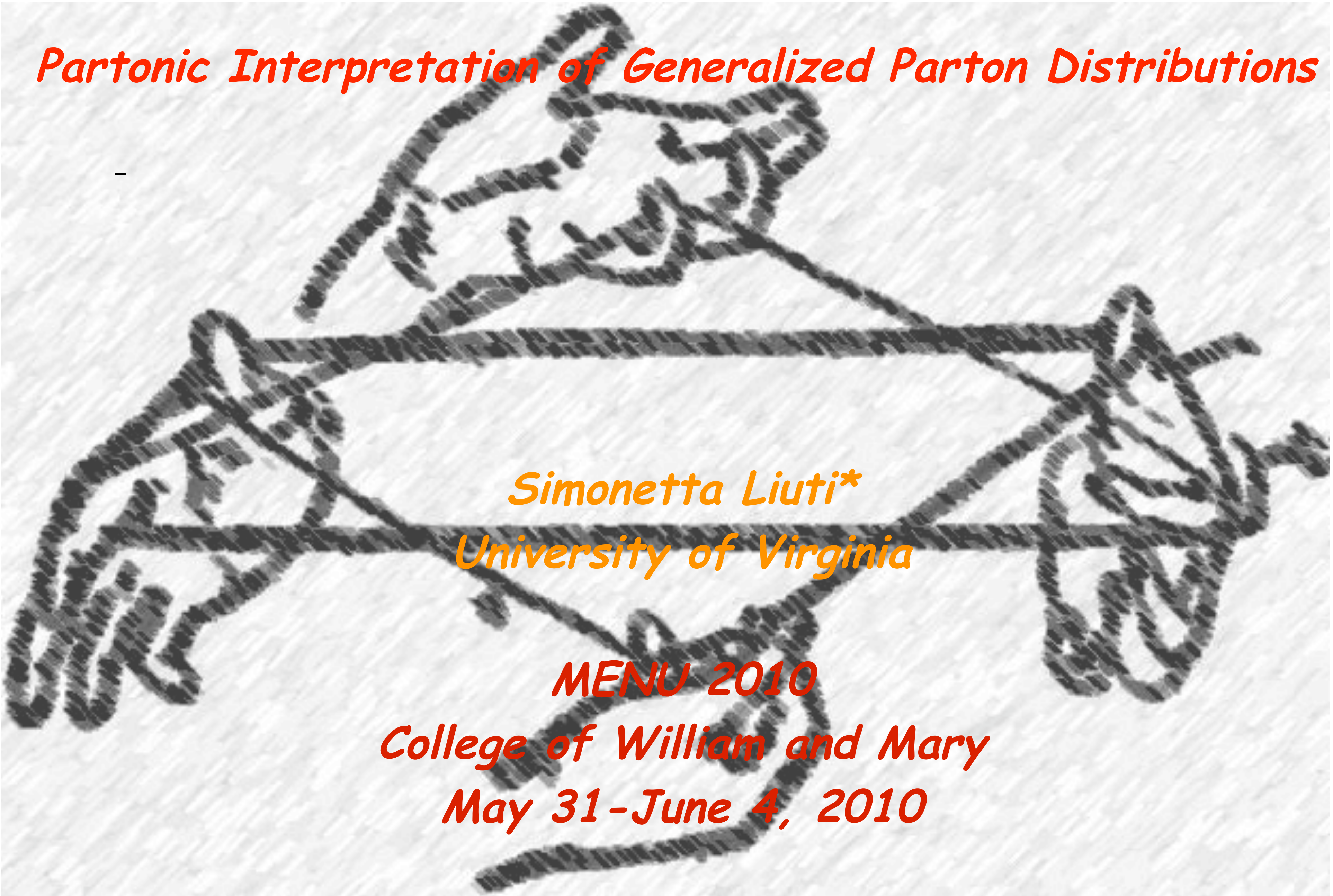

Partonic Interpretation of Generalized Parton Distributions

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MENU 2010
College of William and Mary
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*Work in collaboration with Gary Goldstein

Outline

✓ A new interpretation of the ERBL region based on the analytic properties of GPDs

G. Goldstein and S.L., arXiv:1006.0213 (2010)

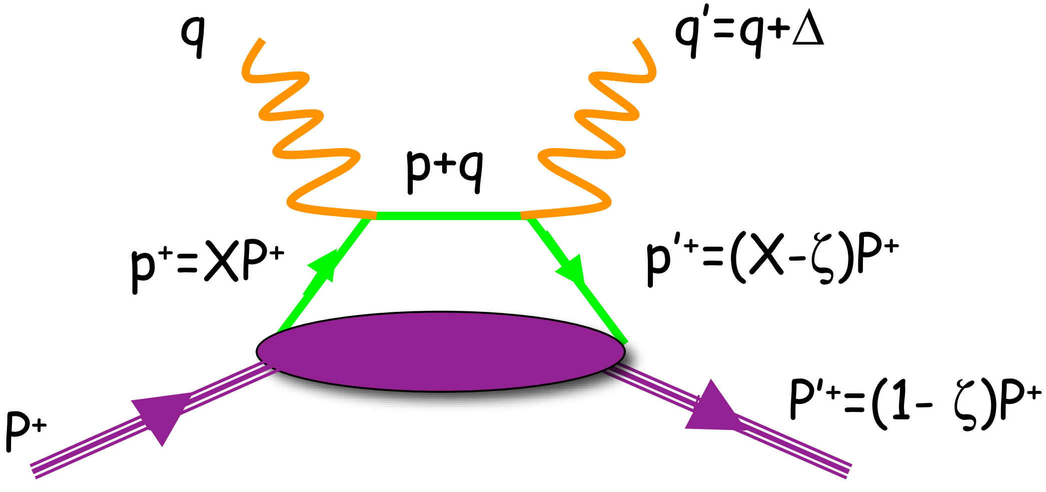
✓ Role and limitations of Dispersion Relations in GPD analyses

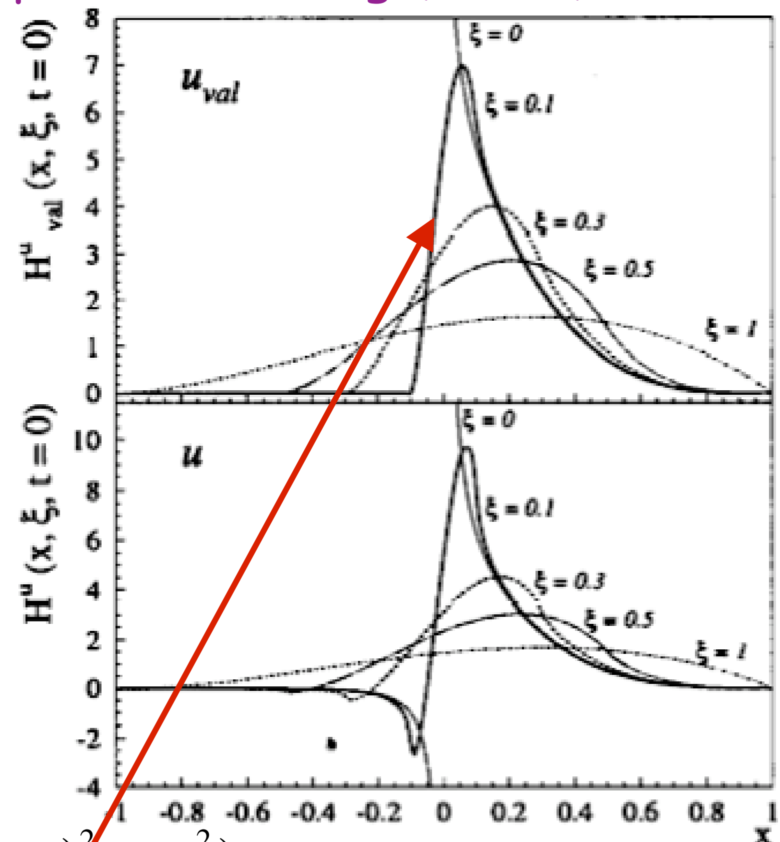
G. Goldstein and S.L., Phys. Rev.D (2009)

✓ Connection between GPDs and TMDs

S.L. and S.K. Taneja, Phys. Rev.D (2005) and in preparation

Off forward Parton Distributions (GPDs) are embedded in soft matrix elements for deeply virtual Compton scattering (DVCS)





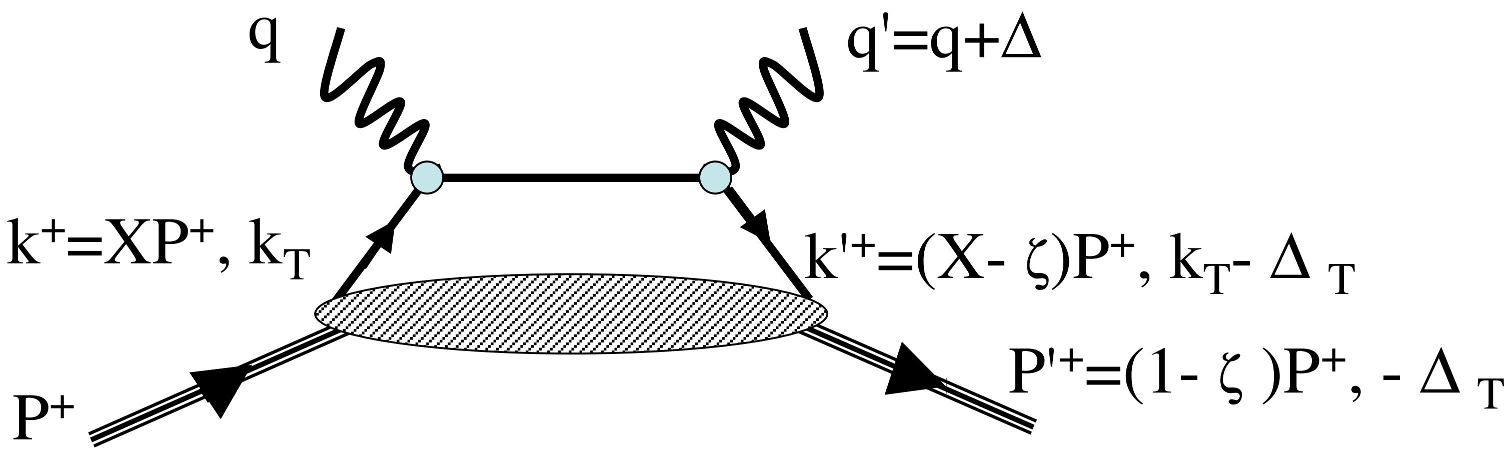
$$\frac{1}{(p+q)^2 - m^2 + i\varepsilon} = PV \frac{1}{(p+q)^2 - m^2} - i\pi\delta((p+q)^2 - m^2)$$

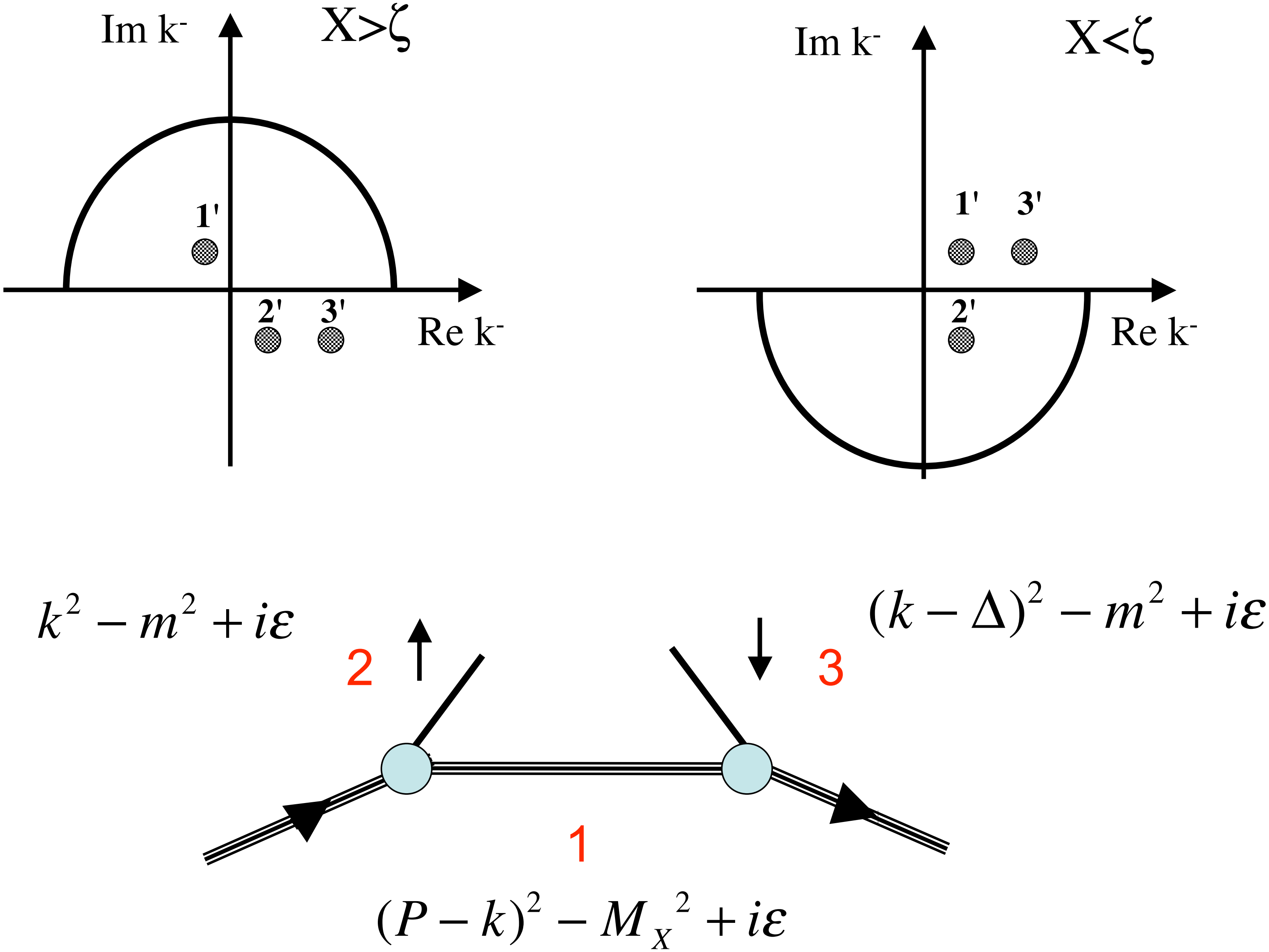
$$\rightarrow \frac{1}{-Q^2 + 2(pq) + i\varepsilon} \rightarrow \frac{1}{-Q^2/2(Pq) + (pq)/(Pq)} = \frac{1}{-\zeta + X}$$

Amplitude

$$\mathcal{F}_q = P.V.\int_{-1+\zeta} dX\, F_q(X,\zeta,t)\left[\frac{1}{\zeta - X} - \frac{1}{X}\right] + i\,\pi e_q^2 F_q(\zeta,\zeta,t)$$

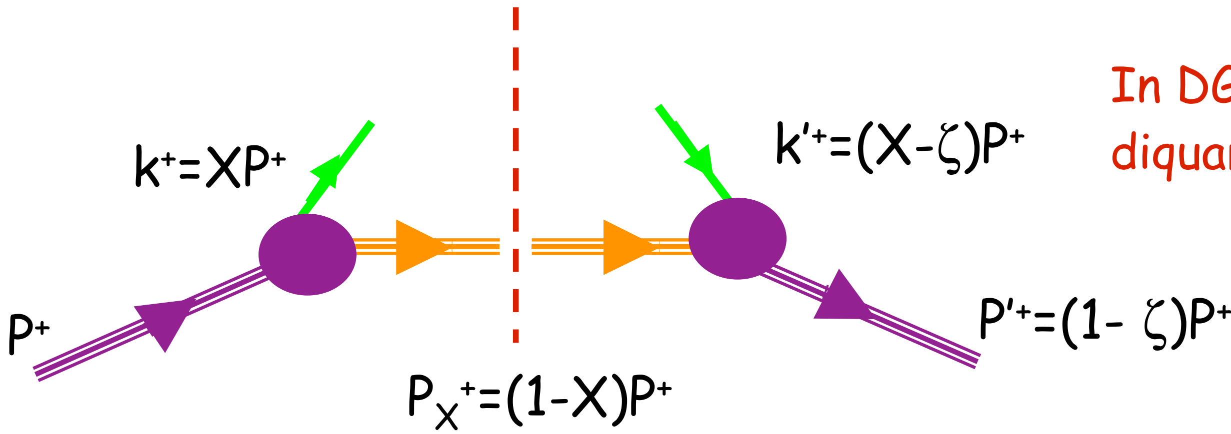
DVCS Kinematics



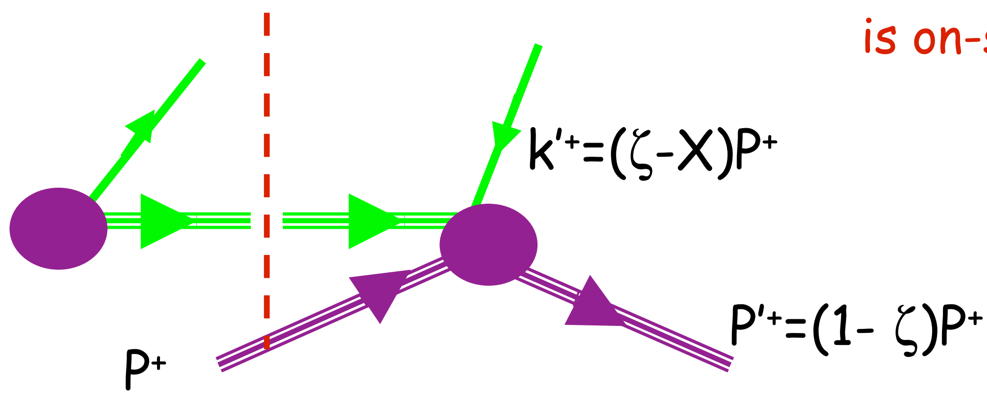


Im k-
X>ζ
1'
2'
3'
Re k-
Im k-
X<ζ
1'
3'
2'
Re k-
$k^2 - m^2 + i\varepsilon$
2
3
$(k-\Delta)^2 - m^2 + i\varepsilon$
1
$(P-k)^2 - M_X{}^2 + i\varepsilon$

$$H(X,\zeta,t) = \sum_n \langle P'|\psi^+|n\rangle\langle n|\psi|P\rangle\,\delta\left[(X-\zeta)P^+ + p_n^+ - P'^+\right]$$



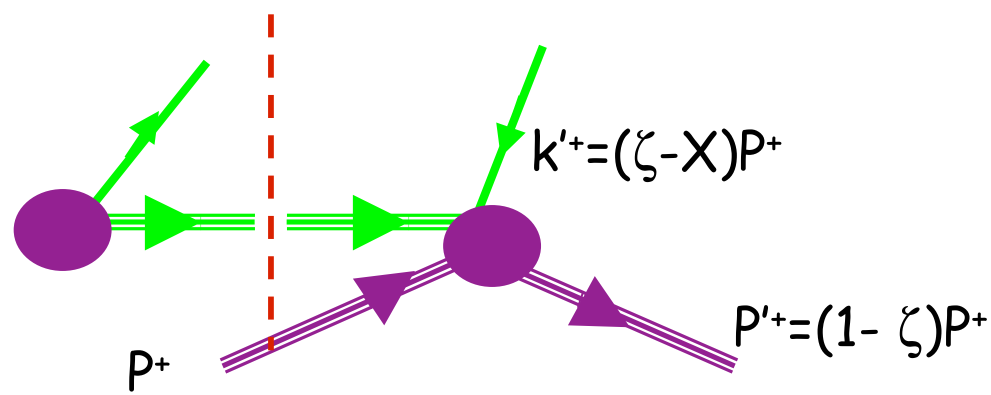
In DGLAP region spectator with diquark q. numbers is on-shell



In ERBL region struck quark, k, is on-shell

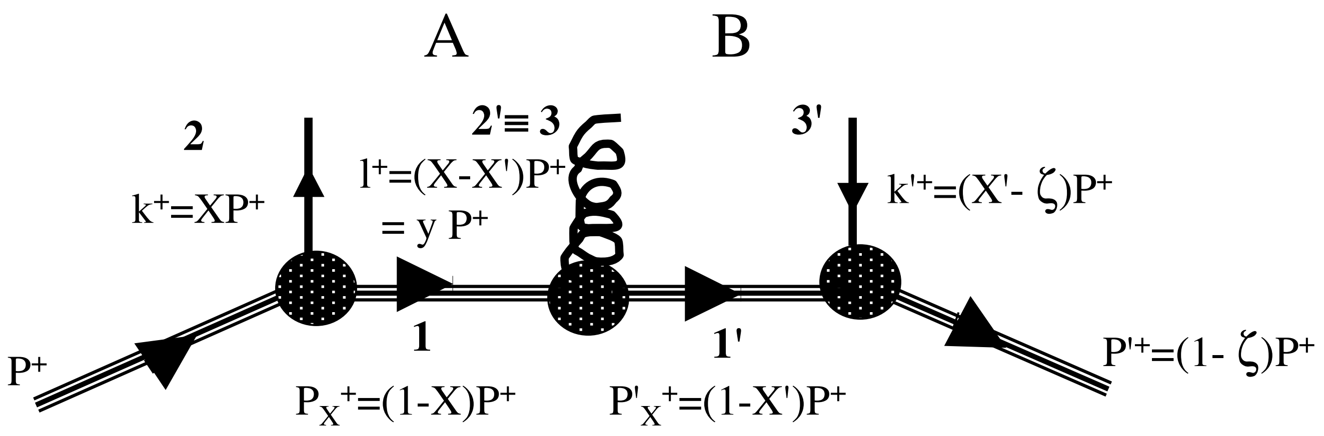
Analysis done for DIS/forward case by Jaffe NPB(1983)

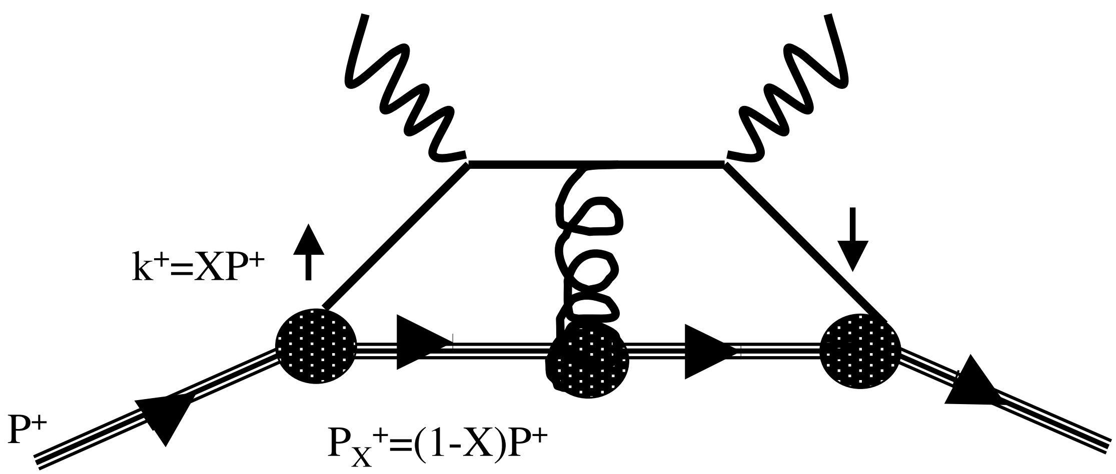
ERBL region corresponds to semi-disconnected diagrams:
no partonic interpretation



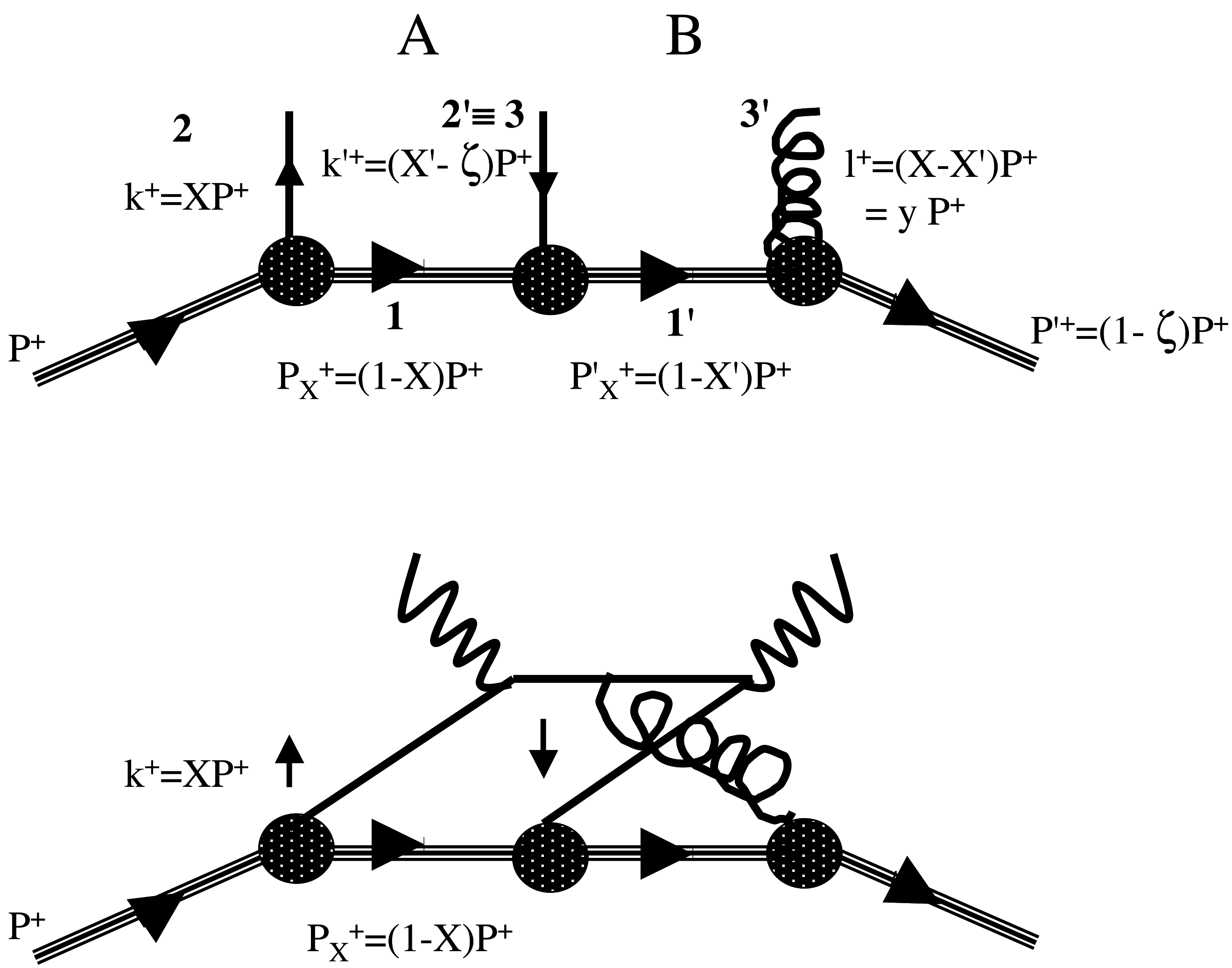
$$\langle P' \mid b^{\dagger}_{\lambda}((X-\zeta)\bar{P}^{+}, -\mathbf{k}_T + \mathbf{\Delta}_T)|n\rangle\langle n|\, d^{\dagger}_{-\lambda}(X\bar{P}^{+}, \mathbf{k}_T)|P\rangle$$
$$= \langle P' \mid b^{\dagger}_{\lambda}((X-\zeta)\bar{P}^{+}, -\mathbf{k}_T + \mathbf{\Delta}_T)|P, n\rangle\langle n|\, d^{\dagger}_{-\lambda}(X\bar{P}^{+}, \mathbf{k}_T)|0\rangle$$

In order to give a partonic interpretation we consider multiparton configurations ⇒ FSI





Planar



Non-Planar

Summary of part 1: GPDs in ERBL region can be described within QCD, consistently with factorization theorems,
only by multiparton configurations

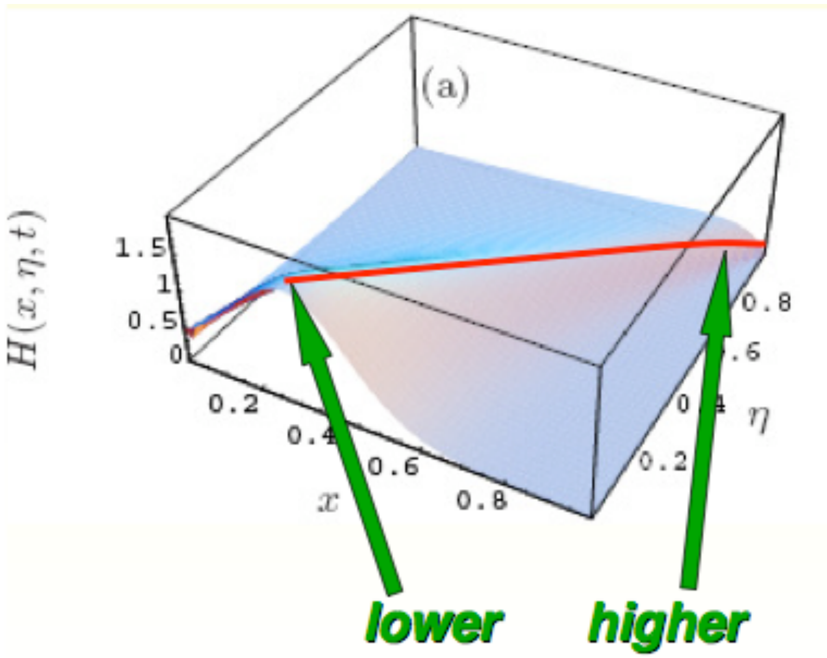
Dispersion Relations G.Goldstein and S.L.,PRD'09,arXiv:0905.4753 [hep-ph]

(Anikin, Teryaev, Diehl, Ivanov, Vanderhaeghen...)

$$\mathrm{Re}T(\nu,t,Q^2) = \frac{2\nu}{\pi}\int_{\nu_{threshold}}^{\infty} d\nu' \frac{\mathrm{Im}T(\nu',t,Q^2)}{\nu'^2-\nu^2},$$

$$\Downarrow \qquad \upsilon = \frac{Q^2}{2\xi}$$

$$\Im m(\mathcal{H}_f(\xi,t)) = H(\xi,\xi,t) \text{ and } \Re e(\mathcal{H}_f(\xi,t)) = \frac{1}{\pi}P.V.\int_{-1}^{+1} dx \frac{H(x,x,t)}{x-\xi}. + C(t)$$

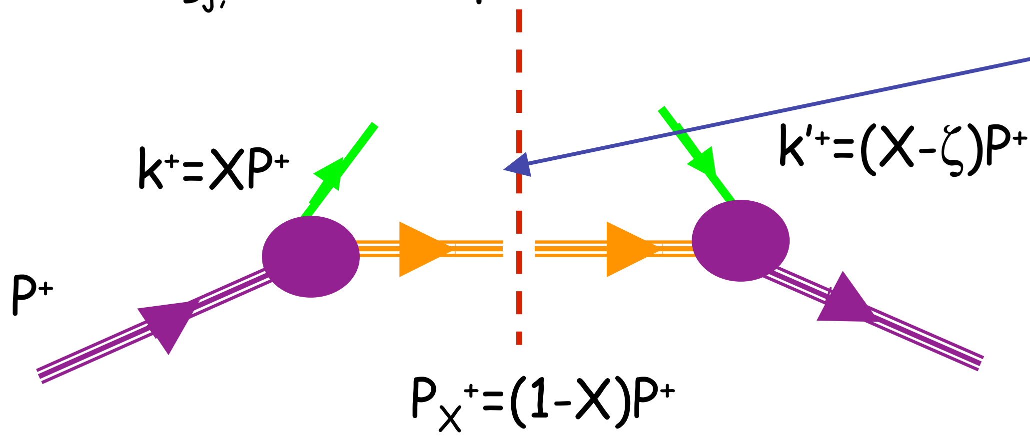


All information contained In the "ridge" x=ξ?

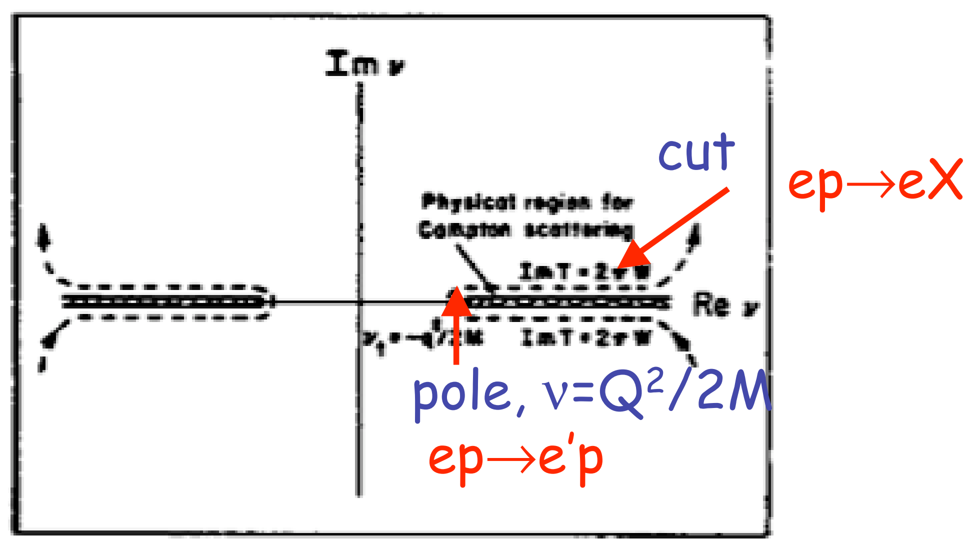
(graph from D. Müller)

Dispersion Theory

The amplitudes are <u>analytic</u> --in the chosen kinematical variables, ν, x_{Bj}, ,s -- except where the intermediate states are on shell



DIS



OPE is seeded in DRs (see e.g. Jaffe's SPIN Lectures)

It is convenient to change variables to $\omega \equiv -2M_T\nu/q^2$ in Eq. (1.16),

$\omega=1/x$

$$T(q^2,\omega) = 4\int_1^\infty \frac{\omega' d\omega'}{\omega'^2-\omega^2} W(q^2,\omega') \quad . \tag{1.18}$$

From DR + Optical Theorem

Note that $T(q^2,\omega)$ is analytic in the circle of radius 1 about $\omega = 0$ and may therefore be expanded in a Taylor series for $|\omega| < 1$:

$$T(q^2,\omega) = 4\sum_{\substack{n \\ \text{even}}} M^n(q^2)\omega^n \tag{1.19}$$

to Mellin moments expansion

with

$$\begin{aligned} M^n(q^2) &= \int_1^\infty d\omega'\, \omega'^{-n-1} W(q^2,\omega') \\ &= \int_0^1 dx\, x^{n-1} W(q^2,x) \quad . \end{aligned} \tag{1.20}$$

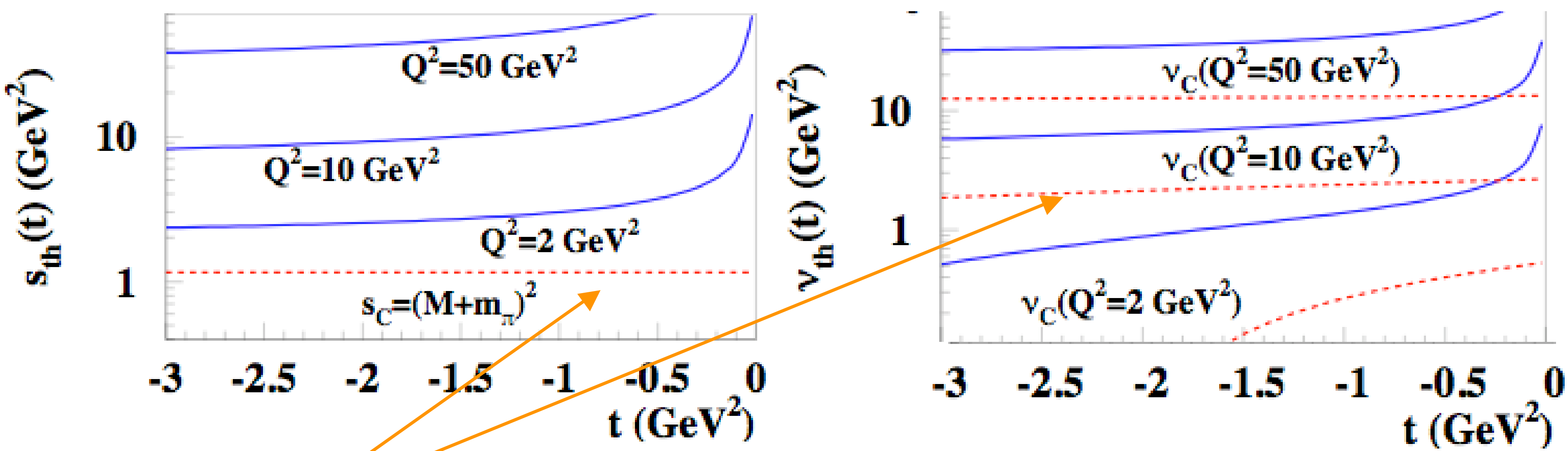
DVCS: Where is the threshold?

G.Goldstein and S.L., PRD'09

Because $t \neq 0$, the quark + spectator's kinematical "physical threshold" does not match the one required for the dispersion relations to be valid

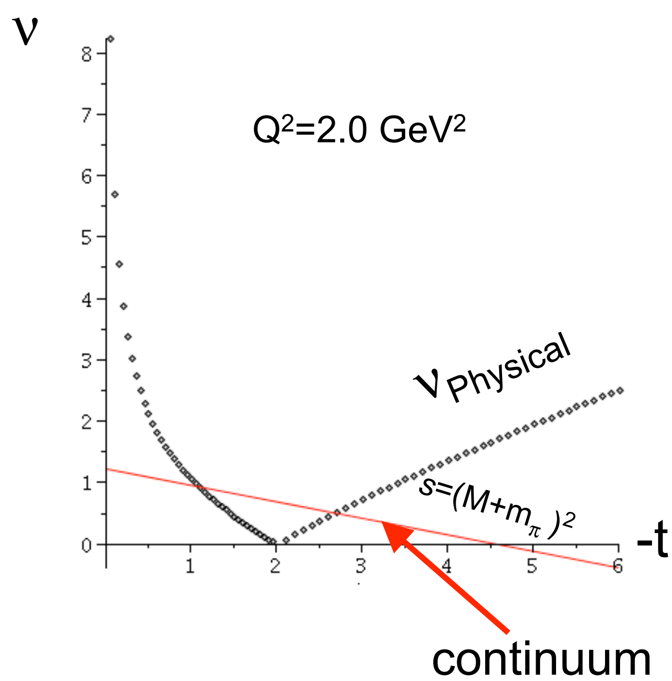
Continuum threshold

$$\nu = (s-u)/4M = (2s + t - 2M^2 + Q^2)/4M = \nu_{Lab} + \frac{t - Q^2}{4M} = \frac{Q^2(\frac{2}{x} - 1) + t}{4M}. \quad (28)$$



- Continuum starts at $s = (M+m_\pi)^2 \Rightarrow$ lowest hadronic threshold.

Physical threshold obtained by imposing $\Delta_T^2 > 0$ (same as $t_{min} = -M^2 \zeta^2/(1-\zeta)^2$)



▪How does one fill the gap? Analytic continuation?…problem for experimental extraction

- ✓ Dispersion relations cannot be directly applied to DVCS because one misses a fundamental hypothesis:

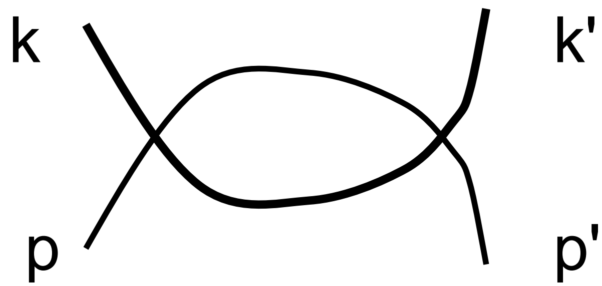
  "all intermediate states need to be summed over"

- ✓ For DVCS one is forced to look into the nature of intermediate states because there is no optical theorem

- ✓ This happens because "t" is not zero and there is a mismatch between the photons initial and final $Q^2 \Rightarrow$ t-dependent threshold cuts out physical states

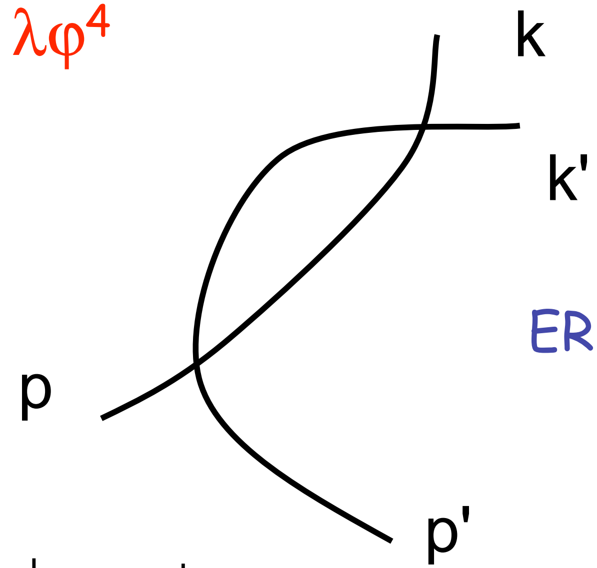
  "counter-intuitively as Q^2 increases the DRs start failing because the physical threshold is farther away from the continuum"

Connected with PARTON INTERPRETATION: Example in $\lambda\varphi^4$



DGLAP

$$|k\,p\rangle = |i\rangle \qquad |k'\,p'\rangle = |f\rangle$$

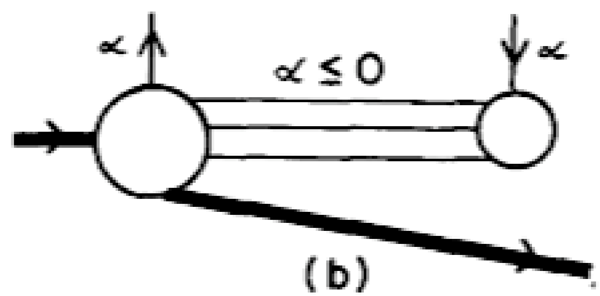


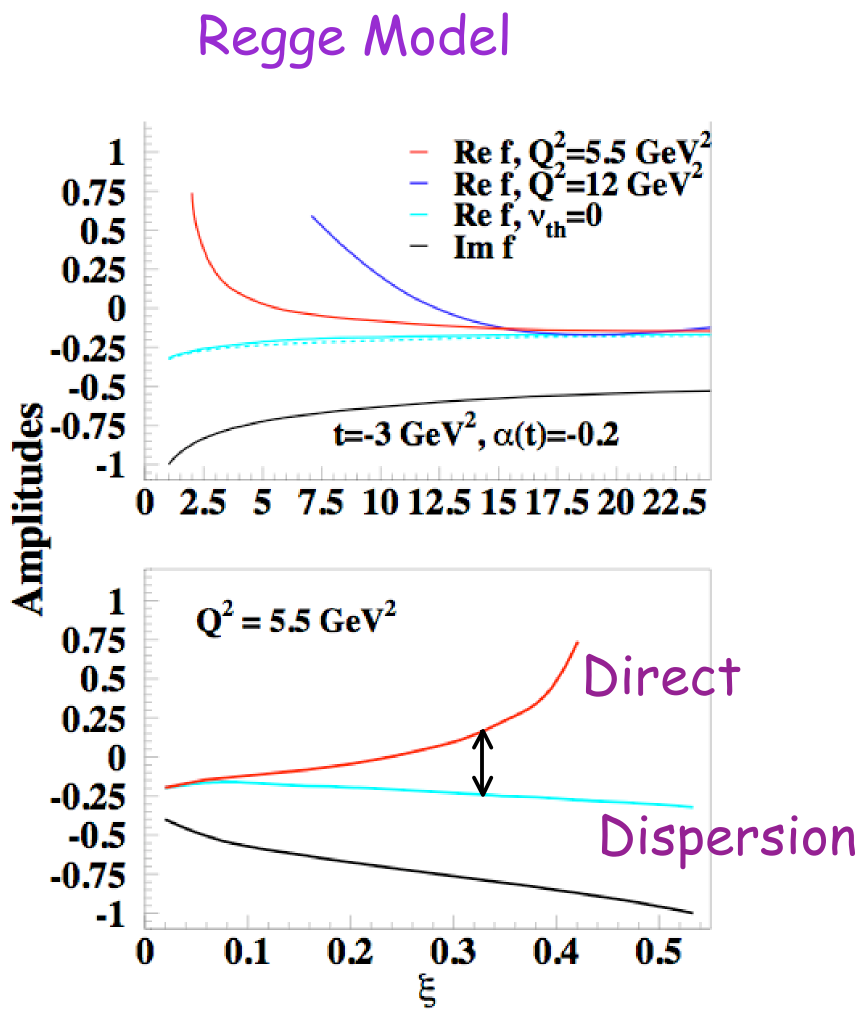
ERBL

$$|p\rangle = |i\rangle$$

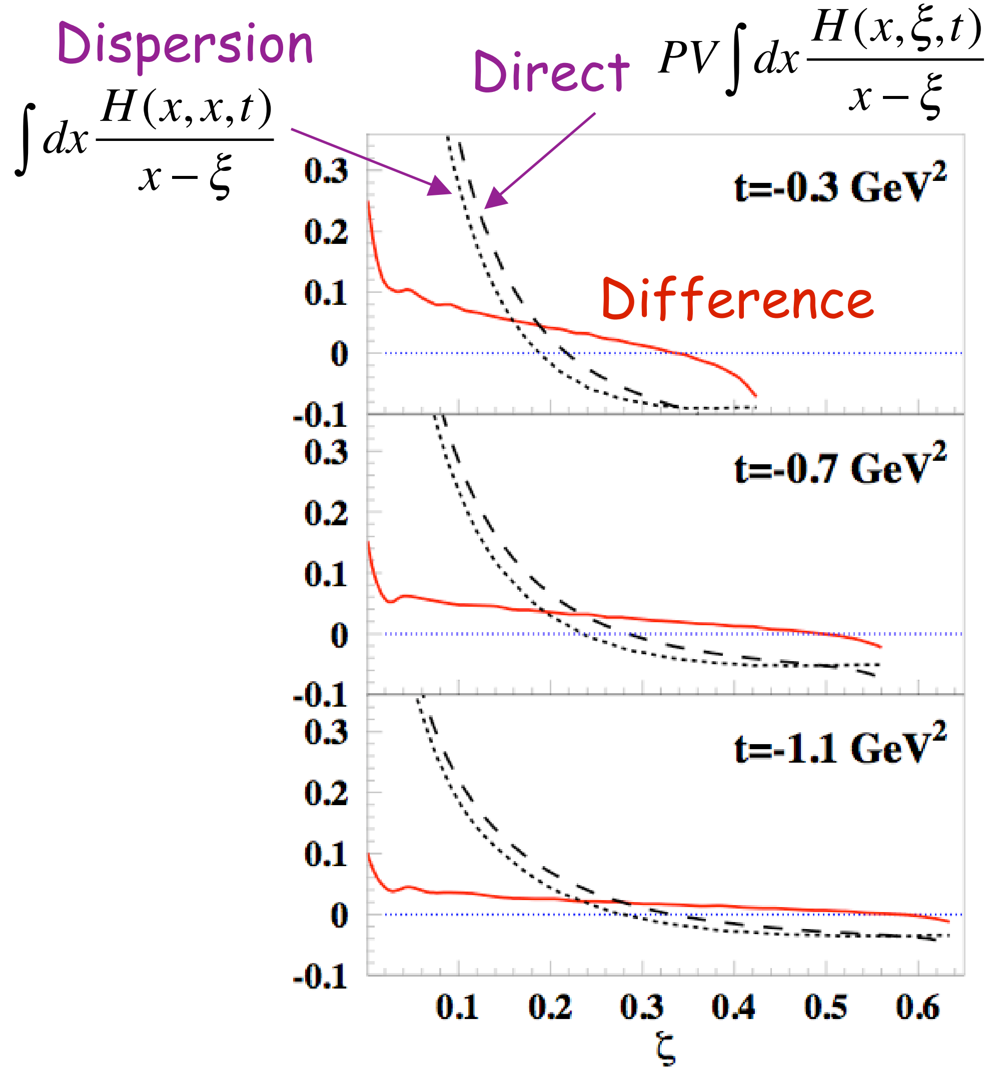
$$|k\,k'p'\rangle = |f\rangle$$

ERBL similar to contributions other than the parton model mentioned by Jaffe (1983)

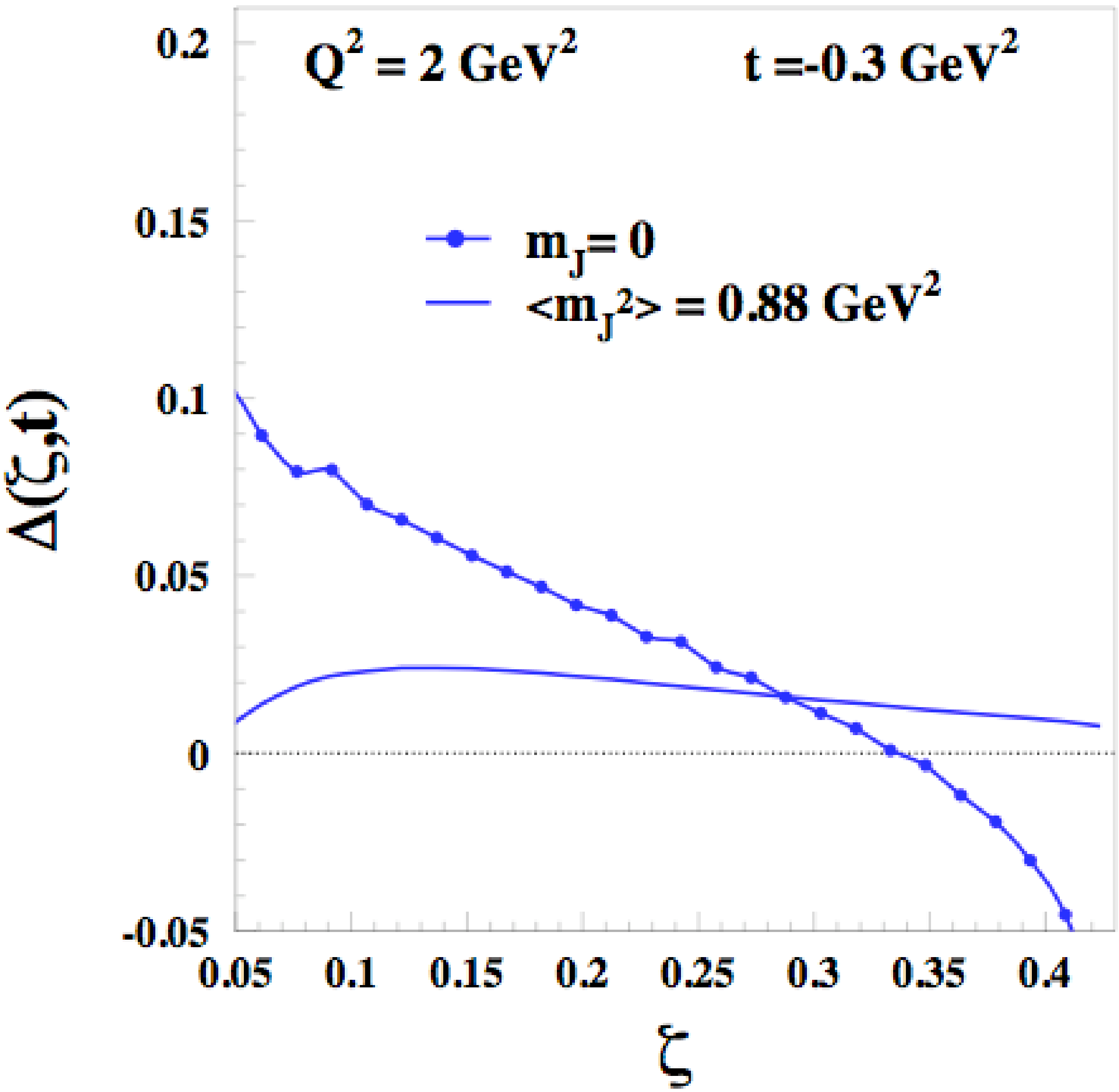




Regge Model
Re f, Q²=5.5 GeV²
Re f, Q²=12 GeV²
Re f, ν_th=0
Im f
t=-3 GeV², α(t)=-0.2
Amplitudes
Q² = 5.5 GeV²
Direct
Dispersion
ξ



Dispersion
$\int dx \frac{H(x,x,t)}{x-\xi}$
Direct
$PV\int dx \frac{H(x,\xi,t)}{x-\xi}$
t=-0.3 GeV²
Difference
t=-0.7 GeV²
t=-1.1 GeV²
ξ



Q^2 = 2 GeV^2
t =-0.3 GeV^2
m_J= 0
<m_J^2> = 0.88 GeV^2
Δ(ζ,t)
ζ
0.2
0.15
0.1
0.05
0
-0.05
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4

When deeply virtual processes involve directly final states
- like in exclusive or semi-inclusive processes - "standard kinematic approximations should be questioned"
(Collins, Rogers, Stasto, 2007, Accardi, Qiu, 2008)

(we write ζ but it is equivalent in ξ), so that

$$H(X,\zeta,t) \to \int dm_J^2 \rho(m_J^2) H(\zeta, \left(1+\frac{m_J^2}{Q^2}\right) \zeta, t)$$

$$H(\xi',\xi',t)/(\xi-\xi')$$

is not the same as

$$\Im m A(\nu',t)/(\nu-\nu')$$

H is calculated off the ridge

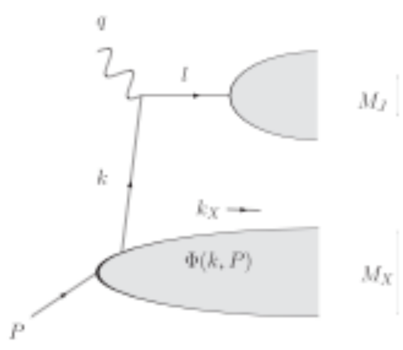
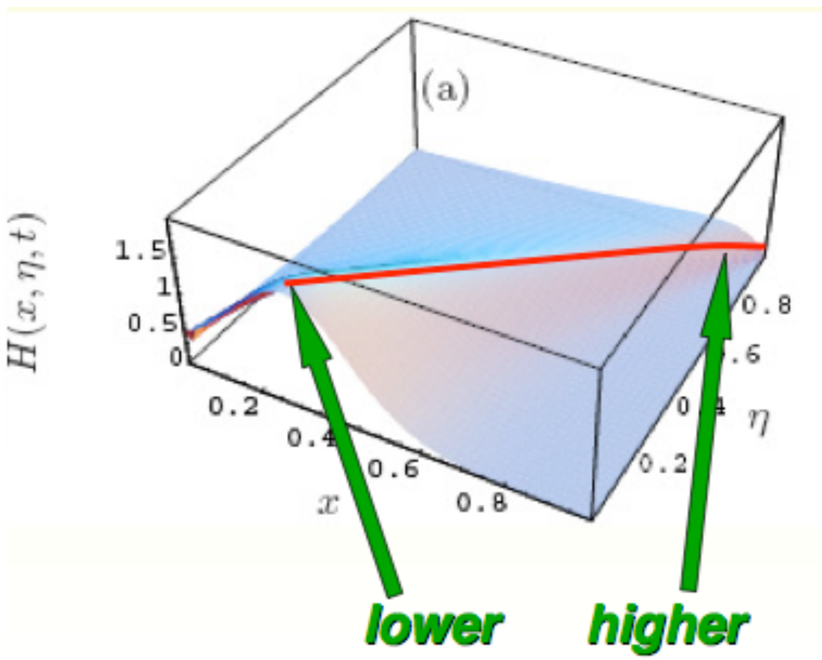


FIG. 2. The amplitude for $\gamma^* p$ scattering into two jets with fixed masses.

Summary of part 2: dispersion relations cannot be applied straighforwardly to DVCS.

The "ridge" does not seem to contain all the information



On the connection between TMDs and GPDs
Liuti and Taneja, PRD (2004) + in preparation

$$\nu W_2(x) = \sum_i e_i^2 x \int d^2\mathbf{k}\ f(x,\mathbf{k}) \quad (11a)$$

$$F(\mathbf{\Delta}) = \sum_i e_i \int d^2\mathbf{k} \int_0^1 dx\, f(x,\mathbf{k},\mathbf{k}+(1-x)\mathbf{\Delta}), \quad (11b)$$

where:

$$f(x,\mathbf{k},\mathbf{k}') = \phi^*(x,\mathbf{k})\phi(x,\mathbf{k}'), \quad (12)$$

with $\mathbf{k}' \equiv \mathbf{k} + (1-x)\mathbf{\Delta}$, is a non-diagonal intrinsic momentum distribution. The diagonal term can be written as:

$$f(x,\mathbf{k}) = \mid \phi(x,\mathbf{k}) \mid^2 . \quad (13)$$

By comparing Eqs.(11a, 11b), with Eqs.(3a, 3b), we find that the relation between the transverse momentum and the transverse separation of quarks inside a hadron is obtained through a non-diagonal distribution in transverse coordinate space, $q(x,\mathbf{b},\mathbf{b}')$:

$$f(x,\mathbf{k}) = \int d^2\mathbf{b} \int d^2\mathbf{b}'\, e^{i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\, q(x,\mathbf{b},\mathbf{b}'), \quad (14)$$

where we define:

$$q(x,\mathbf{b},\mathbf{b}') = \Psi^*(x,\mathbf{b}')\Psi(x,\mathbf{b}), \quad (15a)$$

$$q(x,\mathbf{b},\mathbf{b}) = \mid \Psi(x,\mathbf{b}) \mid^2 \equiv q(x,\mathbf{b}), \quad (15b)$$

Start from localized state in transverse direction
(Soper, '77, Burkardt, 2002)

$$|P^+, R_\perp = 0_\perp, \lambda > \equiv N \int \frac{d^2 P_\perp}{(2\pi)^2} |P^+, P_\perp, \lambda >$$

Define "non-diagonal" impact parameter pdf

$$q(x, \mathbf{b}, \mathbf{b}') = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} < P^+, R_\perp = 0_\perp, \lambda | \overline{\psi}(0^-, \mathbf{b}')\gamma^+\psi(y^-, \mathbf{b}) | P^+, R_\perp = 0_\perp, \lambda > |_{y^+=0}$$

non-diagonal O(x,b,b')

Observe that

$$\psi_+(y^-, \mathbf{b}) = \int \frac{dk^+}{\sqrt{4\pi k^+}} \int \frac{d^2\mathbf{k}}{2\pi} \Sigma_s [u_+(k,s) b_s(k^+, k) e^{-iky} + v_+(k,s) d_s^\dagger(k^+, k_\mathbf{k}) e^{iky}]$$

$$f(x, \mathbf{k}) = N' \Sigma_s \langle P^+, R_\perp = 0_\perp, \lambda | b_s^\dagger(xP^+, \mathbf{k}) b_s(xP^+, \mathbf{k}) | P^+, R_\perp = 0_\perp, \lambda \rangle$$

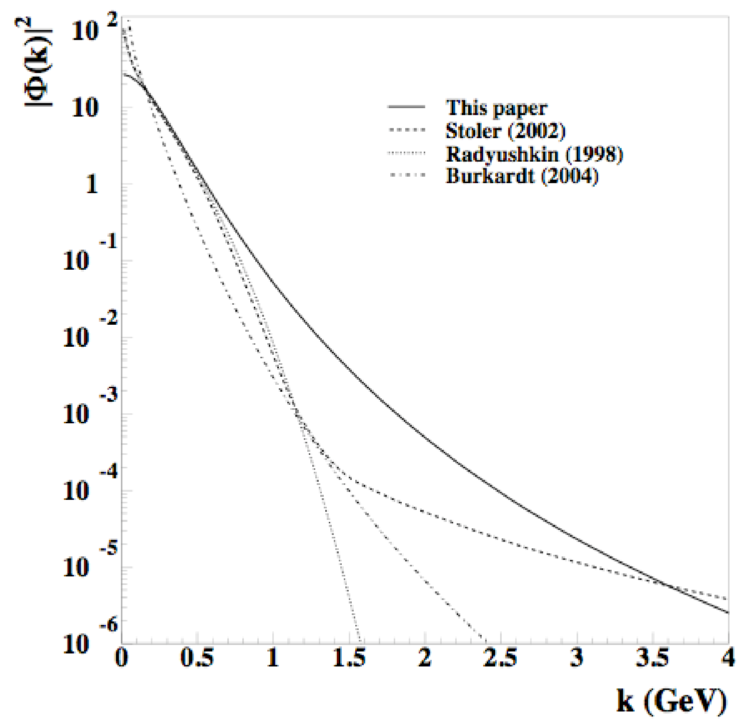
Calculable on lattice?
What measurements?
Connection with Wigner?

$$q(x, \mathbf{b}, \mathbf{b}') = \int \frac{d^2\mathbf{k}}{2\pi} e^{-i(\mathbf{b}'-\mathbf{b})\cdot\mathbf{k}} f(x, \mathbf{k})$$

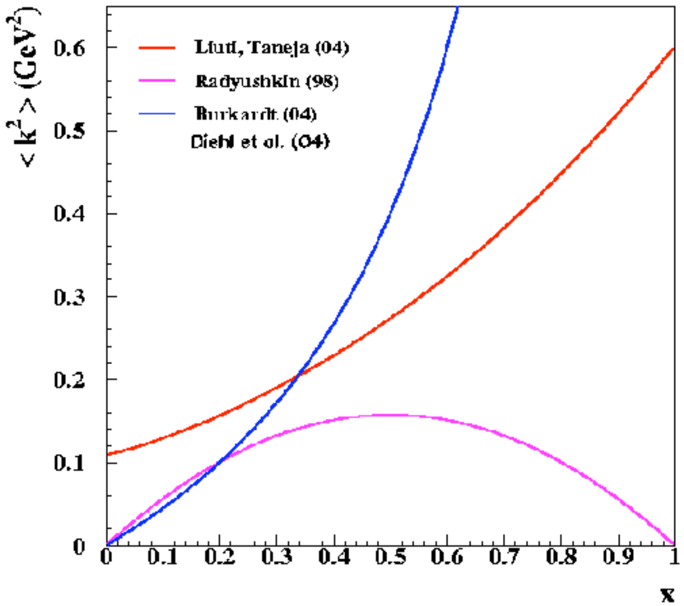
GPD $$H(x,\mathbf{\Delta}) = \int d^2\mathbf{k}\, \phi^*(x,\mathbf{k})\phi(x,\mathbf{k}+(1-x)\mathbf{\Delta})$$

PDF $$\nu W_2(x) = \int d^2\mathbf{k}\; |\; \phi(x,\mathbf{k})\; |^2$$

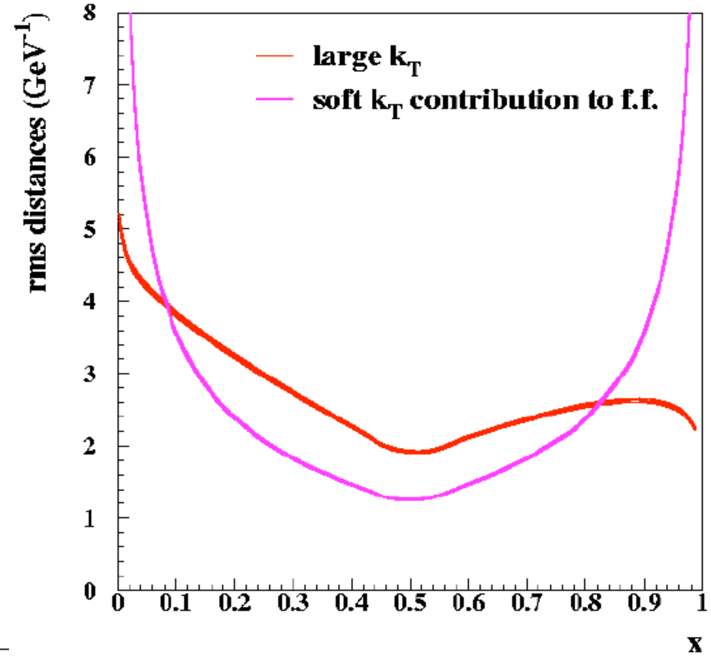
Form Fac. $$F_1(\mathbf{\Delta}^2) = \int d^2\mathbf{k} \int_0^1 dx\, \phi^*(x,\mathbf{k})\phi(x,\mathbf{k}+(1-x)\mathbf{\Delta}),$$



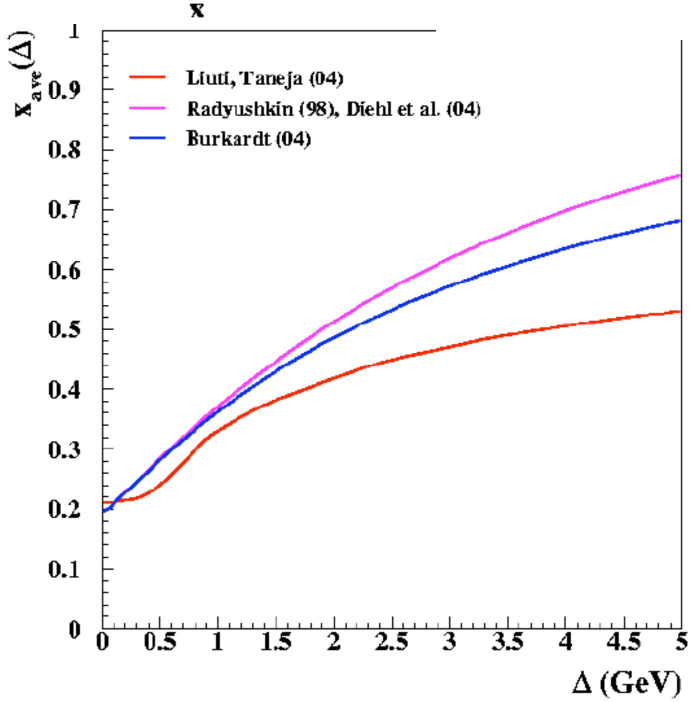
$\langle k_T^2 \rangle$ vs. x



$\langle r^2 \rangle = \langle b^2 \rangle/(1-x)$ vs. x *(M. Burkardt)*



$\langle x \rangle$ vs. Δ



$$R^I_{1(2)} = X^{-\alpha^I - \beta^I_{1(2)}(1-X)^{p^I_{1(2)}} t}$$

Necessary element in GPD parametrizations

S.L. and S.K. Taneja, (2004)

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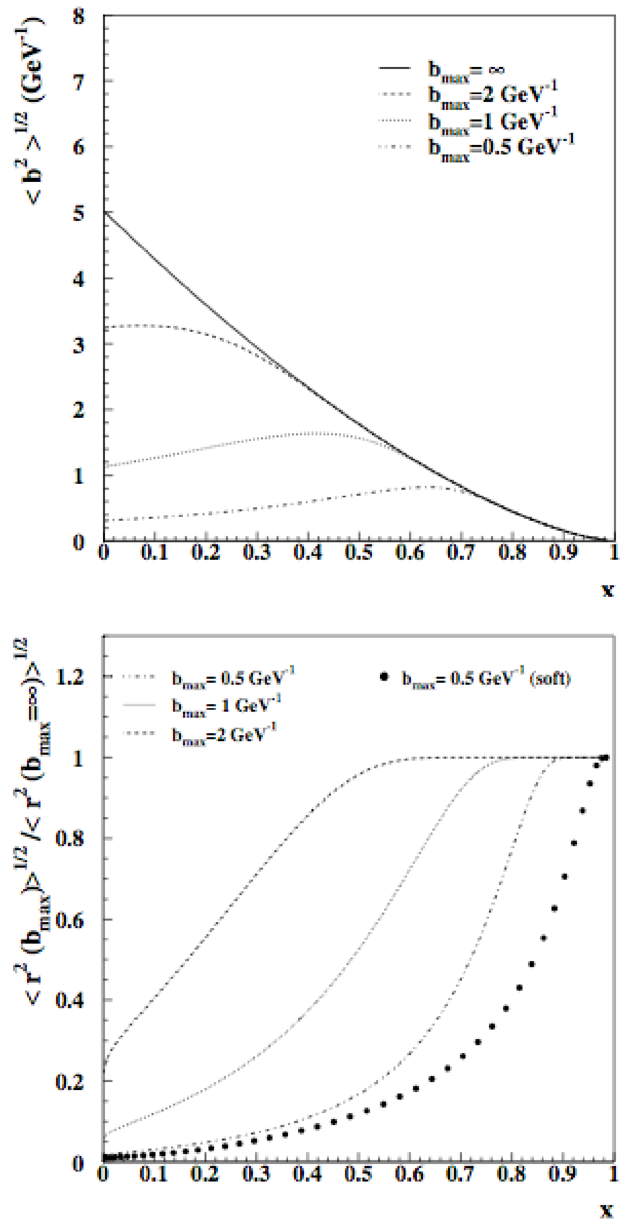
$\sqrt{\langle b^2 \rangle}$ vs. x
correlation



Conclusions

✓ We uncovered a non-trivial partonic interpretation of GPDs
FSI important → underlying connection with TMDs

✓Dispersion relations are not directly applicable: all information is not on the “ridge”. All measurement (real and imaginary parts) are important.

✓ Connection between GPDs and TMDs embedded in k_T dependent quantities (SL and Taneja, 2004)

No damn cat, and no damn cradle..

K. Vonnegut
"Cat's Cradle"