Scale dependence of twist-three contributions to single spin asymmetries

V. Braun, A. Manashov, B. Pirnay

University of Regensburg

ECT, Trento, June 22, 2010

The twist-3 correlation functions are defined as matrix elements of nonlocal operators, e.g.

$$\langle P, s_T|\tilde{s}^\mu\, T_\mu(z_1, z_2, z_3)|P, s_T\rangle = 2P_+^2 \int \mathcal{D}x\; e^{-iP_+(\sum_k x_k z_k)}\; T_{\bar{q}Fq}(x_1, x_2, x_3)\,,$$

$s^2 = -1$, $\tilde{s}^\mu = -\epsilon^{\mu\nu\rho\sigma} s_\nu n_\rho \tilde{n}_\sigma$, $n, \tilde{n}$-light-like vectors, $n\tilde{n} = 1$

$$\int \mathcal{D}x = \int_{-1}^{1} dx_1\, dx_2\, dx_3\, \delta(x_1 + x_2 + x_3)$$

$$T_\mu(z_1, z_2, z_3) = g\bar{q}(z_1 n)\gamma_+ F_{\mu+}(z_2 n) q(z_3 n)\,,$$

The same operator can contribute to different observables, e.g. T_μ contributes to

- structure function $g_2(x)$ $\quad (g_2(x) \sim \int D\xi\, D_x(\vec{\xi})\, T_{\bar{q}Fq}(\vec{\xi}))$

  Shuryak, Vainshtein, 82, Bukhvostov, Kuraev, Lipatov, 83

- Single Spin Asymmetry $\quad (\sim T_{\bar{q}Fq}(-x, 0, x))$

  Efremov, Teryaev, 85, Qiu, Sterman, 91

All relevant correlation functions can be expressed in terms of four operators $\mathfrak{S}^{\pm}, \mathcal{F}^{\pm}$.

$$\mathcal{F}^{\pm}(z) = 2gC_{\pm}^{abc}\tilde{s}^{\rho}\,(1 \mp P_{23} \pm P_{12})F_{+}{}^{\nu,a}(z_1)F^{b}_{+\rho}(z_2)F^{c}_{+\nu}(z_3)\,,$$

$$\mathfrak{S}^{\pm}(z) = \tilde{s}^{\rho}(S^{+}_{\rho}(z_1,z_2,z_3) \pm S^{-}_{\rho}(z_3,z_2,z_1))\,,$$

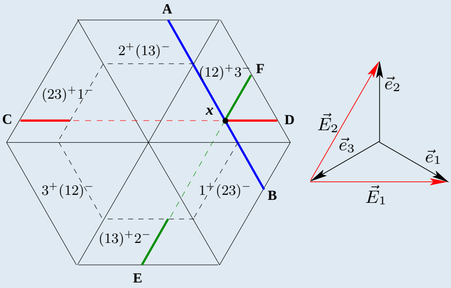
$$S^{\pm}_{\rho}(z) = g\bar{q}(z_1)\Big[F_{\rho+}(z_2) \pm i\gamma_5\widetilde{F}_{\rho+}(z_2)\Big]\gamma_{+}q(z_3)\,.$$

$\mathfrak{S}^{+}, \mathcal{F}^{+}(\mathfrak{S}^{-}, \mathcal{F}^{-})$ are even (odd) under charge conjugation.

$$T_{\bar{q}Fq}(x) = \frac{1}{4}\Big[(1+P_{13})\mathfrak{S}^{+}(x) + (1-P_{13})\mathfrak{S}^{-}(x)\Big]$$

Support properties

$$-1 < x_i < 1 \qquad x_1 + x_2 + x_3 = 0$$



$$\vec{x} = x_1\,\vec{e}_1 + x_2\,\vec{e}_2 + x_3\,\vec{e}_3 = x_1\,\vec{E}_1 + x_2\,\vec{E}_2\,.$$

$$(12)^+3^- \Leftrightarrow x_1, x_2 > 0 \ (x_3 < 0)$$

One-loop evolution.

Evolution operator has a two-particle structure. Schematically

$$\mu\frac{d}{\mu}T_{\bar{q}Fq} \sim \frac{\alpha_s}{2\pi}[H_{12} + H_{23} + H_{31}] \otimes T_{\bar{q}Fq}$$

All two particle kernels are known
Bukhvostov, Frolov, Kuraev, Lipatov,85

(and can be also found in
Braun, A.M, Rohrwild, Nucl.Phys, 09)

$$\mathcal{T}_{q,F}(x,x') = T_{\bar{q}Fq}(-x', x'-x, x) \qquad \Delta\mathcal{T}_{q,F}(x,x') = \Delta T_{\bar{q}Fq}(-x', x'-x, x)$$

$$\Delta T_\mu(z_1,z_2,z_3) = g\bar{q}(z_1 n)\gamma_+\gamma_5 iF_{\mu+}(z_2 n)q(z_3 n)$$

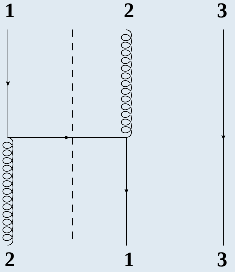
Braun, A.M., Pirnay, Phys.Rev. D80, 09

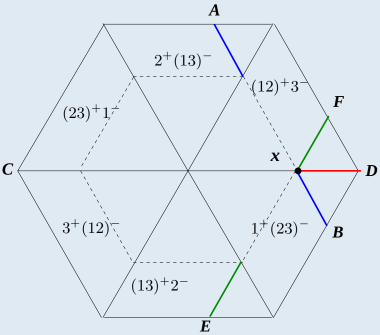
$$\mu\frac{d}{d\mu}\mathcal{T}_{q,F}(x,x) = \frac{\alpha_s}{\pi}\Bigg\{\int_x^1 \frac{d\xi}{\xi}\Bigg[P_{qq}(z)\mathcal{T}_{q,F}(\xi,\xi) + \frac{N_c}{2}\left(\frac{(1+z)\mathcal{T}_{q,F}(x,\xi)-(1+z^2)\mathcal{T}_{q,F}(\xi,\xi)}{1-z} - \mathcal{T}_{\Delta q,F}(x,\xi)\right)\Bigg] - N_c\mathcal{T}_{q,F}(x,x) + \frac{1}{2N_c}\int_x^1\frac{d\xi}{\xi}\Big[(1-2z)\mathcal{T}_{q,F}(x,x-\xi) - \mathcal{T}_{\Delta q,F}(x,x-\xi)\Big]\Bigg\}$$

The two last terms are missing in Kang, Qui, 09, Vogelsang, Yuan, 09

Second term:

Disagreement due to exchange diagram:





Large N_c−limit:

$$\mu\frac{d}{d\mu}\mathcal{T}_{q,F}(x,x) = \frac{\alpha_s N_c}{2\pi}\left\{-\mathcal{T}_{q,F}(x,x) + \int_x^1 \frac{d\xi}{\xi}\left[\left(\bar{P}_{qq}(z)+z\right)\mathcal{T}_{q,F}(x,\xi) - \mathcal{T}_{\Delta q,F}(x,\xi)\right]\right\},$$

First term:

Large $x-$limit

$$\mu \frac{d}{d\mu} \mathcal{T}_{q,F}(x,x) = \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} P_{q,F}^{NS,z\to 1}(z) \mathcal{T}_{q,F}(\xi,\xi)\,,$$

Single Spin Asymmetry

$$P_{q,F}^{NS,z\to 1}(z) = 2C_F \left[\frac{1}{(1-z)_+} + \frac{3}{4}\delta(1-z) \right] - N_c \delta(1-z)\,.$$

Leading twist structure function $F_1(x, Q^2)$

$$P_{qq}^{NS,z\to 1}(z) = 2C_F \left[\frac{1}{(1-z)_+} + \frac{3}{4}\delta(1-z) \right],$$

Polarized structure function $g_2(x, Q^2)$

$$P_{g_2}^{NS,z\to 1}(z) = 2C_F \left[\frac{1}{(1-z)_+} + \frac{3}{4}\delta(1-z) \right] - \frac{N_c}{2} \delta(1-z)\,.$$

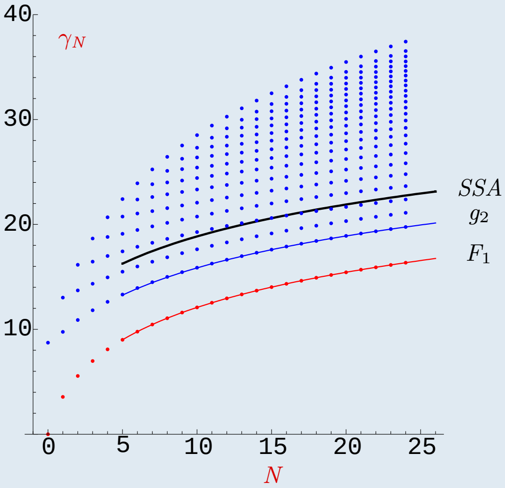
Different $\sim \delta(1-z)$ terms result in suppression of the twist three functions

$$\mathcal{T}_{q,F}(x,x;Q^2)/F_1(x,Q^2) \sim \left(\frac{\alpha_s(Q)}{\alpha_s(\mu_0)}\right)^{2N_c/b_0},$$

$$g_2^{tw.-3}(x,Q^2)/F_1(x,Q^2) \sim \left(\frac{\alpha_s(Q)}{\alpha_s(\mu_0)}\right)^{N_c/b_0}.$$

Spectrum of anomalous dimensions of quark-gluon operators (g_2 function) and twist-2 operators (F_1−function)

$$\gamma_N \sim \log N + c + O(1/N)$$



Summary

- There are six independent three-particle correlation functions: two nonsinglet and four singlet.
- The evolution equations are obtained for both cases.
- One needs concrete models for correlation functions.