TMD factorization breaking

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(Recent work with Piet Mulders)

- Standard PDFs
- TMD PDFs in DIS.
- TMD factorization breaking in pp ➡ hadrons.

TMD PDFs, Trento - June, 22 2010

Standard (Integrated) PDF

- Operator definition:

$$f(x) = \text{F.T.}\, \langle p|\, \bar{\psi}(0, w^-, \mathbf{0}_t) \underline{V_w^\dagger(u_\text{J})} \gamma^+ \underline{V_0(u_\text{J})} \psi(0) |p\rangle$$

$$u_\text{J} = (0, 1, \mathbf{0}_t)$$

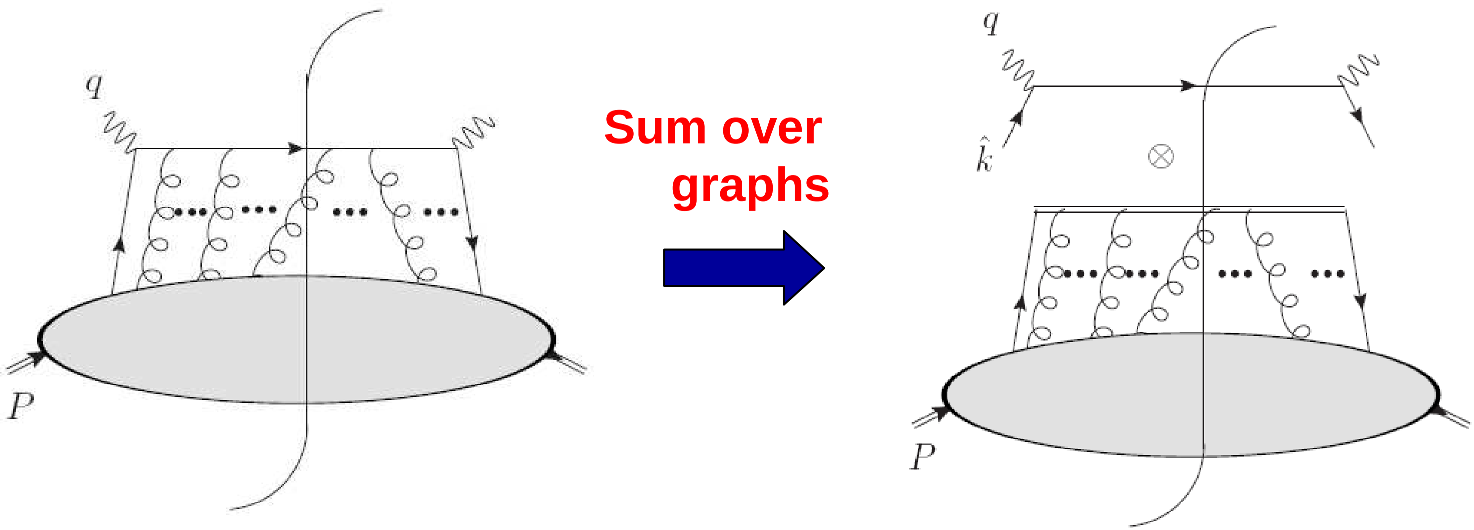
(Ex: DIS)

- Wilson lines enforce gauge invariance.

$$\underline{V_w(n)} = P \exp\left(-igt^a \int_0^\infty d\lambda\, n \cdot A^a(w + \lambda n) \right)$$

$$V_w^\dagger(u_J)\, V_0(n) = P \exp\left(-igt^a \int_0^{w^-} d\lambda\, u_J \cdot A^a(\lambda u_J) \right)$$

Standard (Integrated) PDF

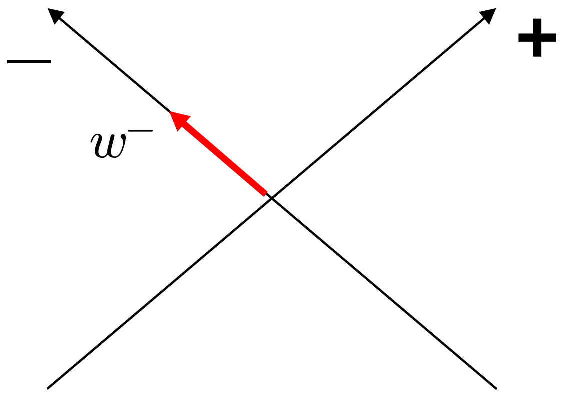


- Eikonal factors, extra Feynman rules.

$$\frac{i}{-l^{+} + i\epsilon} \qquad -igt^{a}u_{J}^{\mu}$$

Standard PDFs: Wilson Lines

- Paths of Wilson lines in coordinate space:



Standard (Integrated)

TMD PDFs:

- Wilson line is ideally determined by requirements of factorization.

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- Extend standard definition *(first try)*:

Link at infinity

$$\Phi^{[+]}(x, \mathbf{k}_t) = \text{ F.T. } \langle p| \, \bar{\psi}(0, w^-, \mathbf{w}_t) \underbrace{\underline{V_w^\dagger(u_J)} \, \underline{\underline{I_{n;w,0}}} \gamma^+ \underline{V_0(u_J)}}_{U[0,w]} \psi(0) \, |p\rangle$$

TMD PDFs:

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- Fields no longer evaluated along light-like separation.

- Connection at infinity.

(Belitsky, Ji, Yuan (2003))
(Boer, Mulders, Pijlman (2003))

TMD PDFs: Wilson Lines

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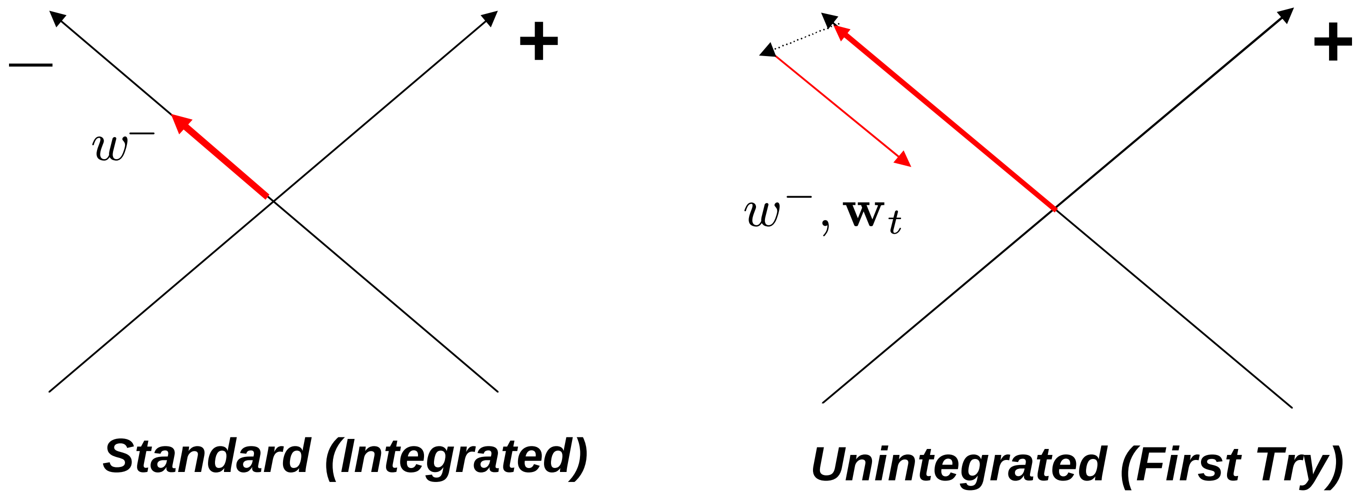

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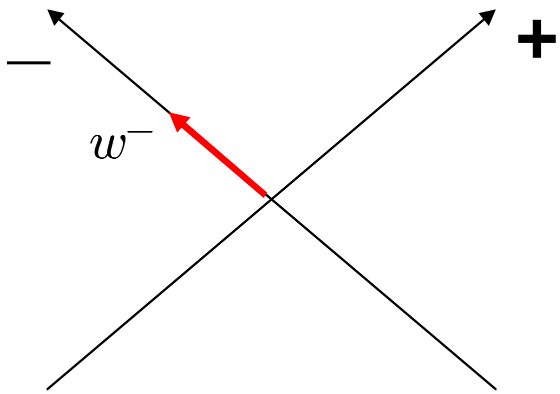

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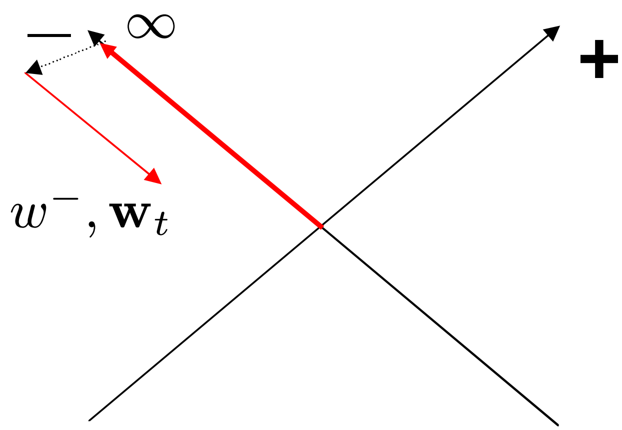
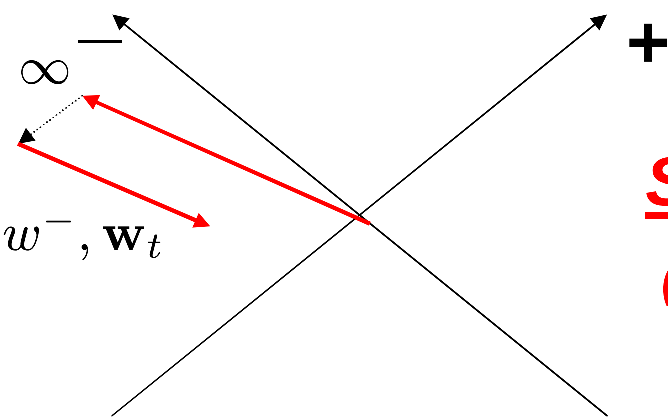


Unintegrated First Try

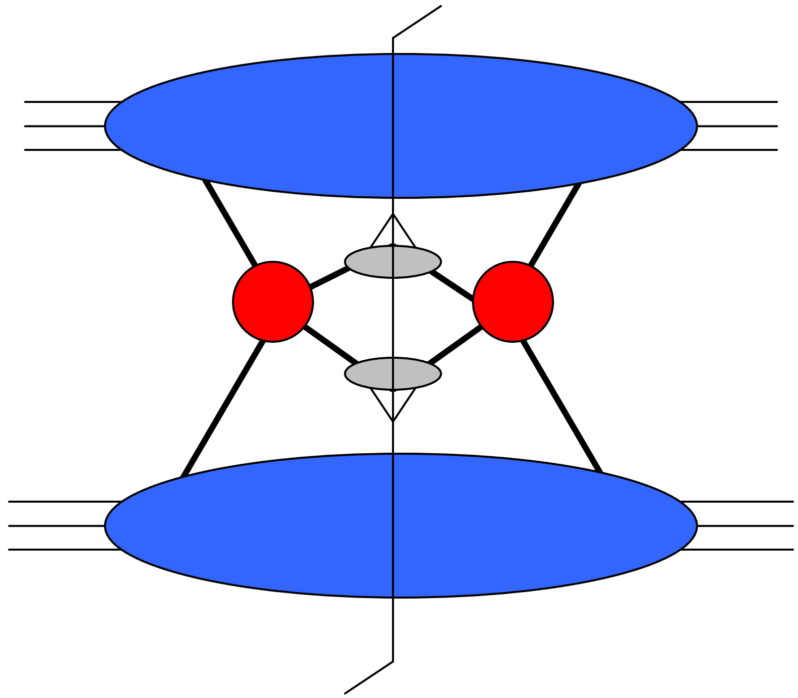


Unintegrated "tilted" Wilson lines

Still more complications!

(See Collins, Cherednikov talks.)

Hadro-Production of Hadrons and TMD-Factorization Breaking



- Factorizes if integrated over parton transverse momentum.

- Cross sections with measured final state transverse momentum?

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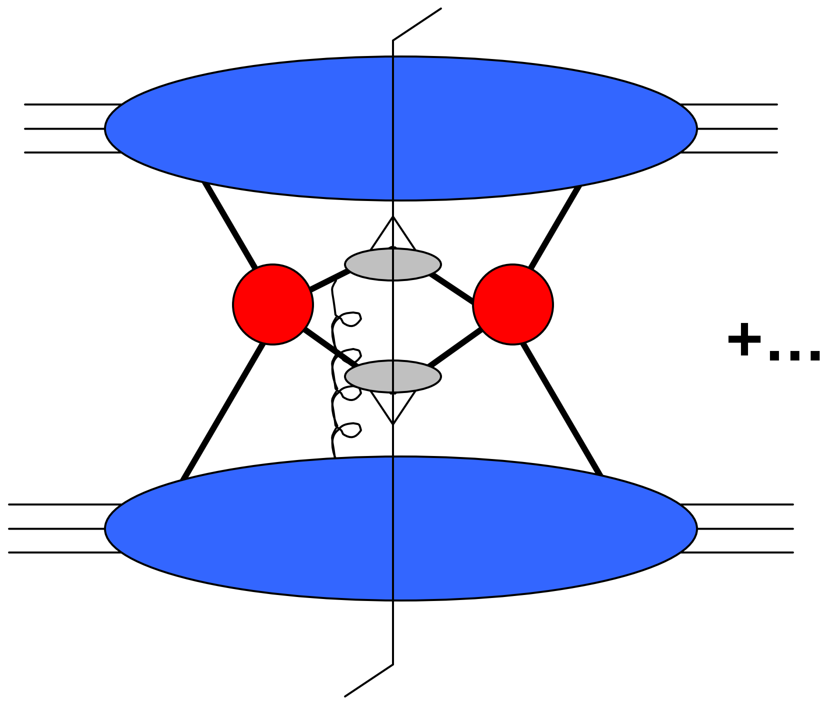


- Factorizes if integrated over parton transverse momentum.
- Cross sections with measured final state transverse momentum?

Breakdown of Universality ("Standard" Factorization Violated)

(Collins, Qiu (2007))

- Use a simple model.

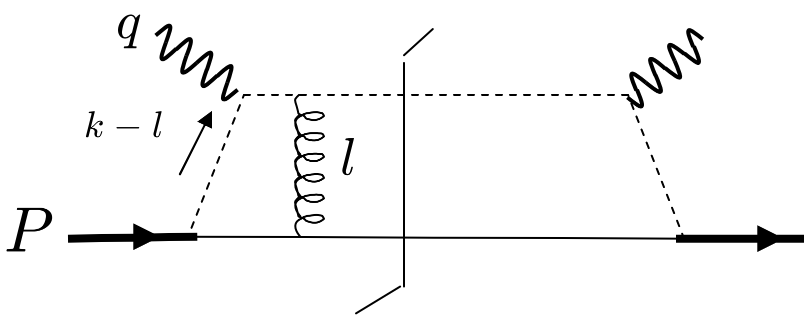
- Scalar quark/polarized hadron model.

- Massive Abelian gauge theory.

- Look at single spin asymmetry.

Wilson line problems in TMD-factorization

SIDIS

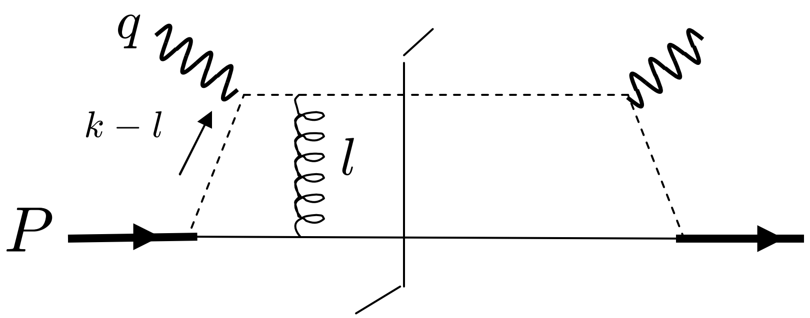


What is the Wilson line?

Abelian Model Theory

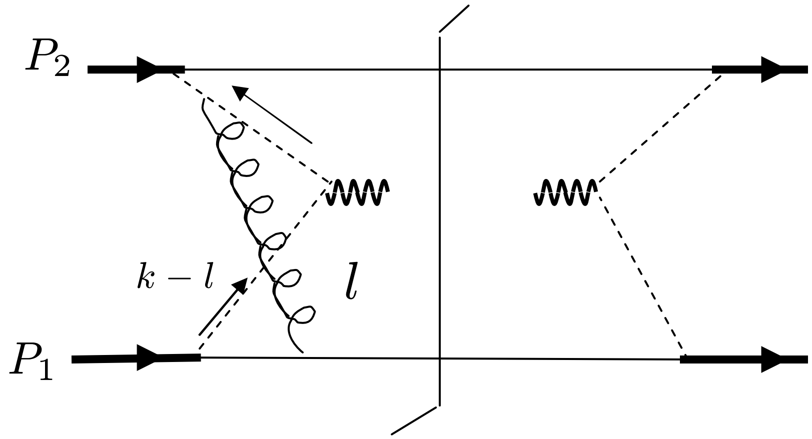
Wilson line problems in TMD-factorization

SIDIS



What is the Wilson line?

Drell-Yan



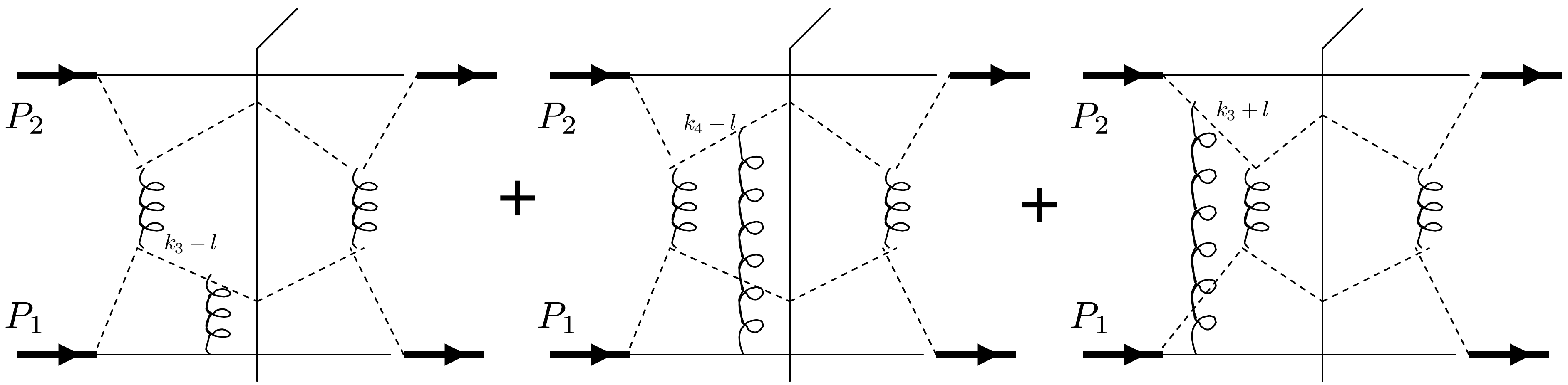
Sign flip!!

(Collins (2002))

Abelian Model Theory

Counter-example to Standard TMD-factorization

- Hadron 1 polarized. What is the Wilson line for hadron 1?



Abelian Model Theory

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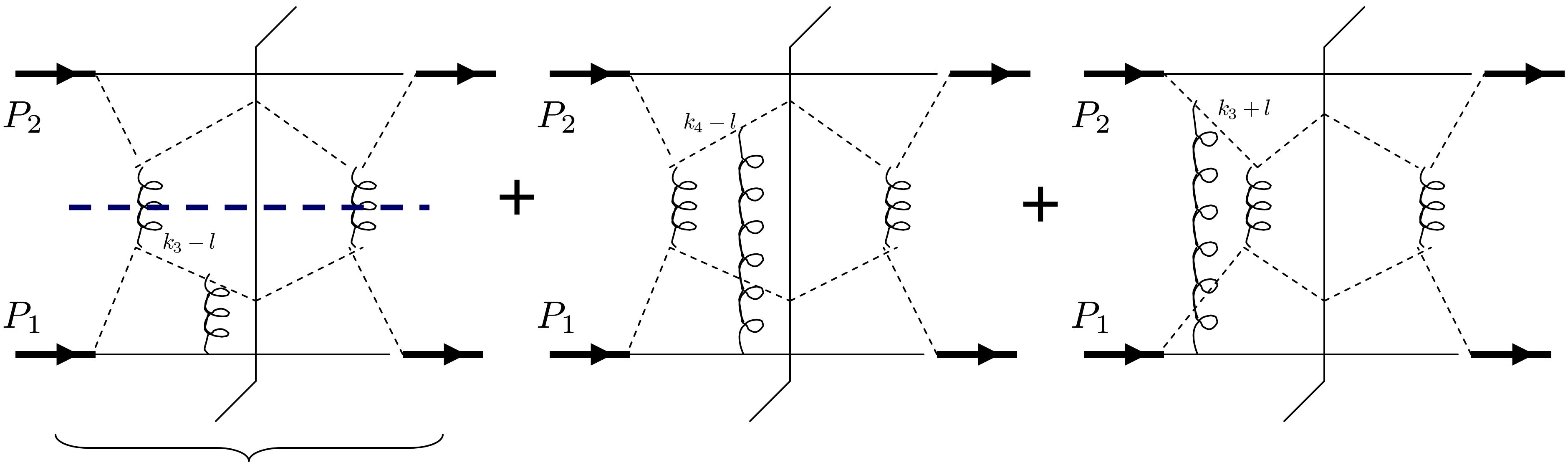

"Standard"

Abelian Model Theory

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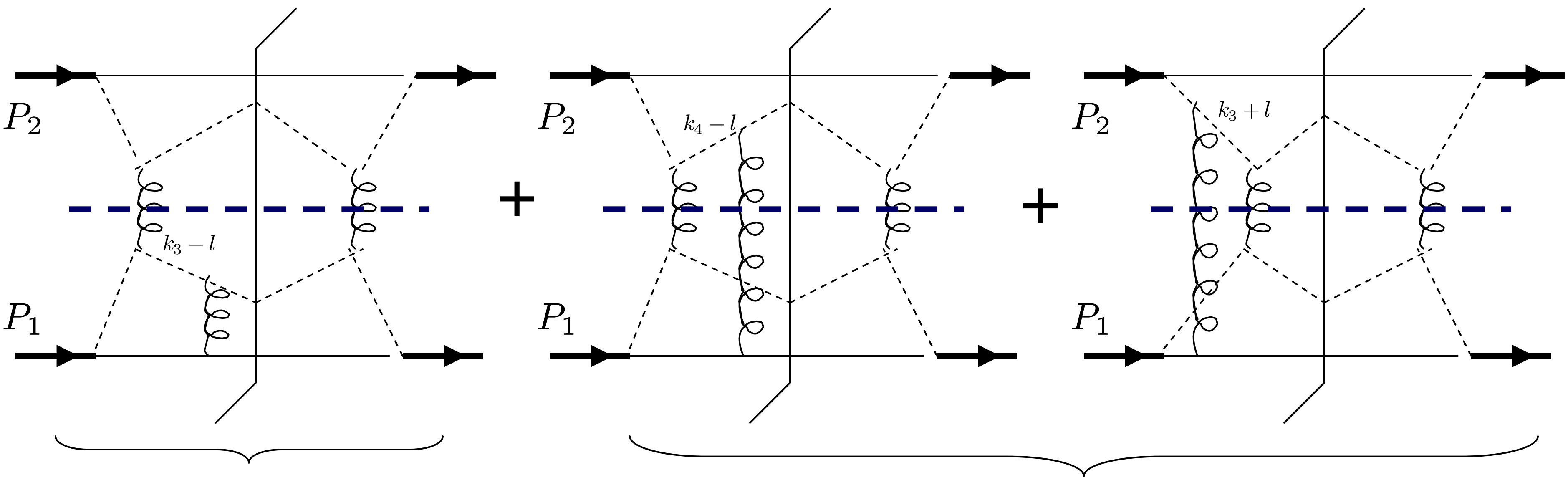


"Standard"

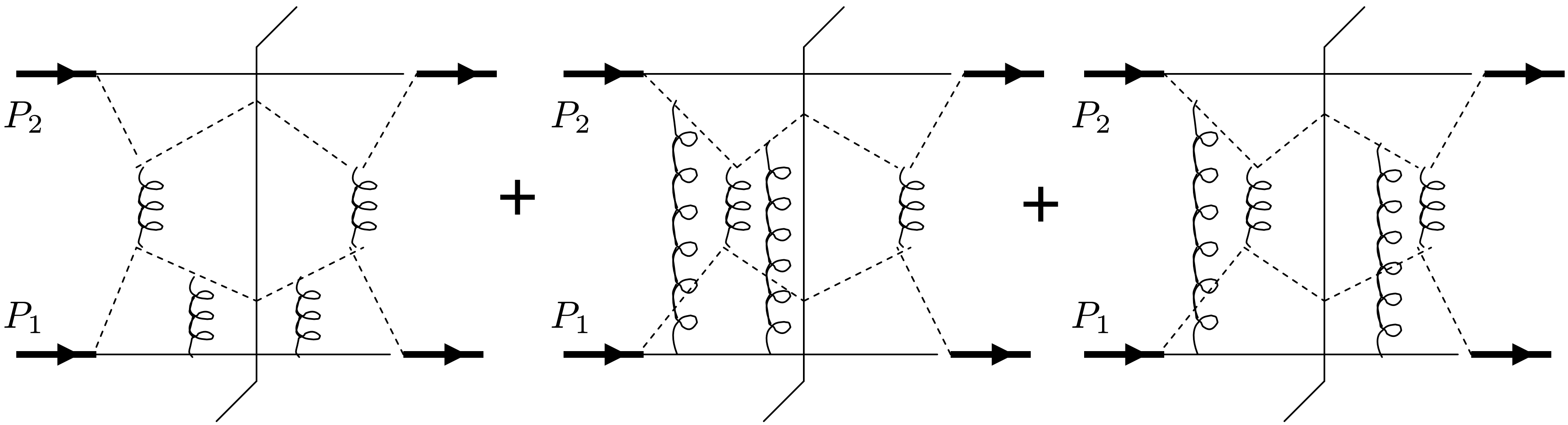
Violates Standard Factorization!!

(Non-universality)

Gluon attaches to "wrong" side of the graph!

Abelian Model Theory

Unpolarized Case: Unintegrated



+ …

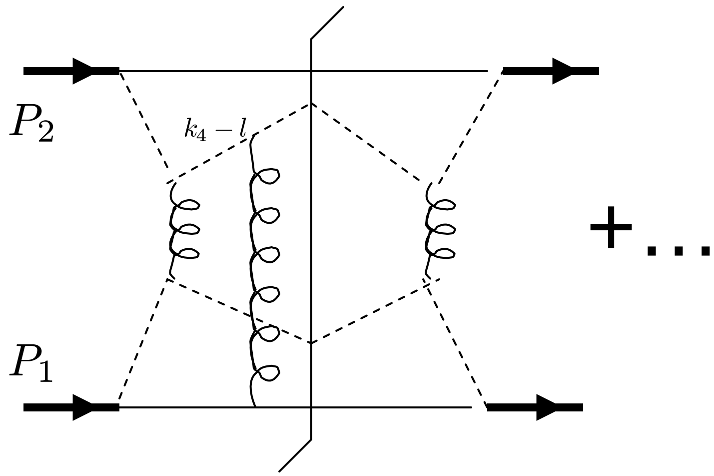
- Without integrating over final state hadron k_T, no general cancellation between cuts!

- Factorization fails even for unpolarized case!

(Collins (2007))

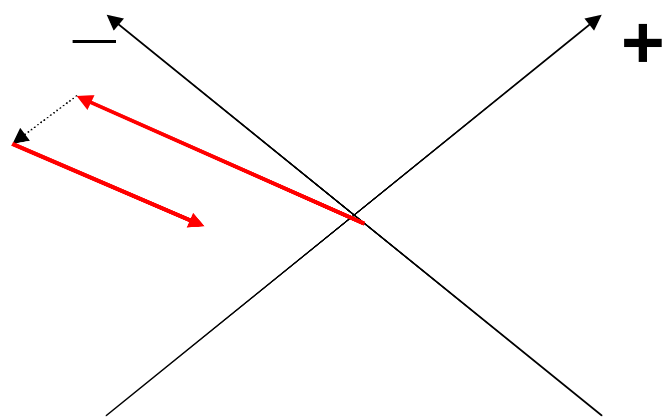
Abelian Model Theory

“Generalized” TMD Factorization



“DIS Like”

Wilson line



Abelian Model Theory

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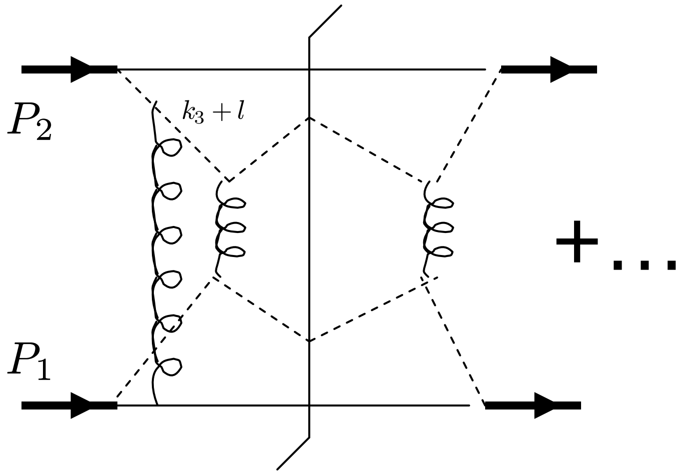
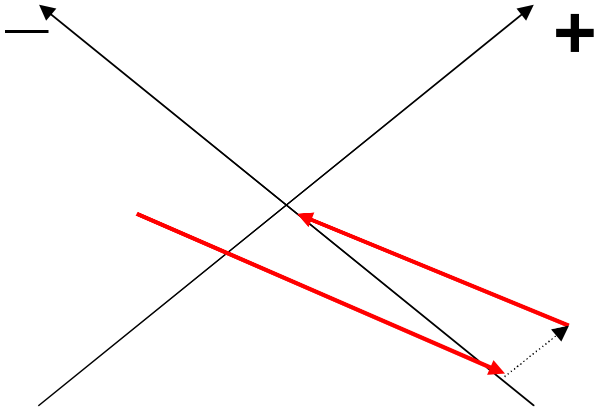


“Drell-Yan Like”



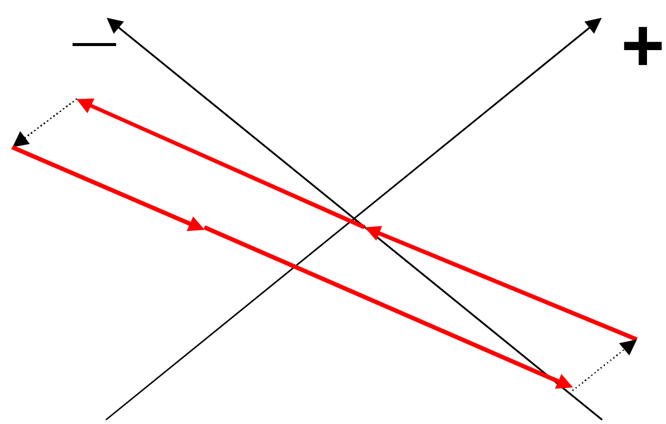
Abelian Model Theory

“Generalized” TMD Factorization

- Together: Wilson loop from upper part of graph.

Sum of all “factorization anomaly” contributions:

Wilson loop



Abelian Model Theory

“Generalized” TMD Factorization

- Modification to standard TMD PDF:

$$\Phi_{P_1}^{[+(\Box)]}(x_1, k_{1T}) \sim \text{F.T.} \langle P_1, s_1 | \, \phi_1^\dagger(0, w^-, \mathbf{w}_t) \, U[0, w] U_{(\Box)} \, \phi_1(0) \, | P_1, s_1 \rangle$$

Standard “hook” Wilson line

Abelian Model Theory

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Standard “hook” Wilson line

Extra Wilson loop insertion!

Abelian Model Theory

“Generalized” TMD Factorization

- “Generalized” TMD-Factorization: Formal factorization formula exists, *but parton distributions are non-universal!*

(!!?)

$$d\sigma \sim \mathcal{H} \otimes \; \Phi_{P_1}^{[+(\Box)]}(x_1, k_{1T}) \; \otimes \; \Phi_{P_2}^{[+(\Box)]}(x_2, k_{2T}) \; \otimes \cdots$$

(Bomhof, Mulders, Pijlman (2004))

No Generalized TMD-factorization!

(TCR, Mulders (2010))

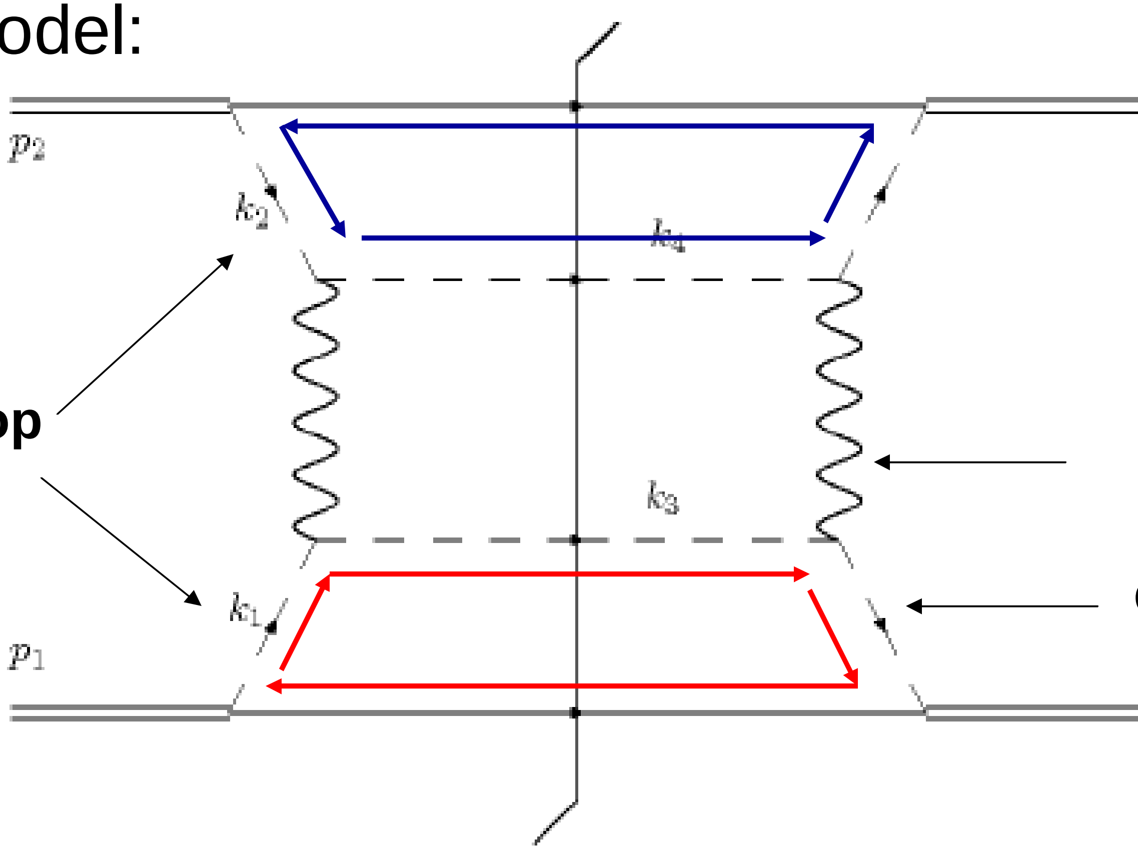
- Counter-example: Again use scalar quark / spinor diquark model.

- Quarks and diquarks now carry color.

- Hard part: exchange of colorless vector boson. No color flowing through hard part!

Generalized TMD-factorization breakdown:

- Spectator Model:



- **“Extra” Wilson loop for each PDF.**

[*Non*-Abelian Model]

Generalized TMD-factorization breakdown:

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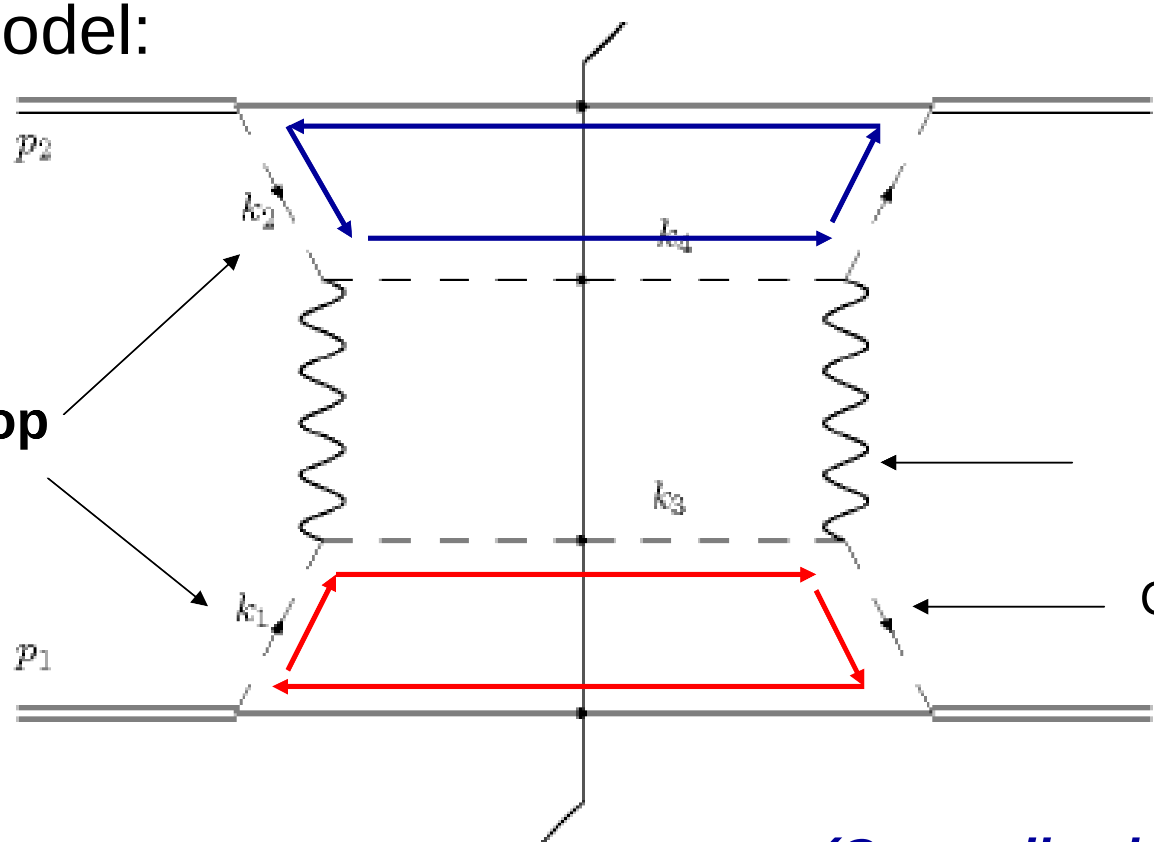


- **“Extra” Wilson loop for each PDF.**

Colorless

Colored

$$\frac{d\sigma}{dq_T^2} \stackrel{!?}{=} \mathcal{H} \otimes \Phi^{[+(\Box)]}_{P_1}(x_1, k_{1T}) \otimes \Phi^{[+(\Box)]}_{P_2}(x_2, k_{2T}).$$

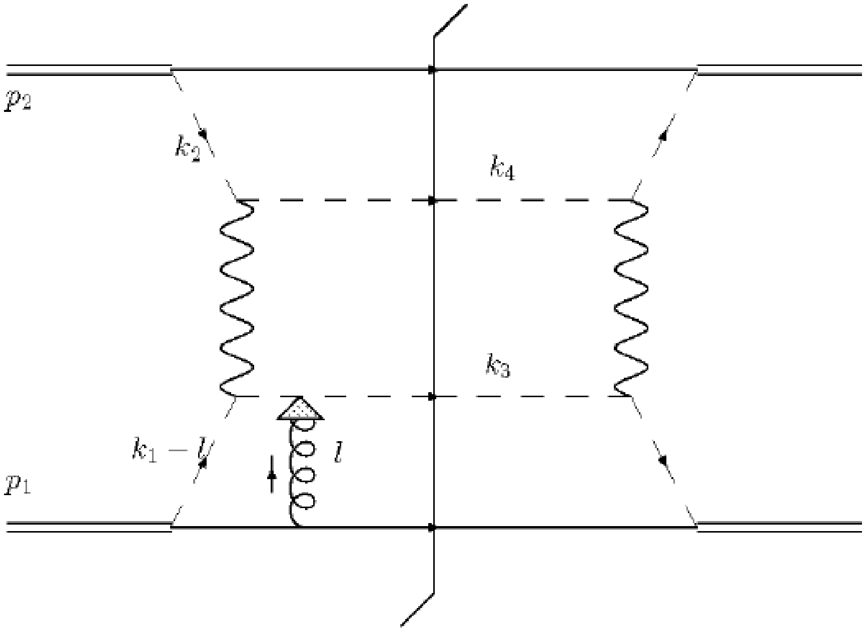
(Generalized factorization formula.)

Color-traced Wilson loops!

***Non*-Abelian Model**

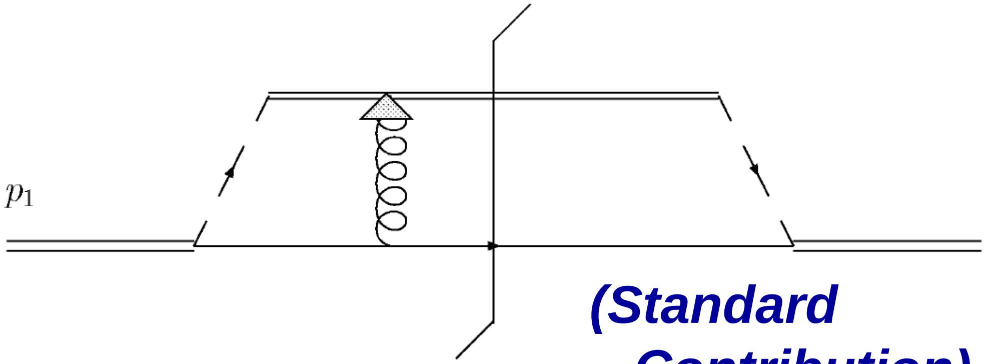
One gluon:

- One collinear gluon:



- Apply Eikonal approx.

- Lower hadron PDF:

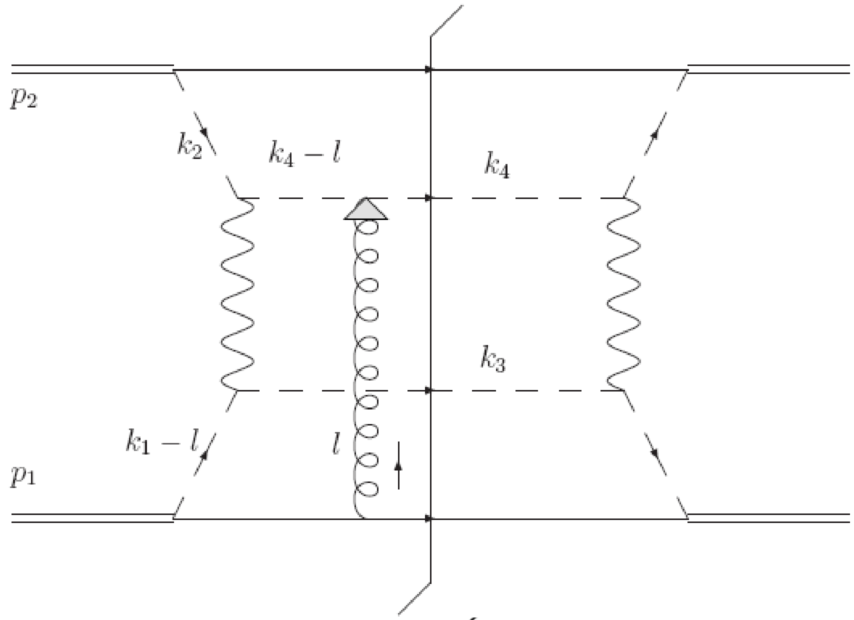


(Standard Contribution)

One gluon:

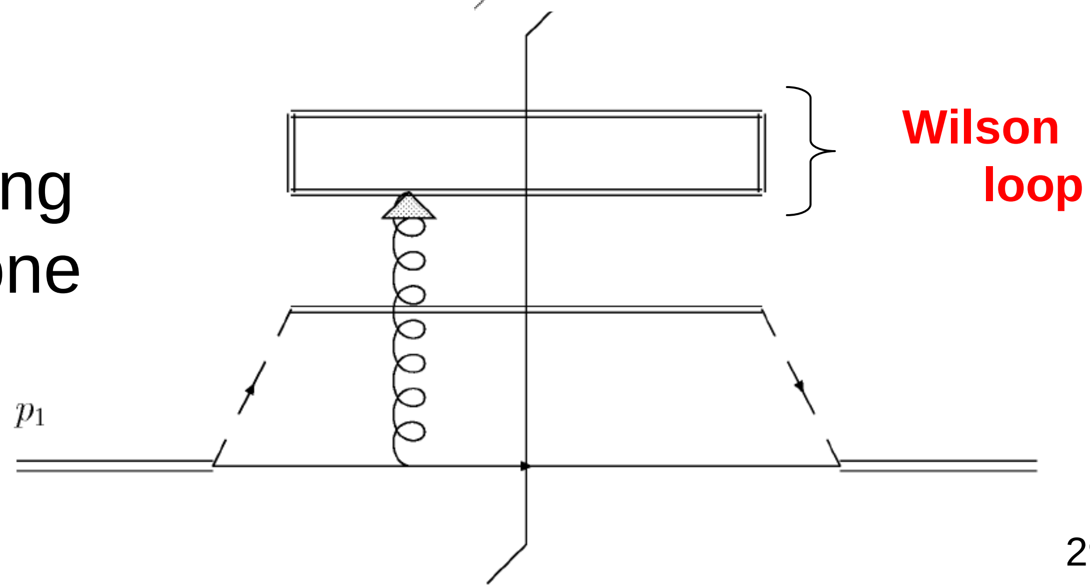
- One collinear gluon:

  (Anomalous Attachment)



- Apply Eikonal approx.

- No factorization violating contribution with just one gluon.



Non-Abelian Model

One gluon:

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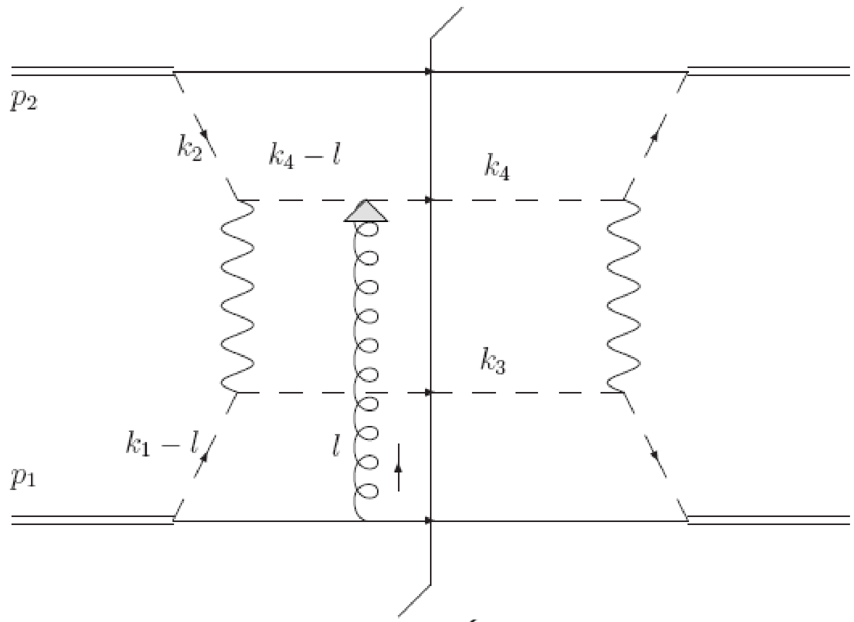

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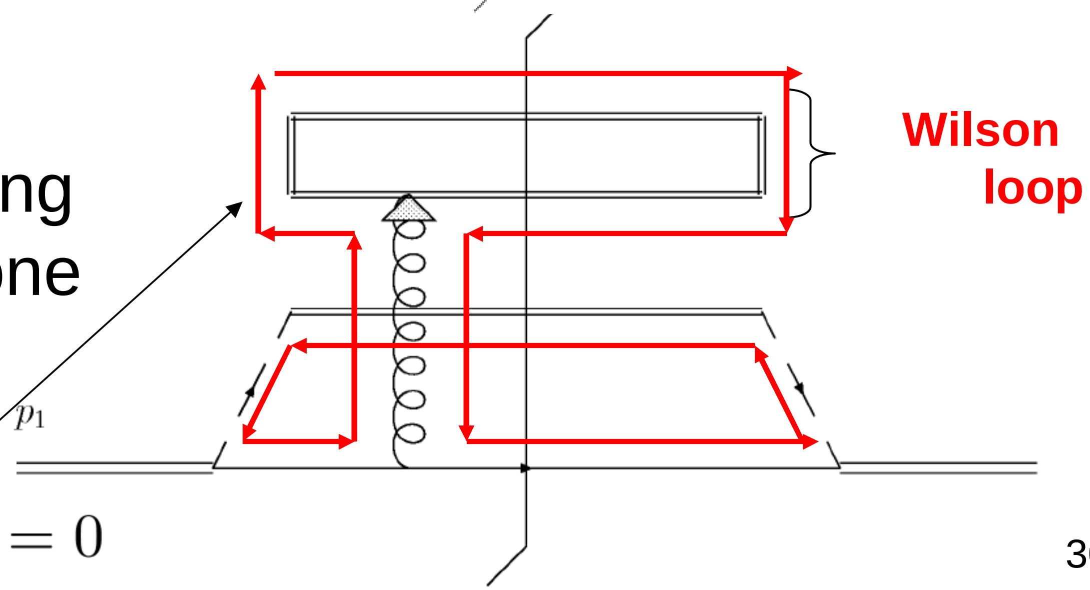
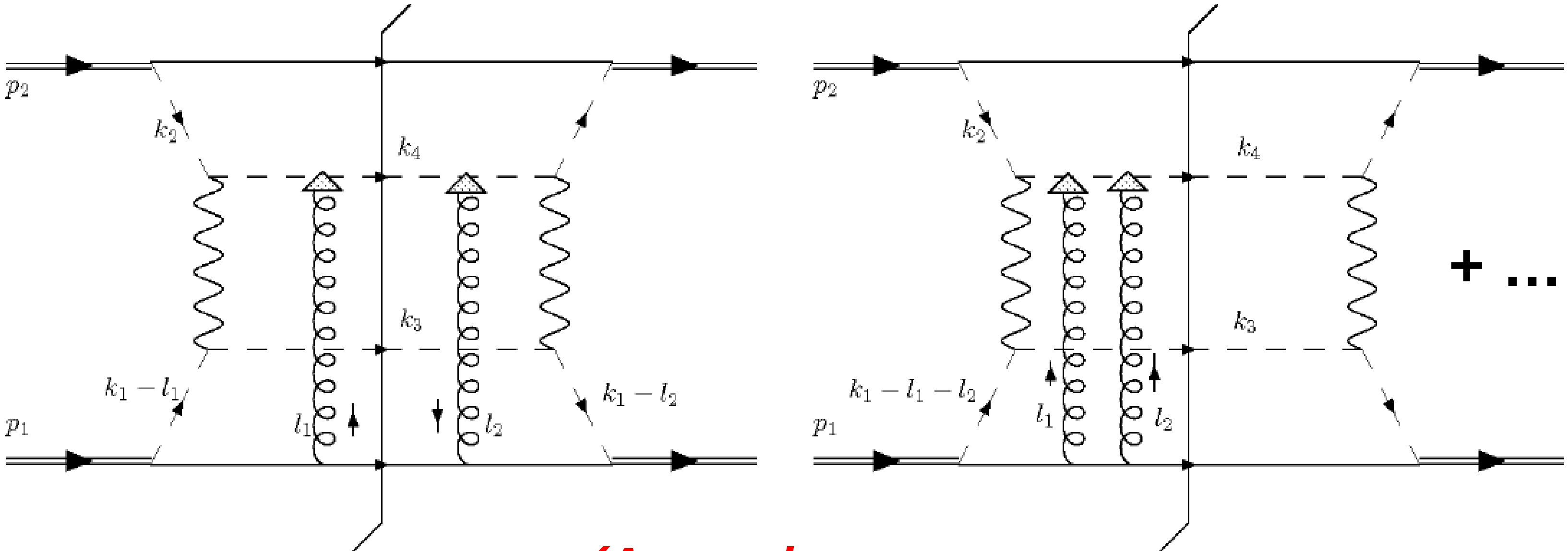


$$\mathrm{Tr}_C\,[t^a] = 0$$

***Non*-Abelian Model**

Two gluons:



(Anomalous Attachments)

- Non-vanishing violation of standard TMD-factorization in both single spin dependence and unpolarized cross section.
- Still consistent with *generalized* TMD factorization.

(Vogelsang, Yuan (2006,2008))

Summary so far:

- One extra gluon is consistent with *standard factorization* for both unpolarized case and single spin dependence.

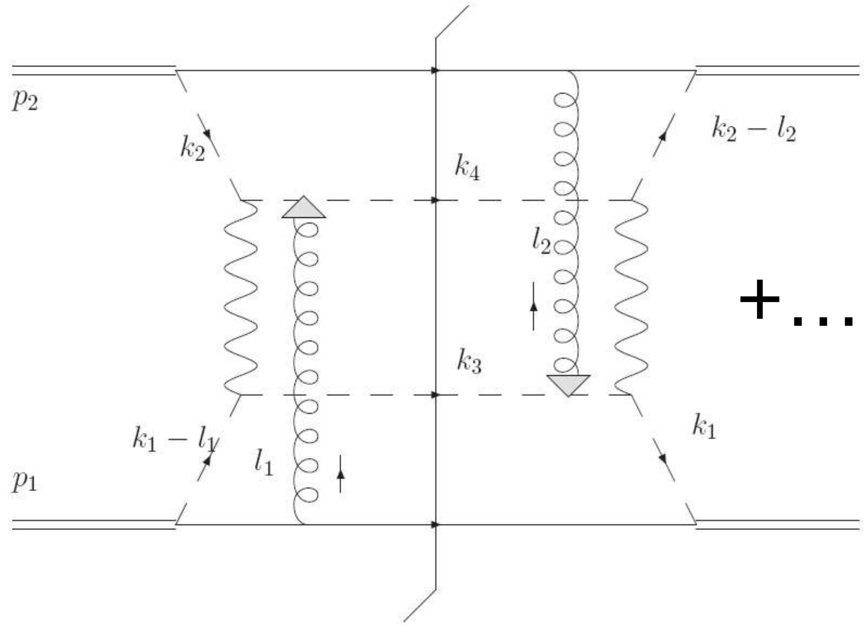
- Two gluons from *same hadron* consistent with generalized TMD factorization for unpolarized case.

- What about extra gluons from *both hadrons simultaneously*?

***Non*-Abelian Model**

Generalized TMD factorization breaking:

- Graphs like:



- ***No SSA or unpolarized contribution.***
  (Cancellation between cuts.)

- ***What about double spin dependence?***
  (No cancellation between cuts.)

- ***How does it contribute to ?:***

$$\frac{d\sigma}{dq_T^2} \overset{!?}{=} \mathcal{H} \otimes \Phi_{H_1}^{[+(\Box)]}(x_1, k_{1T}) \otimes \Phi_{H_2}^{[+(\Box)]}(x_2, k_{2T}).$$

Non-Abelian Model

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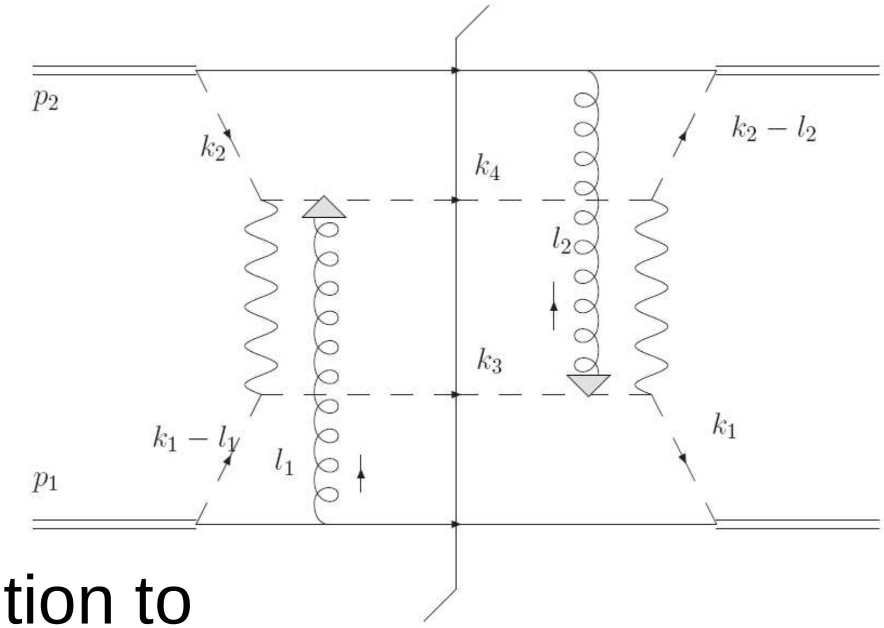
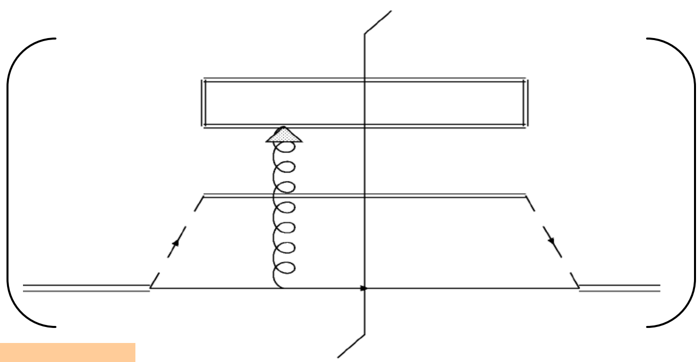
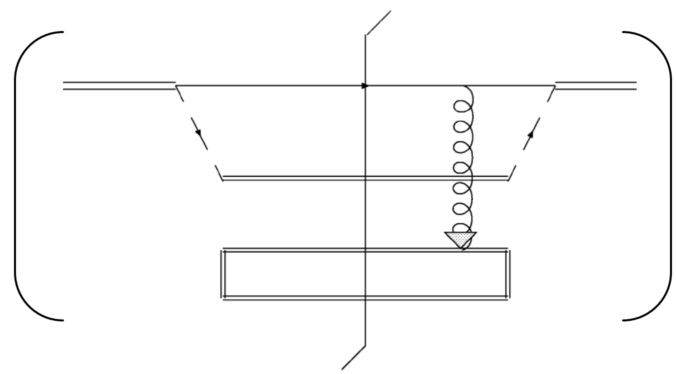
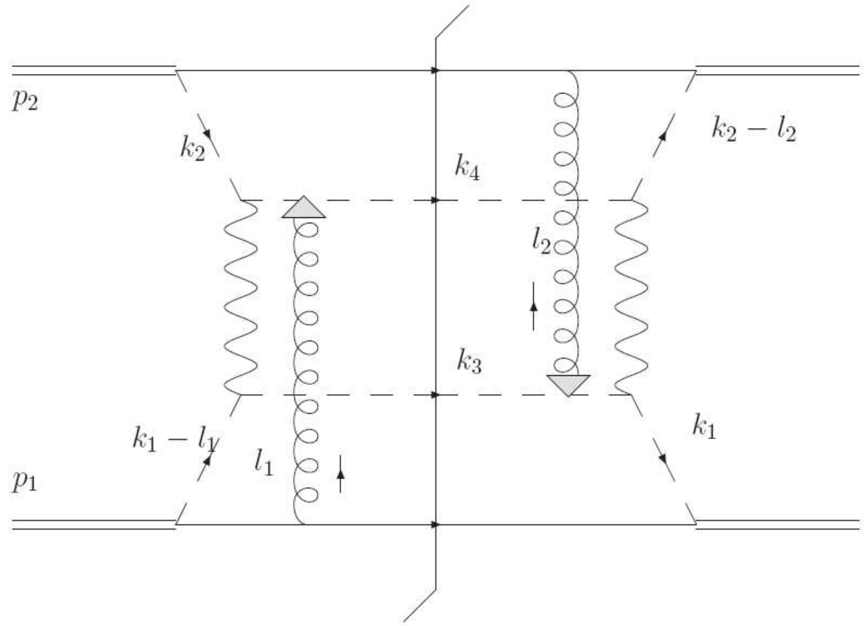


- Corresponds to contribution to "generalized" TMD-factorization formula:

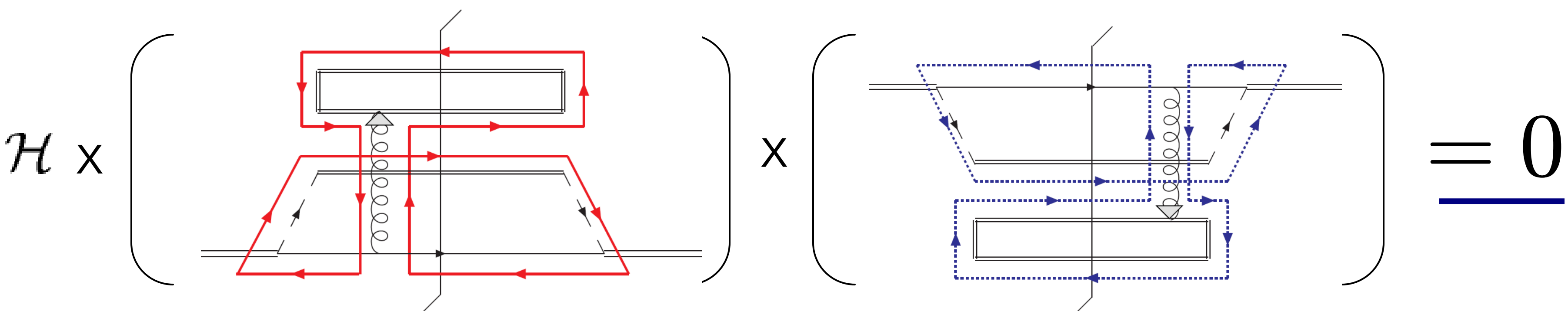
$\mathcal{H}$ x  x 

Generalized TMD factorization breaking:

- Gluons have color.



- Leads to contribution to "generalized" factorization formula:



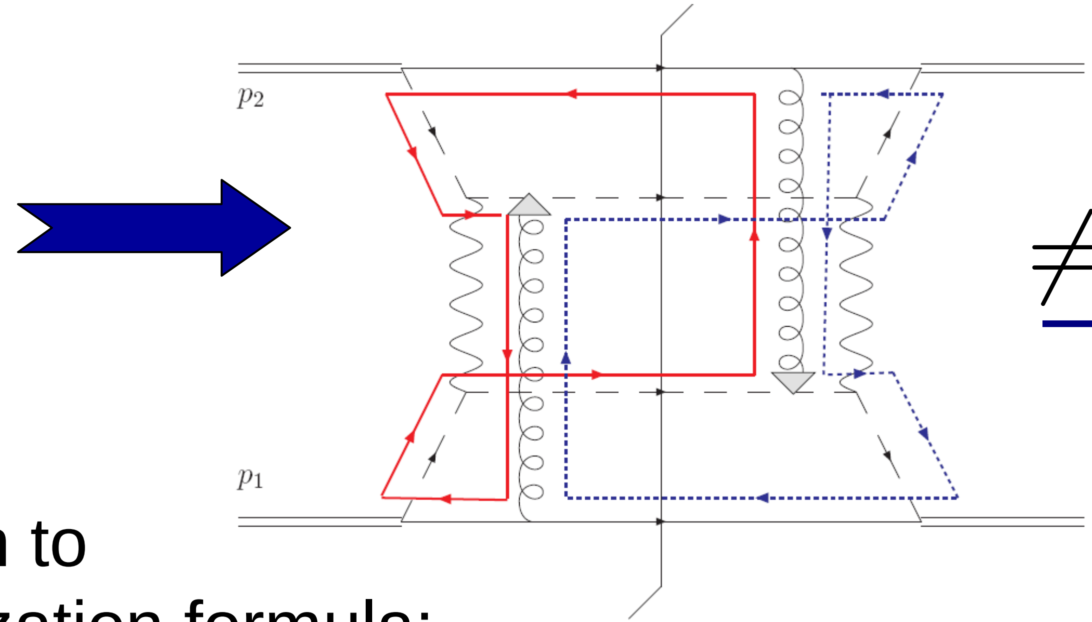
$\mathrm{Tr}_C\,[t^a] = 0$

***Non*-Abelian Model**

Generalized TMD factorization breaking:

- Gluons have color.

Actual color structure



$\neq 0$!

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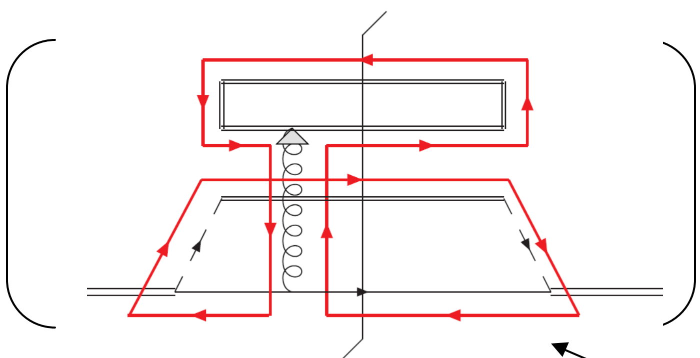
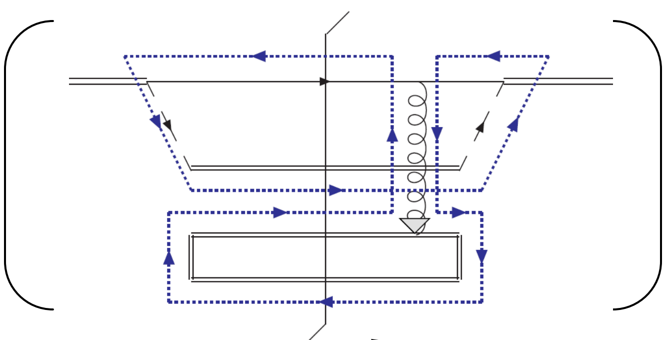
$\mathcal{H}$ x  x  $= 0$

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Non-Abelian Model

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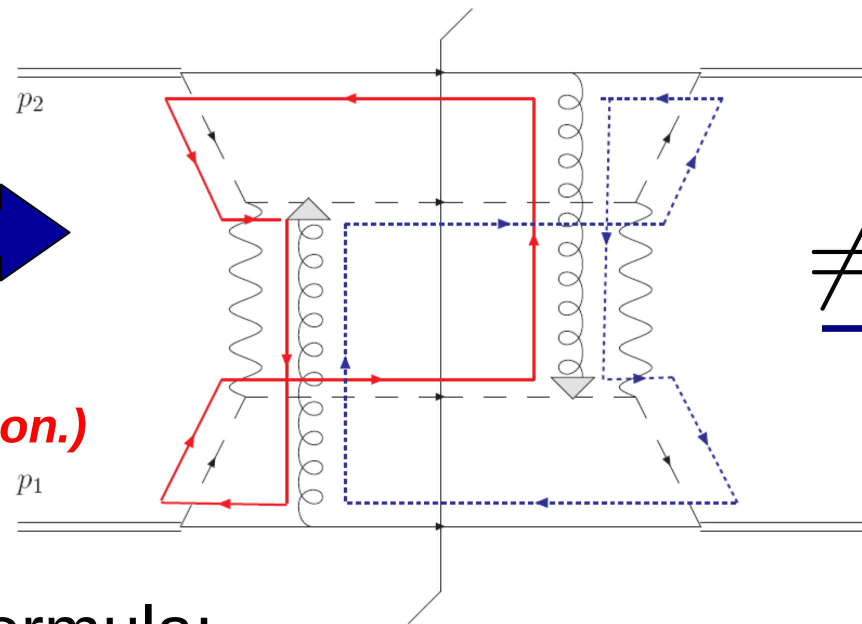
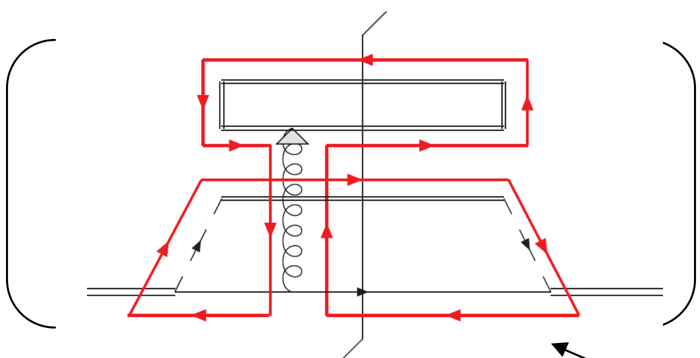
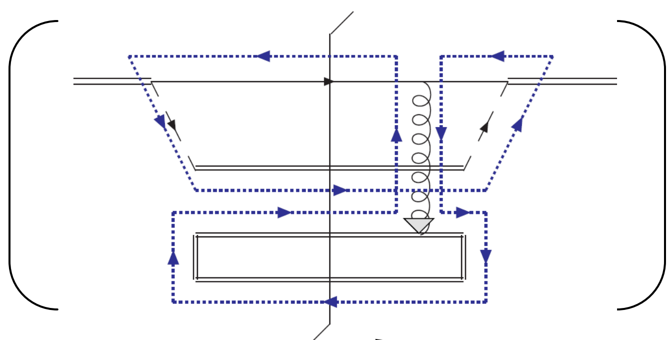


$\neq 0$!

(Cannot extend, e.g., Lu, Ma, Schmidt (2007) to hadro-production.)

- Leads to contribution to "generalized" factorization formula:

$\mathcal{H}$ x  x  $= 0$

$\mathrm{Tr_C}\left[t^a\right] = 0$

Non-Abelian Model

Summary:

- Universality of PDFs and *Standard* Factorization.
  - Process independent definitions for gauge invariant correlation functions.
  - Well-established for *integrated* case.
  - Fails in hadro-production of hadrons with TMD PDFs.

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  - Process still factorizes into hard part and separate correlation functions.
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- Non-universality and *Generalized* Factorization.
  - Process still factorizes into hard part and separate correlation functions.
  - Non-universal because Wilson lines are process dependent.
  - **Even this fails. Correlation functions are not just non-universal. They cannot even by defined!**

Ways forward?

- q_T weighting?
- Recovery of TMD-factorization at small-x?

(Xiao, Yuan (2010))
(Chang, Li (2009))
(Marguet, DIS 2010 proc. and refs.)

- Direct calculations of factorization breaking effects?
- New approach to perturbative QCD: Embrace factorization breaking?

$$\sigma \sim \underbrace{\mathcal{H}}_{\text{Hard}} \otimes \underbrace{\Phi(P_1, P_2, \ldots)}_{\text{Soft and Collinear}} \otimes \text{Evolution}$$

(Work in Progress)