

TMDs in photon-pair production and DY

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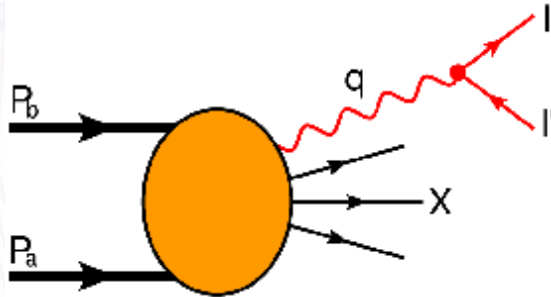
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The Drell-Yan process

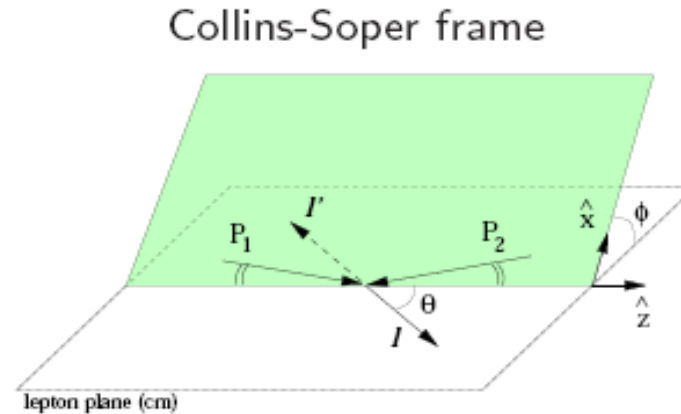
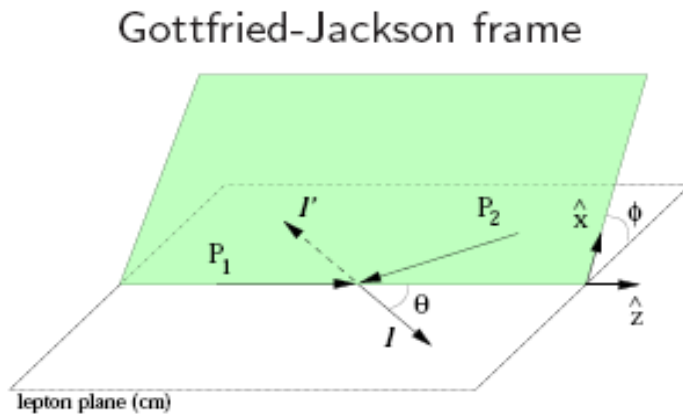
Kinematics (lepton pair produced by one decaying gauge boson):



$$\frac{d\sigma}{d^4l d^4l'} = \frac{d\sigma}{d^4q d^4l} \propto \frac{\delta^+(l^2)\delta^+((q-l)^2)}{4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Disentangling the δ -functions \rightarrow Dilepton rest frame

Gottfried-Jackson frame and Collins-Soper frame:



Lepton angles:

$$d^4l \rightarrow d\Omega = d\phi d \cos \theta$$

Diff. CS including angular dependences:

$$\frac{d^6\sigma}{d^4q d\Omega} = 2 \frac{d^6\sigma}{dy dQ^2 d^2\vec{q}_T d\Omega}$$

Angular structure functions

Separation of the leptonic part (generated by one photon):

$$\frac{d\sigma}{d^4q d\Omega} \propto L_{\mu\nu} W^{\mu\nu} \quad \text{with: } L_{\mu\nu} = 4 \left(l_\mu l'_\nu + l_\nu l'_\mu - \frac{Q^2}{2} g_{\mu\nu} \right) \longrightarrow \text{Limited number of structure function}$$

Hadronic Tensor:
$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \langle a, b | J^\mu(0) J^\nu(x) | a, b \rangle$$

Parameterization constraint by **current conservation**, **hermiticity** and **parity**

Decomposition into 4 + 8 + 8 + 28 = 48 structure functions $F(x_a, x_b, q_T^2, Q^2)$

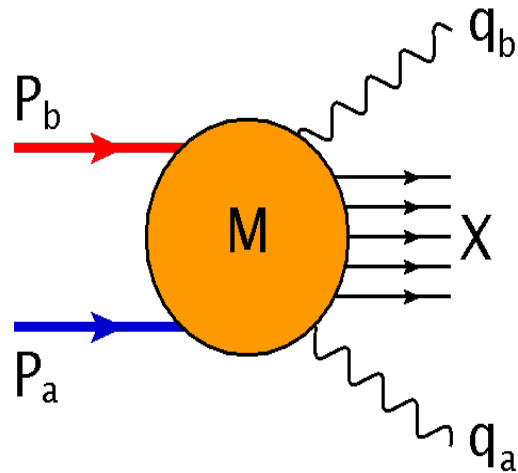
[Arnold, Metz, M.S., PRD 79, 034005] **e.g. unpolarized Drell-Yan**

$$\frac{d\sigma_{UU}}{dx_a dx_b d^2q_T d\Omega} = \frac{\alpha^2 s}{2Q^2 F} \left((1 + \cos^2 \Theta) F_{UU}^1 + (1 - \cos^2 \Theta) F_{UU}^2 + \sin 2\Theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \Theta \cos 2\phi F_{UU}^{\cos 2\phi} \right)$$

Classification of structure functions helpful for data analysis

→ **Parton model**: 24 leading twist structure functions

Diphoton production



Two highly energetic real photons produced with

$$q \equiv q_a + q_b$$

$$\frac{d\sigma}{d^4 q_a d^4 q_b} = \frac{d\sigma}{d^4 q d^4 q_a} \propto \frac{\delta^+(q_a^2) \delta^+((q - q_a)^2)}{2 \times 4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Convenient choice: Diphoton rest frame \rightarrow **Collins-Soper frame**

$$\frac{d^6 \sigma}{dy dQ^2 d^2 \vec{q}_T d\Omega_a}$$

Unfortunately: No separation into **hadronic – photonic** parts possible!
 \rightarrow **all** angular modulations are allowed, in principle.

$$\frac{d^6 \sigma}{dy dQ^2 d^2 \vec{q}_T d\Omega_a} = \sum_{l=0}^{\infty} \sum_{m=-l}^l C_{lm}(y, Q^2, q_T^2) Y_{lm}(\Omega_a)$$

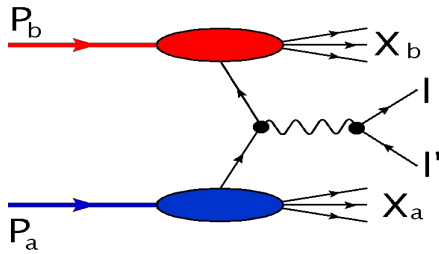
$$C_{00} = \frac{d^4 \sigma}{dy dQ^2 d^2 q_T}, \dots$$

However, we can calculate the cross section in the **parton model**.

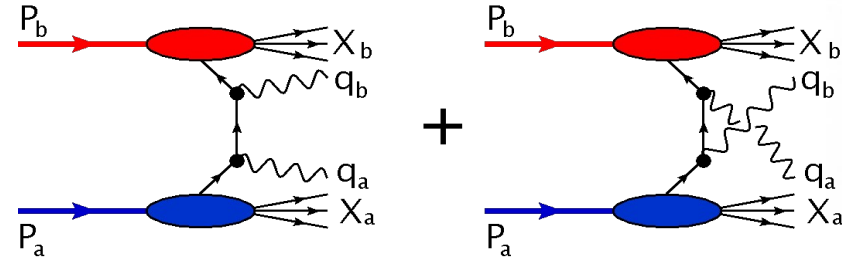
TMD tree-level formalism

Parton model tree-level at $O(\alpha_s^0)$:

Drell-Yan dilepton production:



Diphoton production:



Only relevant at very small q_T : $\Lambda_{QCD} \sim q_T \ll Q$

$$\left(\frac{d\sigma}{d^4q d\Omega} \right) \propto \int d^2k_{aT} \int d^2k_{bT} \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T) \text{Tr} \left[\Phi(x_a, \vec{k}_{aT}) H(x_a, x_b, q_a, q_b) \bar{\Phi}(x_b, \vec{k}_{bT}) H^\dagger \right] + \mathcal{O}\left(\frac{M}{Q}\right)$$

k_T - correlator:
$$\Phi_{ij}(x, \vec{k}_T) = \int \frac{dz^- d^2z_T}{(2\pi)^2} e^{ik \cdot z} \langle P, S | \bar{q}_j(0) \mathcal{W}^{?/DY}[0; z] q_j(z) | P, S \rangle \Big|_{z^+=0}$$

→ can be parameterized in terms of TMDs according to quark / nucleon spin

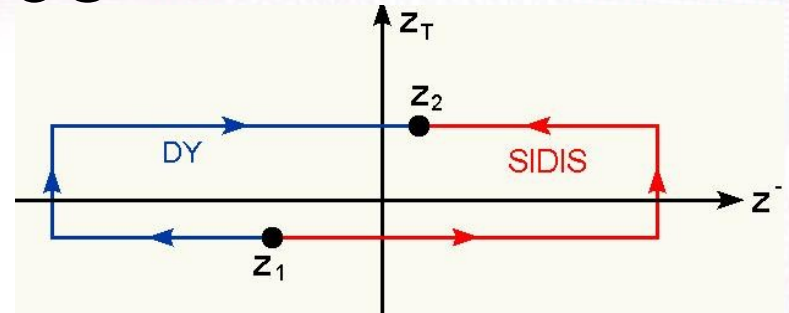
Main result of the TMD tree-level formalism:

$$\left(\frac{d^6\sigma^{hh \rightarrow \gamma\gamma X}}{dy dQ^2 d^2q_T d\Omega} \right) (\Lambda \sim q_T \ll Q) = \frac{2}{\sin^2\theta} \left(\frac{d\sigma^{hh \rightarrow l^+l^- X}}{dy dQ^2 d^2q_T d\Omega} \right) (\Lambda \sim q_T \ll Q | e_q \rightarrow e_q^2)$$

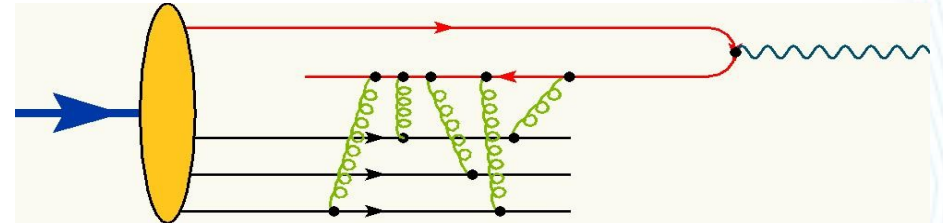
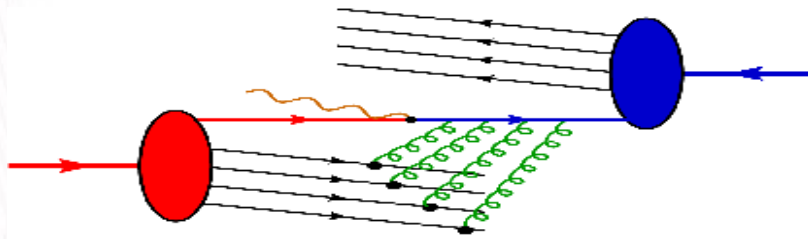
Wilson lines

Wilson line process-dependent in DY/SIDIS:

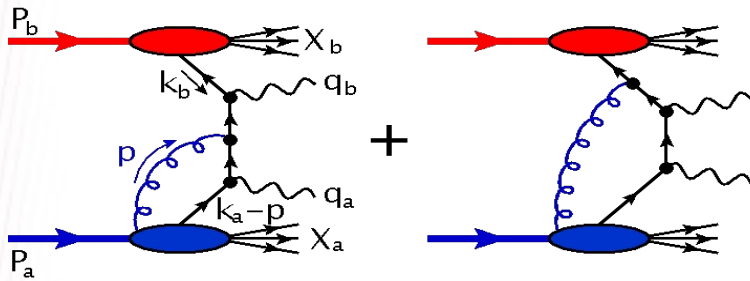
$$\mathcal{W}[z_1; z_2] = \mathcal{P}e^{-ig \int_{z_1}^{z_2} ds \cdot A(s)}$$



Physics: **Initial** / **Final** state interactions



Wilson line in diphoton production:



+ crossed

Check for A^+ , $A_T^i(z^- = -\infty)$

Diagrams topologically different to DY, **but** cancellations between diagrams

$$\mathcal{W}^{\gamma\gamma}[0; z] \Big|_{z^+=0} = 1 - ig \int_0^{-\infty} d\lambda A^+(\lambda n) - ig \int_0^{z_T} d\vec{y}_T \cdot \vec{A}_T(-\infty, 0, \vec{y}_T) - ig \int_{-\infty}^0 d\lambda A^+(\lambda n + z_T) + \mathcal{O}(g^2) = \mathcal{W}^{DY}[0; z] \Big|_{z^+=0}$$

Example: Sivers effect

k_T – correlator for unpolarized quarks:

$$\frac{1}{2} \text{Tr}[\Phi(x, \vec{k}_T) \gamma^+] = f_1(x, \vec{k}_T^2) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} f_{1T}^\perp(x, \vec{k}_T^2)$$

Sivers function \rightarrow time-reversal odd \rightarrow sign switch:

$$f_{1T}^\perp \Big|_{DIS} = -f_{1T}^\perp \Big|_{DY}$$

Can be determined from SIDIS data of a transverse target SSA (HERMES, COMPASS):

$$A_{UT}^{Siv} \sim \frac{f_{1T}^\perp \otimes D_1}{f_1 \otimes D_1}$$

k_T – deconvolution through Gaussian ansatz

$$f(x, \vec{k}_T^2) = f(x) \exp \left[-\vec{k}_T^2 / \langle k_T^2 \rangle \right]$$

Fit of the Sivers function to data:

[Anselmino et al., EPJA 39, 89],

[Schweitzer, M.S., 0805.2137 and in prep.]

$$f_{1T}^{\perp,q}(x) = A^q f_1^q(x)$$

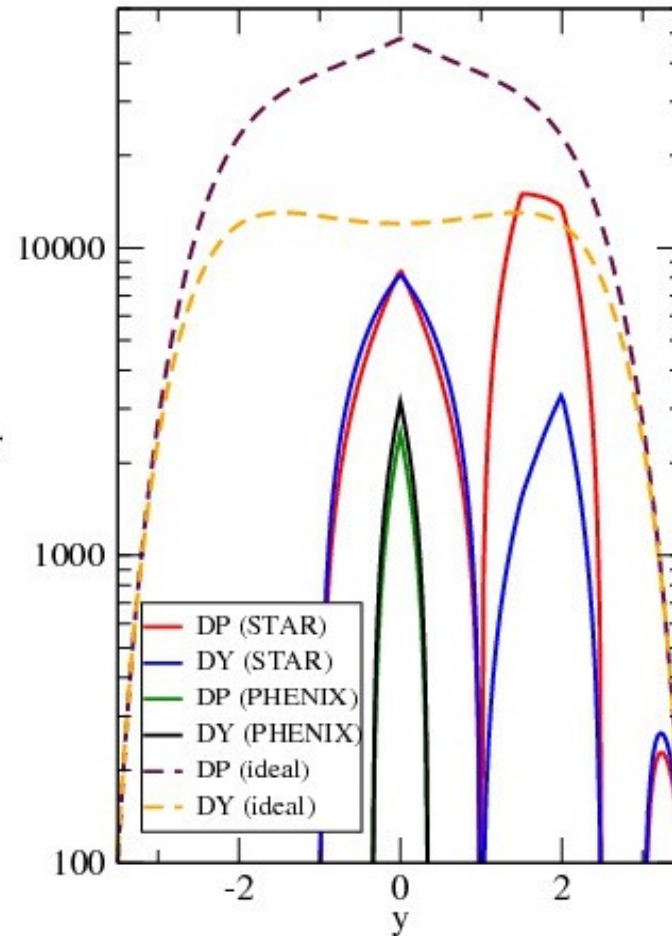
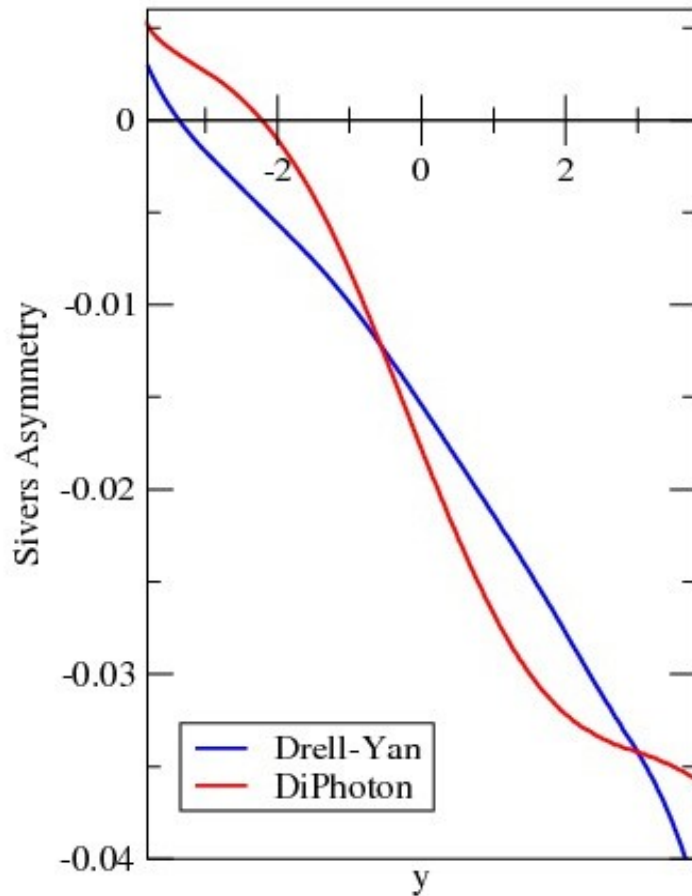
- non-zero sea-quark Sivers functions.
- Take into account finalized HERMES proton data and COMPASS deuteron data
- small statistical errors, $\chi^2 / \text{d.o.f.} \sim 1.07$

Sivers effect in Diphoton/DY process:

$$A_{TU}^{Siv, DP/DY} \sim \frac{2 / \sin^2 \theta f_{1T}^\perp \otimes \bar{f}_1}{2 / \sin^2 \theta f_1 \otimes \bar{f}_1}$$

Sivers Asymmetry vs. pair rapidity $y = (\eta_a + \eta_b)/2$

Bins[GeV]: $4 < Q < 10$, $0 < q_T < 1$, $|\eta| < 1$ and $3 < \eta < 4$ (STAR), $|\eta| < 0.35$ (PHENIX)



Sivers Asymmetry roughly equal in DY and DP

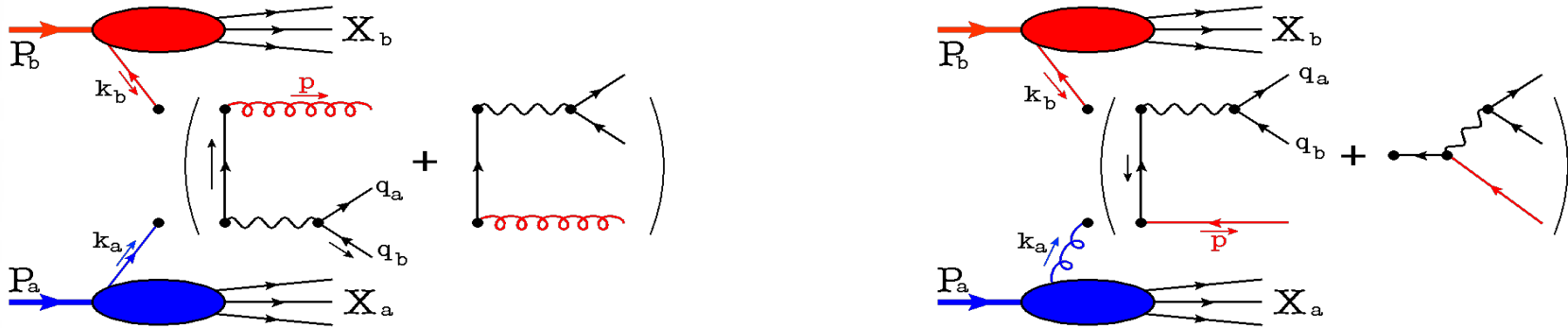
$$e_q^2 \rightarrow e_q^4 \quad \text{u-quark dominance}$$

Rapidity enhancement seems to cancel in SSA

Photon pair production rate up to 5 times larger for large rapidity differences

High - q_T behaviour

At large $q_T \sim Q \rightarrow$ transverse momentum generated by gluon radiation



Unpolarized angular dependencies in Drell-Yan:

$$\frac{d\sigma_{UU}}{dx_a dx_b d^2q_T d\Omega} = \frac{\alpha^2 s}{2Q^2 F} \left((1 + \cos^2 \theta) F_{UU}^1 + (1 - \cos^2 \theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right)$$

Collinear parton model applicable [Boer, Vogelsang, PRD74, 014004]

$$\frac{F^i}{x_a x_b} \sim \int \frac{d\xi_a}{\xi_a} \int \frac{d\xi_b}{\xi_b} \delta\left((\xi_a - x_a)(\xi_b - x_b) - \frac{q_T^2}{s}\right) \sum_{q, \bar{q}} e_q^2 \left(f_1^q(\xi_a) f_1^{\bar{q}}(\xi_b) \hat{F}_{q\bar{q}}^i + f_1^q(\xi_a) f_1^g(\xi_b) \hat{F}_{qg}^i + f_1^g(\xi_a) f_1^{\bar{q}}(\xi_b) \hat{F}_{g\bar{q}}^i \right)$$

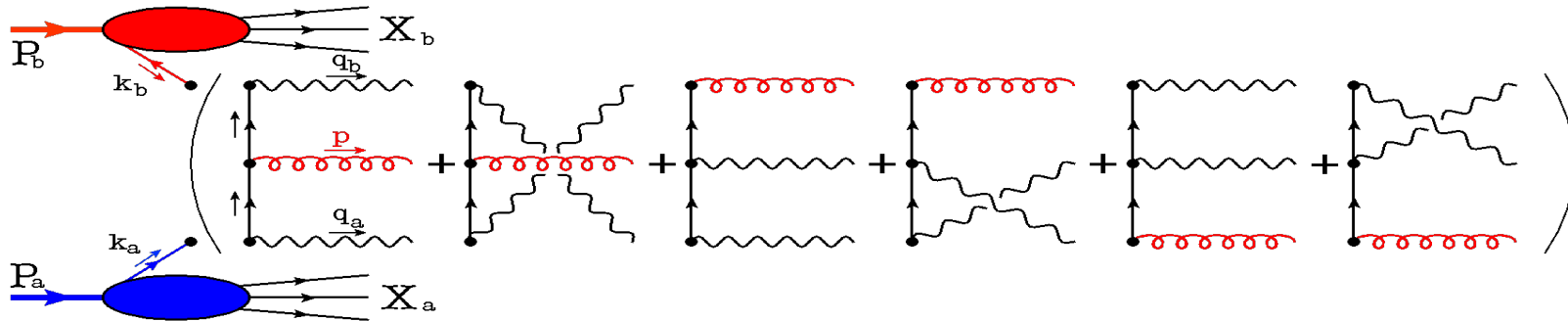
Partonic structure functions automatically obey angular decomposition

Lam-Tung relation in pQCD
$$F_{UU}^2 = 2F_{UU}^{\cos 2\phi} + \mathcal{O}(\alpha_s^2)$$

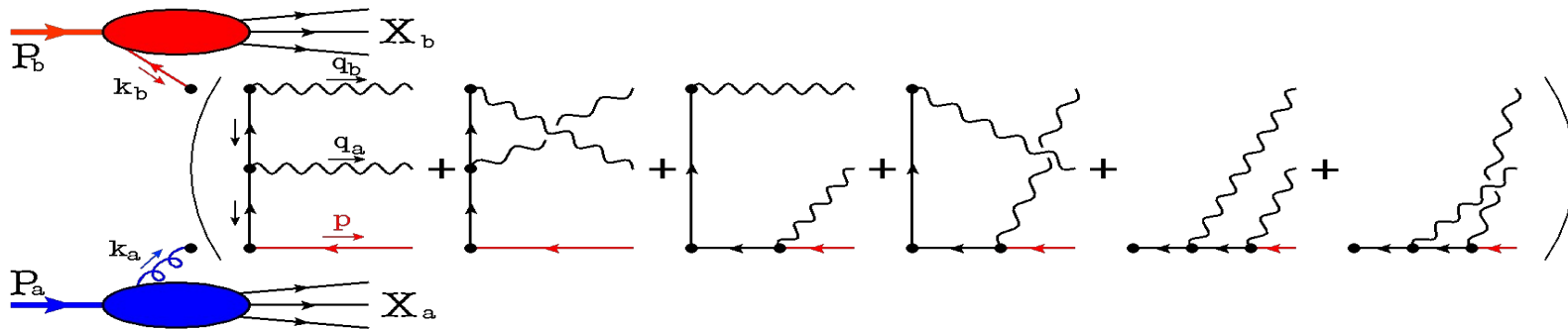
Similar to Callan-Gross relation in DIS

High - q_T of the photon pair

Same strategy as for DY:
quark – antiquark scattering:



quark – gluon scattering:



However: No model-independent angular decomposition!

Diphoton angles enter the partonic cross section in numerator and denominator
 → All angular dependencies are allowed.

Analytical results

What happens if q_T becomes smaller? → Expansion in $1/q_T$...

Drell-Yan:
$$d\sigma_{UU} \sim \left((1 + \cos^2 \Theta) F_{UU}^1 + (1 - \cos^2 \Theta) F_{UU}^2 + \sin 2\Theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \Theta \cos 2\phi F_{UU}^{\cos 2\phi} \right)$$

Order $1/q_T^2$:
$$F_{UU}^1 \propto \frac{\alpha_s}{q_T^2} \left[-C_F(2 \ln \frac{q_T^2}{Q^2} + 3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq} \otimes \bar{q} + P_{qg} \otimes g)(x_2) + \{x_1 \leftrightarrow x_2\} \right]$$

Order $1/q_T$:
$$F_{UU}^{\cos \phi} \propto \frac{\alpha_s}{q_T} \left[q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}_{qg} \otimes g)(x_2) - \{x_1 \leftrightarrow x_2\} \right]$$

Order $1/q_T^0$:
$$F_{UU}^2 = 2F_{UU}^{\cos 2\phi} = -C_F(2 \ln \frac{q_T^2}{Q^2} + 3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq} \otimes \bar{q} + P'_{qg} \otimes \bar{q})(x_2) + \{x_1 \leftrightarrow x_2\}$$

DiPhoton production:

leading order follows "TMD - rule"
$$\sigma^{DP} = \frac{2}{\sin^2 \theta} \sigma^{DY} (e_q \rightarrow e_q^2) + \mathcal{O}(1/q_T)$$

Order $1/q_T$:

$$\sigma^{DP} \sim (\sin 2\theta \cos \phi) \frac{\alpha_s}{q_T} \left[\frac{4}{\sin^4 \theta} q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}_{qg} \otimes g)(x_2) + \frac{2}{\sin^2 \theta} q(x_1)(\tilde{P}_{qq} \otimes \bar{q} + \tilde{P}_{qg}^a \otimes g)(x_2) - \{x_1 \leftrightarrow x_2\} \right]$$

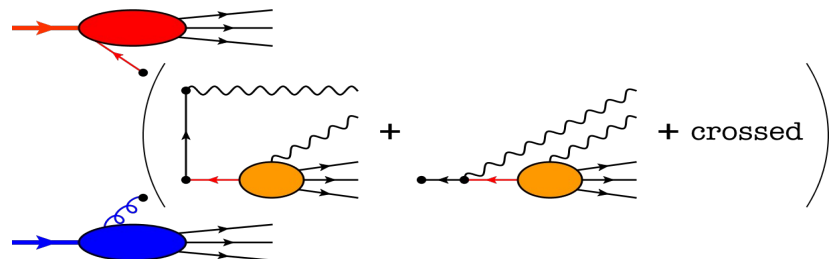
Order $1/q_T^0$:

quark – antiquark contr. to diphoton production
$$\sigma_{q\bar{q}}^{DP} \propto \frac{(4 - \cos^2 \theta \sin^2 \theta) \cos 2\phi + (3 \sin^2 \theta + 2 \cos^2 \theta)}{\sin^4 \theta} F_{UU, q\bar{q}}^{2, DY}$$

quark – gluon scattering → collinearly divergent → need photon fragmentation function

Isolation of direct photons

Hide collinear divergence in photon fragmentation function:



- Potentially endangers TMD-factorization
- Photon FF unknown

Circumvent the problem → **Isolation** [Frixione PLB 429,369; Frixione, Vogelsang NPB 568, 60]

Define "cone" in rapidity – azimuthal angle space:

$$\mathcal{C}_\gamma(R_0) \equiv \left\{ (\eta, \phi) \mid \sqrt{(\eta - \eta_\gamma)^2 + (\phi - \phi_\gamma)^2} \leq R_0 \right\}$$

1. "Traditional" **Criterion**: allow certain percentage of hadronic energy inside the cone

$$E_T(R_0) \leq \epsilon q_{T\gamma}$$

- Boost-invariant criterium.
- Infra-red safe.
- Allows certain contribution from fragmentation photons.

2. "Improved" **Criterion**: dynamically generated cone $R < R_0$

$$E_T(R) \leq \epsilon_\gamma q_{T\gamma} f(R)$$

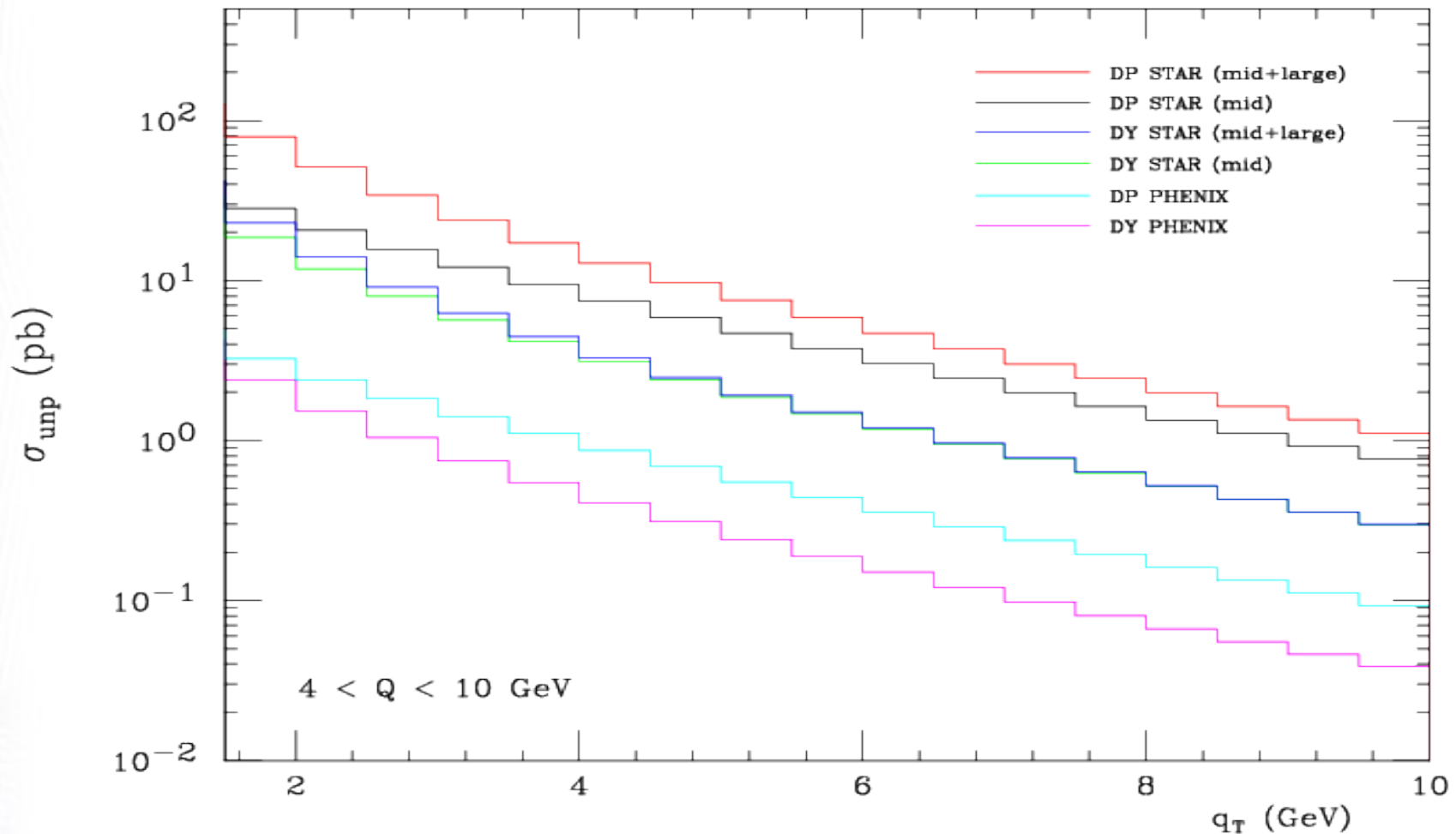
$$\lim_{R \rightarrow 0} f(R) = 0$$

- Boost-invariant criterium.
- Infra-red safe.
- Cuts out *all* fragmentation photons.
- Experimentally harder → needs high resolution in η and ϕ .

Numerical results

Predictions for the q_T -tail
for STAR and PHENIX including isolation:

$$\sigma_{\text{unp}} = \int_{\text{cuts}} dy dQ^2 dq_T d\varphi_q d\Omega \frac{d\sigma}{dy dQ^2 dq_T d\varphi_q d\Omega}$$



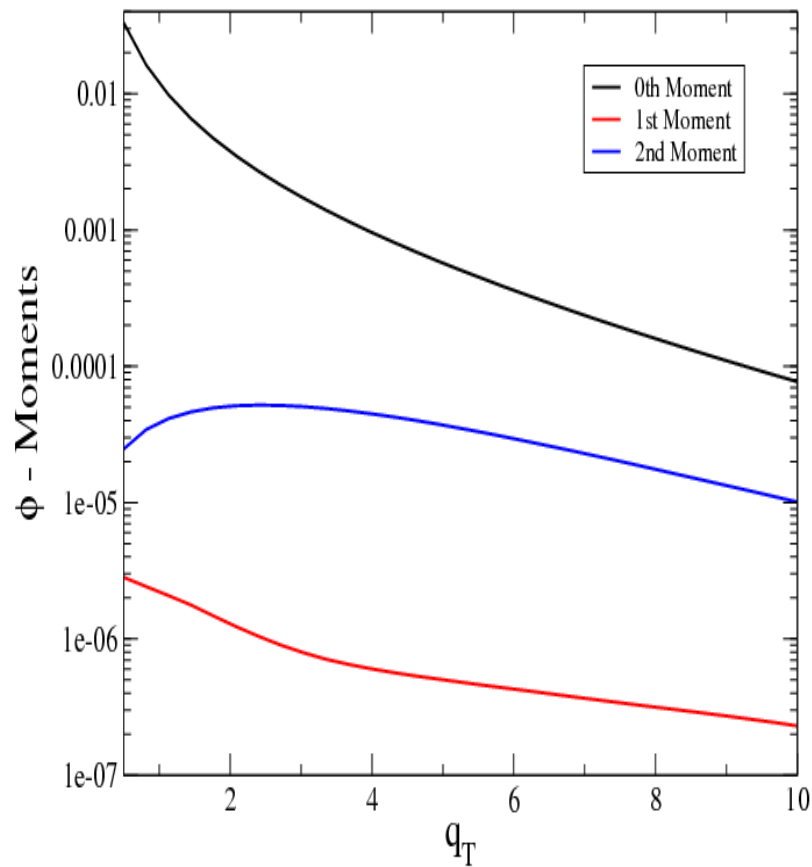
Also at larger q_T \rightarrow Diphoton production rate about 5 times larger than Drell-Yan

Define phi moments:

$$\langle \cos(n\phi) \rangle = \int_0^{2\pi} d\phi \cos(n\phi) \frac{d\sigma}{dy dQ^2 d^2q_T d\Omega}$$

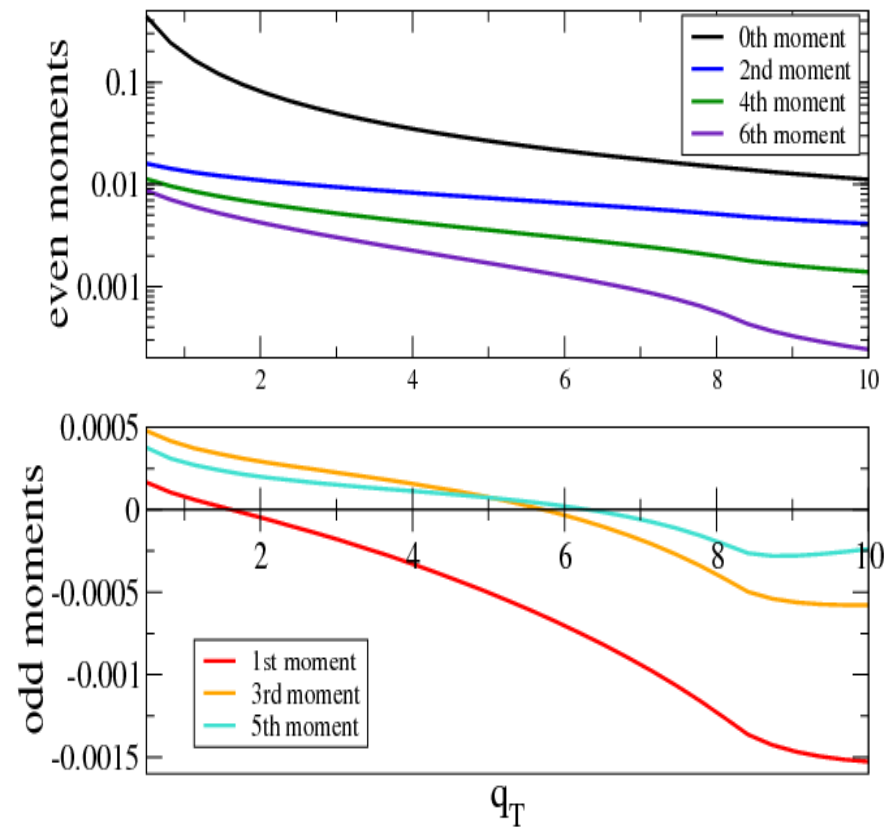
ϕ - Moments of the unpol. Drell-Yan Cross Section vs. q_T

CS at [GeV]: $S = 200^2, Q = 10, y = 0.1, \theta = \pi/4$



ϕ - Moments of the unpol. Diphoton Cross Section vs. q_T

CS at [GeV]: $S = 200^2, Q = 10, y = 0.1, \theta = \pi/4$



Summary:

- **Drell-Yan cross section can be decomposed model-independently into angular structure function, not possible for photon pair production**
- **TMD-factorization at low q_T : Photon pair production similar to Drell-Yan**
- **Sivers effect similar in Photon pair production, but higher production rate**
→ simultaneous measurement
- **Collinear factorization at larger q_T : all azimuthal modulations possible for photon pair production in contrast to lepton pair production**
- **Expansion to smaller q_T : Azimuthal behaviour partly recovered**
→ photon fragmentation or Isolation needed.