

TMD 2010 — Workshop on Transverse Momentum Distributions
ECT, Trento, Italy, 21–25 June 2010*

Relations among TMDs, and orbital angular momentum

Peter Schweitzer

University of Connecticut

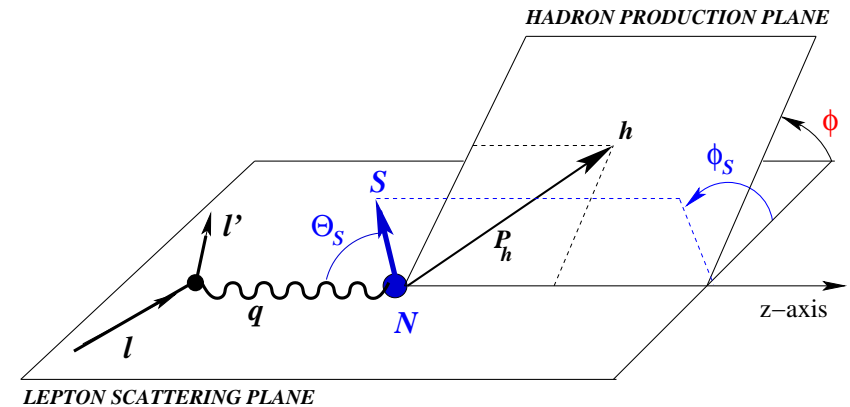
works with H.Avakian, A.Efremov, A.Metz, T.Teckentrup, O.Teryaev, F.Yuan, P.Zavada

Overview:

- motivation
- approximations for TMDs in QCD?
- useful relations among TMDs from models?
- access information from TMDs on orbital angular momentum (OAM)?
- conclusions

Motivation

in SIDIS



$$\frac{d\sigma}{d\phi} = 8 \cdot \left(\frac{M}{Q}\right)^0 + 8 \cdot \left(\frac{M}{Q}\right)^1 + 2 \cdot \left(\frac{M}{Q}\right)^2 = \mathbf{18 \text{ structure functions}}$$

\Updownarrow
8 TMDs
(factorization!)

\Updownarrow
16 TMDs
(factorization?)

& frag. funct.

- explain structure functions with TMDs (k_T -factoriz., $P_{h\perp} \ll Q$)?
- each **structure function** \rightarrow different aspect of nucleon structure! Which?
- what can we learn from TMDs about OAM?

models help to address such questions, **how to use models?**

- **directly!**

produce numbers (needed: JLab 12, EIC, ...)

(issues: evolution, credibility of model, range of reliability

Boffi, Efremov, Pasquini, PS, 2009. Bacchetta et al, 2010)

- **indirectly!**

explore **relations** among TMDs supported in (several!) models.

Less model-sensitivity, and more reliability(?)

- **conceptually!**

source of '**intuition**' & '**inspiration**'

e.g., Gauss Ansatz (works, but why?)

e.g., intuition on nucleon structure (s, p, d-wave)

e.g., how to access **orbital angular momentum** with TMDs

- **issues**

definition, where is soft-factor, evolution, theoretical consistency

2. Approximations for TMDs in QCD?

(to best of our understanding: in QCD no exact relations)

Wandzura-Wilczek approximation: $g_T^q(x) = \int_x^1 \frac{dy}{y} g_1^q(y) + \tilde{g}_T^q(x)$ (1977)

$\tilde{g}_T^q(x) = \langle \bar{q}gq \rangle + m_q \langle \bar{q}q \rangle$ suppressed in instanton vacuum Balla, Polyakov, Weiss 1997
small in experiments SLAC, JLab (Accardi et al, 2009)
negligible on lattice Göckeler et al, 2000

Analog Wandzura-Wilczek-type approx. for TMDs?

Avakian et al, 2007; Metz, PS, Teckentrup 2009; Accardi et al 2009

Important examples:

- $g_{1T}^{\perp(1)}(x) \approx x \int_x^1 \frac{dy}{y} g_1^q(y) \rightarrow A_{LT}^{\cos(\phi-\phi_S)}$ Kotzinian, Parsamyan, Prokudin, 2006
- $h_{1L}^{\perp(1)}(x) \approx -x^2 \int_x^1 \frac{dy}{y^2} h_1^q(y) \rightarrow A_{UL}^{\sin(2\phi)}$ Avakian et al, 2007

no contradiction to data, but large error bars
(HERMES, JLab, COMPASS)

test at CLAS 12:

Can predict $A_{UL}^{\sin(2\phi)} \propto h_{1L}^{\perp(1)}(x) H_1^{\perp}$ with $h_{1L}^{\perp(1)}(x) \approx -x^2 \int_x^1 \frac{dy}{y^2} h_1(y)$

HERMES, COMPASS, JLab, (Belle): $A_{UT}^{\sin(\phi+\phi_s)} \propto h_1(x) H_1^{\perp}$

Robust prediction!

(Avakian et al, 2007)

projections H.Avakian

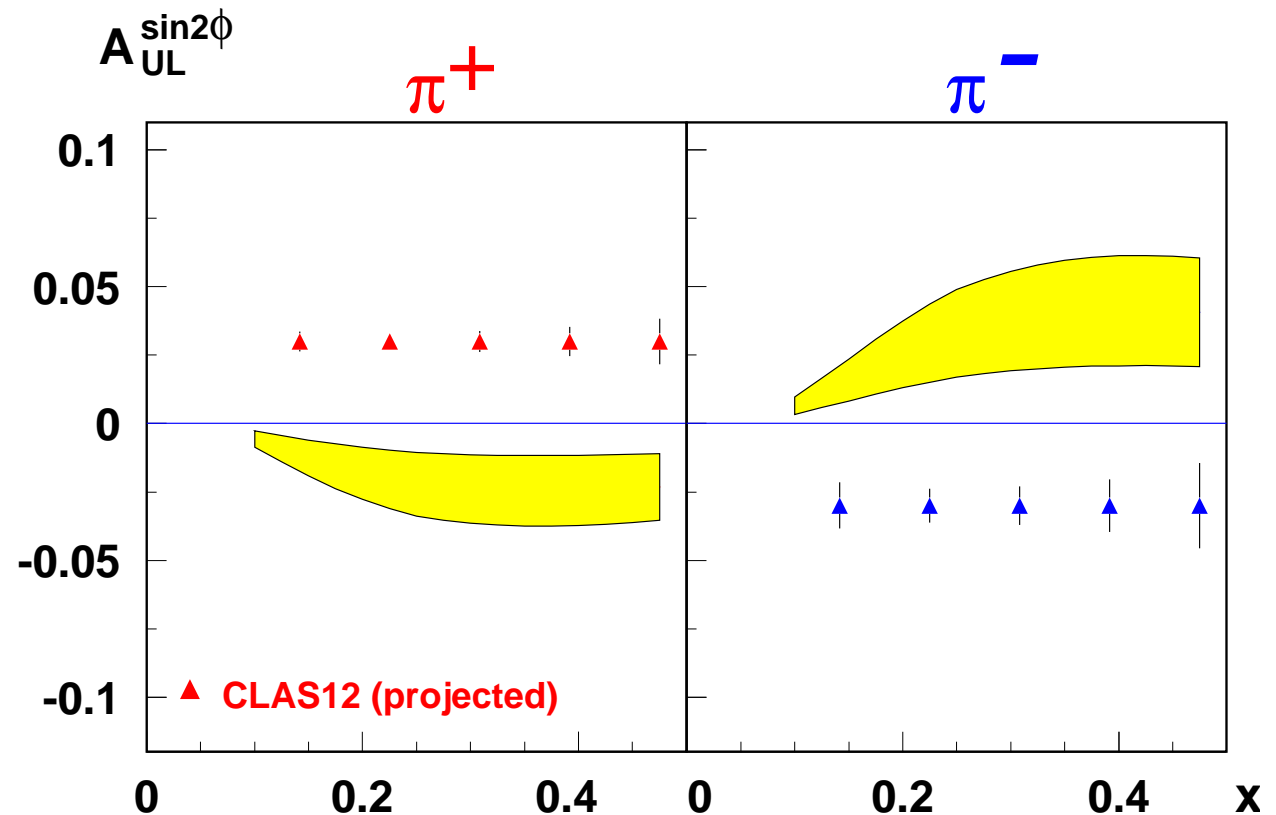
error bands will decrease

(uncertainty of $H_1^{\perp}(z, K_T)$)

Efremov, Goeke, PS, 2006

Anselmino et al, 2007

Will see, and learn!



If WW-type approximations work

- have to understand case by case
- welcome, especially twist-3 (assuming factorization). E.g. monster SSA:

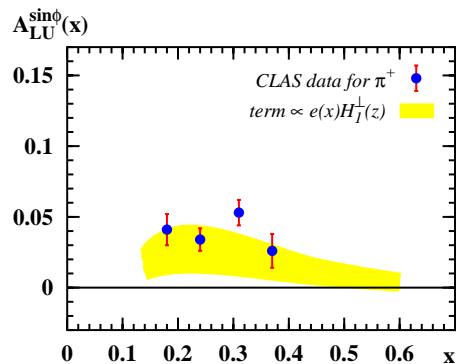
$$A_{UT}^{\sin\phi_S} \propto \frac{M}{Q} \left(f_T D_1 + h_1 \tilde{H} + h_T H_1^\perp + g_{1T} \tilde{G}^\perp + h_T^\perp H_1^\perp + f_{1T}^\perp \tilde{D}^\perp \right) = \text{seen}$$

Word of caution: Necessary condition: (LO \neq 0) \gg (NLO)

But it may happen that LO = 0 ... !

Nice example:

$$A_{LU}^{\sin\phi_h} \propto \frac{M}{Q} \left(\tilde{e} H_1^\perp + f_1 \tilde{G}^\perp + \tilde{g}^\perp D_1 + h_1^\perp \tilde{E} \right) \propto \langle \bar{q} g q \rangle + m_q \langle \bar{q} q \rangle \stackrel{?}{\approx} 0$$



at CLAS, HERMES \neq 0 , compatible with $\frac{\langle \bar{q} g q \rangle}{\langle \bar{q} q \rangle} = \text{small}$

glimpse on $e^q(x)$, 'transverse forces' (Burkardt, 2008)?

there are doubts (talk by Harut Avakian)

more data + studies needed, **in progress**

3. Relations in models

We have to specify what 'model' means.

here: **quark model** = model without gauge-field degrees of freedom

e.g.

- spectator models (Jakob, Mulders, Rodrigues, 1997; ...)
- light-cone constituent quark models (Barbara Pasquini et al)
- parton model with intrinsic angular motion (Petr Zavada, and et al)
- non-relativistic limit (Courtoy et al; Efremov, PS, Teryaev, Zavada)
- chiral quark soliton model (Diakonov et al, Wakamatsu)
- other models

Are there relations among TMDS in such models?

Yes!

There must be!

LIRs

general unintegrated correlator $\Phi(P, p, S, n_-)$ described by **32 independent** “amplitudes”: $\underbrace{A_1-A_{12}}_{\text{from } P, S, p}$ and $\underbrace{B_1-B_{20}}_{\text{and } n_-}$ due to gauge link **absent in quark models!**

(Tangerman, Mulders 1994, Kundu, Metz 2001;
Goeke, Metz, Poblitsa, Polyakov 2003)

more TMDs than amplitudes \longrightarrow **'LIRs'**

T-even: (6 twist-2) + (8 twist-3) \Leftrightarrow (9 amplitudes) + (5 LIRs) :

$$g_T(x) \stackrel{\text{LIR}}{=} g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x),$$

$$h_L(x) \stackrel{\text{LIR}}{=} h_1(x) - \frac{d}{dx} h_{1L}^{\perp(1)}(x),$$

$$g_L^{\perp}(x) \stackrel{\text{LIR}}{=} -\frac{d}{dx} g_T^{\perp(1)}(x),$$

$$h_T(x) \stackrel{\text{LIR}}{=} -\frac{d}{dx} h_{1T}^{\perp(1)}(x),$$

$$(h_T - h_T^{\perp})(x, p_T) \stackrel{\text{LIR}}{=} h_{1L}^{\perp}(x, p_T).$$

hold in any quark model!

- valuable cross check for models!
- useful approximations in nature?
We do not know, will see.
- What we know:
LIRs \Leftrightarrow 'WW-type approximation'
(Metz, PS, Teckentrup, 2009)

Can there be other model relations?

quark model: B_i absent, A_i ($i=4,5,12$ T-odd, absent) \rightarrow 9 T-even amplitudes

- If all 9 A_i distinct \rightarrow only LIRs
- If some A_i related \rightarrow further quark model relations! **Depends on model!**

Some **fascinating(!)** relations found, **in many (not all) quark models**

many $\stackrel{\text{def}}{=}$

- spectator model Jakob et al, 1997
- spectator model Meissner, Metz, Goeke 2007
- MIT bag model Avakian, Efremov, PS, Yuan, 2008
- light front constituent model Pasquini, Cazzaniga, Boffi 2008
- covariant parton model with quark orbital motion Efremov et al 2008
- some spectator model versions of Bacchetta, Conti, Radici 2008

not all $\stackrel{\text{def}}{=}$

- other spectator model versions of Bacchetta, Conti, Radici 2008

look at bag model

no gluons, hence:

$$\left((6 \text{ twist-2}) + (8 \text{ twist-3}) \right) \text{ TMDs} = (9 A_i) + (5 \text{ LIRs}) \checkmark$$

moreover:

9 linear + 2 non-linear relations

complete set of model relations (Avakian, Efremov, PS, Yuan, 2010)

supported by many (not all) models!

Bag model

- 3 non-interacting quarks inside spherical cavity
- quark-wave-functions with s - and p -waves
- use SU(6) spin-flavour-symmetry:
 - unpolarized functions $f_1^q = N_q f_1 \dots$ with $N_u = 2, N_d = 1$
 - polarized functions $g_1^q = P_q g_1 \dots$ with $P_u = \frac{4}{3}, P_d = -\frac{1}{3}$

Linear relations I-III: connect unpolarized and polarized TMDs, $D^q = \frac{P_q}{N_q}$

$$\begin{aligned}
 (I) \quad & \mathcal{D}^q f_1^q(x, k_\perp) + g_1^q(x, k_\perp) = 2h_1^q(x, k_\perp) && \text{Jaffe, Ji, 1991} \\
 (II) \quad & \mathcal{D}^q e^q(x, k_\perp) + h_L^q(x, k_\perp) = 2g_T^q(x, k_\perp) && \text{Signal, 1995} \\
 (III) \quad & \mathcal{D}^q f^{\perp q}(x, k_\perp) = h_T^{\perp q}(x, k_\perp) && \text{new!}
 \end{aligned}$$

Linear relations IV-VI: connect two polarized TMDs

$$\begin{aligned}
 (IV) \quad & g_{1T}^{\perp q}(x, k_\perp) = -h_{1L}^{\perp q}(x, k_\perp) && \text{Jakob et al 1997} \\
 (V) \quad & g_T^{\perp q}(x, k_\perp) = -h_{1T}^{\perp q}(x, k_\perp) && \text{new!} \\
 (VI) \quad & g_L^{\perp q}(x, k_\perp) = -h_T^q(x, k_\perp) && \text{new!}
 \end{aligned}$$

Linear relations VII-IX: connect three polarized TMDs

$$\begin{aligned}
 (VII) \quad & g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) && \text{pretzelosity-relation} \\
 (VIII) \quad & g_T^q(x, k_\perp) - h_L^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) && \text{new!} \\
 (IX) \quad & h_T^q(x, k_\perp) - h_T^{\perp q}(x, k_\perp) = h_{1L}^{\perp q}(x, k_\perp) && \text{new! (LIR)}
 \end{aligned}$$

Non-linear relations: connect polarized chiral-odd (or even) TMDs

$$h_1^q(x, k_\perp) h_{1T}^{\perp q}(x, k_\perp) = -\frac{1}{2} [h_{1L}^{\perp q}(x, k_\perp)]^2,$$

$$g_T^q(x, k_\perp) g_T^{\perp q}(x, k_\perp) = \frac{1}{2} [g_{1T}^{\perp q}(x, k_\perp)]^2 - g_{1T}^{\perp q}(x, k_\perp) g_L^{\perp q}(x, k_\perp).$$

Remarks

- Lin. relations (I-III) connecting unpol. and pol. TMDs: **'weak'** only in simplest models with SU(6) symmetry (bag, ...)
- SU(6) not sufficient: spect. model **Jakob et al** (only for $m_a \rightarrow m_s \Leftrightarrow N_c \rightarrow \infty$)
In nature? Maybe very roughly.
- Lin. rel. IV-IX (connect only polarized TMDs): **stronger** do not need SU(6) (valid in model by Petr Zavada)
→ valid in larger class of models! **(in many, not all models)**
- the **'new!'** among them, are not so new: valid in **Jakob et al 1997!**

Warning:

quark-model relations hold only in quark models!
as expected, spoiled by gluons:

- **Quark target model** (explicit gluons!)
light-cone Hamiltonian approach
step closer to real world (there are gluons)

Meissner, Metz, Goeke, 2007
A. Mukherjee, ...

none of these quark model relations holds!

⇒

indication that even if relations were true at some (initial) scale,
spoiled by evolution

- **QCD**
no relations at all!
approximately satisfied? Interesting to see!

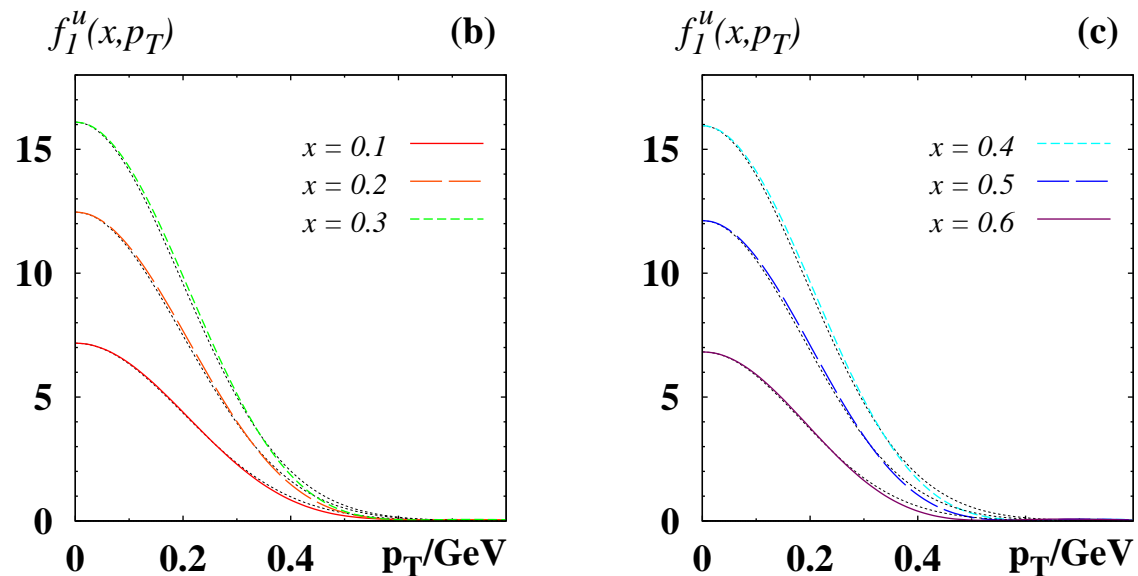
Lessons from models I : p_T -dependence

bag model: relativistic, non-Gaussian p_T , but(!):
(Avakian, Efremov, PS, Yuan, arXiv:1001.5467)

Gaussian $f_1^q(x, p_T) = f_1^q(x, 0) \exp(-p_T^2 / \langle p_T^2(x) \rangle)$ **correct** at $p_T = 0$!

correct also in some region $p_T > 0$ due to continuity

Expect deviations at some point. At which?

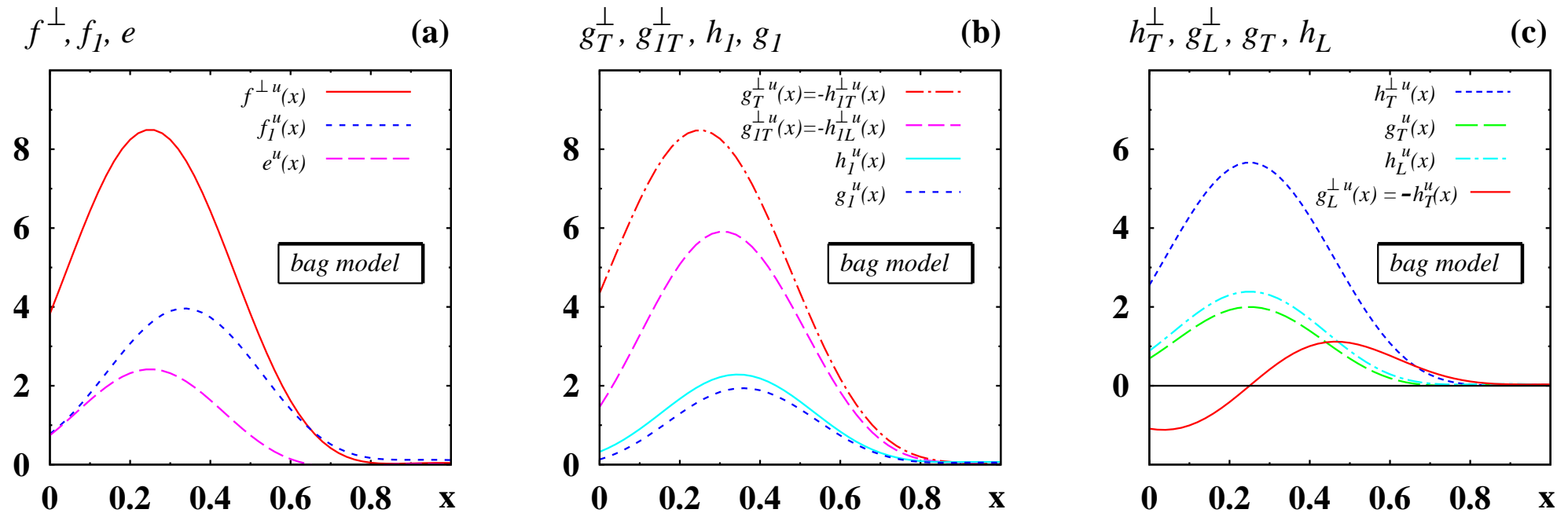


colored lines: exact model result
black-dotted: Gaussian approx.

similar for other TMDs

Gauss works!

Lessons from models II : relative size of TMDs in bag model



especially $f^\perp(x)$, $g_{1T}^\perp(x)$, $h_{1T}^\perp(x)$ large, but no positivity constraints

in cross sections moderated: $\frac{p_T^i}{M} g_{1T}^\perp$, $\frac{p_T^i p_T^j}{M^2} h_{1T}^\perp$ and in model $\langle p_T \rangle \ll M$

But why so large?

Instructive: **non-relativistic** ("symbolic") limit

Lessons III: non-relativistic limit (Efremov, PS, Teryaev, Zavada 2009, byproduct)

(i) covariant Zavada parton model,

(ii) $\lim_{\text{non-rel}} G(\vec{p}) \rightarrow \delta^{(3)}(\vec{p})$

$$N_u = \frac{1}{2}(N_c + 1), \quad N_d = \frac{1}{2}(N_c - 1),$$

$$P_u = \frac{1}{6}(N_c + 5), \quad P_d = -\frac{1}{6}(N_c - 1),$$

$$\lim_{\text{non-rel}} f_1^q(x, p_T) = N_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} g_1^q(x, p_T) = P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h_1^q(x, p_T) = P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} g_{1T}^{\perp q}(x, p_T) = N_c P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h_{1L}^{\perp q}(x, p_T) = -N_c P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T),$$

$$\lim_{\text{non-rel}} h_{1T}^{\perp q}(x, p_T) = -\frac{N_c^2}{2} P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T).$$

delta-functions smeared out in realistic non-relativistic models
(Courtoy, Fratini, Scopetta, Vento)

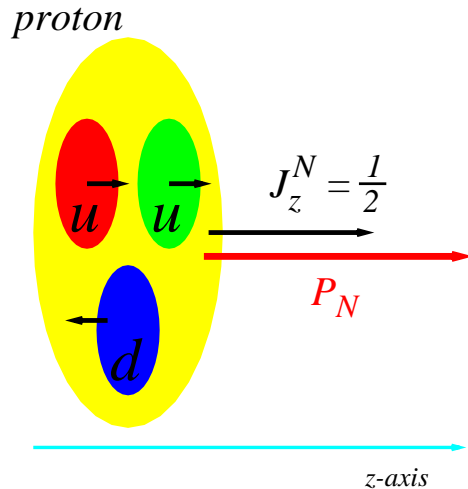
$$\Rightarrow g_1^q(x) = h_1^q(x) \checkmark \quad \& \quad h_{1T}^{\perp(1)q}(x) = 0 \checkmark$$

$$g_{1T}^{\perp q}(x) = N_c g_1^q(x) \checkmark \quad \& \quad h_{1T}^{\perp q}(x) = -\frac{1}{2} N_c^2 h_1^q(x) \checkmark$$

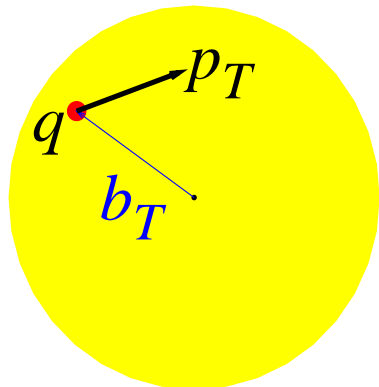
'enhancement' since $M = N_c m_q$ chosen to compensate dimension of p_T in $\frac{p_T^j}{M}, \frac{p_T^j p_T^k}{M^2}$

4. Spin structure of nucleon

naive picture:



longitudinally pol. nucleon
moving fast in z -direction



nucleon moving towards us

realistic picture: GPDs

$$\frac{1}{2} = J_z^N = \underbrace{S_z^Q + L_z^Q}_{J_z^Q} + J_z^g \quad ?$$

$$J_z^q = \int dx \int d^2 b_T (H^q(x, b_T) + E^q(x, b_T))$$

(Ji, 1997)

$$S_z^q = \int dx \int d^2 b_T \widetilde{H}^q(x, b_T)$$

GPD(x, b_T) in impact parameter space (M.Burkardt)

$$\Rightarrow J^q - S^q = L^q \quad (\text{issues in gauge theory!})$$

GPD \Rightarrow OAM $\stackrel{?}{\Leftarrow}$ TMDs
expect connection!?

Promising candidate h_{1T}^\perp

- Definition: (j transverse to +) (Mulders, Tangerman 1995)

$$\frac{1}{2} \text{tr}[i\sigma^{+j}\gamma_5 \phi(x, \vec{p}_T)] = S_T^j h_1 + S_L \frac{p_T^j}{M_N} h_{1L}^\perp + \frac{(p_T^j p_T^k - \frac{1}{2} \vec{p}_T^2 \delta^{jk}) S_T^k}{M_N^2} h_{1T}^\perp + \frac{\varepsilon^{jk} p_T^k}{M_N} h_1^\perp$$

- inequalities $|h_{1T}^{\perp q}(x, p_T)| + |h_1^q(x, p_T)| \leq f_1^q(x, p_T)$ (Bacchetta et al. 1999)

- describes non-sphericity of “transverse spin distribution” (G. Miller, Burkardt) (“pretzel”, “baggle”, “peanut”)

- requires nucleon wave-function components with $\Delta L = 2$ (M. Burkardt, 2007)

- some (not all) quark models: (Avakian et al, Bacchetta et al,

$$h_{1T}^{\perp(1)q}(x, p_T) = g_1^q(x, p_T) - h_1^q(x, p_T)$$

Efremov et al, Jakob et al,
Pasquini et al, She et al)

“measure-of-relativity”

notation $h_{1T}^{\perp(1)q}(x, p_T) \equiv \frac{p_T^2}{2M^2} h_{1T}^{\perp q}(x, p_T)$, $h_{1T}^{\perp(1)q}(x) = \int dp_T h_{1T}^{\perp(1)q}(x, p_T)$

relation model-dependent ...

- not valid in quark-target model $h_{1T}^{\perp q} = 0, h_1^q - g_1^q \neq 0$ (Meißner, Metz, Goeke, 2007)
- not supported in some versions of spectator models (Bacchetta et al 2008)

... but inspiring

- known in light-cone SU(6) quark-diquark model (Ma and Schmidt, 1998)

$$h_1^q(x) - g_1^q(x) = L_z^q(x), \quad \int dx L_z^q(x) = L_z^q$$

direct calculation in light-cone SU(6) quark-diquark model (She, Zhu, Ma, 2009)

$$h_{1T}^{\perp(1)q}(x, p_T) = g_1^q(x, p_T) - h_1^q(x, p_T) \quad \text{pretzelosity-relation!}$$

- light-cone SU(6) quark-diquark model She, Zhu, Ma, 2009

$$L_z^q = - \int dx h_{1T}^{\perp(1)q}(x)$$

first connection of TMDs and OAM! **But model!**
 take different model: you get different result (?) let's see:

- bag model uses SU(6)

$$L_z^q = - \int dx h_{1T}^{\perp(1)q}(x)$$

Avakian, Efremov, PS, Yuan, 2010

- covariant parton model no SU(6)-symmetry,

$$L_z^q = - \int dx h_{1T}^{\perp(1)q}(x)$$

Efremov, PS, Teryaev, Zavada, 2010

- non-relativistic limit $\lim_{\text{non-rel}} h_{1T}^{\perp q}(x, p_T) = -\frac{N_c^2}{2} P_q \delta\left(x - \frac{1}{N_c}\right) \delta^{(2)}(\vec{p}_T)$

$$0 = -0$$

trivial but consistent byproduct in op. cit.

Questions:

- How can L_z^q and pretzelocity be related?

$$\text{OAM} = \langle N(S_L) | \dots | N(S_L) \rangle$$

$$\text{pretzelocity} = \langle N(S_T) | \dots | N(S_T) \rangle$$

$$\text{rotation: } |N(S_L)\rangle = U_{90^\circ} |N(S_T)\rangle$$

$$\nexists \hat{O}_{\text{OAM}} = \hat{O}_{\text{pretzelocity}}, \text{ relation at the level of matrix-elements}$$

- Does the result depend on choice of OAM definition?

Here (no-gauge-field theory) for L_z^q no ambiguity

(Jaffe-Manohar = Ji, M. Burkardt and H. BC, 2009)

- What are model limitations? Valid in models with $L \geq 2$ (d -wave, ...)?

→ Cédric Lorcé, Barbara Pasquini, ...

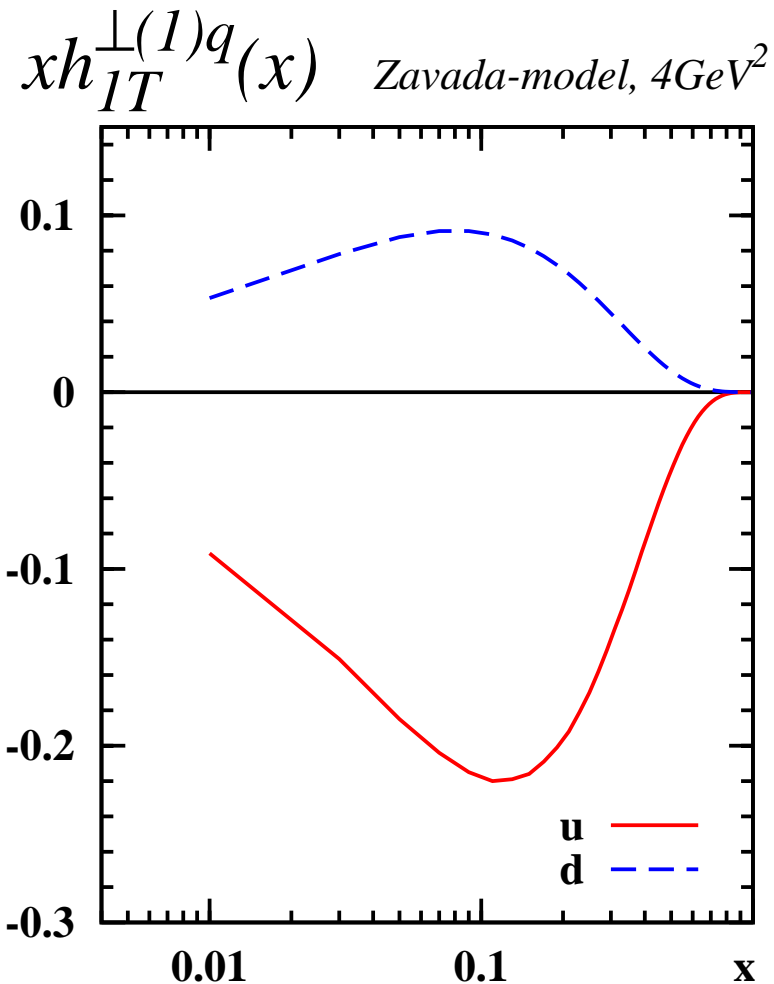
- What happens when we have gluons?

No relation! (Meißner, Metz, Goeke, 2007)

Jaffe-Manohar vs. Ji matters (Burkardt, BC, 2009)

- How does it look like?

covariant parton model with intrinsic motion (→ Petr Zavada)
 at $Q^2 = 4 \text{ GeV}^2$ describes q, \bar{q} (leaves room for gluons)
 (Efremov, PS, Teryaev, Zavada 2009)



→ talk by Petr Zavada

opposite sign to transversity

Glimpse (through parton-model-glasses)

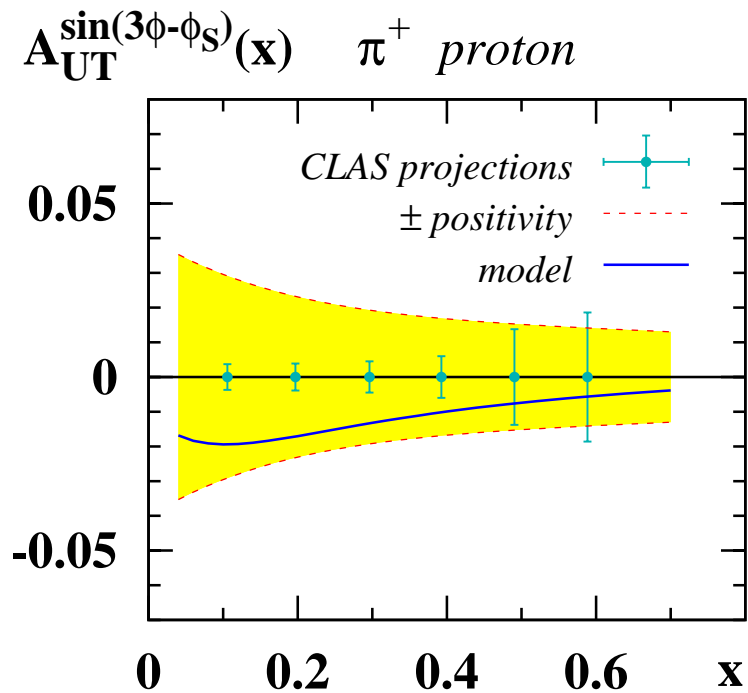
on $(-1) \times \text{OAM} ???$ Will see ...

- Can we access pretzelosity? in semi-inclusive DIS

$$A_{UT}^{\sin(3\phi-\phi_S)} = \frac{h_{1T}^\perp H_1^\perp}{f_1 D_1} \sim 0 \text{ within error bars}$$

preliminary COMPASS (deuteron)
HERMES (proton)

Zavada-model: Efremov, PS, Teryaev, Zavada



covariant parton model with
 rotationally symmetric parton motion
 $G(Pp/M) = G(p^0)$ in rest frame,
 Interesting because $h_1^u > g_1^q$

→ sizeable $h_{1T}^{\perp(1)q}(x)$

positivity bound Bacchetta et al, 1999

projections CLAS12 H.Avakian

Conclusions

- in QCD: **WW-type approximations** (?)
much work, interesting questions, how large is $\langle \bar{q}gq \rangle$
- in quark models (powerful test) : **LIRs (!)** in QCD: \Leftrightarrow WW-type (work?)
- **quark model relations**: help intuition; used carefully they work(!)

 $\text{sign}(g_{1T}^\perp) = -\text{sign}(h_{1L}^\perp)$ supported by all models, lattice, Hall-A & B
Jakob et al, 1997 (\rightarrow talk by Harut Avakian)
- **why** do quark model relations work, in many (not all) models?
Cédric Lorcé & Barbara Pasquini (\rightarrow talk by Cédric Lorcé)
- first connection **TMD vs. OAM** (in models, gauge fields absent)

- “dual” picture of OAM

in (naive, happy) quark-model world:

$$\begin{aligned}
 L_z^q &= \int dx \int d^2 b_T \text{GPDS}^q(x, b_T) && \text{(Ji sum rule, QCD)} \\
 &= - \int dx \int d^2 p_T h_{1T}^{\perp(1)q}(x, p_T) && \text{ (“pretzelosity sum rule”, models)}
 \end{aligned}$$

certainly not that simple in QCD (but used carefully: useful for intuition)
 will learn: present & **future data** (JLab12, EIC, ...) on **excl. + SIDIS** (!)
 (spin decomp. in quark models: Thomas, Wakamatsu, gluons, evol.)

- “dual” picture of OAM

in (naive, happy) quark-model world:

$$\begin{aligned} L_z^q &= \int dx \int d^2 b_T \text{GPDS}^q(x, b_T) && \text{(Ji sum rule, QCD)} \\ &= - \int dx \int d^2 p_T h_{1T}^{\perp(1)q}(x, p_T) && \text{ (“pretzelosity sum rule”, models)} \end{aligned}$$

certainly not that simple in QCD (but used carefully: useful for intuition)
will learn: present & **future data** (JLab12, EIC, ...) on **excl. + SIDIS** (!)

Thank you!!