Calculation of alpha decay using complex energy basis

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Many-Body Open Quantum System: From Atomic Nuclei to Quantum Dots
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Goals

- Obtain a microscopic description of the α decay in the Shell Model framework using complex energy basis.
  - Reproduce experimental reduce width
- Calculation of spectrosocpic factor in the region of ^{100}Sn.
  - Comparing α decay spectrosocpic factor of ^{104}Te versus ^{212}Po
  - Shows the trend of the spectroscopy factor in Tellurium isotopes.

Very short review about α reduce width calculation in ^{212}Po(g.s.)

About reduce width calculations

1959 J. O. Rasmussen. Phys. Rev. **113**, 1593. $\gamma^2 \leftarrow WKB + T_{1/2}^{exp}$
Wentzel-Kramers-Brillouin

1960 H. J. Mang. Phys. Rev. **119**, 1069.
- Good agreement with experiment for relative γ^2
- Absolute γ^2 of the order of 10^{-4}

1960 K. Harada. Prog. Theor. Phys. **26**, 667. Shows that configuration mixing enhance γ^2

1979 I. Tonozuka, A. Arima. Nucl. Phys. A **323**
- $\gamma^2_{pure}/\gamma^2_{exp} = 3 \times 10^{-5}$
- $\gamma^2_{GH}/\gamma^2_{exp} = 3 \times 10^{-4}$
- $\gamma^2_{7\hbar\omega}/\gamma^2_{exp} = 1.3 \times 10^{-2}$
- $\gamma^2_{13\hbar\omega}/\gamma^2_{exp} = 4.3 \times 10^{-2}$

1985 G. Dodig-Crnkovic, F. A. Janouch, R. J. Liotta. Nucl. Phys. A **501**, 533. Proton-neutron in the framework of Multistep Shell Model: $\gamma^2/\gamma^2_{exp} \approx 10^{-1}$

Very short review about α reduce width calculation in ^{212}Po(g.s.)

About reduce width calculations

1992 K. Varga, R. G. Lovas, R. J. Liotta. Phys. Rev. Lett. **69**, 37. Shell Model + Cluster model.

- Reproduce the experimenal decay width and incertainty $\pm$10%
- The amount of $\alpha + ^{208}Pb$ clustering in ^{212}Po is around 30%

2010 A. Astier, et al. Phys. Rev. Lett. **104**, 42701. Enhance $E1$ transitions $\rightarrow \alpha$ clustering in ^{212}Po

Single particle absolute alpha decay width in ^{212}Po

Model Hamiltonian

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$

- $V(r)$ =W-S + Coulomb
  - $r_0 = 1.315 fm, \; a = 0.65 fm$ (R. M. DeVries, Phys. Rev. Lett. **37**, 481, 1976)
  - V_0=143.77 MeV ($E_\alpha = Q$ + (screening $\approx$ 0.3 %) = 8.817 MeV)
- Boundary condition $u_\alpha(r \to \infty) \to H^+_{l=0}(\chi, kr)$
  - H^+_l outgoing Coulomb-Hankel function
  - $E_\alpha = \frac{\hbar^2}{2\mu} k^2$. Complex energy: $E_\alpha = E - i \, \frac{\Gamma_{sp}}{2}$

Current expression

Single particle absolute alpha decay width in ^{212}Po

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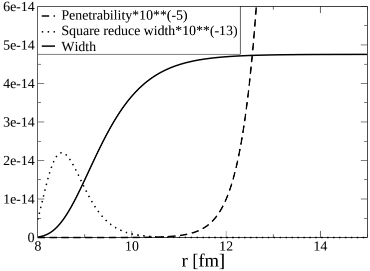
Current expression

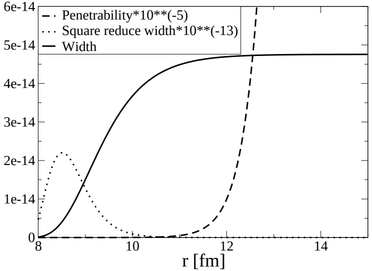
$$\Gamma_{sp} = \frac{\hbar^2 Re(\kappa)}{\mu} \frac{|u_\alpha(r)|^2}{|H^+_0(\chi, kr)|^2}$$

Current expression for the absolute width in ^{212}Po



Width, Penetrability, reduce width

Current expression for the absolute width in ^{212}Po



Width, Penetrability, reduce width

$$\Gamma_{sp} = 2\,\gamma^2\,P(r)$$

$$P(r) = \frac{Re(k)\,r}{|H_0^+(\chi, kr)|^2} \qquad \gamma^2 = \frac{\hbar^2}{2\,\mu\,r}|u_\alpha(r)|^2$$

Single particle decay width in ^{212}Po: direct diagonalization

Using the same model Hamiltonian

$$H\,u_\alpha(r) = \left(E - i\,\frac{\Gamma_{sp}}{2}\right)\,u_\alpha(r)$$

Comparation

Single particle decay width in ^{212}Po: direct diagonalization

Using the same model Hamiltonian

$$H\ u_\alpha(r) = \left(E - i\ \frac{\Gamma_{sp}}{2}\right)\ u_\alpha(r)$$

Comparation

	$\Gamma[MeV \times 10^{-13}]$	$T_{1/2}[\mu sec]$
Current Expresion	0.4756	0.009593
Diagonalization	0.4740	0.009627
Experimental	0.0153	0.299

Experimental Spectroscopic factor

Single particle decay width in ^{212}Po: direct diagonalization

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Experimental Spectroscopic factor

$$S_{exp} = \frac{\Gamma_{exp}}{\Gamma_{sp}} = 0.3 \times 10^{-1}$$

Spectroscopy Factor from overlap function

Absolute width

$$\Gamma = 2\,\gamma^2_{\alpha L}\;P_L$$

Reduce width

$$\gamma_{\alpha L} = \sqrt{\frac{\hbar^2 R}{2\mu}} F_L(R)$$

Preformation factor

$$F_L(R) = \int \phi^*_{JM}\left[\phi_\alpha(x_\alpha) Y_L(\hat{R}) \Psi_j(x_k)\right]_{JM} dx_k dx_\alpha d\Omega_R$$

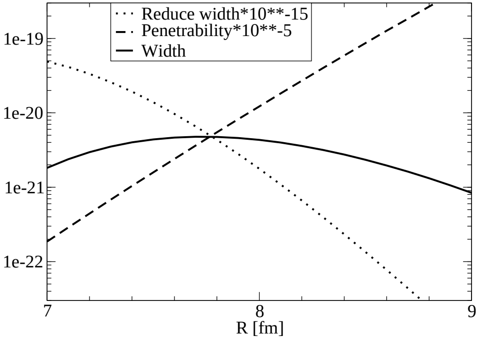
Spectroscopic factor

$$\boxed{S = \int\; R^2\; F_L^2(R)\; dR}$$

Example of absolute Width Calculation in ^{212}Po $\rightarrow \alpha$ + ^{208}Pb

Pure configuration

- Mother wave function: $|^{212}Po\rangle = |^{210}Pb\rangle \otimes |^{210}Po\rangle$
- Model space: $\epsilon^{proton}_{0h_{9/2}} = -3.784\ MeV$ $\quad \epsilon^{neutron}_{1g_{9/2}} = -3.926\ MeV$



Formation amplitud equation

Woods-Saxon basis

$$F(R) = \left[\sqrt{8}\right]\left[\sqrt{4\pi}\right] \int d^3\rho_1 \int d^3\rho_2 \int d^3\rho_3 \left[\left(\frac{8\beta}{\pi}\right)^{9/4} e^{-4\beta(\rho_1^2+\rho_2^2+\rho_3^2)}\right] \left[(-)^{l_n}\frac{\hat{j_n}}{4\pi\sqrt{2}} R_{j_n}(r_1)R_{j_n}(r_2)P_{l_n}(cos(\theta_{12}))\right] \left[(-)^{l_p}\frac{\hat{j_p}}{4\pi\sqrt{2}} R_{j_p}(r_3)R_{j_p}(r_4)P_{l_p}(cos(\theta_{34}))\right]$$

Harmonic oscilator basis

$F(R) = \sum_N C_N \left(\frac{8\beta-b}{8\beta+b}\right)^{N_{max}-N} \phi_N(R)$ (b, size parameter h. o.)

Formation amplitud with mix wave function: bound single particle

Correlated wave function

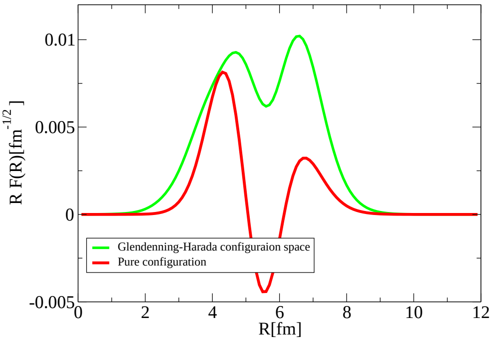
- Wave functions
  - Four-particle w.f.: $|^{212}Po\rangle = |^{210}Pb\rangle \otimes |^{210}Po\rangle$
  - Two-particle w.f.: $|\Psi_{2i,J}\rangle = \sum_{b\leq a} X_{ab,J}\,|ab,JM\rangle$
- Interaction
  - Matrix elememts: $\langle ab,JM|V|cd,JM\rangle = -G_J f(ab,J)f(cb,J)$
  - Form factor: $f(ab,J) = \frac{(-)^{l_a}}{\sqrt{1+\delta_{ab}}}\langle j_a||Y_J||j_b\rangle\, I(ab)$
  - w.f. amplitudes: $X_{ab,J} = N_J \frac{f(ab,J)}{\epsilon_a+\epsilon_b-E_J}$
- Model space

π state	π energy [MeV]	ν state	ν energy [MeV]
$0h_{9/2}$	-3.784	$1g_{9/2}$	-3.926
$1f_{7/2}$	-3.541	$0i_{11/2}$	-2.797
$0i_{13/2}$	-1.844	$2d_{5/2}$	-2.072
		$0j_{15/2}$	-1.883

Formation amplitud with mix wave function: bound s.p. states

Glendening-Harada model space

- Protons: $0h_{9/2}$ $1f_{7/2}$ $0i_{13/2}$
- Neutrons: $1g_{9/2}$ $0i_{11/2}$ $2d_{5/2}$ $0j_{15/2}$



Formation amplitud with mix wave function: unbound single particle

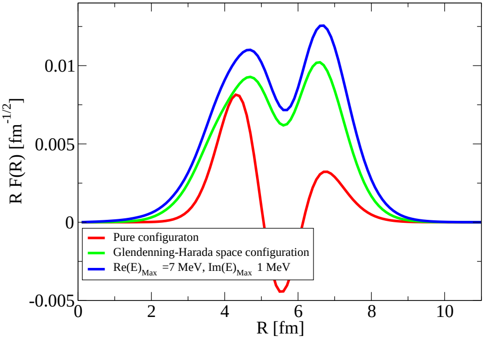
Model space

π state	π energy [MeV]	ν state	ν energy [MeV]
$0h_{9/2}$	-3.784	$1g_{9/2}$	-3.926
$1f_{7/2}$	-3.541	$0i_{11/2}$	-2.797
$0i_{13/2}$	-1.844	$2d_{5/2}$.	-2.072
$2p_{3/2}$	-0.684	$0j_{15/2}$	-1.883
$1f_{5/2}$	-0.512	$3s_{1/2}$	-1.438
$2p_{1/2}$	0.498 - i 0.115 10^{-11}	$2d_{3/2}$	-0.781
$1g_{9/2}$	4.035 - i 0.135 10^{-7}	$1g_{7/2}$	-0.768
$0i_{11/2}$	5.442 - i 0.102 10^{-7}	$1h_{11/2}$	2.257 - i 0.026
$0j_{15/2}$	5.968 - i 0.118 10^{-7}	$0j_{13/2}$	5.419 - i 0.010
$2d_{5/2}$	6.755 - i 0.186 10^{-2}		

Formation amplitud with mix wave function: unbound s.p. states

Continuum model space: bound and resonances

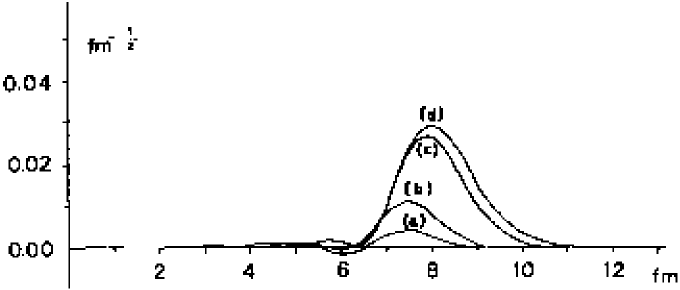
- $Re(E_{max})$= 7.0 MeV
- $Im(E_{max})$= 1.0 MeV



Formation amplitud with mix wave function: harmonic oscilator basis

Model space

- **a** Pure configuration
- **b** Glendening-Harada model space
- **c** Orbits up to N=7
- **d** Orbits up to N=13

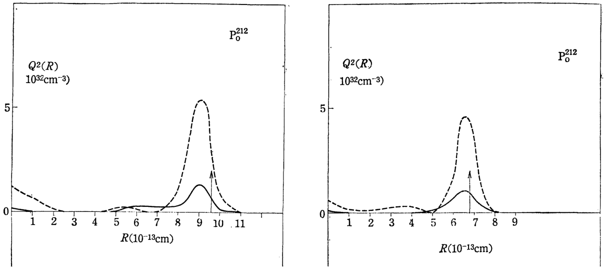


From I. Tonozuka and A. Arima. Nuclear Physics **A323**, 45, 1979

Formation amplitud: position of the peak

Comparation between H.O. and W.S basis

- The overal shape are different
- The amplitud are very similar
- The position of the maximum are different



From K. Harada. Theoretical Physics **???**, 430, 1963
Left(right) h.o. size parameter $b = 0.11 fm^{-2}$($b = 0.22 fm^{-2}$). The arrow indicates the nuclear radius $\sqrt{5/3 < r^2 >}$

Spectroscopic factor in ^{212}Po

Comparation with experimental spectroscopic factor

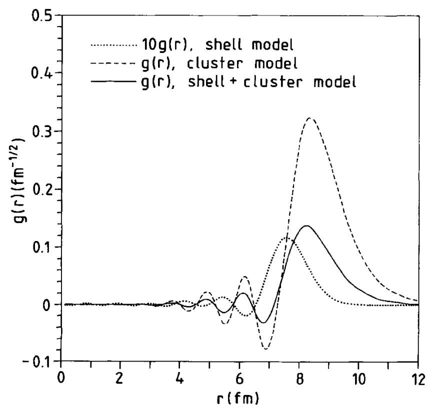
Model space	S/S_{exp}
pure configuration	1.9×10^{-2}
Glendenning-Harada	6.7×10^{-2}
$Re(E_{max}) = 7.0\ MeV$	1.04×10^{-1}

Comparation with combined Shell and Cluster Model

Formalism	S
Experimental	0.0322
SM + Cluser	0.025
W.S. SM	$0.00323 - i\ 0.006 \times 10^{-3}$

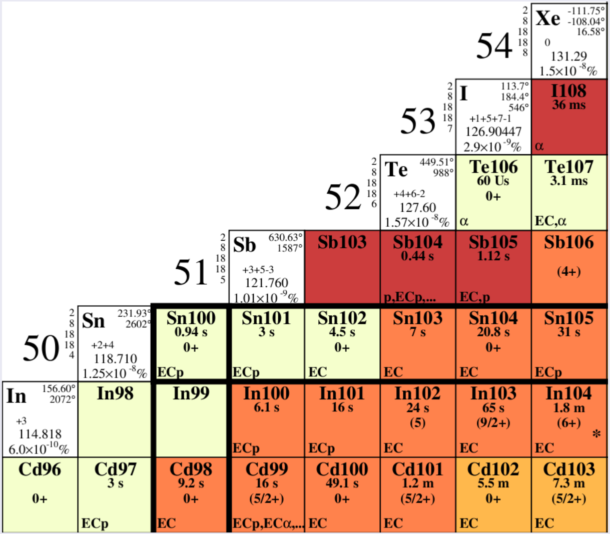
Formation factor from combined Shell and Cluster Model

From K. Varga, R. G. Lovas, R. J. Liotta. PRL 69, 37, 1992



Alpha decay in ^{104}Te

ie.lbl.gov/toi/pdf/chat.pdf



Alpha decay in ^{104}Te

Correlated wave function

- Model space
  - Neutron W.S. parameters $\rightarrow$ lowest s.p. level and S_n in ^{101}Sn from www.nndc.gov
  - Proton W.S. parameters $\rightarrow$ lowest s.p. level of ^{101}Sb(Antinomy) from R. Grzywacz (ORNL)
  - S_p extrapolated from www.nndc.gov
- Wave functions
  - Four-particle w.f.: $|^{104}Te\rangle = |^{102}Te\rangle \otimes |^{102}Sn\rangle$
  - Two-particle w.f.: $|\Psi_{2i,J}\rangle = \sum_{b\leq a} X_{ab,J} \, |ab, JM\rangle$
- Separable interaction parameters
  - Neutron strength $G_n \rightarrow S_{2n}$ from www.nndc.gov
  - Proton strength $G_p \rightarrow S_{2p}$ extrapolated from www.nndc.gov
  - Form factor parameters as the W.S. mean-field

Alpha decay in ^{104}Te

Model space: first major shell

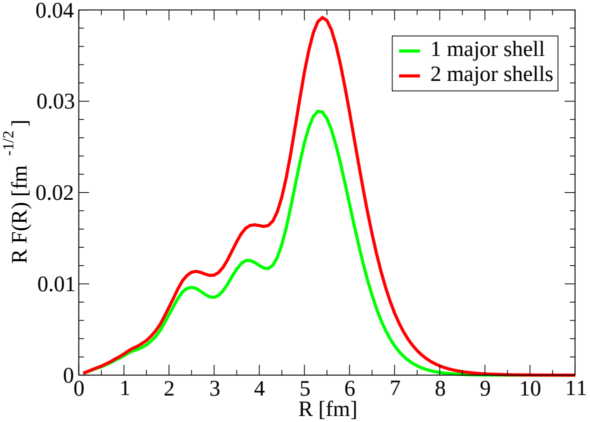
State	ν-Exp.	ν-Num.	π-Exp.	π-Num.
$0g_{7/2}$	-10.850	-10.830	2.646	(2.669, -0.207$\times$ 10^{-7}
$1d_{5/2}$	-10.678	-10.674	2.846	(2.869, -0.963$\times$ 10^{-5}
$2s_{1/2}$	-9.30	-9.074	6.096	(4.150, -0.595$\times$ 10^{-2}
$1d_{3/2}$	-9.20	-8.927	6.196	(4.393, -0.166$\times$ 10^{-2}
$0h_{11/2}$	-7.30	-5.793	5.646	(7.280, -0.110$\times$ 10^{-2}

Four particle amplitudes

proton neutron	$(0g_{7/2})^2$	$(1d_{5/2})^2$	$(2s_{1/2})^2$	$(1d_{3/2})^2$	$(0h_{11/2})^2$
$(0g_{7/2})^2$	0.549	0.391	0.115	0.172	-0.211
$(1d_{5/2})^2$	0.429	0.306	0.090	0.135	-0.165
$(2s_{1/2})^2$	0.114	0.081	0.024	0.036	-0.044
$(1d_{3/2})^2$	0.162	0.115	0.034	0.051	-0.062
$(0h_{11/2})^2$	-0.179	-0.127	-0.037	-0.056	0.069

Alpha decay in ^{104}Te

Formation amplitude



Alpha decay in ^{104}Te

Spectroscopy factor ^{104}Te

Model Space	S
1 Mayor Shell	0.0103 - i 0.015 $\times 10^{-3}$
2 Mayor Shells	0.0197 - i 0.073 $\times 10^{-2}$

Compararion of spectroscopy of ^{104}Te and ^{212}Po

Alpha decay in ^{104}Te

Spectroscopy factor ^{104}Te

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1 Mayor Shell	0.0103 - i 0.015 $\times 10^{-3}$
2 Mayor Shells	0.0197 - i 0.073 $\times 10^{-2}$

Compararion of spectroscopy of ^{104}Te and ^{212}Po

$$\frac{S(^{104}Te)}{S(^{212}Po)} = 4.4$$

Formation Amplitude Using BCS Wave Functions

Wave functions

- Mother and daughter wave functions:
  $\Phi = \Psi(protons) \times \Psi(neutrons)$
- BCS wave function: $\Psi = \prod_{\nu}(u_\nu + v_\nu a^\dagger_\nu a^\dagger_{\bar{\nu}})|0\rangle$

Formation amplitude

$$F(R) = \sum_{n,p} X_n^{BCS} X_p^{BCS} F^{np}(R)$$

Coefficients

- $X_\nu^{BCS} = (-)^{l_\nu}\sqrt{\frac{2j_\nu+1}{2}} c_\nu u^f_\nu v^i_\nu$
- $c_\nu = \prod_{\nu'}(u^i_{\nu'} u^f_{\nu'} + v^i_{\nu'} v^f_{\nu'})^{\left(\frac{2j_{\nu'}+1}{2} - \delta_{\nu\nu'}\right)}$

Formation Amplitude Using BCS Wave Functions

Model space

- First major shell: $0g_{7/2}\ 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2}\ 0h_{11/2}$
- $V_n(N,Z) = V_n^0 - 24\frac{N-Z}{A}$
- $V_p(N,Z) = V_p^0 + 24\frac{N-Z}{A}$

  $V(N,Z)$ from F. D. Becchetti, G. W. Greenlees,PR **182**,1190, 1969

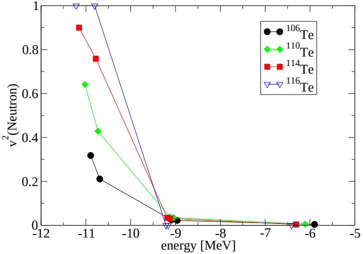
Interaction

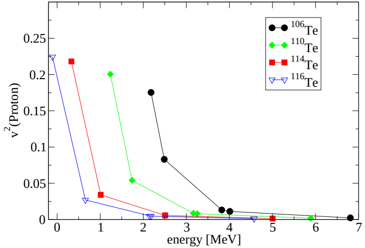
- $G_n(N,Z) = 18.95 - 0.078\frac{N-Z}{A}$
- $G_p(N,Z) = 17.90 - 0.176\frac{N-Z}{A}$

  $G(N,Z)$ from W.N., M. A. Rilley, J. D. Garrett, NPA **512**, 61, 1990, eq. (6)

Formation Amplitude Using BCS Wave Functions

Occupation probability in Tellurium isotopes

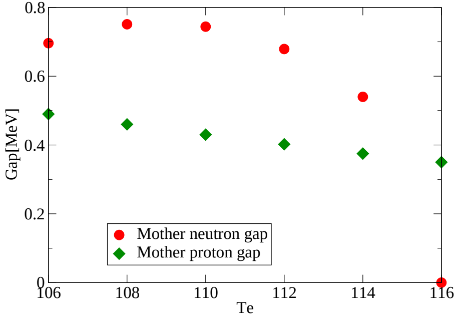




$(0g_{7/2}\ 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2}\ 0h_{11/2})$

Formation Amplitude Using BCS Wave Functions

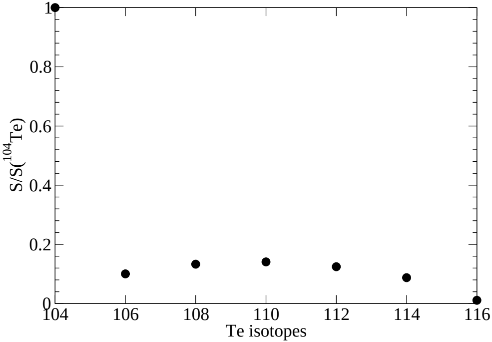
Gap in tellurium isotopes



$(0g_{7/2}\ 1d_{5/2}\ 2s_{1/2}\ 1d_{3/2}\ 0h_{11/2})$

Formation Amplitude Using BCS Wave Functions

Spectrocopic factor in tellurium isotopes



Conclusions

Results and perspective

- The calculated $S(^{212}Po)$ in the shell model framework is ten times smaller than the experimental one
  - Inclusion of the proton-neutron interaction and tensor force
- The $S(^{104}Te)$ is much bigger than $S(^{212}Po)$
  - The calculation of the absolute decay in $S(^{104}Te)$ is still missing
    $\Gamma = S\,\Gamma_{sp}$
- The S of the Tellurium isotopes are around ten time smaller than $S(^{104}Te)$