High-momentum components

how to (and how *not* to) measure them

Ingo Sick

A short look to other fields

- atoms
- liquids
- nucleons (?)

Main topic: nuclei

Reason for interest:

high-k = signature of physics beyond mean-field
  short-range correlations
not covered by the phenomenology involved in MF
directly related to underlying V_{NN}

Emphasis of talk

not so much: how to measure high-k components
rather
what have learned from past attempts
how *not* to try

Atoms

High-quality wave functions calculable

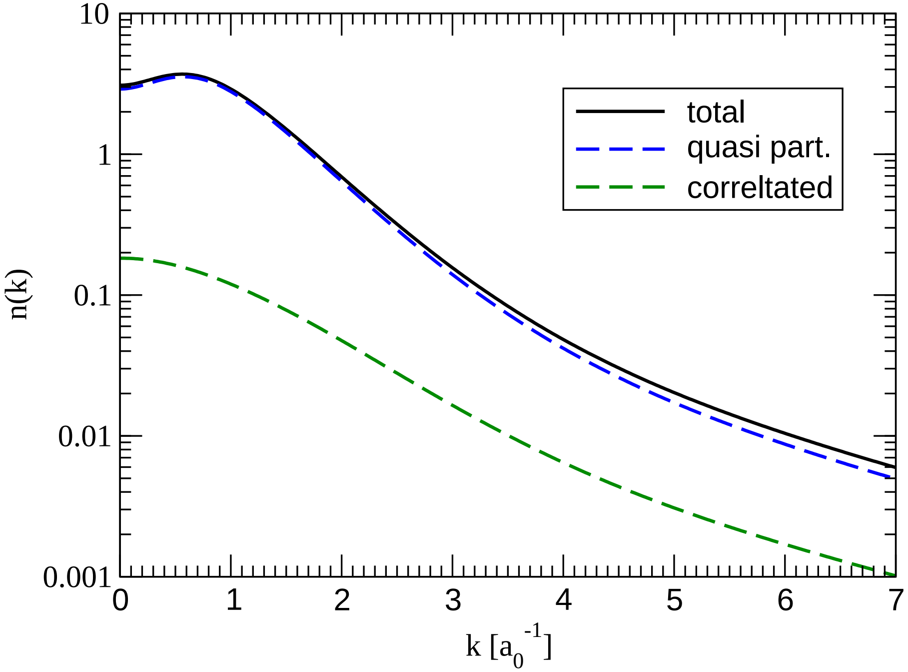
strongly dominated by mean-field aspects
effect of e-e correlations present, but small

Typical measurements

Compton profile, (γ,e), positron annihilation, (e^+,2γ), seldom (e,2e)

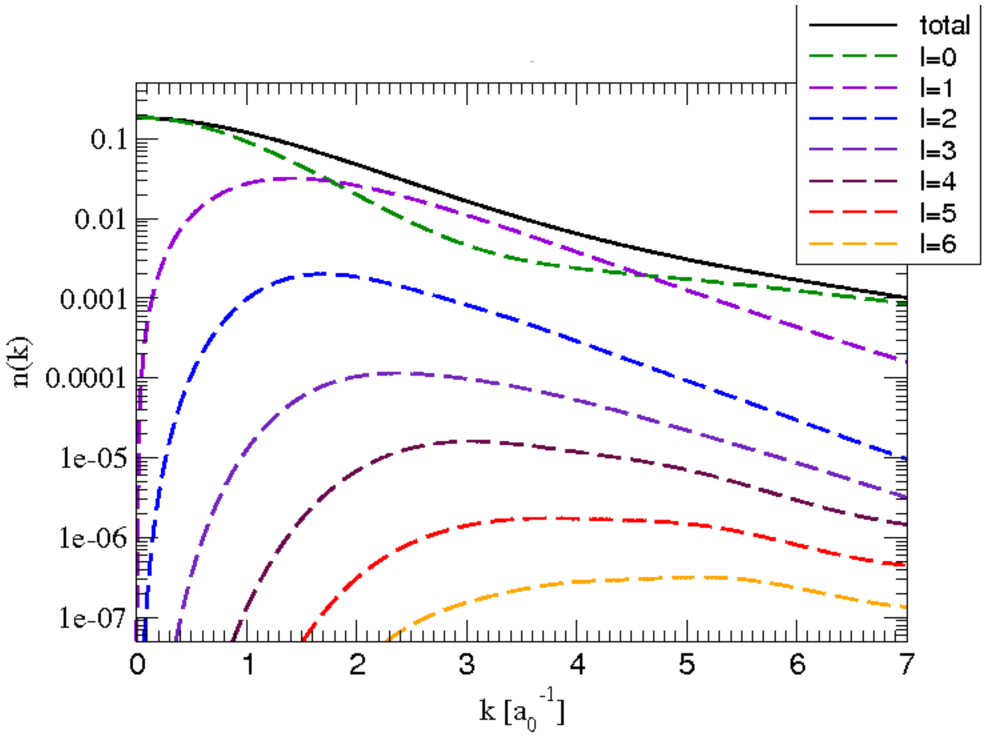
For atom like *e.g.* Neon

calculation Barbieri *et al.*



Find:
small effect of correlations

Contributions of individual shells



high-*k* tail from 1s-state only

high-*k* from confinement to nuclear neighborhood

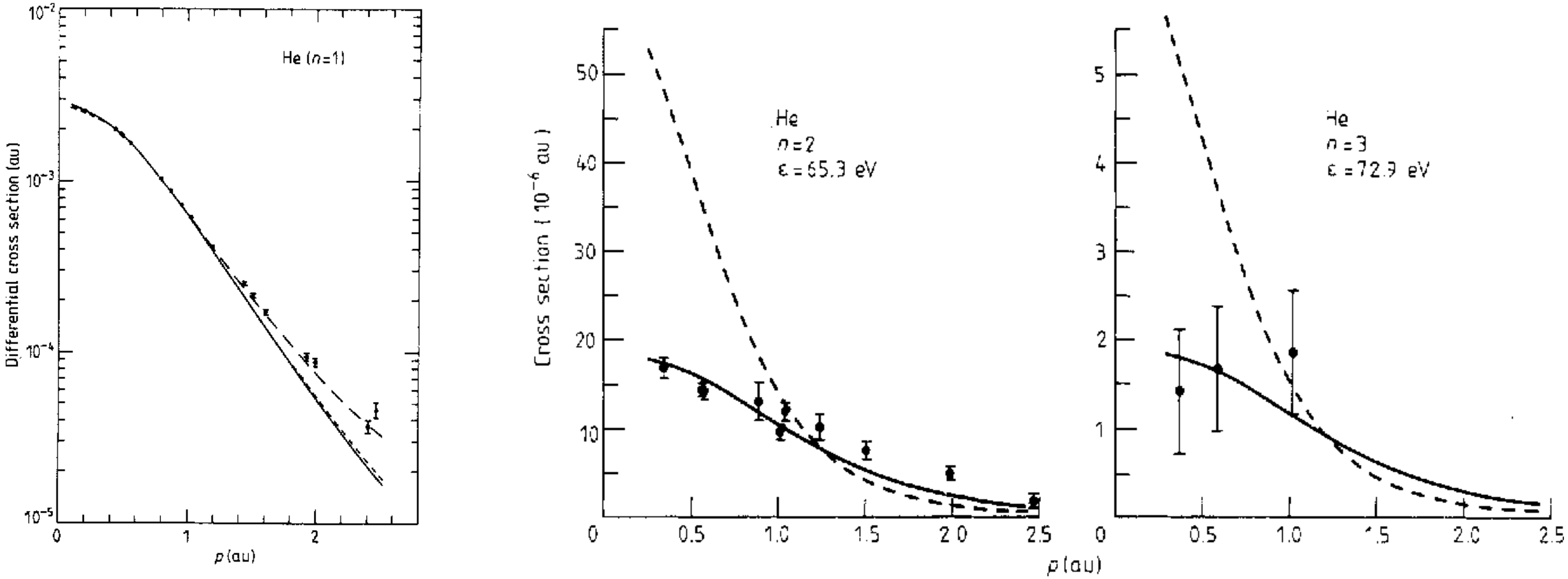
not due to e-e correlations

High-k in Helium

studied via (e,2e), Cook *et al.*

small effect of correlations (——— → - - -)

$\ll$ than Coulomb distortion (- - - → — — —)

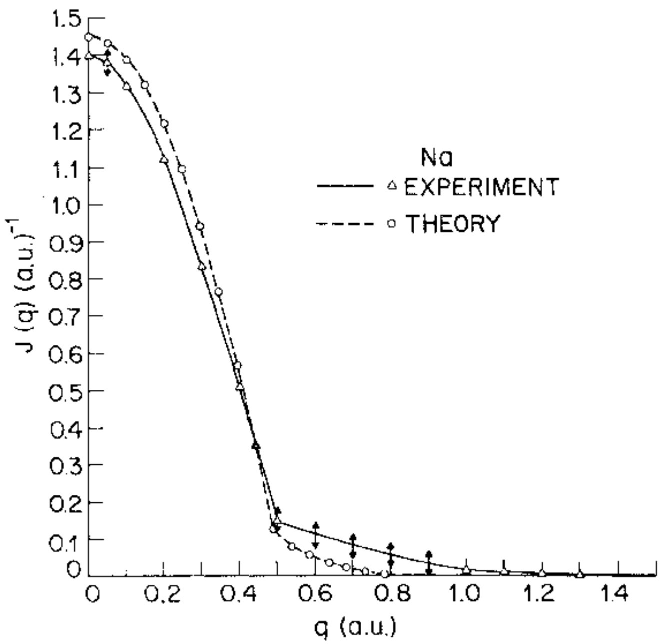


correlation effects visible in transitions to $He^+(2s)$ and $He^+(3s)$

(see discussion of $S(k, E)$ below)

Main interest in atomic high-k

not so much: e-e correlations
rather: molecular structure of solid
(lattice leading to high-k tail)
correlated conduction electrons
see Compton profile of Na (core-e removed)

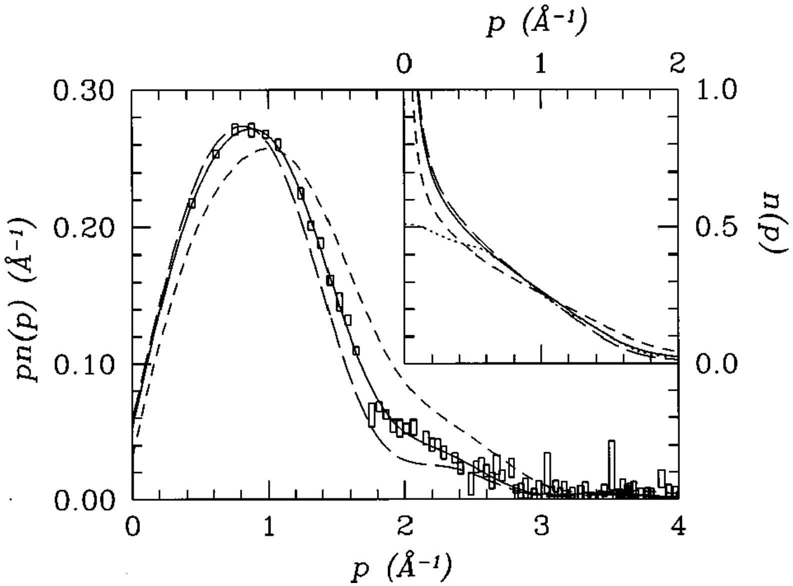


Liquids

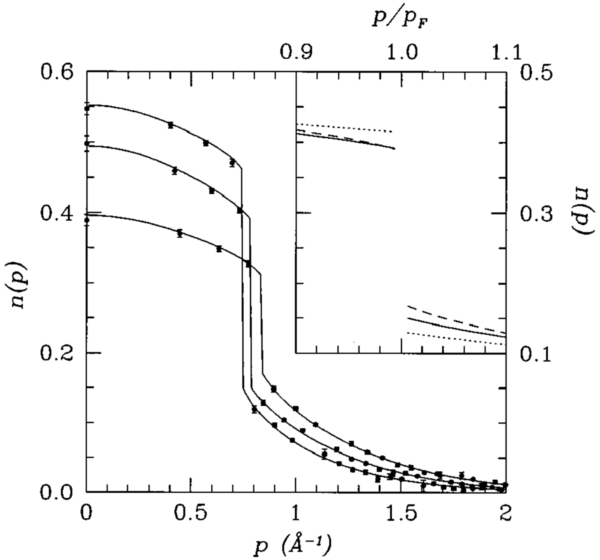
prototypes: L^4He, L^3He, mixtures
strong correlations due to repulsive core of He-He interaction
Lennard-Jones type potential $r^{-12} - r^{-6}$
sophisticated calculations, *e.g.* Diffusion Monte-Carlo

Examples: Moroni *et al.*

^{4}He

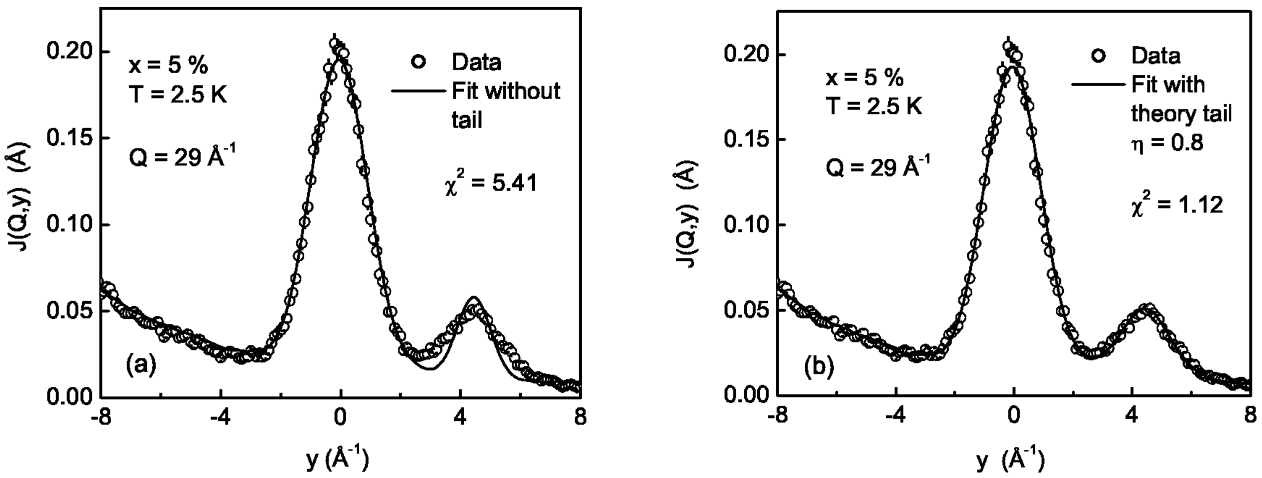


^{3}He

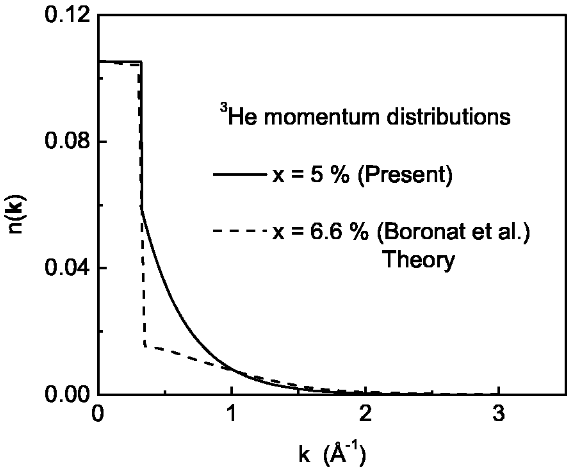


Data: from (n,n'), ~1eV, Diallo *et al.*

quasi-elastic scattering



Better agreement with tail of $n(k)$



Main interest to condensed matter physics:

not high-k

rather % Bose condensate $\rightarrow \delta(k=0)$ peak

should occur for superfluid L^{4}He

$\delta(y=0)$ not visible on q.e.-peak. Reason: FSI

Detailed studies of FSI-effects (of interest to nuclear physics!)

main effect: folding of IA (n,n') response

width of folding function proportional to σ_{tot} of He-He interaction

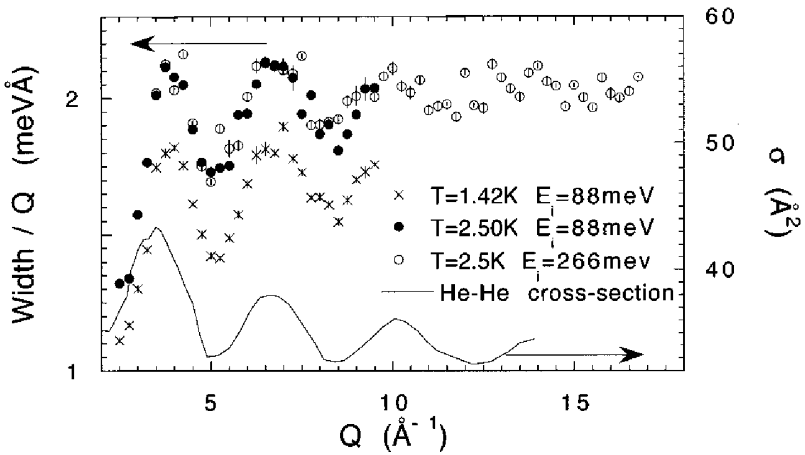
smears out δ-function peak

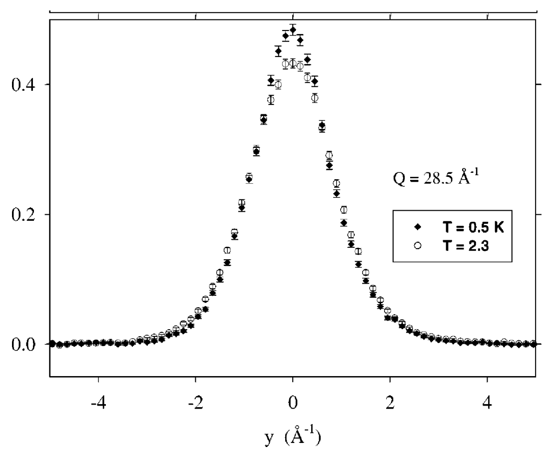
FSI-theory can be verified

σ_{tot} is oscillating function of recoil-He energy

$\rightarrow$ folding width oscillating function of q

nicely observed in data $\rightarrow$ see effect of BC





High-k tail in nucleon structure functions?

know virtually nothing
DIS data at large x obscured by resonances
interpretation based on constituents with mass $x \cdot m_N$ murky anyway

theoretical predictions??
  none I am aware of
  finite lattice spacing not helpful
asymptotic freedom $\rightarrow$ minimal high-k??

  would be interesting!

Nuclei

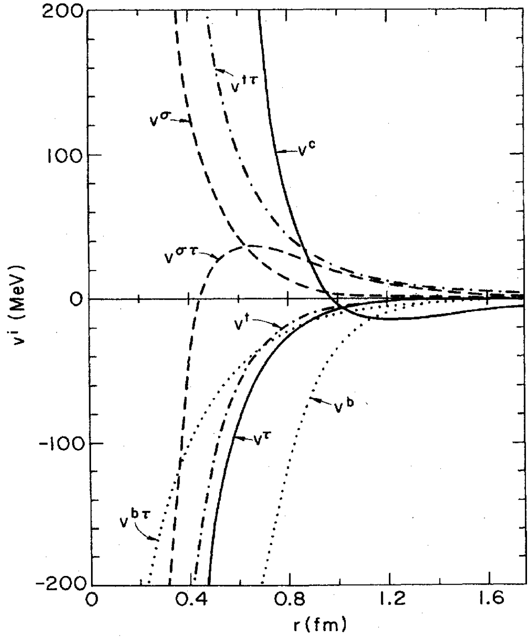
Important high-k components

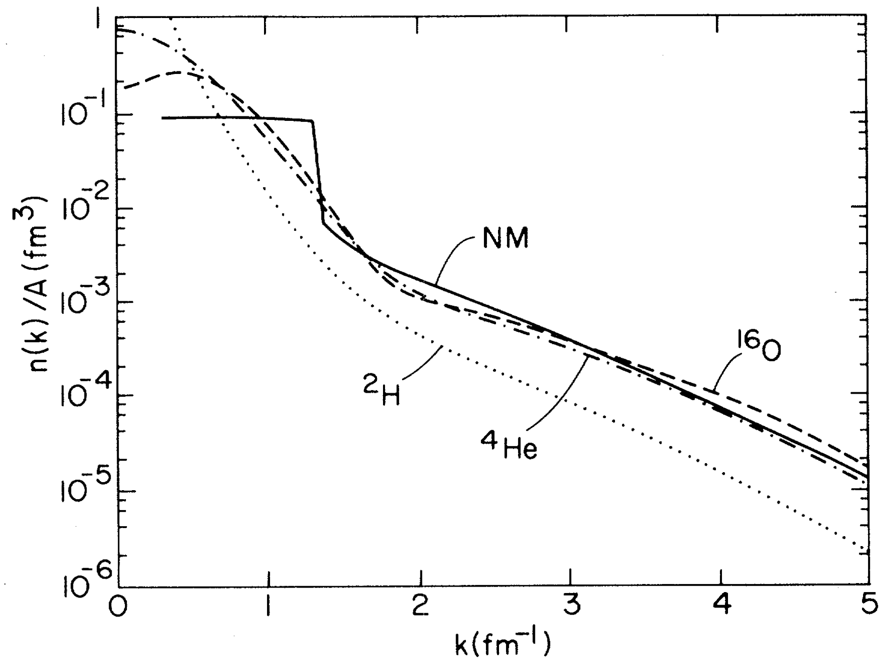
V_{NN} in some channels strongly repulsive at small r

channel dependence complicates exact solution of Schrödinger equation

core leads to high-k tail of $n(k)$

rather universal for nuclei A=2...∞





→ search for high-k popular theme. And *many* failures!

Generic approaches used

Approach #1

with projectile dump energy in nucleus, but little momentum

observe backward-going high-momentum "liberated" nucleon

Reasoning

nucleon must have had high-k before reaction

Typical processes: (x,p), (γ,p), (π,p)

Approach #2

bombard nucleus with probe

observe strength of process at energy subthreshold on nucleon, allowed on nucleus

Reasoning

could only work if nucleon before reaction had large k opposite to momentum of probe
then above threshold in CM of probe-nucleon system

Typical processes: (x,K), (x,$\bar{p}$), (x,π), (e,e') at $x > 1$

Approach #3

(e,e'p) at large q

Reasoning

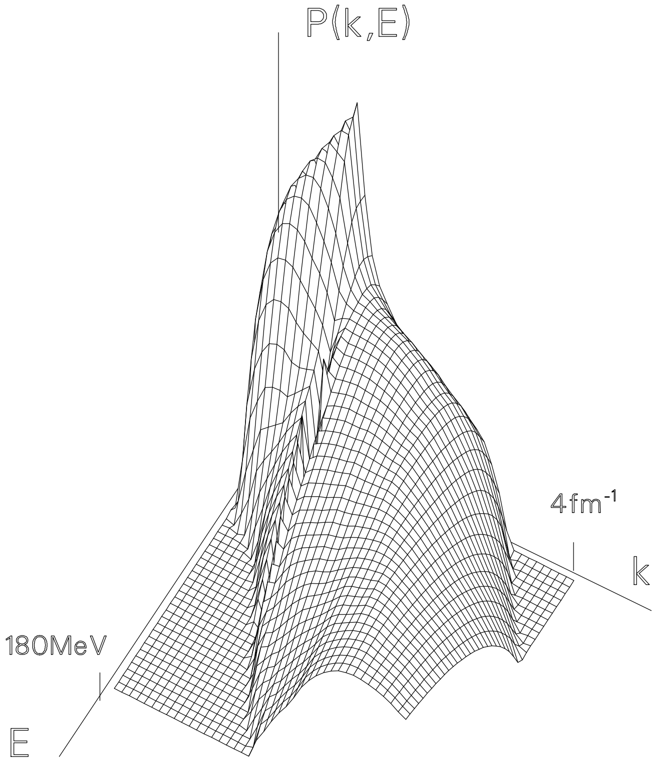
can reconstruct k and E from measured momenta of e, e', p

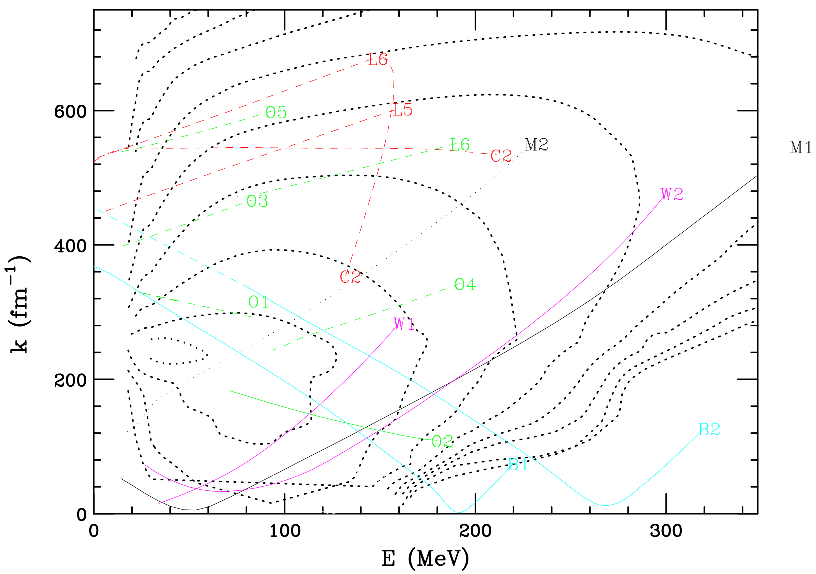
Common pitfalls

1. Use of $n(k)$ instead of $S(k,E)$

in all processes have to conserve momentum *and* energy
high-k-strength occurs at large E, not low E
when hit high-k nucleon correlated partner with $-k$ also "freed "
costs energy $E \sim (-k)^2/2m_N$

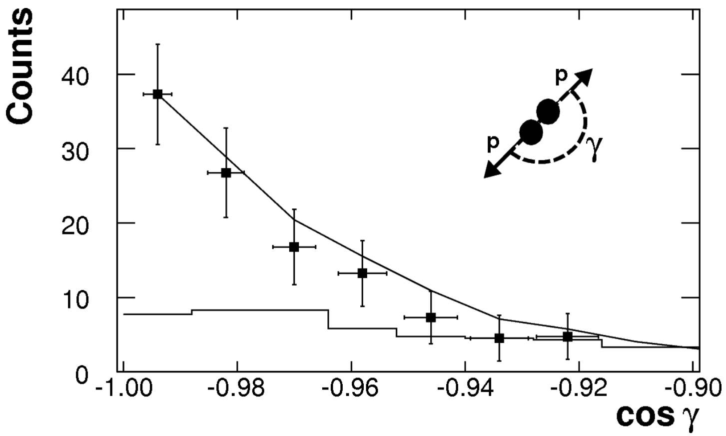
$\Rightarrow$ must discuss data in terms of $S(k,E)$, *not* $n(k)$





Theoretical picture confirmed by experiment

$^{12}C(e,e'2p)$ Shneor *et al.*



correlated nucleon is back-to-back with high-k nucleon

accounts (together with not-observed (e,e'pn)) for all high-k strength

np/pp from Wiringa *et al.*

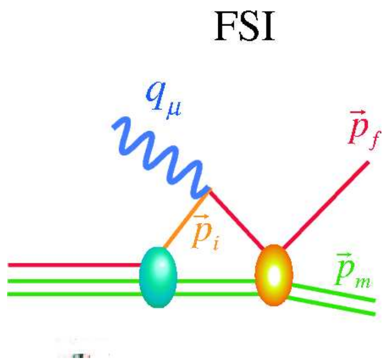
Consequences of large E

high-k strength moved to large energy-loss where covered up by low-k strength

for examples see below

2. Multi-step processes not negligible

probability high-k small, very spread out in E
multi-step processes, even if not dominant, have similar probability
also move strength to larger energy loss
there cover up large-k/large-E strength
important for intermediate off-shell states



Treatment of FSI using DWBA inadequate

Im(V) supposed to account for absorption
works only for (essentially) elastic channels
nucleon is not "swallowed up" but reappears
interacting nucleon moved to larger energy loss/different momentum
there can simulate high-k/high-E-strength

FSI must be treated with approach like Glauber

need to follow fate of interacting nucleon(s)

FSI = *additive* contribution, *not* multiplicative

fraction of large low-k/E strength moved to region of large-k/E

$\Longrightarrow$ "removal" of FSI via cross-section ratios is an illusion

3. Low momentum transfer to nucleon maximizes FSI!

initial and final-state of high-k N must be orthogonal

in limit of momentum transfer = 0:

FSI (which orthogonalizes) *cancels entirely* high-k contribution (Amado+Woloshyn, 1977)

cannot use for *quantitative* study unless have total control of FSI

... which is more difficult than predicting $S(k, E)$

4. Kinematical 2N-region $\not\Longrightarrow$ correlations!

often assumed without addressing specific mechanism:

kinematical region needing mass $2 \cdot m_N$ $\rightarrow$ correlations $\rightarrow$ high-k

much more likely

2N means FSI with intermediary (off-shell) N

..... and even more so in region corresponding to mass $3 \cdot m_N$

What do we know even *without* measuring high-k?

1. $n(k)$ from exact calculations for A=3÷11,16,∞

can today solve Schrödinger equation for best NN-potentials
  Faddeev, CBF, AFMC, GFMC, ..
calculations are phenomenally successful
  explain many observables

in particular explain binding energy

Koltun sumrule $\quad BE/A = \langle E \rangle - \langle T \rangle$

$\pm$ 1MeV $\qquad \sim$50MeV

$\langle T \rangle$ quite accurate $\rightarrow$ can trust $n(k)$ at large k

2. $S(k, E)$ for A=3,4 and ∞

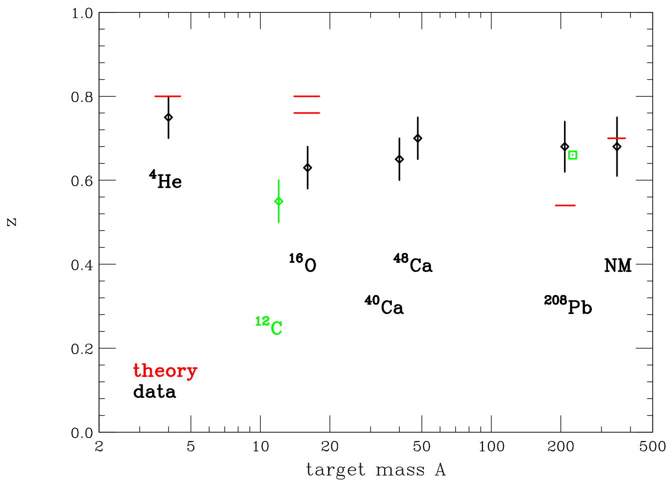
calculated using exact methods
  situation similar to the one for $n(k)$
for other A can "interpolate" (as basically $\equiv$)
  using LDA for correlated part

3. **Integrated correlated (high-k) strength**

occupation s_{MF} of mean-field orbits measured

$1{-}s_{MF}$ yields integrated correlated strength

agrees well with theoretical predictions



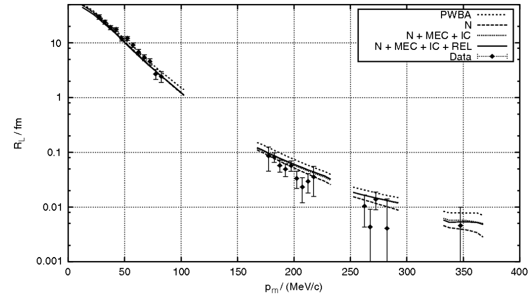
4. **$n(k)$ at large k constrained by elastic $F(q)$'s**

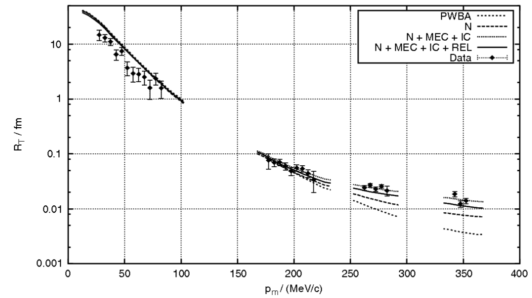
$F(q)$ dominated by overlap $\psi(k \sim -q/2) \cdot \psi(k \sim +q/2)$

theoretical $F(q)$ at large q close to experiment, fall-off with q correct

for $q/2$ much larger than k_F

fall-off of $n(k)$ with k must be ±Ok





5. **Large-k fall-off same as for deuteron**

know quite well from experiment

...... we know a lot. Lets not behave as if nothing were known!

Minimum requirement for high-k-study

start from results of microscopic ("exact") theory
    we know it must be close to reality
use spectral function $S(k, E)$, and *not* $n(k)$
    i.e. respect energy conservation
do PWIA calculation of cross section
    is straightforward to do
    does not have difficulties of FSI calculation

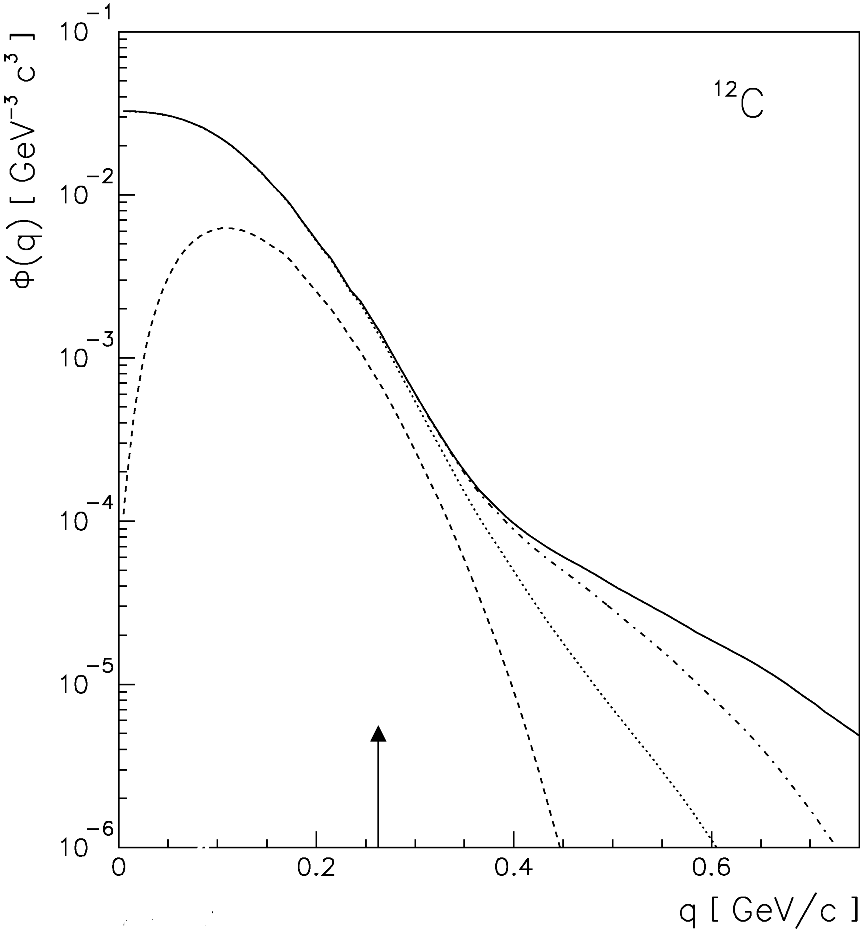
If σ_{PWIA} is much smaller/larger than σ_{exp}
then know that process *not* suitable for study of high-k
    then FSI/multistep dominates
    data good for FSI/multistep study *only*
If σ_{exp} is close, then it pays to deal quantitatively with FSI

Most important

keep away from above-listed "simpleminded reasonings"
    we today know their fallacies
    high-k have been studied for ~50 years, much is known
    particularly on how *not* to try

Illustrations

Large k are at large E



$E < 30 MeV$ - - -
$E < 100 MeV$ .....
$E < 200 MeV$ -.-.-.
$E < 500 MeV$ ———-

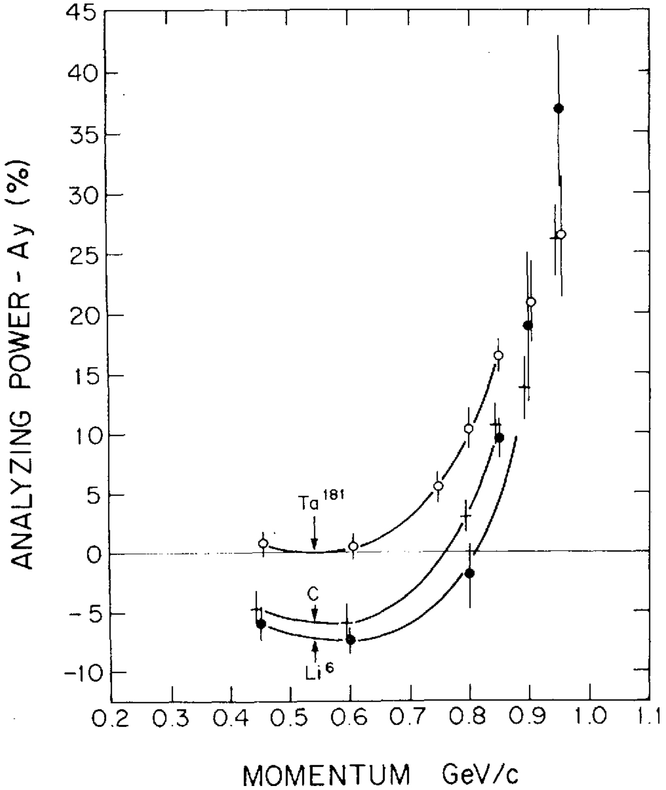
at low E find only mean-field strength

Backward high-momentum protons not from high-k

Typical reaction p + A $\rightarrow$ p' + X, with p' backward

measure analyzing power (Frankel *et al.*)

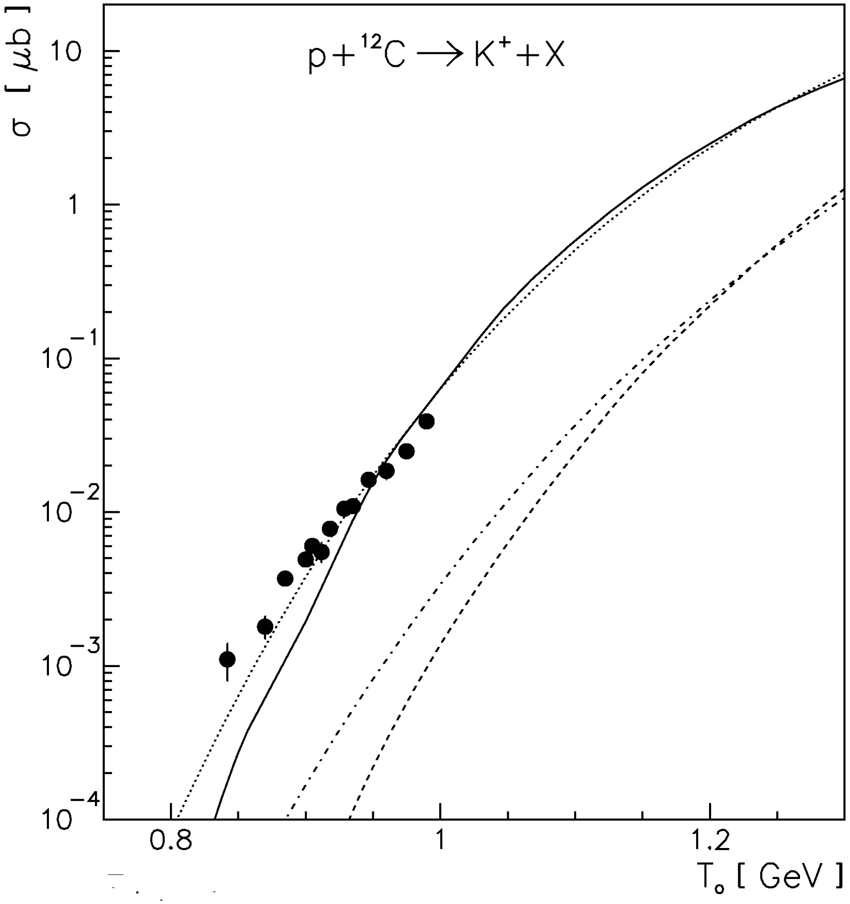
should be large and negative for 2-body mechanism



but is positive for high momenta

remember Amado+Woloshyn!

Subthreshold K-production



- - - - One-step, with $S(k, E)$

——— Two-step, with $S(k, E)$

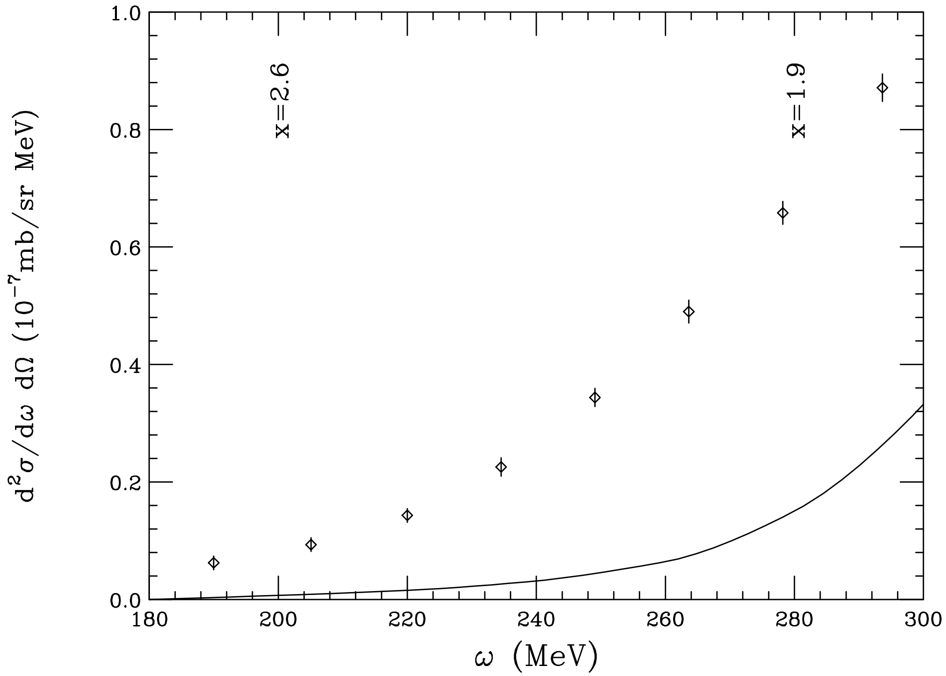
σ_{PWIA} order of magnitude too small

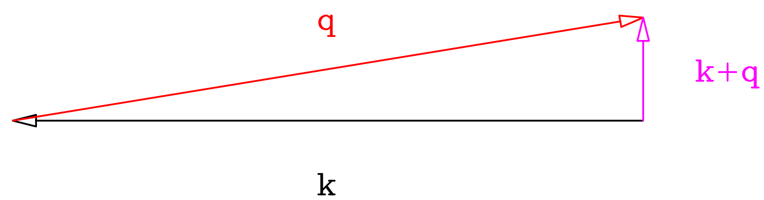
high-k strength shifted by typically 100MeV

two-step is close to data (Sibirtsev *et al.*)

^{3}He(e,e') in threshold region, $x \sim 2 \div 3$

PWIA calculation using full Faddeev $\boldsymbol{S(k,E)}$



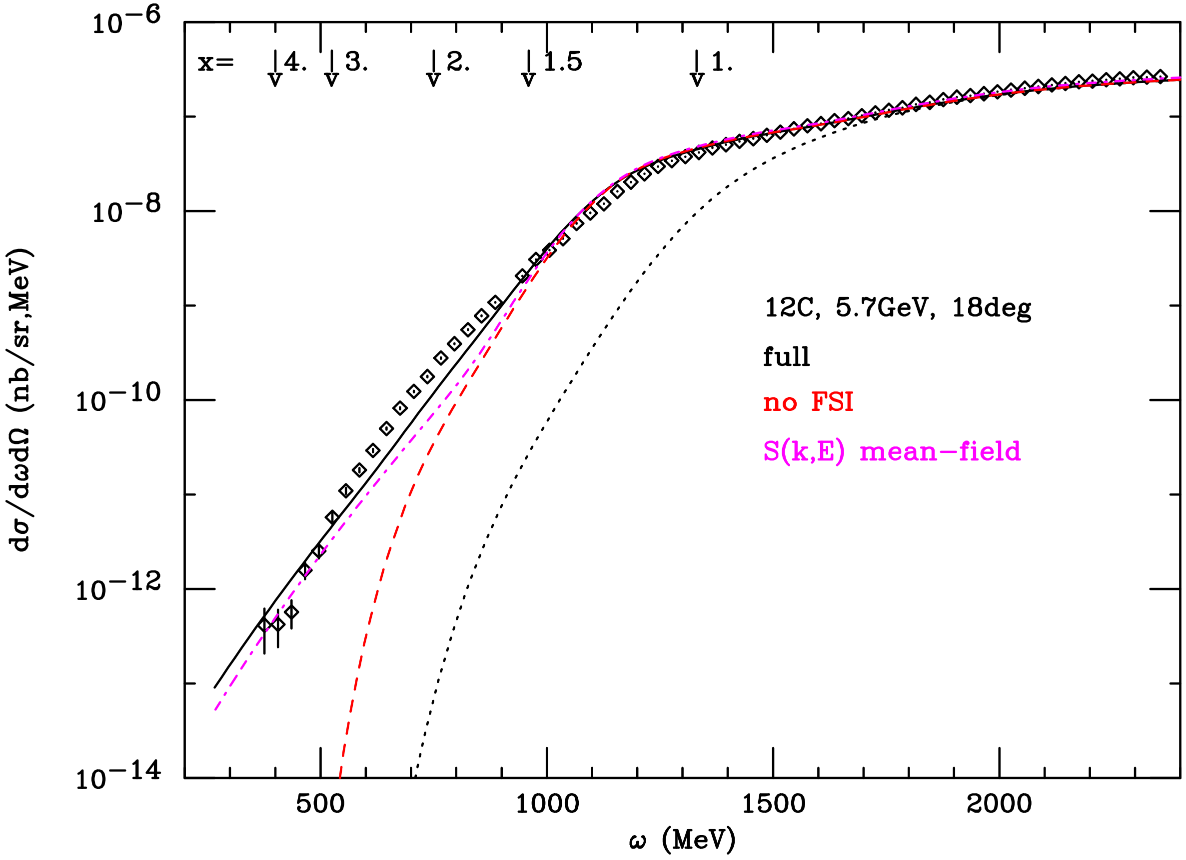


$\boldsymbol{\sigma_{PWIA}}$ **at large x order of magnitude too small**
need FSI to get close to data

mea culpa: **I originally proposed this approach in 1974**
.... but I have learned a few things since!
the first $S(k,E)$ (^{3}He, Dieperink *et al.*, 1976) taught us a lot

Recent (e,e') at $x \sim 2 \div 3$

5.7GeV, 18°, ^{12}C



——— full $S(k,E)$ and FSI

- - - - - full $S(k,E)$, no FSI

-.-.-.-.-. no high-k, with FSI

effect of FSI: folding of IA result
remember FSI in L^{4}He

folding function from particle-$S(k,E)$
Benhar *et al.*

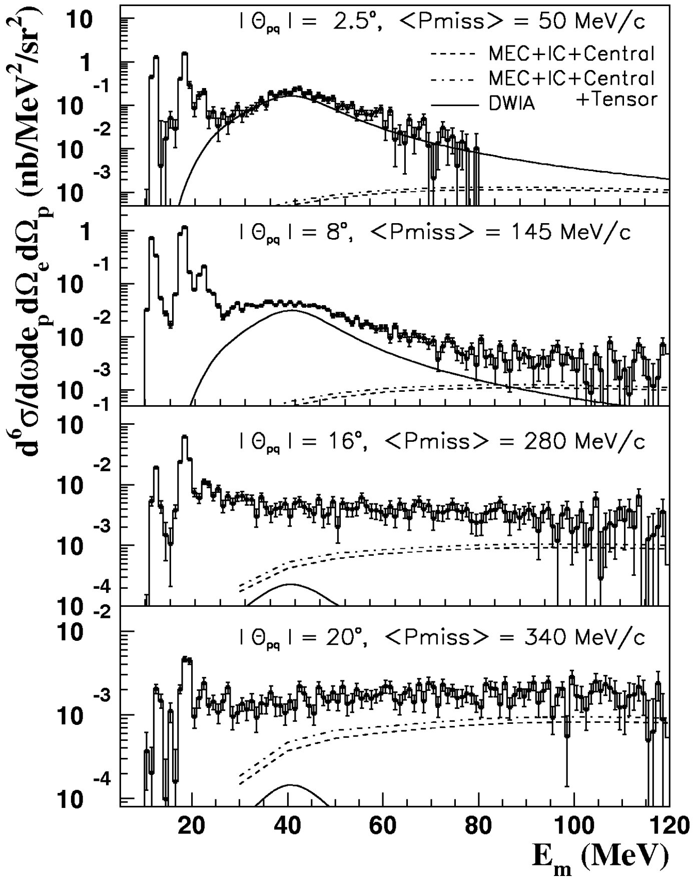
σ_{PWIA} at large x much too small
effect of large-k minimal, FSI dominates (Benhar *et al.* 1989)

cross section ratios $\frac{\sigma_A}{\sigma_{A'}}$ *a la JLab* $\Rightarrow$ ratios of FSI, not $n(k)$! (Egiyan *et al.*)

(e,e'p) at large q: the best tool known *provided that*:

- FSI brought under control
- q ($\sim$ recoil proton momentum) large enough to apply Glauber
- kinematics such as to minimize Δ-excitation

example for maximal FSI: $^{16}O(e,e'p)$ at $Q^2 \sim 0.8$ (Liyanage *et al.*)

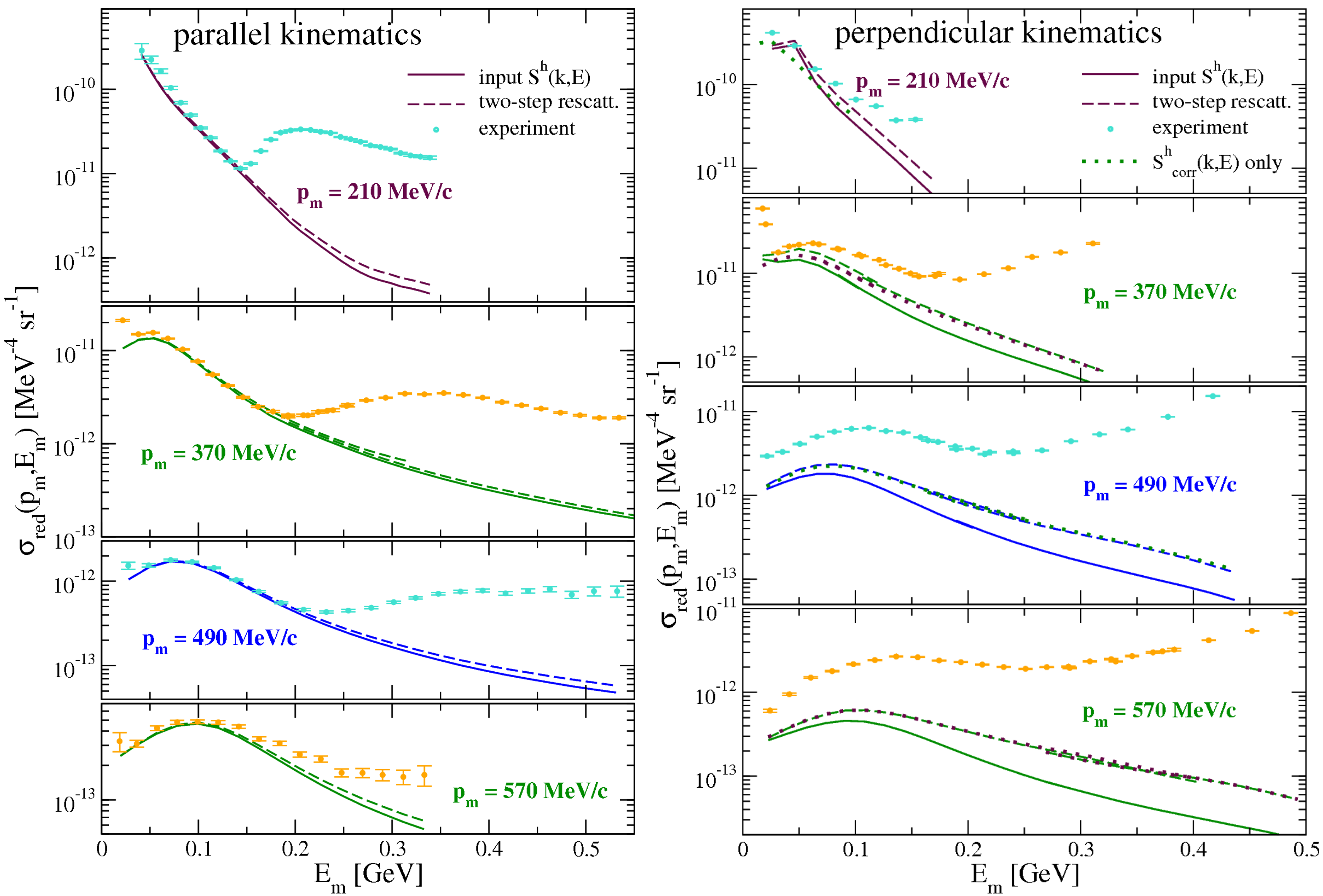


perpendicular kinematics maximizes
multi-step processes

multistep $\longrightarrow$ strength at large E
cannot even see 1s-peak at $k > 250 MeV/c$
let alone the high-k/E-strength
(weaker, more spread out)

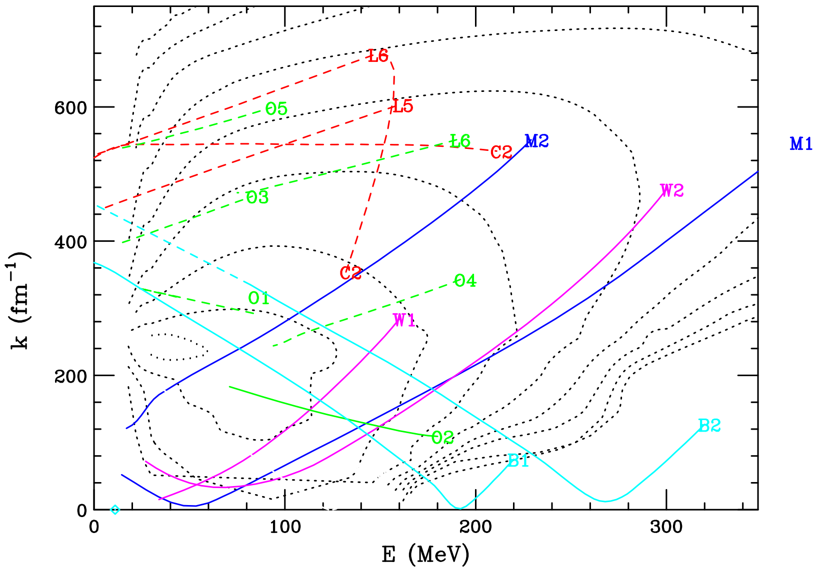
(e,e'p) with minimized FSI:
parallel kinematics, $\vec{q}$ parallel to $\vec{k}$

effect multistep reactions in Glauber (Barbieri *et al.*)



Independent evaluation of role of FSI

Compare σ_{exp} and σ_{theo} for all (e,e'p) experiments
most done in perpendicular kinematics
(selected with idea to correct for FSI using DWBA)

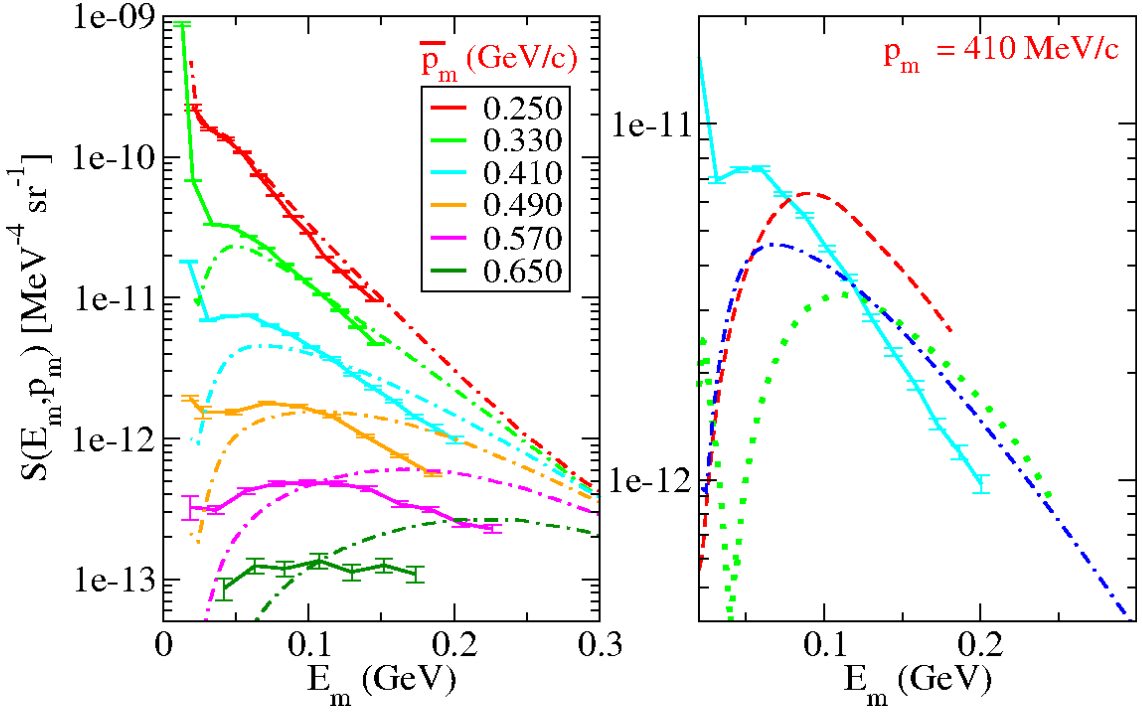


Find

for almost all data $\sigma_{exp} \gg \sigma_{theo}$ in correlated region
exception: data taken in $\pm$ parallel kinematics

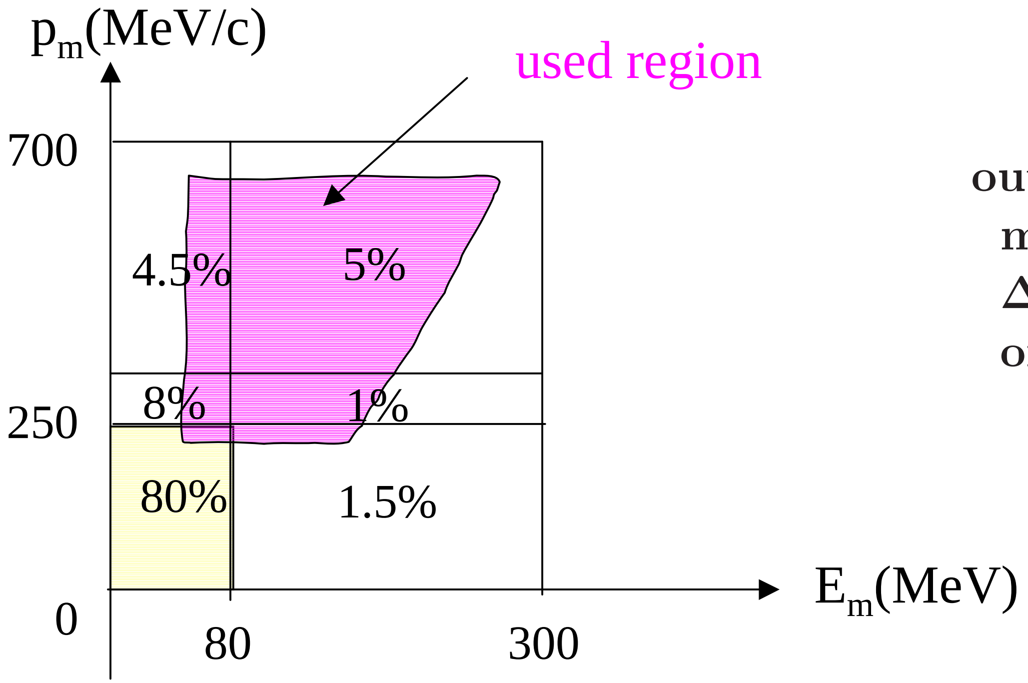
$\Longrightarrow$ JLab experiment Rohe *et al.*, 2004
as close as practical to parallel kinematics

Results: Spectral function



Find ± satisfactory correspondence with theory
in detail: find shift of $S(k, E)$ to smaller E
at present not understood

Integrated strength

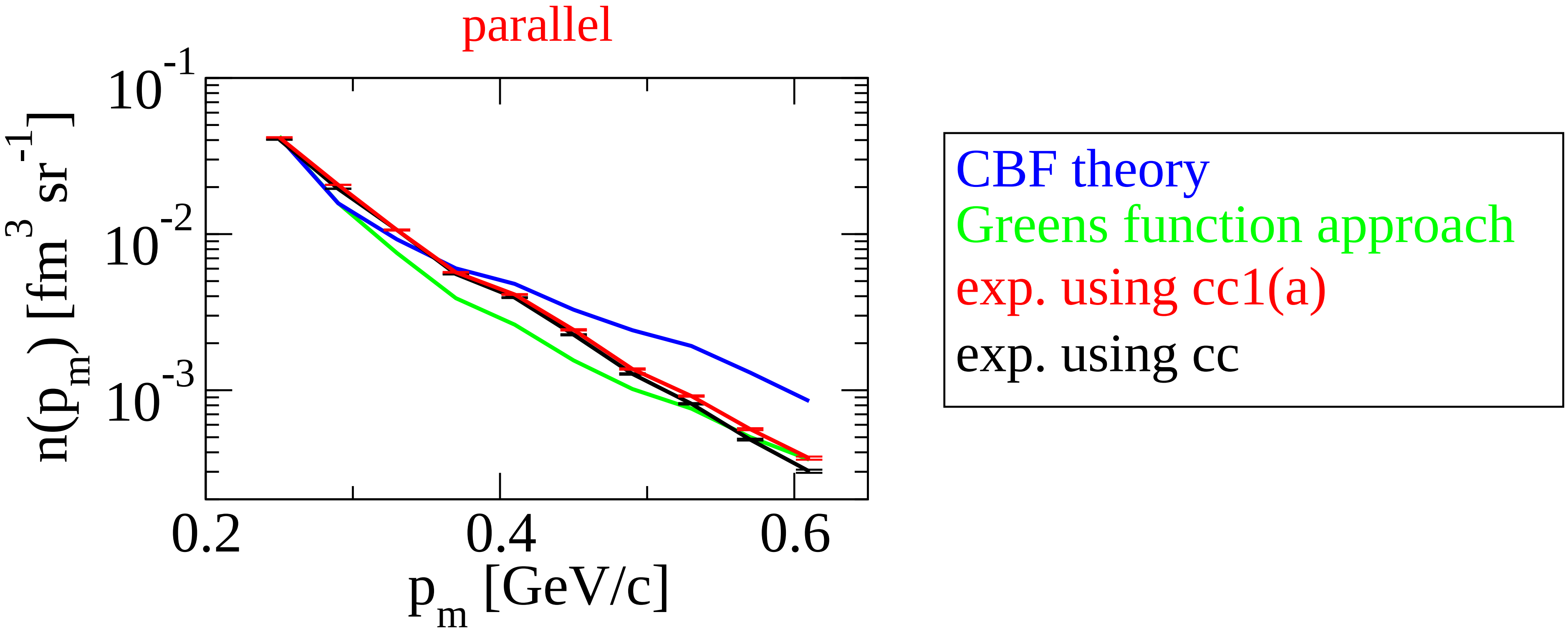


outside used region:
mean-field dominates
Δ too important
or no data

# of correlated protons in ^{12}C	used	total
integral over S from experiment	0.59	
integral over S from CBF	0.64	1.32
integral over S from SGGF	0.61	1.27

→ good agreement
→ can believe total from theory
→ 20%, integrated over $\boldsymbol{k}, \boldsymbol{E}$

Momentum distribution



measure believable high-*k*-tail for first time
find rather good agreement with theory

but both data and theory could stand some improvement

Final insight

Don't even think about measuring $n(k)$ at large k!

every measuring process must conserve momentum *and* energy
large k always involve large E
large k and large E are inseparable
can only measure *together* !

Think only about measuring $S(k, E)$

if one measures $S(k, E)$ over large enough a region in E

then one can obtain $n(k)$ from an integral over $S(k, E)$

Some references

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