Full spectral function in the dispersive optical model

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Outline

1. Green function or propagator theory
2. DOM for quasiparticle properties
3. What about DOM for complete spectral function?
4. Nonlocality issues
5. First try for ^{40}Ca: results
6. Things to do

GFT or propagator theory

- GFT: powerful many-body technique used in many branches of physics.

- Central object is s.p. propagator:

$$G(r, r'; E) = \langle \Psi_0^N | a_r \frac{1}{E-H+i\eta} a^\dagger_{r'} + a^\dagger_{r'} \frac{1}{E+H-i\eta} a_r | \Psi_0^N \rangle$$

- Matrix object in single-particle space: $G(r, r'; E) \rightarrow \hat{G}(E)$, obeying Dyson equation

$$\hat{G}(E) = \hat{G}^0(E) + \hat{G}^0(E)\hat{\Sigma}(E)\hat{G}(E)$$

- Most natural framework for studying spectroscopic factors.

GFT or propagator theory

- Lehman representation
  $G(r, r'; E) = \sum_m \frac{\psi_m(r)\psi_m^*(r')}{E - E_m^{N+1} + i\eta} + \sum_n \frac{\psi_n(r)\psi_n^*(r')}{E + E_n^{N-1} - i\eta}$

- Overlap function

  $\psi_n(r) = \langle \Psi_n^{N-1} | a_r | \Psi_0^N \rangle$

- Spectroscopic factor

  $S_n = \int dr |\psi_n(r)|^2$

GFT or propagator theory: ab initio approach

- Theoretical predictions based on constructing suitable approximations for the selfenergy (e.g. FRPA).

- Exact selfenergy is complex, nonlocal, energy-dependent s.p. potential

$$\Sigma(r, r'; E) = \Re\Sigma(r, r'; E) + i\,\Im\Sigma(r, r'; E)$$

- Real and imaginary part are connected through dispersion relation

$$\Re\Sigma(r, r'; E) = \Sigma_s(r, r') + \frac{1}{\pi}\mathcal{P}\int dE'\,\frac{\Im\Sigma(r, r'; E)}{E - E'}$$

GFT or propagator theory:ab initio approach

- For systems with strong short-range repulsion : use subtracted dispersion relation:

$$\Re\Sigma(r, r'; E) = \Re\Sigma(r, r'; E_F) + \frac{1}{\pi}\mathcal{P}\int dE' \Im\Sigma(r, r'; E)\left(\frac{1}{E-E'} - \frac{1}{E_F-E'}\right)$$

- $\Re\Sigma(r, r'; E_F) \rightarrow$ "HF-like" contribution", energy-independent.
- Remainder $\rightarrow$ Dynamic contribution $\rightarrow$ determined by $\Im\Sigma(r, r'; E)$

GFT or propagator theory: spectral distribution

- Solution to Dyson equation:

$$\hat{G}(E) = [(\hat{G}^0(E))^{-1} - \hat{\Sigma}(E)]^{-1} = [E - \hat{T} - \hat{\Sigma}(E)]^{-1}$$

- This amounts to inversion of a complex matrix (after discretization of coordinate space)

- Spectral function (e.g. below E_F)

$$S(r;E) = \frac{1}{\pi}\Im G(r,r;E) = \sum_n |\psi_n(r)|^2 \delta(E + E_n^{N-1})$$

DOM for QP properties

- Instead of ab-initio, one can also try to parametrize the selfenergy

- Elastic nucleon scattering on nucleus is traditionally analyzed in terms of optical model potential
- Real and imaginary part fitted to scattering data.
- In principle, $\Im\hat{\Sigma}(E)$ for $E > 0$ is the microscopic OMP, so fits can be used.
- But then care must be taken that real and imaginary part are connected through dispersion relation $\rightarrow$ dispersive optical model (DOM)

DOM for QP properties

- Pioneered by Mahaux and Sartor in 1980's.
- Once selfenergy is modelled, one has access to valence-hole QP properties by solving Schrodinger equation (dropping the imaginary part) $\rightarrow$ single-particle energies, spectroscopic factors, etc.
- DOM model (see talk by Robert Charity):

$$\left(-\frac{1}{2}\nabla^2 + V_{HF}(r;E) + \Delta V(r;E)\right)\psi_L(r) = E\psi_L(r)$$

- Dynamical part assumed local, follows dispersion relation

$$\Delta V(r;E) = \frac{1}{\pi}\mathcal{P}\int dE' W(r;E)\left(\frac{1}{E-E'} - \frac{1}{E_F-E'}\right)$$

- Imaginary part: volume and surface contributions, WS- type

Nonlocality issues

- Works fine but: the microscopic HF-type potential $\Re\Sigma(r, r'; E_F)$ is nonlocal and energy-independent.
- For fitting convenience, one uses rather a local but energy-dependent potential in OMP or DOM analyses.
- Perey-Buck trick: one can replace a nonlocal static potential by an equivalent local energy-dependent one.
- But this is disastrous for getting the spectral function out of the selfenergy, since this relies on on the fact that all energy dependence is governed by dispersion relations.
- Solution: go back to nonlocal potential

Nonlocality issues

- We take the fitted DOM potential for ^{40}Ca, remove $V_{HF}(r;E)$ and replace it with

  $$V_N(r,r') = v_{WS}([r+r']/2)H([r-r'])$$

- Gaussian nonlocality governed by length parameter β:

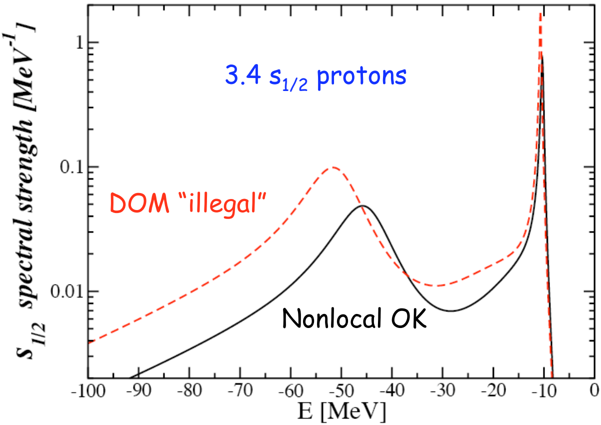
  $$H([r-r'] \sim \exp\left\{-[r-r']^2/\beta^2\right\}$$

- It can also be shown that the imaginary part should then receive a correction through
  $W(r;E) \rightarrow (m/\tilde{m}(r;E))W(r;E)$
- Effective mass $\tilde{m}(r;E) = 1 - \frac{d}{dE}V_{HF}(r;E)$.

Application to ^{40}Ca

- Now we can calculate the proton spectral function through inversion of a complex matrix. We project onto (*lj*) channels and discretize the *r*-axis.
- The new parameters in V_N were determined by a manual fit to the *sd* single particle energies and the charge radius.
- One can clearly see that if one doesn't take care of nonlocality, nonsense results are obtained...

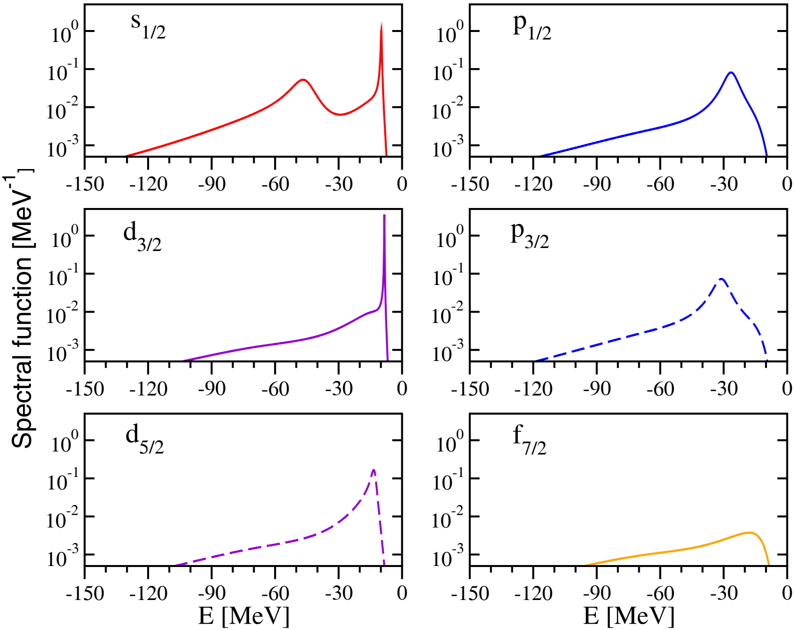
Spectral strength in ^{40}Ca



Nucleon correlations

Application to ^{40}Ca

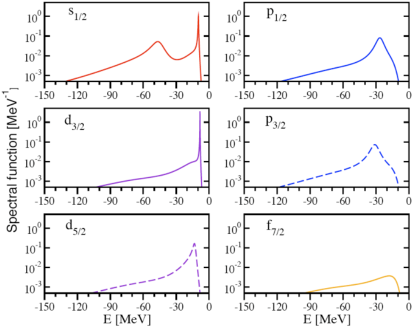
- Spectral functions look o.k. Note the position of the broad $0s1/2$ distribution at about -50 MeV. The QP equation with a local potential has problems with this, placing it at a much too low energy if the $1s1/2$ energy is reproduced correctly.


Spectral function [MeV^{-1}]
$s_{1/2}$
$p_{1/2}$
$d_{3/2}$
$p_{3/2}$
$d_{5/2}$
$f_{7/2}$
10^{0}
10^{-1}
10^{-2}
10^{-3}
-150 -120 -90 -60 -30 0
E [MeV]
E [MeV]

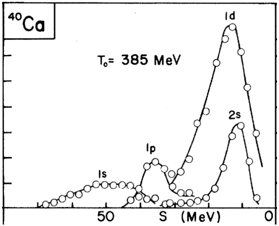
Below ϵ_F

^{40}Ca spectral function

Recent theoretical development: nonlocal self-energy below the Fermi energy
Van Neck, Charity, Sobotka,WD 2009



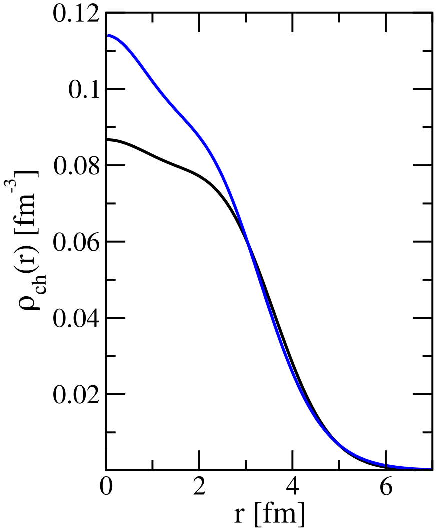
Old (p,2p) data from Liverpool



Application to ^{40}Ca

- Total integrated spectral strength=20.603 protons.
- Not constrained by fit, but could be.
- Also: no state-dependence [(lj)-dependence] included in the fit.

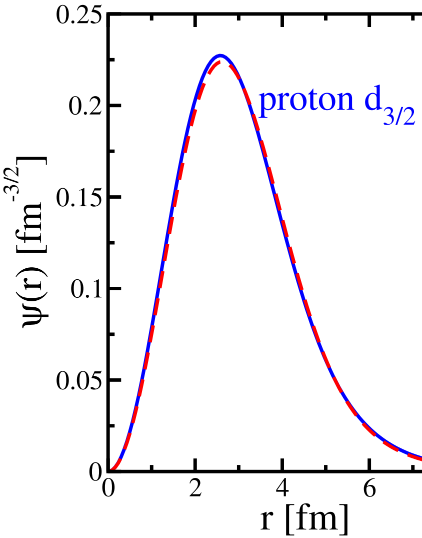
Black=DOM (rms=3.46 fm); Blue= exp. charge density (rms=3.45 fm)
Too much central strength! Lack of short-range correlations?

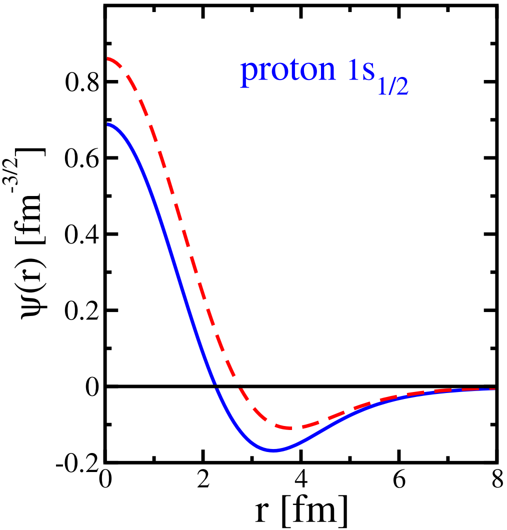


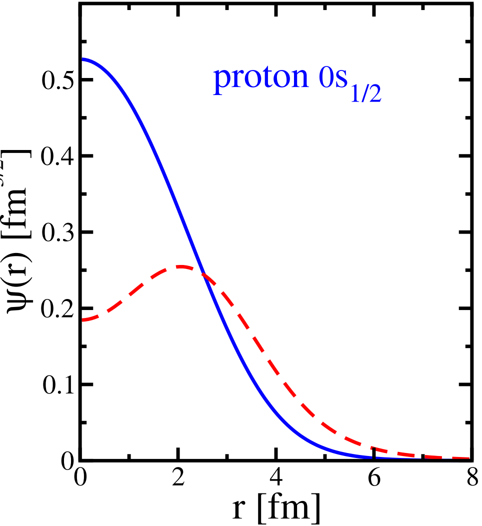
Application to ^{40}Ca

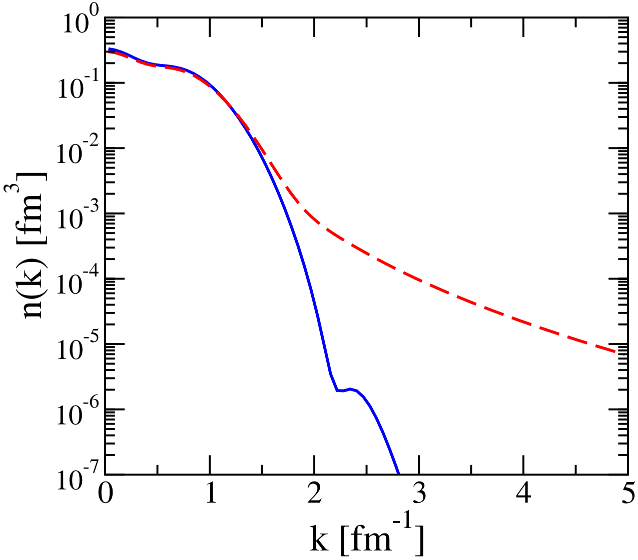
- QP orbitals (obtained by setting $\Im\Sigma = 0$) also look o.k..
- Rms radii ca. 0.1 fm smaller than (e,e'p) BSWF.
- Recoil effect? (A versus A-1)
- Comparison with natural orbitals can be done, since we have access to the density matrix.

Blue=QP wave function; red= natural orbital

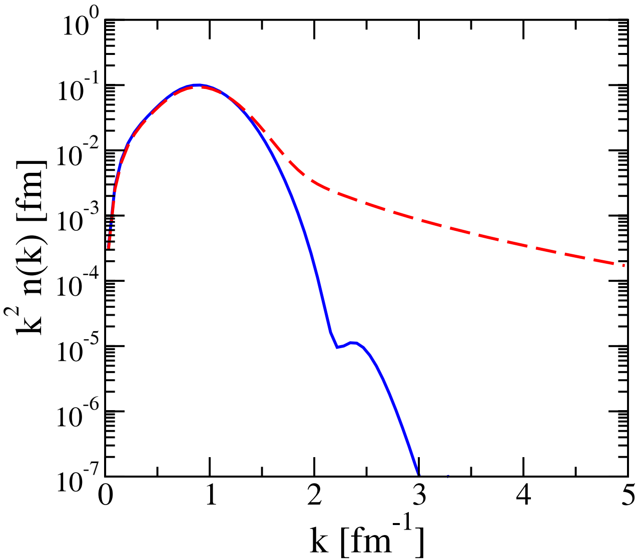



proton 1s1/2
ψ(r) [fm-3/2]
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
r [fm]


proton 0s1/2
ψ(r) [fm^3/2]
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
r [fm]



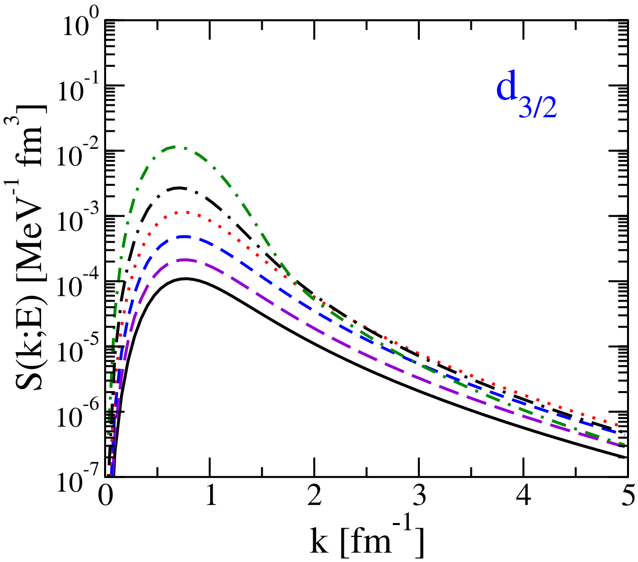
Momentum distribution in red, versus quasihole contribution in blue



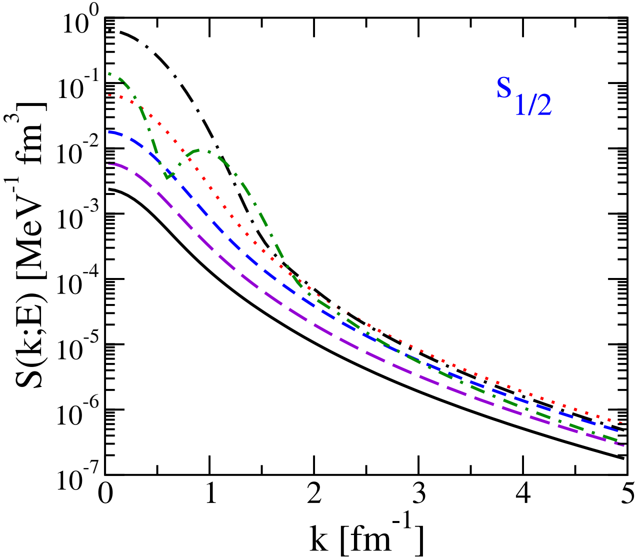
Only 2% high-momentum strength!

Application to ^{40}Ca

- Points again to lack of short-range correlations in present model, consistent with too much central density.
- Energy through Koltun sumrule: only -2.7 per particle.
- Probable difficulty: WS geometry parameters do not depend on energy in our model.
- At large negative removal energies, the geometry should shrink.
- This does not happen: shape remains the same when looking at S(k;E) at various energies (-150 to -25 MeV in steps of 25 MeV). No high-momentum components when going deeper in the removal domain.



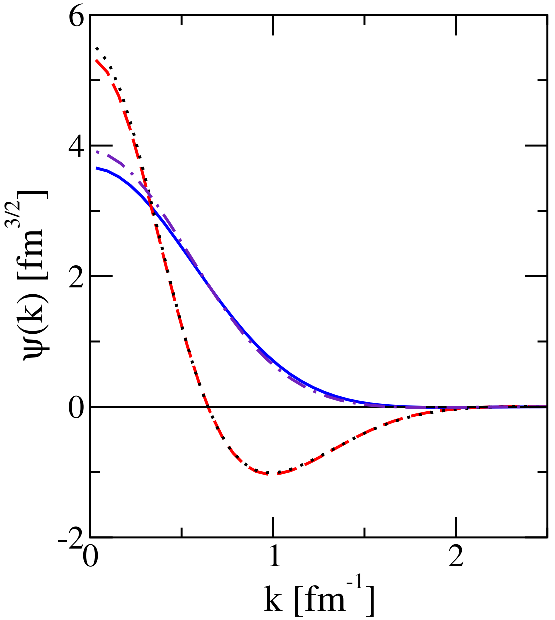
d3/2
S(k;E) [MeV-1 fm3]
k [fm-1]
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0
1
2
3
4
5



10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
1
2
3
4
5
S(k;E) [MeV^{-1} fm^{3}]
k [fm^{-1}]
$s_{1/2}$

Summary and things to do

- Replace treatment of nonlocality by an equivalent local energy-dependent potential with explicitly nonlocal potential allows to extract full spectral function in DOM model.
- Yields one-body density matrix, natural orbitals, charge density, momentum distribution, etc.
- Inclusion of short-range correlations. At this moment no high momentum/removal energy spectral strength included. Energy-dependent geometric parameters of the potential wells?
- Fit all data with nonlocal potentials to begin with...
- Consider state-dependent selfenergy parametrizations.
- Positive energy part of spectral function?
- Thanks.



6
4
2
0
-2
0
1
2
ψ(k) [fm^3/2]
k [fm^-1]