

How far can we push the impulse approximation picture ?

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 - The low momentum transfer regime

Single particle properties in interacting many-body systems

- ★ **Overlaps** are well (and *uniquely*) defined quantities for interacting many-body systems

$$\chi_n(\mathbf{r}_1) = \int d^3 r_2 \dots d^3 r_A \Psi_n^{A-1}(\mathbf{r}_2 \dots \mathbf{r}_A)^\dagger \Psi_0^A(\mathbf{r}_1 \dots \mathbf{r}_A)$$

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- ★ If $|\Psi_n^{A-1}\rangle$ is a bound state χ_n carries information on single particle dynamics
- ★ Within the mean field picture $\chi_n \rightarrow \phi_n^{\text{MF}}$, ϕ_n^{MF} being the n -th single particle orbital

- ★ The **spectroscopic factors** are defined as

$$Z_n = \int \frac{d^3k}{(2\pi)^3} |\hat{\chi}_n(\mathbf{k})|^2$$

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- ★ Assuming that the ground state of the target be fully understood, the extraction of the spectroscopic factors from nuclear cross sections

$$\sigma_A \propto |\langle \Psi_f | \hat{O} | \Psi_0^A \rangle|^2$$

implies assumptions on the structure of **both** the operator inducing the transition and the target final state.

Spectroscopic factors in uniform nuclear matter

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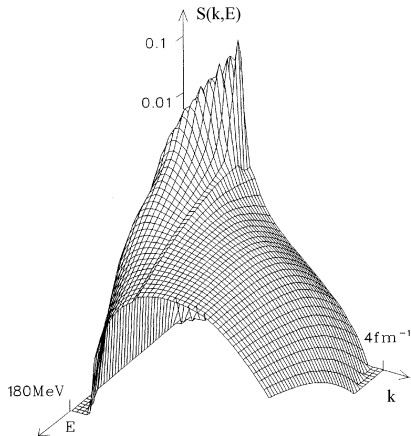
- ★ Note: Z_k *does not* coincide with the occupation number of the state $|\Phi_{\mathbf{k}}^{1h}\rangle$, $n(\mathbf{k})$, given by

$$n(\mathbf{k}) = \langle \Psi_0 | a_{\mathbf{k}}^\dagger a_{\mathbf{k}} | \Psi_0 \rangle = \sum_n |\langle \Phi_{\mathbf{k}}^n | a_{\mathbf{k}} | \Psi_0 \rangle|^2 = \int dE P(\mathbf{k}, E)$$

where $\{|\Phi_{\mathbf{k}}^n\rangle\}$, is the complete set of (A-1)-nucleon states of momentum \mathbf{k}

Spectral function of infinite nuclear matter

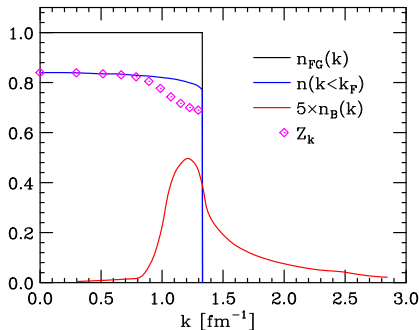
- ★ Results obtained using CBF perturbation theory and the U14+TNI hamiltonian (OB, A. Fabrocini and S.Fantoni, AD 1989)



Momentum distribution and spectroscopic factors

- ★ The momentum distribution can be split into quasi particle (pole) and and correlation (continuum) contributions (OB, A. Fabrocini and S. Fantoni, AD 1990)

$$n(\mathbf{k}) = \int dE P(\mathbf{k}, E) = Z_k + \int dE P_B(\mathbf{k}, E) = Z_k + n_B(\mathbf{k})$$

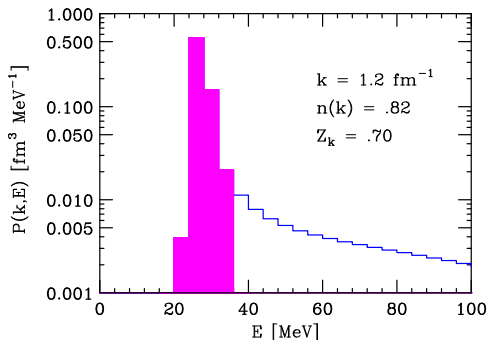


Occupation probability and spectroscopic factor

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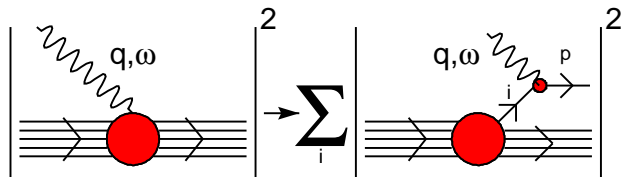
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- ▷ integration over the peak region yields Z_k
- ▷ integration over the whole energy range yields $n(k)$

The impulse approximation (IA) paradigm*

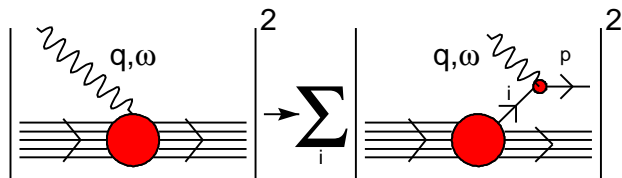
- ★ At momentum transfer \mathbf{q} such that $|\mathbf{q}|^{-1} \lesssim d$, d being the average nucleon-nucleon separation distance, replace



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- ★ At momentum transfer \mathbf{q} such that $|\mathbf{q}|^{-1} \lesssim d$, d being the average nucleon-nucleon separation distance, replace



- ★ As a result, the x-section for knock out of a nucleon of momentum \mathbf{p} and energy E_p takes the simple form

$$\sigma_A \propto P(\mathbf{p} - \mathbf{q}, \omega - E_p)$$

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Assumptions involved in the IA picture

- ▶ The operator inducing the transition is written as the sum of operators acting on individual *nucleons*

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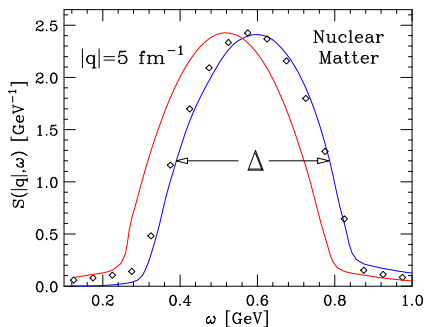
- ▶ The final state is written in the factorized form

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- ▶ Final state interactions (FSI) between the knocked out nucleon and the spectator particles are neglected

Momentum distribution vs spectral function

- ★ Minimal use of the underlying assumptions lead to expression of the IA nuclear cross section in terms of the spectral function (OB, A. Fabrocini and S. Fantoni, AD 2001)
- ★ The definition in terms of the momentum distribution involves a more extended use of the same assumptions, leading to the disappearance of the effect of the removal energy distribution.



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- ★ **Current conservation** is violated.

- ★ Consider the transition matrix element of the process

$$e + A \rightarrow e' + p + (A - 1)_\alpha$$

$$M_\alpha(\mathbf{p}, \mathbf{q}) = \langle \Psi_{\alpha\mathbf{p}}^{(-)} | \hat{O}(\mathbf{q}) | \Psi_0 \rangle$$

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- ★ Isolate the contributions responsible for final state interactions in the nuclear hamiltonian

$$H_A = H_0 + H_{FSI} \quad , \quad H_0 = T_1 + H_{A-1}$$

where T_1 is the kinetic energy of the struck nucleon

★ Scattering state

$$|\Psi_{\alpha\mathbf{p}}^{(-)}\rangle = \Omega_{\mathbf{p}}^{(-)}|\Phi_{\alpha\mathbf{p}}\rangle$$

$$|\Phi_{\alpha\mathbf{p}}\rangle = |\mathbf{p}\rangle \otimes |\varphi_{\alpha}\rangle$$

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★ Distortion (Möller) operator

$$\Omega_{\mathbf{p}}^{(-)} = \lim_{t \rightarrow \infty} e^{iH_A t} e^{-iH_0 t} = \lim_{t \rightarrow \infty} \widehat{T} e^{-i \int_0^{\infty} dt' H_{FSI}(t')}$$

$$H_{FSI}(t) = e^{iH_0 t} H_{FSI} e^{-iH_0 t}$$

High energy (Glauber) approximation

- (A) **Eikonal approximation** : the outgoing proton moves along a straight trajectory in the direction of \mathbf{p} , with constant velocity \mathbf{v}
- (B) **Frozen approximation** : the spectator nucleons are seen as a collection of fixed scattering centers

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- ★ Under assumptions (A) & (B), the distortion operator can be rewritten in coordinate space as ($R \equiv \{\mathbf{r}_1, \dots, \mathbf{r}_A\}$)

$$\Omega_{\mathbf{p}}^{(-)}(R) = P_z \frac{1}{A} \sum_{i=1}^A \left[1 - \sum_{j>i} \Gamma_p(i,j) + \sum_{k>j>i} \Gamma_p(i,j)\Gamma_p(1,k) - \dots \right]$$

- ★ The z -ordering operator P_z prevents the occurrence of backward scattering.

High energy approximation (continued)

- ★ FSI interactions are driven by the coordinate space t -matrix Γ_p , related to the NN scattering amplitude f_p through

$$\Gamma_p(i,j) = \theta(z_j - z_i) \gamma_p(|\mathbf{b}_j - \mathbf{b}_i|)$$

$$\gamma_p(b) = -\frac{i}{2} \int \frac{d^2 k_t}{(2\pi)^2} e^{i\mathbf{k}_t \cdot \mathbf{b}} f_p(k_t)$$

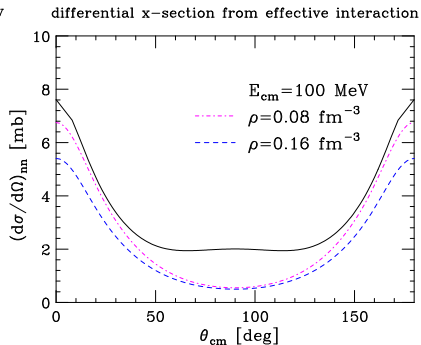
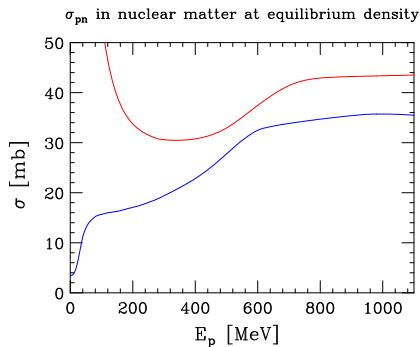
- ★ At large p , the *measured* free space $f_p(k_t)$ is generally parametrized in the form

$$f_p(k_t) = i \sigma (1 - i\alpha) e^{-\frac{1}{2} \frac{k_t^2}{B}}$$

where σ is the total NN cross section

High energy approximation (continued)

- ★ **WARNING**: NN scattering in the nuclear medium expected to be appreciably modified by, e.g., Pauli blocking and dispersive effects
- ★ Medium modifications **consistently** calculable (S. Pieper & V.R. Pandharipande, AD 1992, OB & M. Valli, AD 2007)

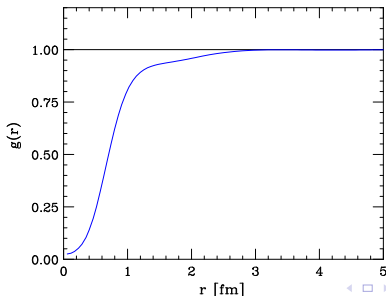


Local Density Approximation (LDA)

$$\begin{aligned}\Omega_{\mathbf{p}}^{(-)}(\mathbf{r}) &= \frac{1}{\rho(\mathbf{r})} \int d^3\mathbf{r}_1 \dots d^3\mathbf{r}_A |\Psi_0(\mathbf{r}_1 \dots \mathbf{r}_A)|^2 \\ &\times \frac{1}{A} \sum_{i=1}^A \left[1 - \sum_{j>i} \gamma_{\mathbf{p}}(\mathbf{b}_i - \mathbf{b}_j) \theta(z_i - z_j) + \dots \right] \delta(\mathbf{r} - \mathbf{r}_i) \\ g(\mathbf{r}_1, \mathbf{r}_2) &= \frac{\rho(\mathbf{r}_1, \mathbf{r}_2)}{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)} \approx g_{NM} \left[|\mathbf{r}_1 - \mathbf{r}_2|, \rho_A \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) \right]\end{aligned}$$

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Transition amplitude including FSI effects

- ★ Recall: within PWIA

$$M_{\alpha}(\mathbf{p} - \mathbf{q}) = \int d^3 r_1 e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}_1} \chi_{\alpha}(\mathbf{r}_1)$$

- ★ In the presence of FSI

$$\chi_{\alpha}(\mathbf{r}) \rightarrow \psi_{\alpha\mathbf{p}}(\mathbf{r}) = \Omega_{\mathbf{p}}^{(-)}(\mathbf{r})\chi_{\alpha}(\mathbf{r})$$

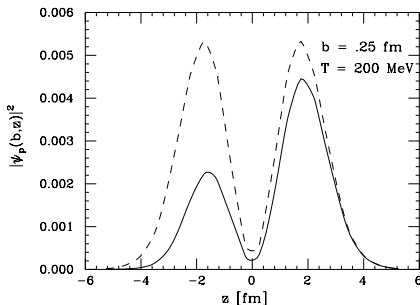
- ★ Z_{α} is reduced by a transparency factor $T_{\alpha\mathbf{p}}$

$$Z_{\alpha} \rightarrow \tilde{Z}_{\alpha} = \int \frac{d^3 k}{(2\pi)^3} |\psi_{\alpha\mathbf{p}}(\mathbf{k})|^2 = T_{\alpha\mathbf{p}} Z_{\alpha}$$

- ★ The momentum distributions $|\psi_{\alpha\mathbf{p}}(\mathbf{k})|^2$ is shifted with respect to $|\chi_{\alpha}(\mathbf{k})|^2$

Electron induced knock out of a p-shell proton from oxygen

★ $|\psi_p^{\mathbf{p}}(\mathbf{r})|^2$ vs $|\chi_p(\mathbf{r})|^2$



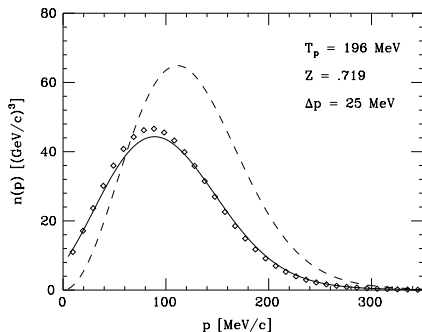
★ Spectroscopic factor and transparency

$$Z_p = \int d^3 r |\chi_p(\mathbf{r})|^2 = .62 \quad , \quad T_p = \frac{1}{Z_p} \int d^3 r |\psi_p^{\mathbf{p}}(\mathbf{r})|^2 = .72$$

Knock out of a p-shell proton from oxygen (continued)

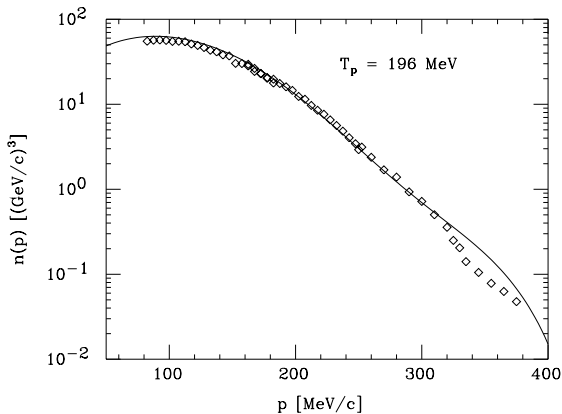
- ★ Momentum distribution:

$$|\widehat{\psi}_p^{\mathbf{p}}(|\mathbf{p} - \mathbf{q}|)|^2 \approx \widetilde{Z}_p |\chi_p(|\mathbf{p} - \mathbf{q}| + \Delta_p)|^2$$

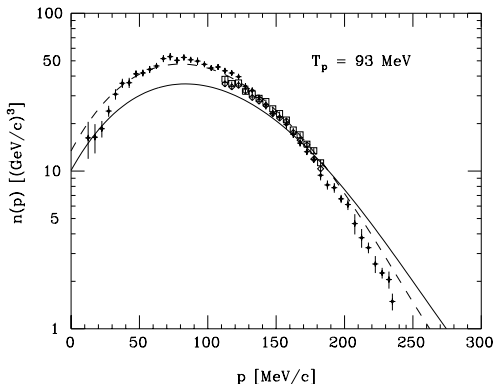




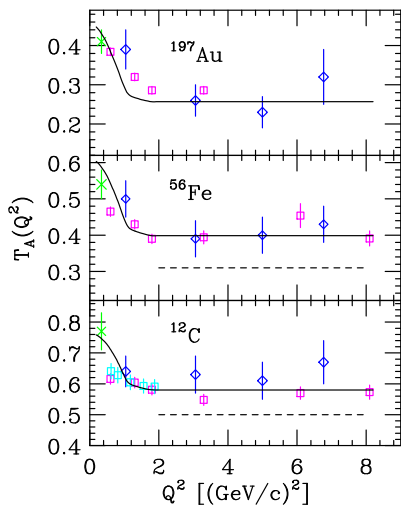
$$Z = 0.62, \quad \tilde{Z}_p = 0.72$$



- ▶ solid line: $\chi(\mathbf{r}) = \langle {}^{15}\text{N}(3/2)^- | a_{\mathbf{r}} | {}^{16}\text{O} \rangle$, $Z = 0.62$
- ▶ dashed line: $\chi(\mathbf{r}) = \sqrt{Z} \phi_{\text{WS}}(\mathbf{r})$, $Z = 0.56$



Nuclear transparency (no FSI $\rightarrow T_A \equiv 1$)



- ▶ Nuclear transparency obtained from

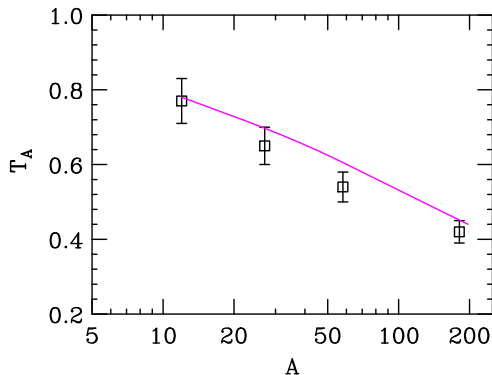
$$T_A = \frac{1}{A} \int d^3r \rho_A(\mathbf{r}) |\Omega_{\mathbf{p}}^{(-)}(\mathbf{r})|^2$$

compared to MIT-Bates, SLAC and JLab data (D. Rohe *et al*, Phys. Rev. C 72(05)054602)

- ▶ Complicated pattern of correlation effects, leading to a sizable enhancement of the transparency

How low can the proton energy be ?

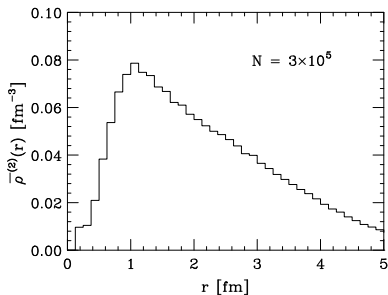
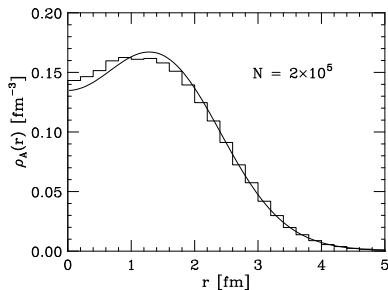
- ★ Compare theory to the A-dependence of nuclear transparency to a 200 MeV proton, measured at MIT



- ★ The high energy approximation appears to work down to surprisingly low energy

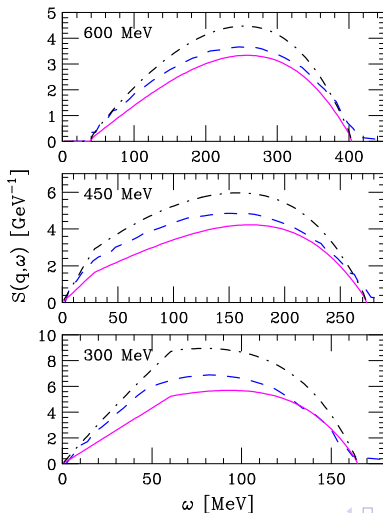
Implementing FSI in Monte Carlo simulations

- ★ Distribution of the primary vertex and two-nucleon density obtained sampling the probability distributions associated with the ^{16}O wave function of Pieper *et al*



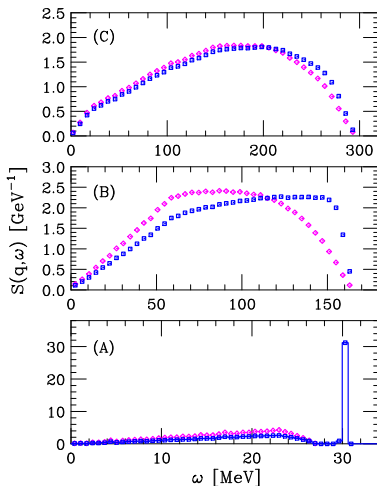
How far down can we go in momentum transfer ?

- ★ Compare **IA** ($P(\mathbf{p}, E)$, no FSI) to **correlated Hartree-Fock** response (A. Ankowski, OB and N. Farina, AD 2010)



Onset of collective excitations

- ★ Compare **correlated Hartree-Fock** to **correlated Tamm Dancoff** (OB and N. Farina, AD 2009): (A), (B), (C) = 0.3, 1.2, 2.4 fm⁻¹



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- The IA can be seen as the zero-th order of a systematic approximation scheme, to be improved upon, *consistently* including effects such as FSI, antisymmetrization of the final state and coupling to collective excitations.