Few-body nuclear reactions in transition operator framework

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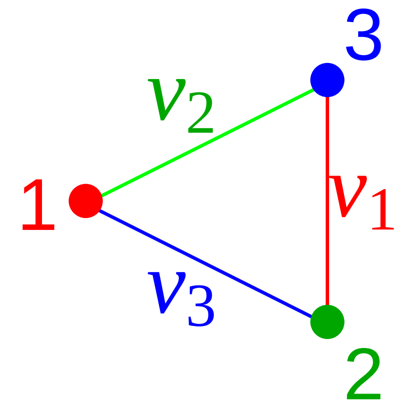
Few-body scattering

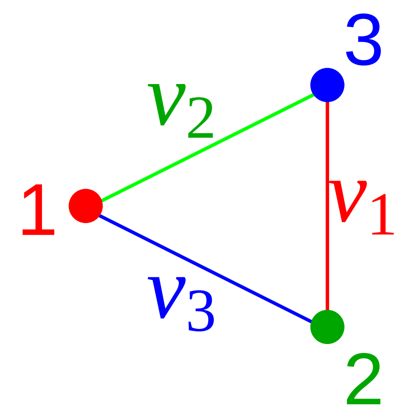
- Three-particle scattering equations
- Three-body direct nuclear reactions
- Extensions: four-body scattering, e.m. reactions

Three-body system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-body system



Hamiltonian $H_0 + \sum_\alpha v_\alpha$

- Faddeev equations

$$(E - H_0 - v_\alpha)|\psi_\alpha\rangle = v_\alpha \sum_\sigma \bar{\delta}_{\alpha\sigma}|\psi_\sigma\rangle$$

$$|\Psi\rangle = \sum_\alpha |\psi_\alpha\rangle$$

Alt-Grassberger-Sandhas equations

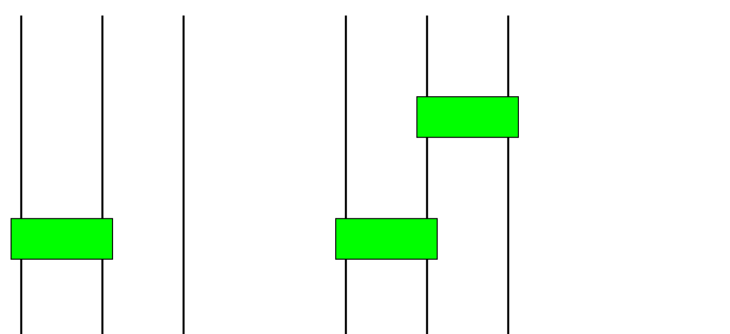
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

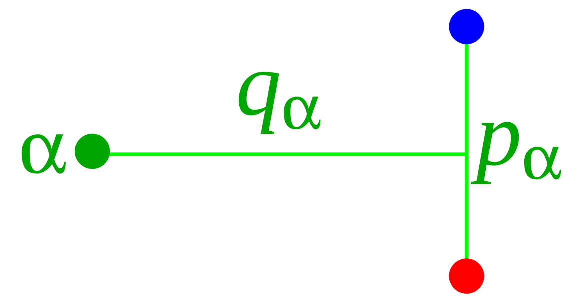
$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$



AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_\sigma G_0 U_{\sigma\alpha}$$



- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Gaussian integration, spline interpolation, Padé summation
  [PRC 67, 034001 (2003); 74, 064001 (2006)]

Coulomb: screening and renormalization

$$w_R(r) = w_C(r)\; e^{-\left(\frac{r}{R}\right)^n}$$

- screening: standard scattering theory

$$v_\sigma \longrightarrow v_\sigma + w_{\sigma R}: \quad T_\sigma,\, U_{\beta\alpha} \longrightarrow T_\sigma^{(R)},\, U_{\beta\alpha}^{(R)}$$

Coulomb: screening and renormalization

$$w_R(r) = w_C(r)\, e^{-\left(\frac{r}{R}\right)^n}$$

- screening: standard scattering theory

  $v_\sigma \to v_\sigma + w_{\sigma R}: \quad T_\sigma,\, U_{\beta\alpha} \to T_\sigma^{(R)},\, U_{\beta\alpha}^{(R)}$

- nature: Coulomb is screened at large distances
- large R:
  physical observables insensitive to screening,
  screened and full Coulomb physically indistinguishable
- renormalization:

$$w_R \xrightarrow[R\to\infty]{} w_C \quad \text{but} \quad T_R\, e^{2i\varphi_R} \xrightarrow[R\to\infty]{} T_C \qquad \left(\begin{matrix}\text{on-shell,}\\ \text{as distribution}\end{matrix}\right)$$

[*J. R. Taylor, Nuovo Cim. B23, 313 (1974)*]

Practical realization

- Calculation of short-range part using
standard scattering theory (Faddeev/AGS)
for nuclear + screened Coulomb interaction

$$U_{\beta\alpha}^{(R)} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\sigma \bar{\delta}_{\beta\sigma} T_\sigma^{(R)} G_0 U_{\sigma\alpha}^{(R)}$$

+ renormalization (fast R-convergence)

$$U_{\beta\alpha}^{(C)} = \delta_{\beta\alpha} T_{\alpha C}^{\text{c.m.}} + \lim_{R\to\infty} e^{i\varphi_{\beta R}} [U_{\beta\alpha}^{(R)} - \delta_{\beta\alpha} T_{\alpha R}^{\text{c.m.}}] e^{i\varphi_{\alpha R}}$$

Practical realization

- Calculation of short-range part using
standard scattering theory (Faddeev/AGS)
for nuclear + screened Coulomb interaction

$$U^{(R)}_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\sigma \bar{\delta}_{\beta\sigma} T^{(R)}_\sigma G_0 U^{(R)}_{\sigma\alpha}$$

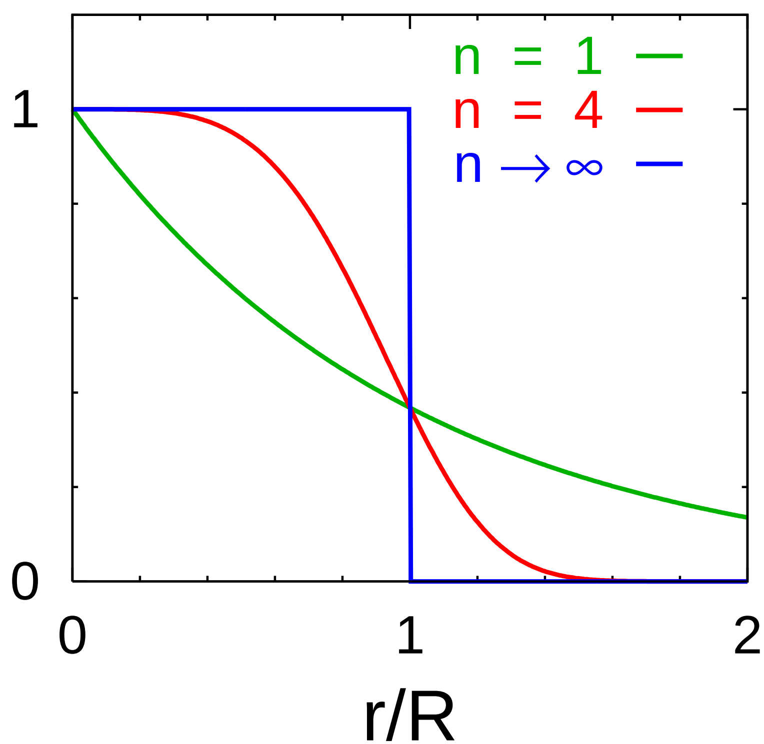
+ renormalization (fast R-convergence)

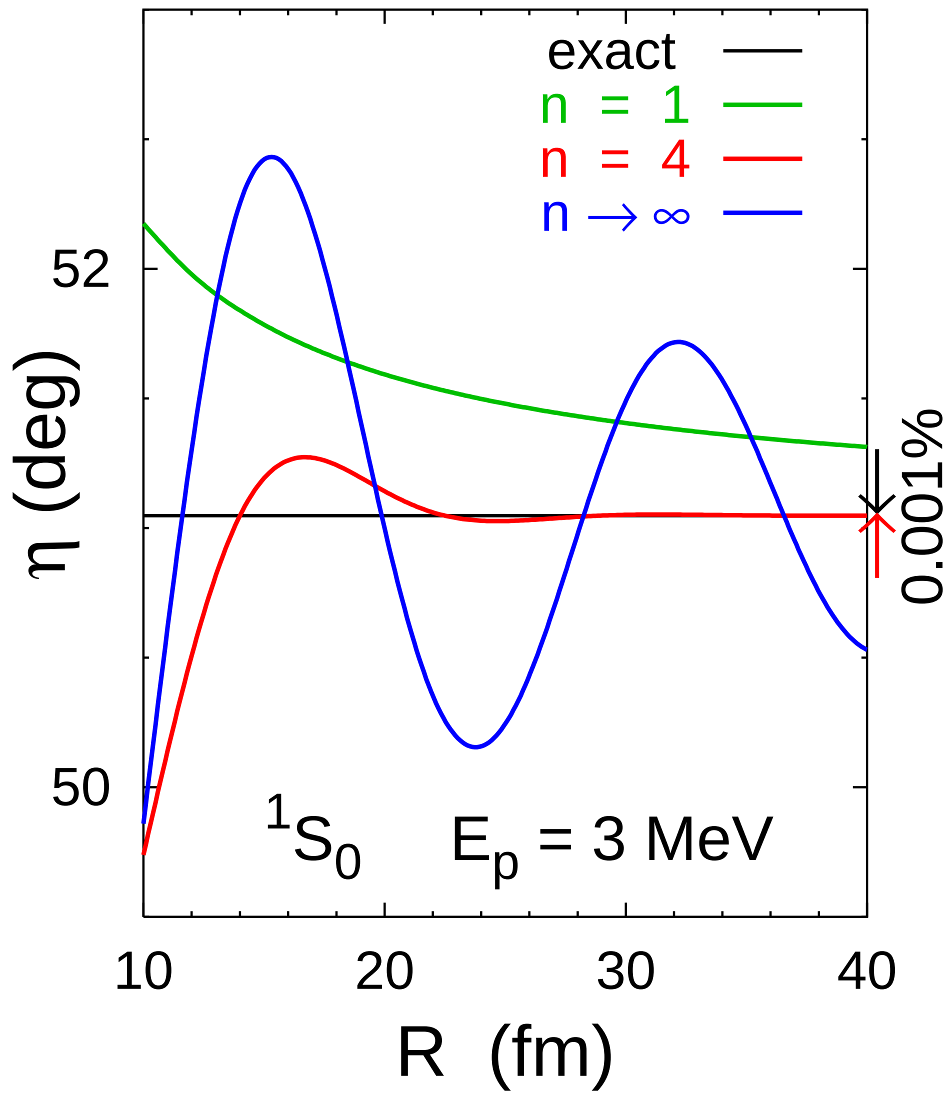
$$U^{(C)}_{\beta\alpha} = \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha C} + \lim_{R\to\infty} e^{i\varphi_{\beta R}} [U^{(R)}_{\beta\alpha} - \delta_{\beta\alpha} T^{\text{c.m.}}_{\alpha R}] e^{i\varphi_{\alpha R}}$$

- Additional difficulties:
quasi-singular nature of screened Coulomb potential
slow partial-wave convergence
- Success of the method depends strongly on the choice of screening function [PRC 71, 054005 (2005)]

Screened Coulomb potential

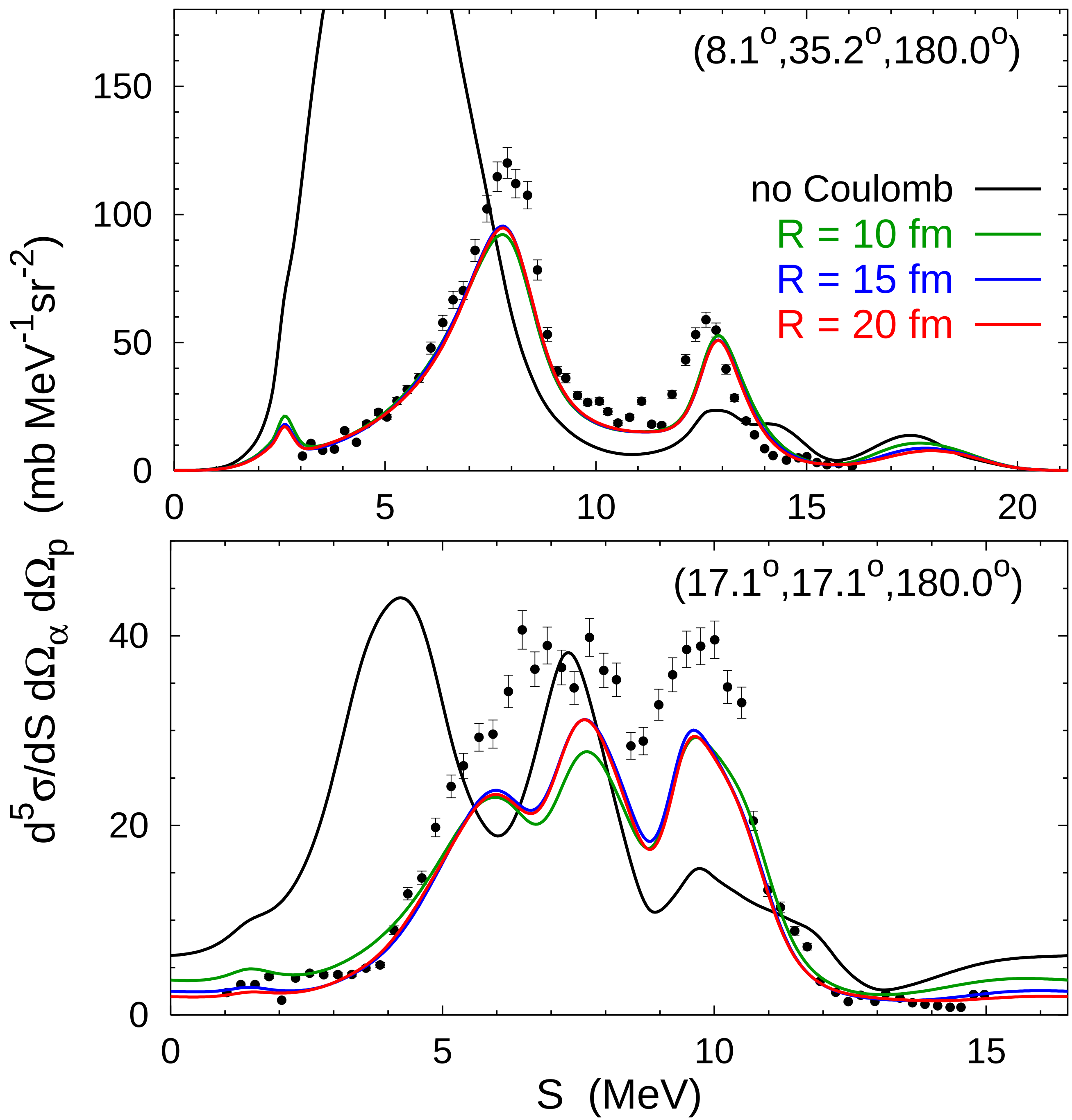
$$\frac{w_R(r)}{w_C(r)} = e^{-\left(\frac{r}{R}\right)^{\mathrm{n}}}$$



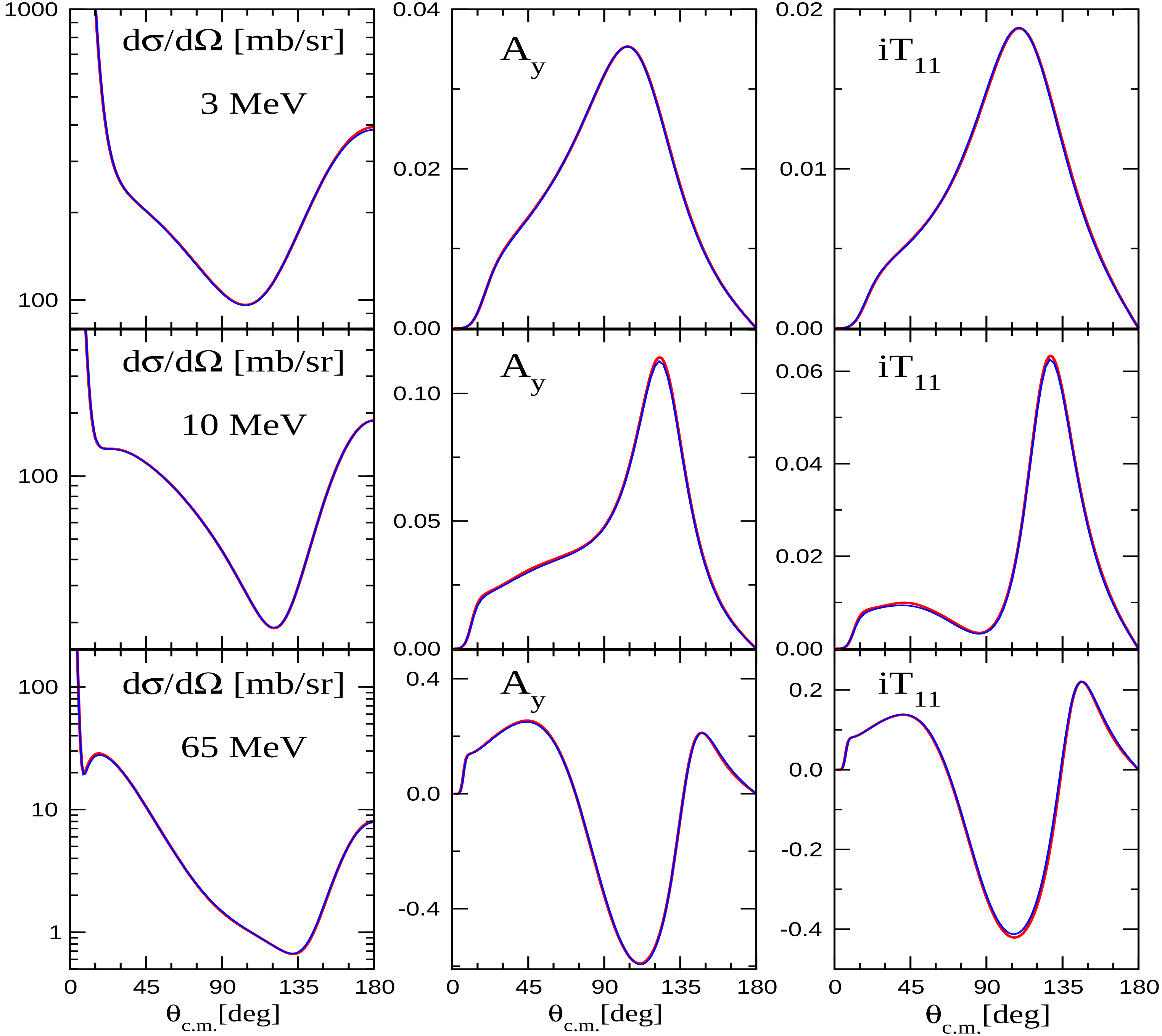


optimal choice: $3 \leq \mathrm{n} \leq 8$

Convergence with R: α-d breakup at $E_\alpha = 15$ MeV



Comparison with configuration-space results



p-*d* elastic scattering:

Kohn variational principle (Pisa)

screening and renormalization

[PRC 71, 064003 (2005)]

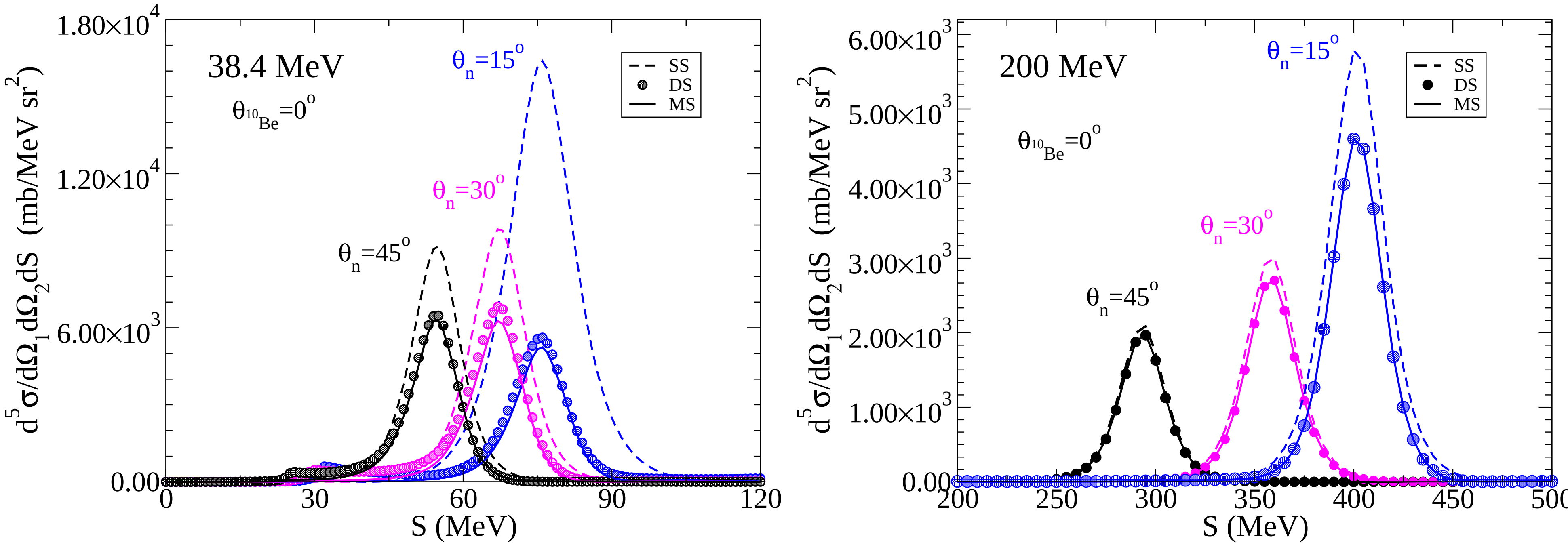
Application to 3-body nuclear reactions

$$\left.\begin{array}{l} p+(nA) \\ d+A \end{array}\right\} \longrightarrow \left\{\begin{array}{l} n+(pA) \\ p+(nA) \\ d+A \\ p+n+A \end{array}\right.$$

with $A = {}^{4}\mathrm{He},\ {}^{10}\mathrm{Be},\ {}^{12}\mathrm{C},\ {}^{14}\mathrm{C},\ {}^{16}\mathrm{O},\ {}^{28}\mathrm{Si},\ {}^{40}\mathrm{Ca},\ {}^{58}\mathrm{Ni},\ \ldots$

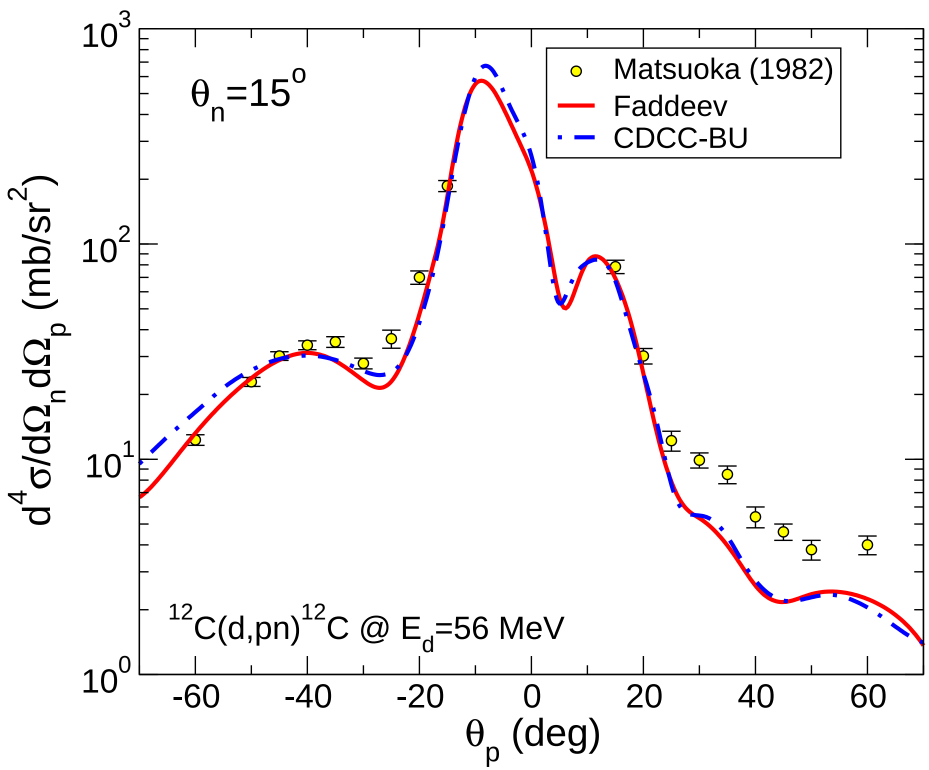
- Validity test of approximate nuclear reaction methods: PWIA, DWBA, Glauber, CDCC, ...
- Novel dynamic input: nonlocal or energy-dependent potentials, ...

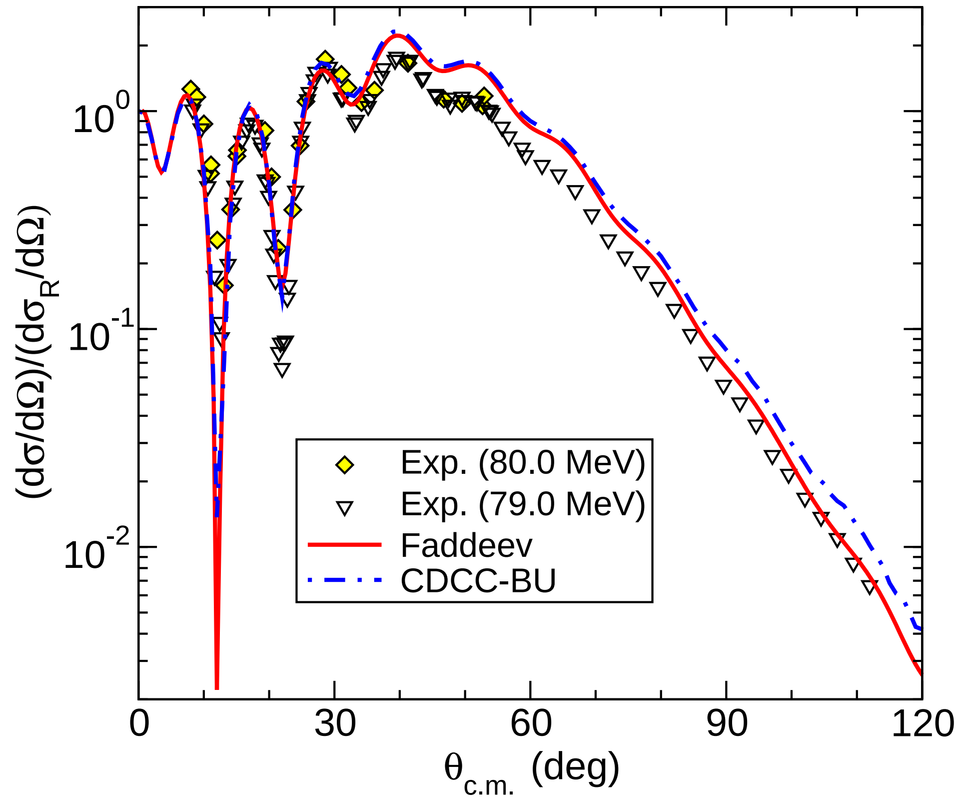
Convergence of scattering series: $^{1}\mathrm{H}(^{11}\mathrm{Be},{}^{10}\mathrm{Be}\,n)p$



[R. Crespo *et al*, PRC 77, 024601 (2008)]

CDCC test: $^{12}C(d,pn)^{12}C$ & $^{58}Ni(d,d)^{58}Ni$

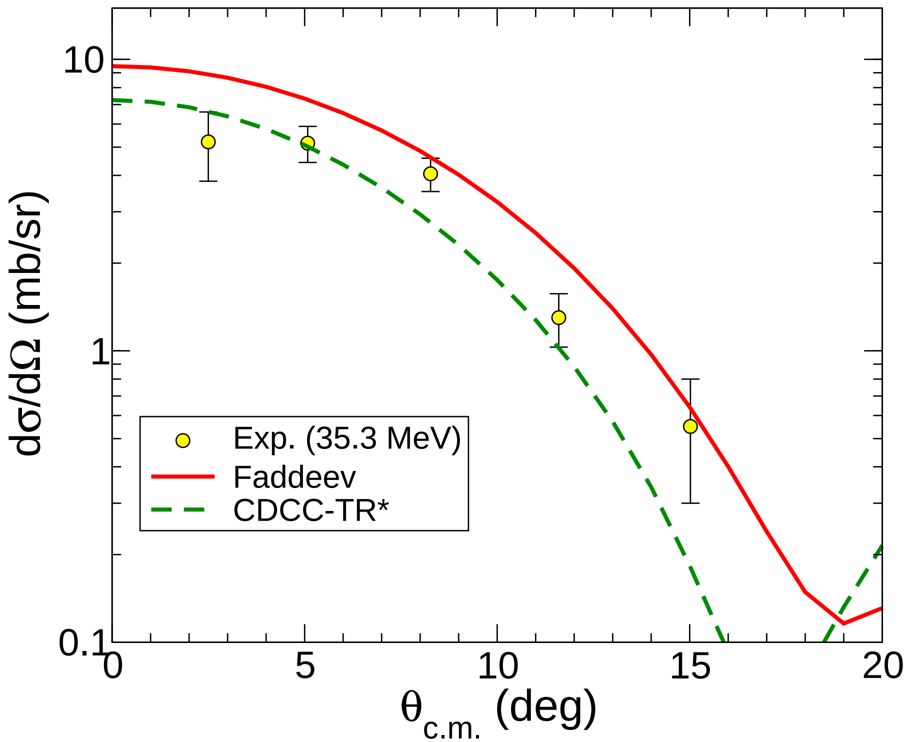


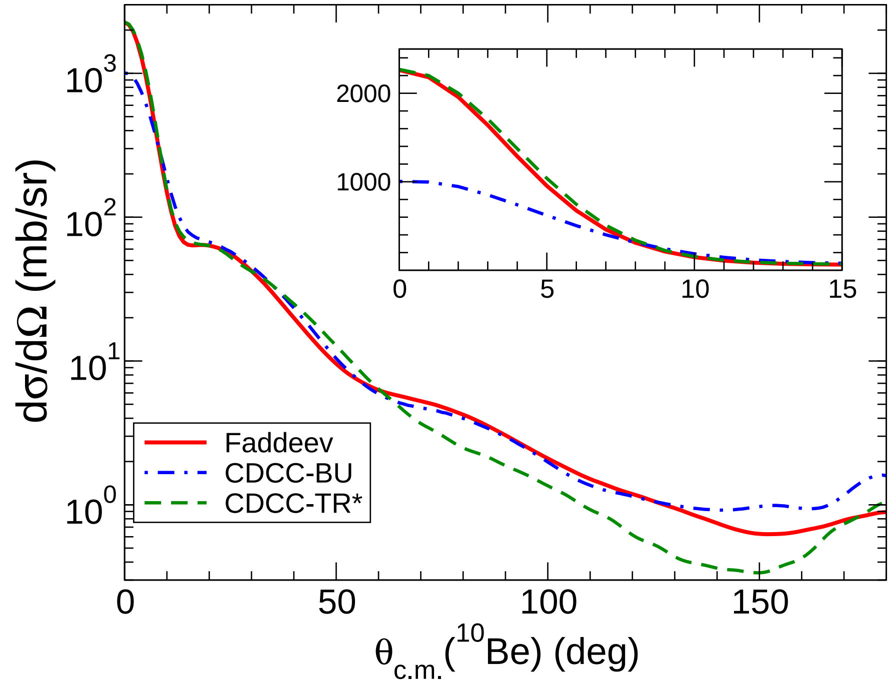


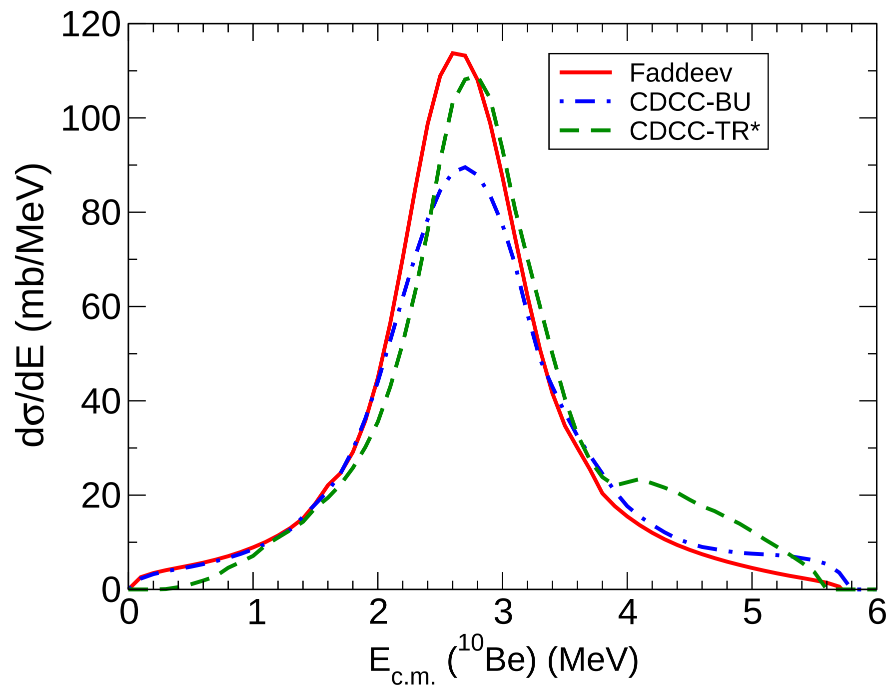
CDCC: A. M. Moro & F. M. Nunes

[PRC 76, 064602 (2007)]

CDCC test: $^{1}\mathrm{H}(^{11}\mathrm{Be},^{10}\mathrm{Be})d$ & $^{1}\mathrm{H}(^{11}\mathrm{Be},^{10}\mathrm{Be})np$



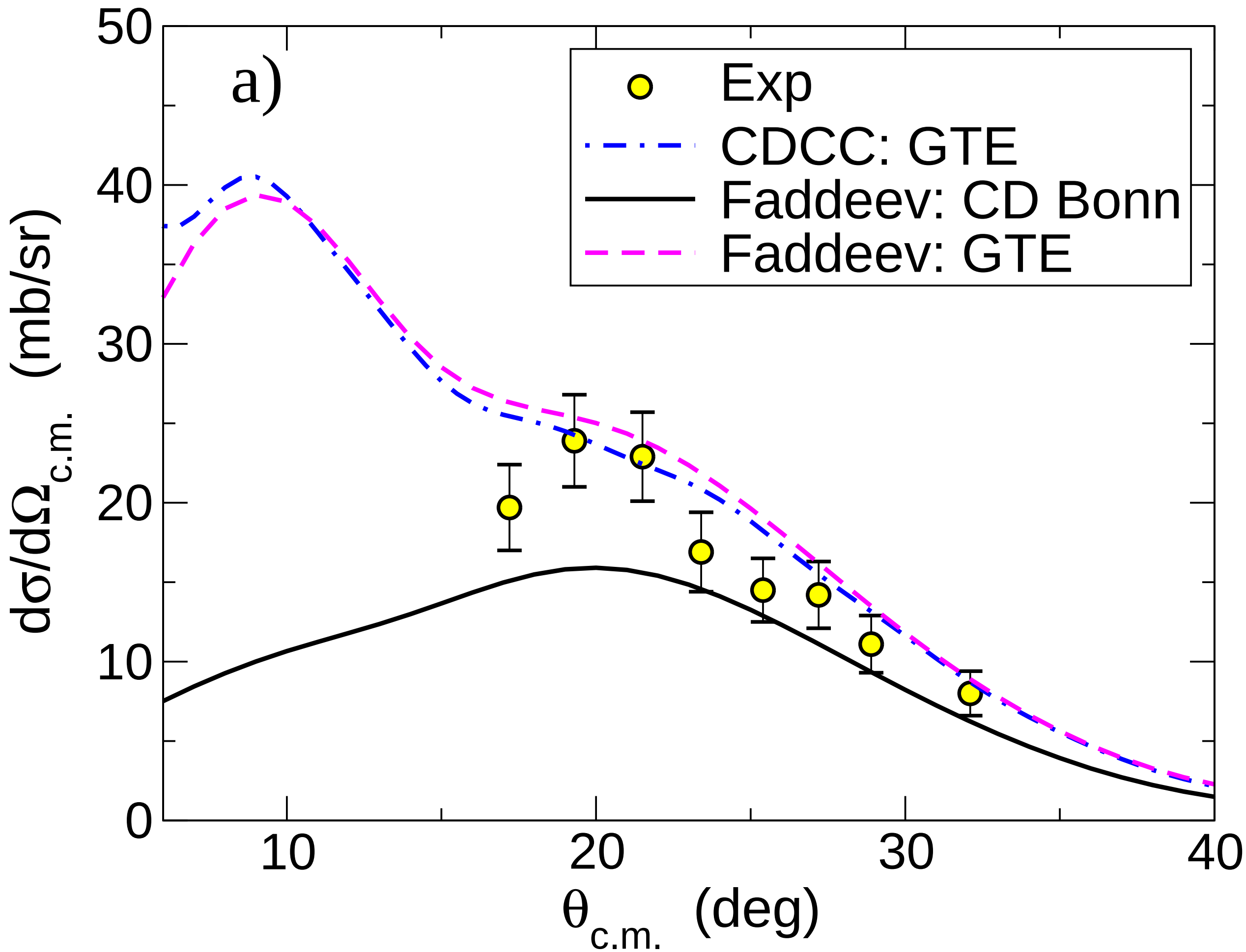




$E/A = 38\ \mathrm{MeV}$

CDCC:
A. M. Moro & F. M. Nunes

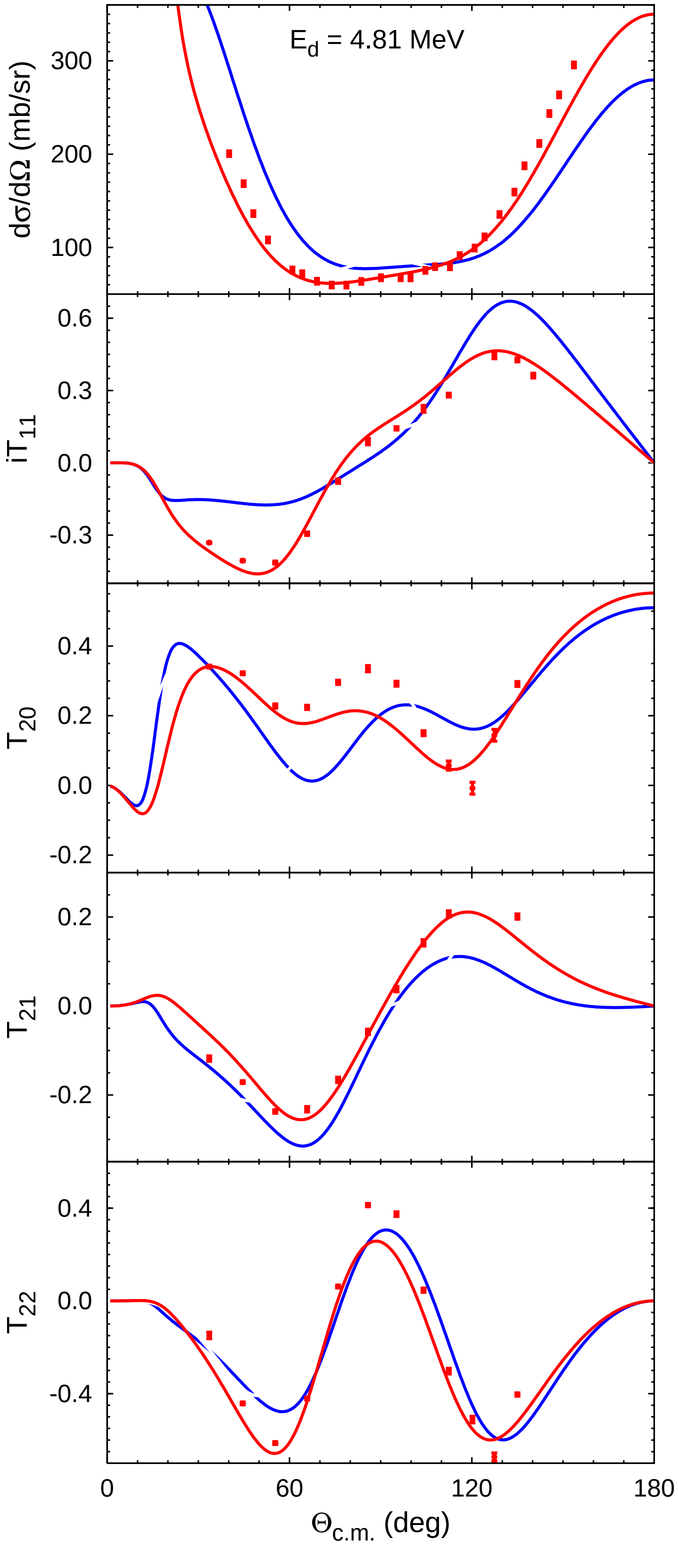
CDCC and NN potential test: $^{1}\mathrm{H}(^{11}\mathrm{Be}, n\,^{10}\mathrm{Be})p$



$E/A = 63.7$ MeV

CDCC: A. M. Moro

[E. Cravo *et al*, PRC 81, 031601 (2010)]



$\vec{d} + \alpha$ elastic scattering

$E_d = 4.81$ MeV

N-α S-wave:

$V_{N\alpha} \to V_{N\alpha} + |b\rangle\Gamma\langle b|$

Pauli forbidden bound state projected out

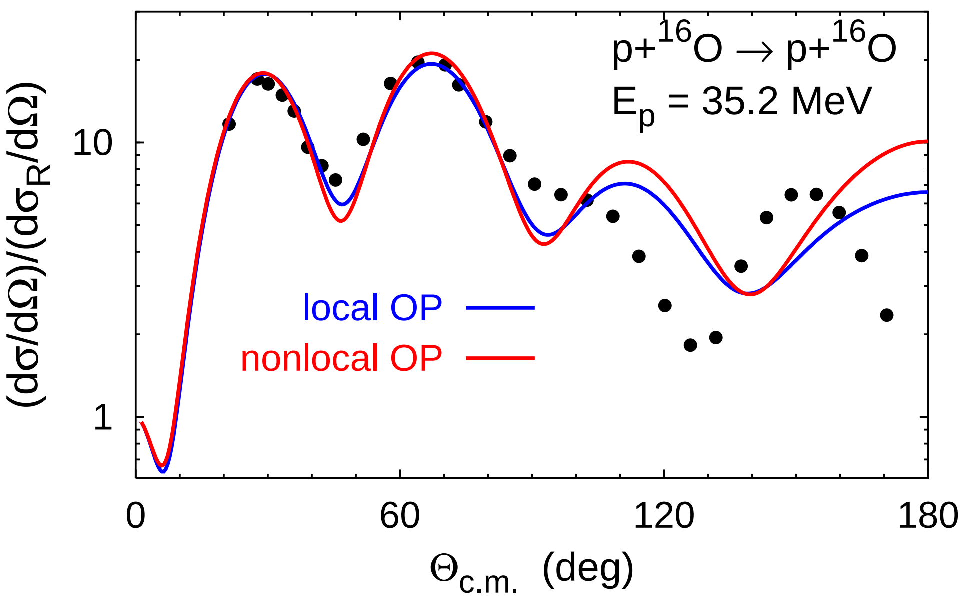
$V_{N\alpha}$ local repulsive

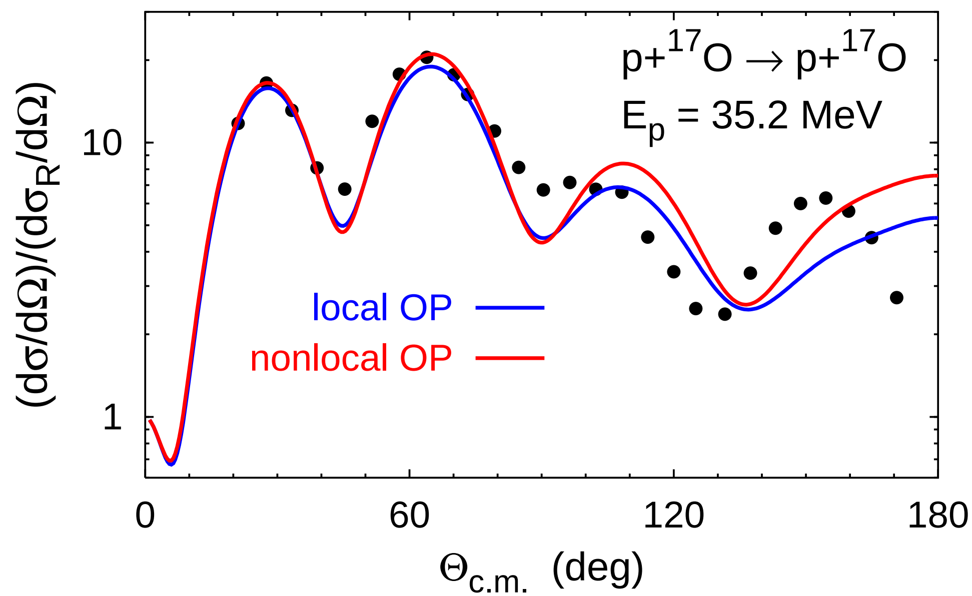
[PRC 74, 064001 (2006)]

Nonlocal optical potential: proton elastic scattering

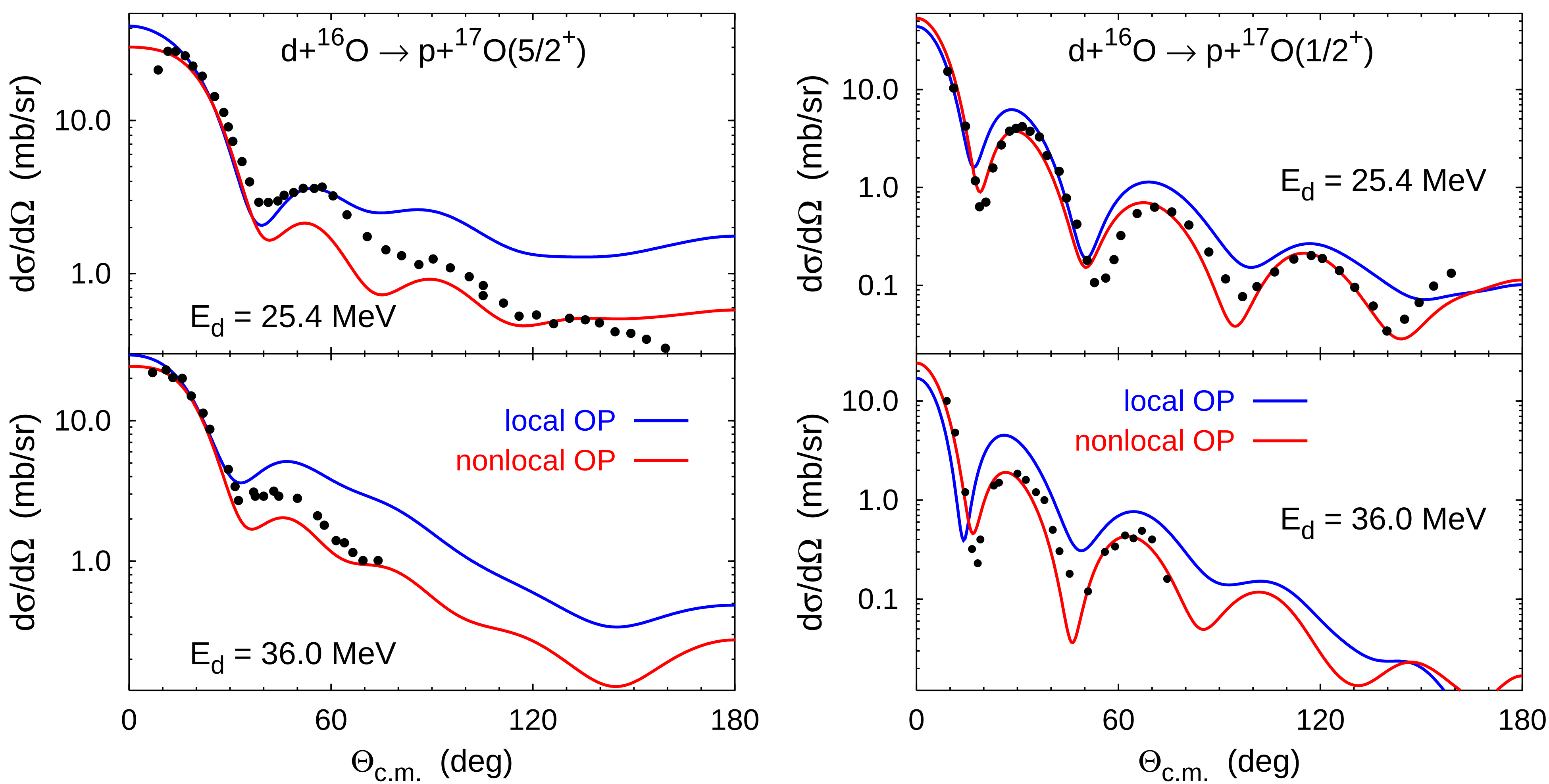
$$V_N(\mathbf{r}',\mathbf{r}) \sim e^{-(\mathbf{r}'-\mathbf{r})^2/\beta^2} V((\mathbf{r}'+\mathbf{r})/2)$$

[M. M. Giannini *et al.*,
Ann. Phys. (NY) 102, 458 (1976) & 124, 208 (1980)]



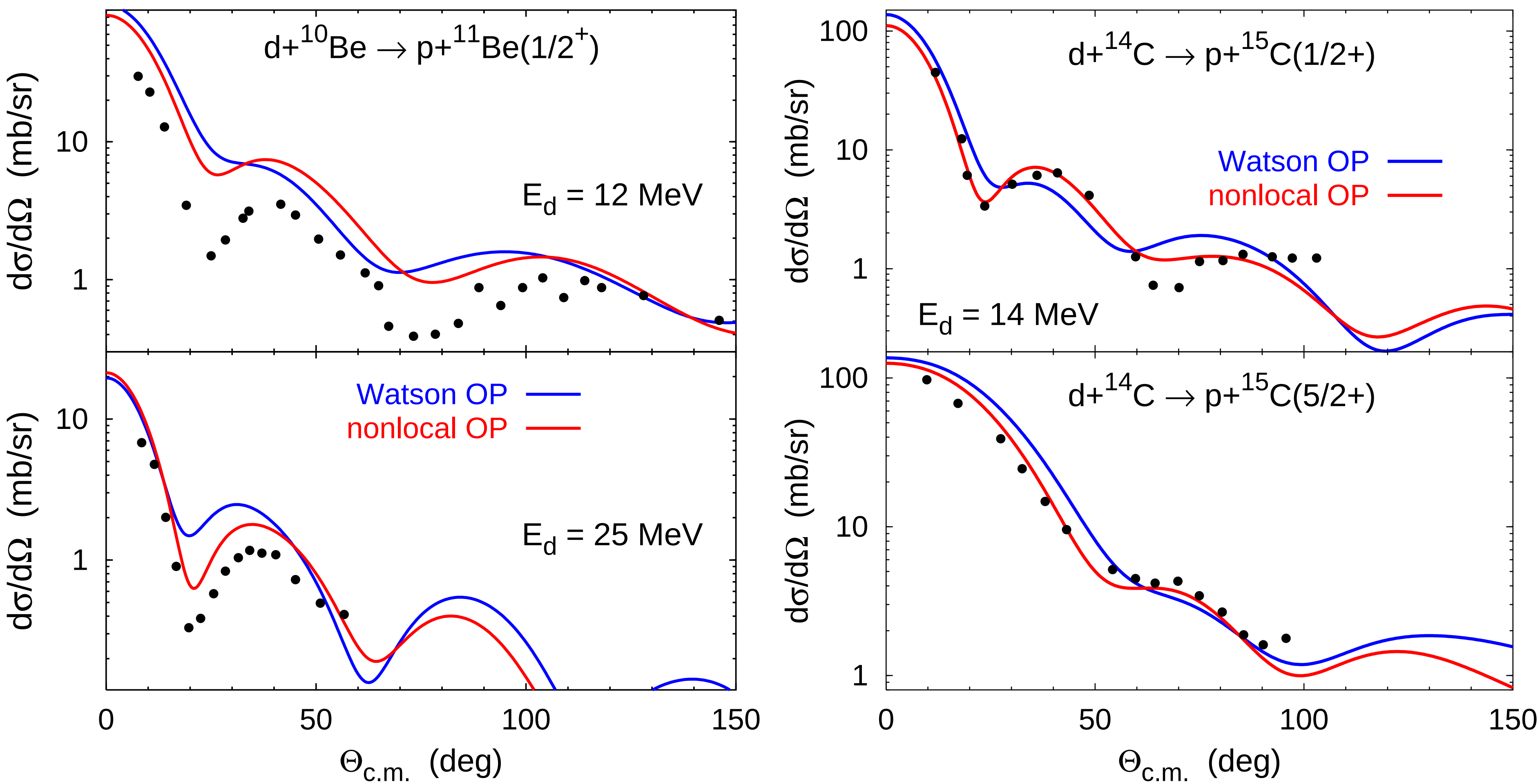


Nonlocal OP: transfer reactions $^{16}\mathrm{O}(d,p)^{17}\mathrm{O}$



[PRC 79, 021602 (2009)]

Nonlocal OP: $^{10}Be(d,p)^{11}Be$ & $^{14}C(d,p)^{15}C$



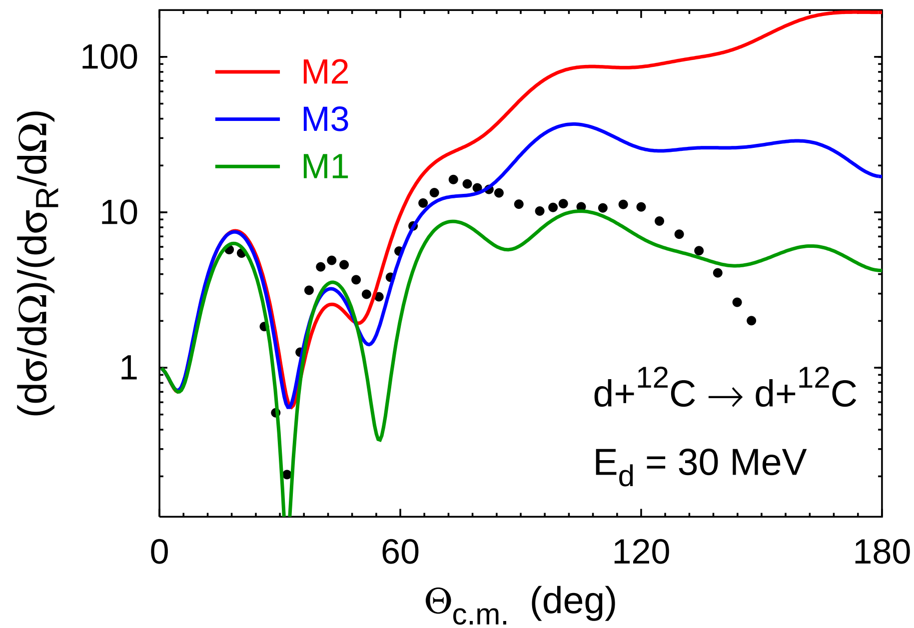
[PRC 79, 054603 (2009)]

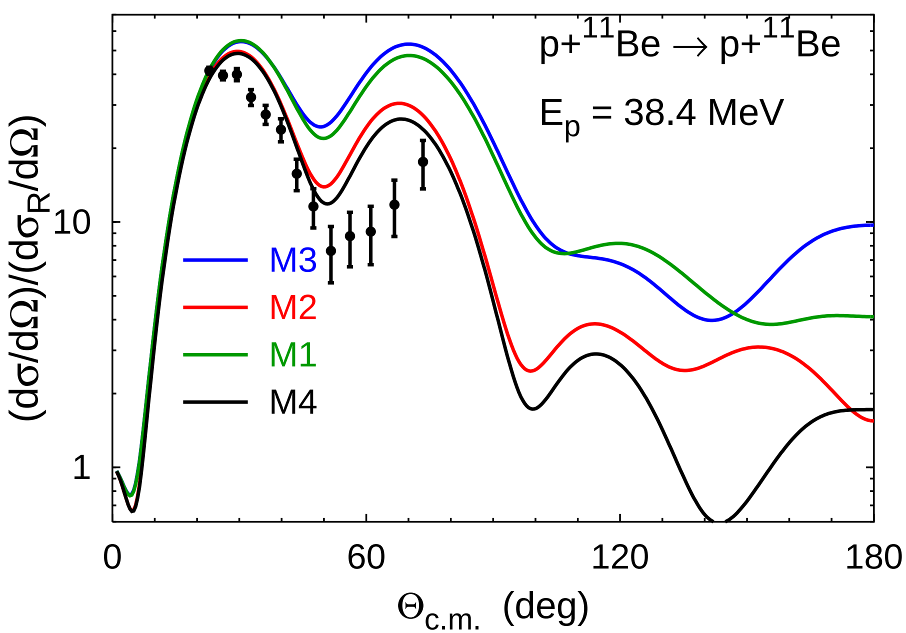
Energy-dependent optical potential

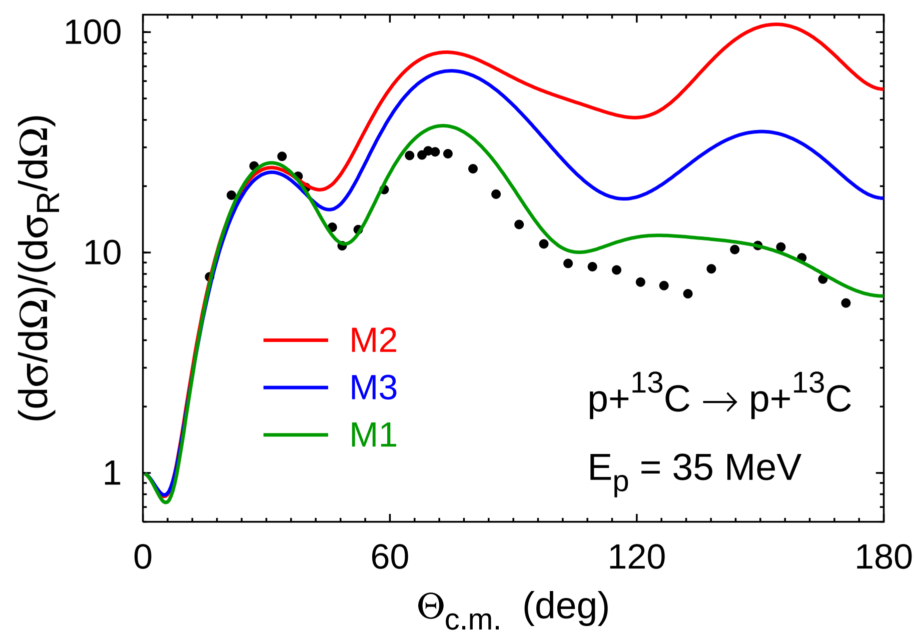
- non-hamiltonian theory
- A^*: effective E-dependent 2- and 3-body forces
- charge-exchange reactions $p + (An) \to n + (Ap)$:
  $E_{NA} < 0$: real NA potential supporting bound states
  $E_{NA} > 0$: complex NA potential describing scattering
- AGS: $T_{NA} = V_{NA} + V_{NA} G_0 T_{NA}$
  with $E_{NA} = E - q^2/2M \; \in (-\infty, E]$
  [PRC 79, 014606 (2009)]

Energy-dependent OP: elastic scattering

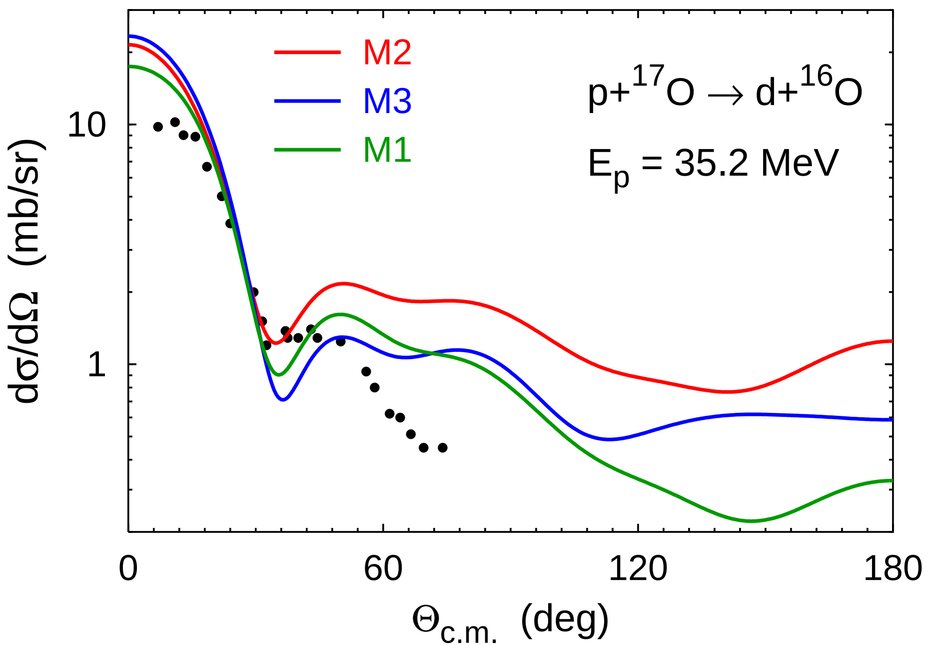
M1: E-independent OP
M2: E-dependent OP
M3: E(L)-dependent OP
M4: E-dependent OP for nA

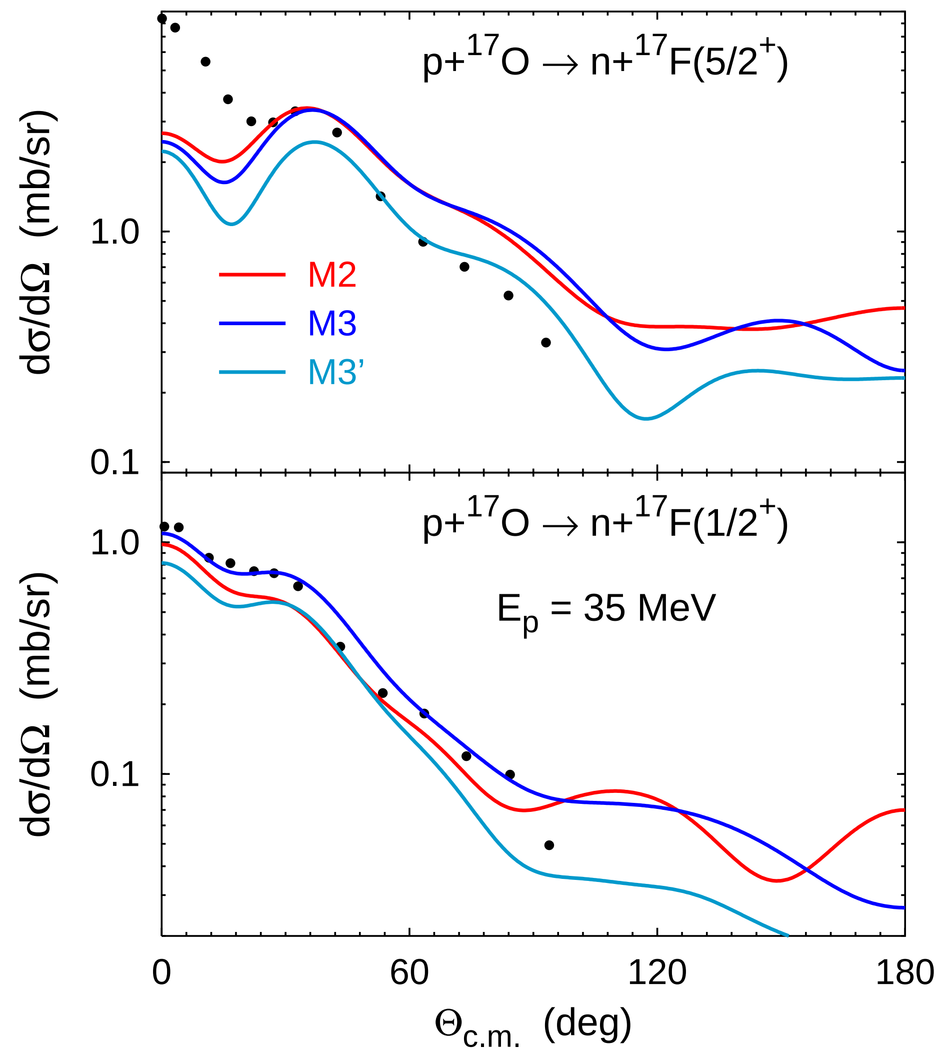






Energy-dependent OP: transfer and charge exchange





4N scattering: symmetrized AGS equations

two-cluster 1+3 and 2+2 transition operators

$$\mathcal{U}_{11} = -(G_0 T G_0)^{-1} P_{34} - P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{11} + U_2 G_0 T G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 T G_0)^{-1}(1 - P_{34}) + (1 - P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 T G_0)^{-1} - P_{34} U_1 G_0 T G_0 \, \mathcal{U}_{12} + U_2 G_0 T G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 - P_{34}) U_1 G_0 T G_0 \, \mathcal{U}_{12}$$

$$U_j = P_j G_0^{-1} + P_j T G_0 U_j$$

$$P_1 = P = P_{12} P_{23} + P_{13} P_{23}$$

$$P_2 = \tilde{P} = P_{13} P_{24}$$

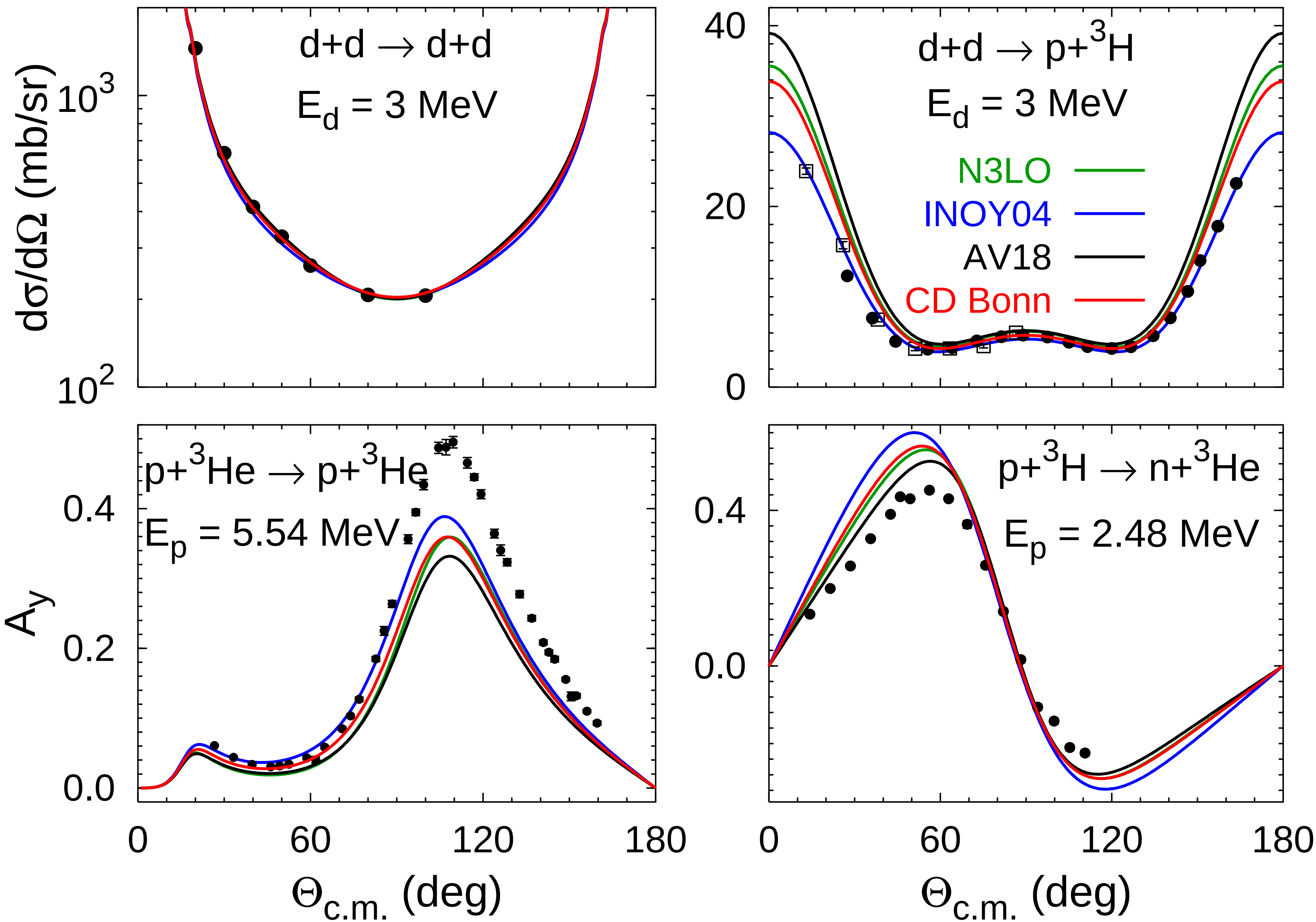
$$T = v + v G_0 T$$

scattering amplitude $\quad \mathcal{T}_{fi} = S_{fi} \langle \mathbf{p}_f \phi_f | \mathcal{U}_{fi} | \mathbf{p}_i \phi_i \rangle$

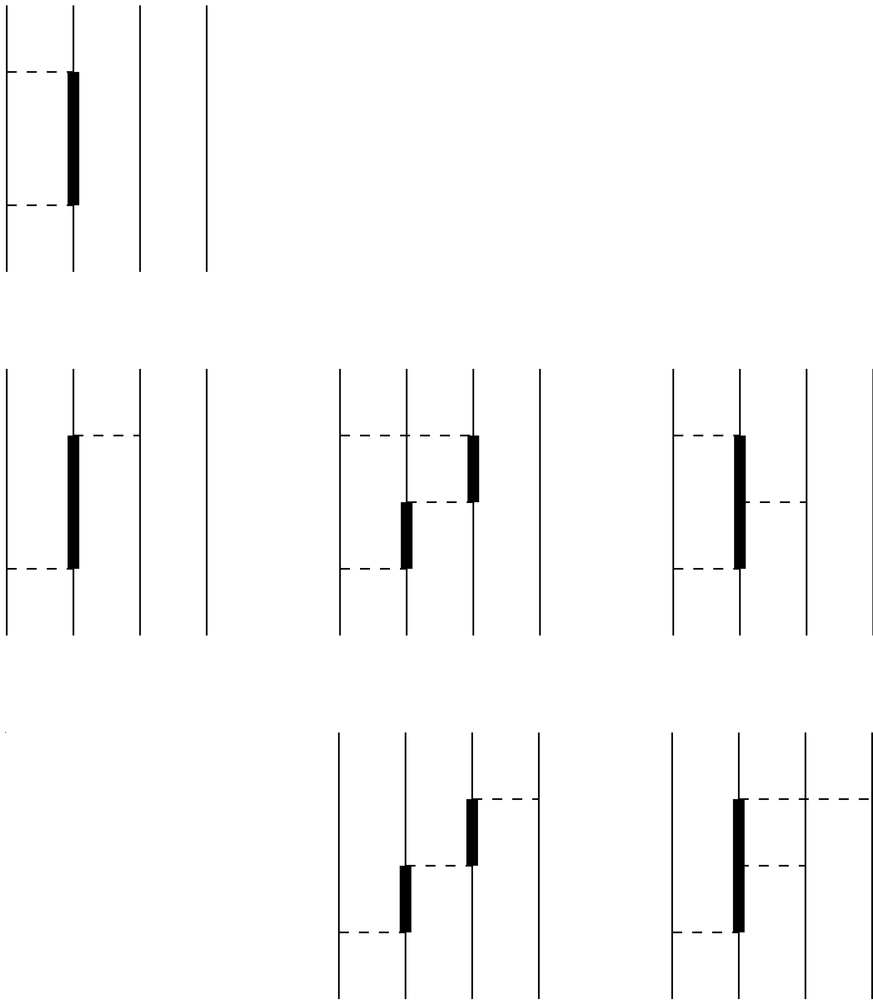
$$|\phi_j\rangle = G_0 T P_j |\phi_j\rangle$$

[PRL 98, 162502 (2007)]

4N elastic, transfer, and charge exchange reactions



Excitations: effective 2-, 3-, and 4-body forces



[PLB 660, 471 (2008)]

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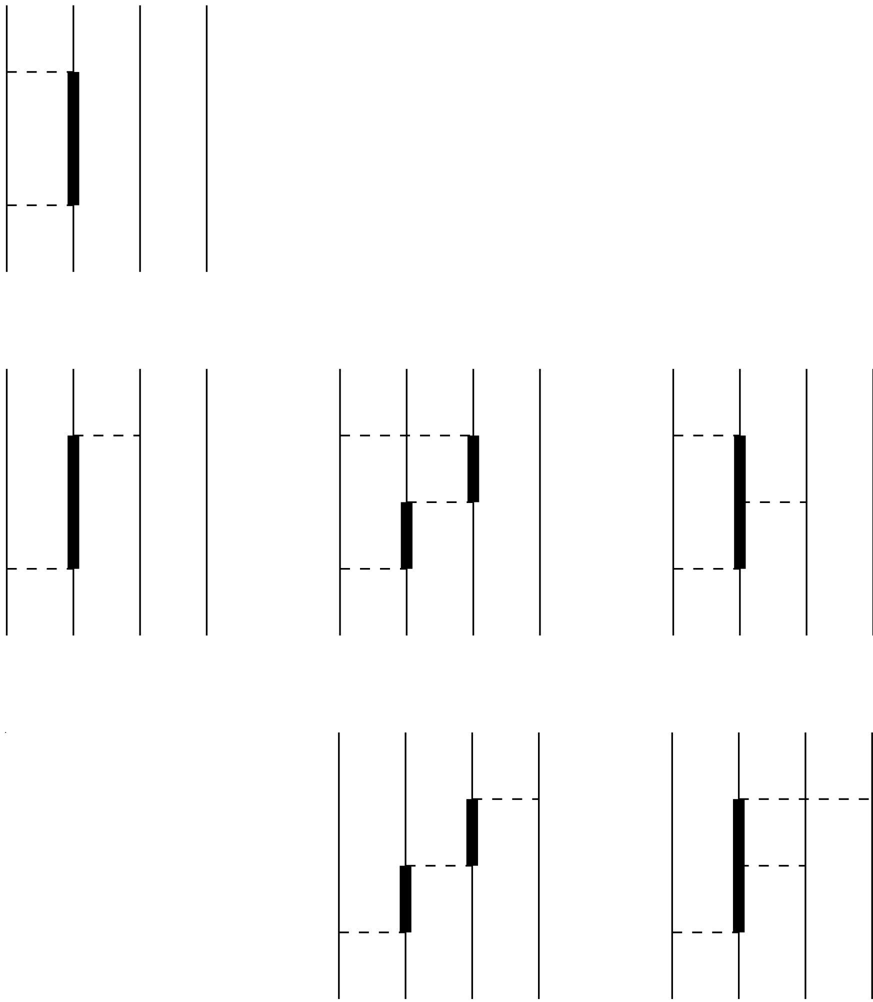
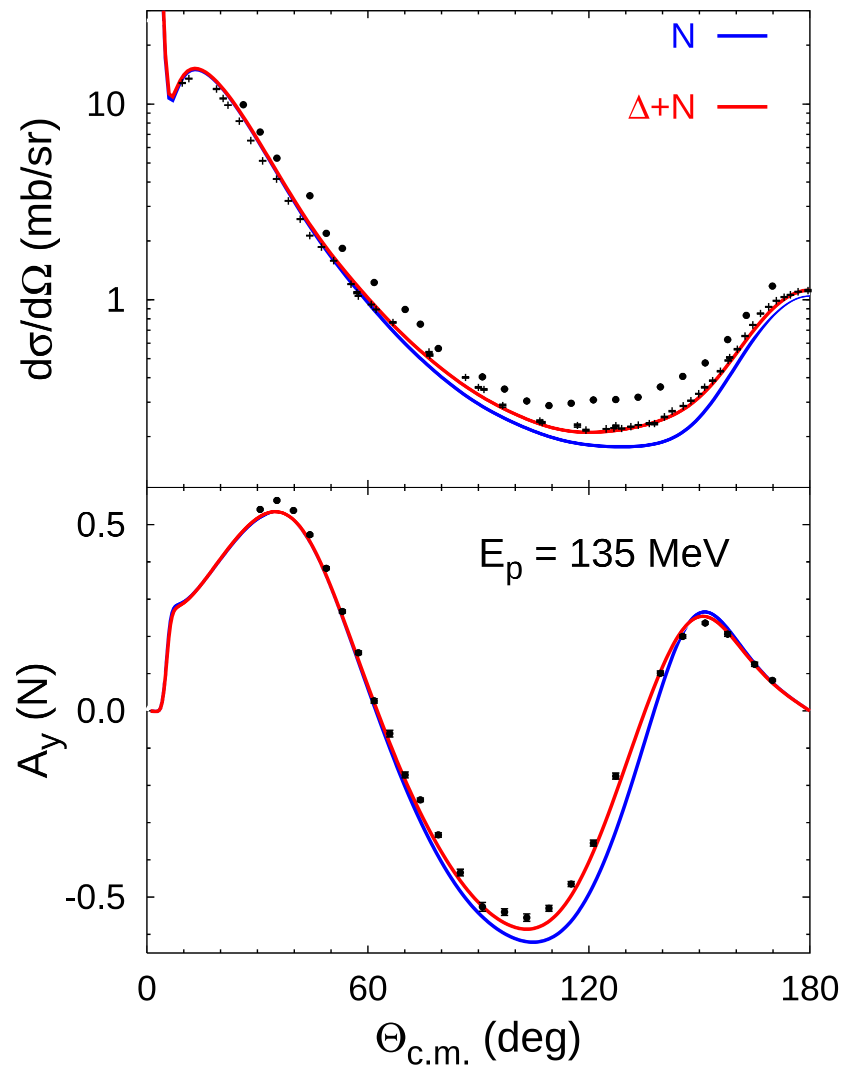
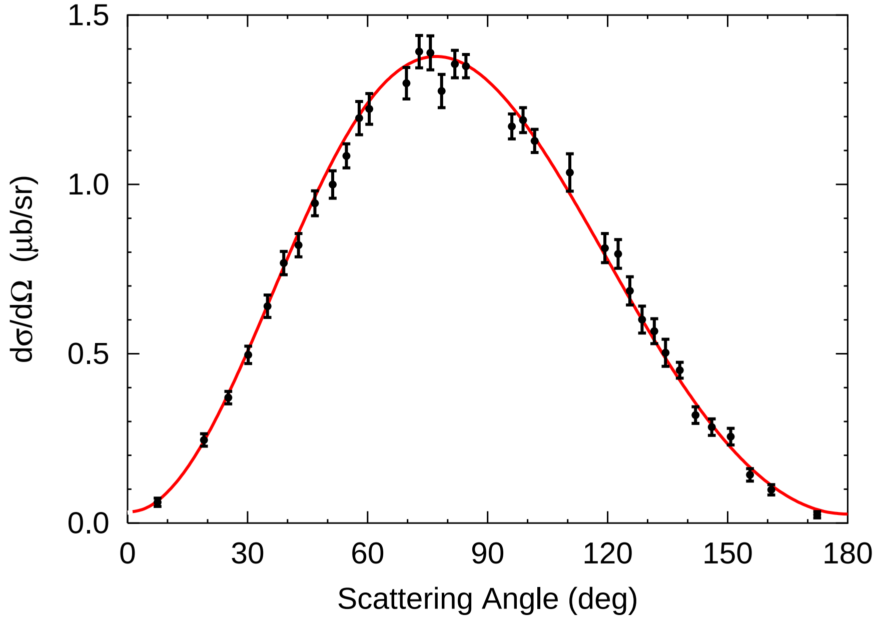


[PLB 660, 471 (2008)]



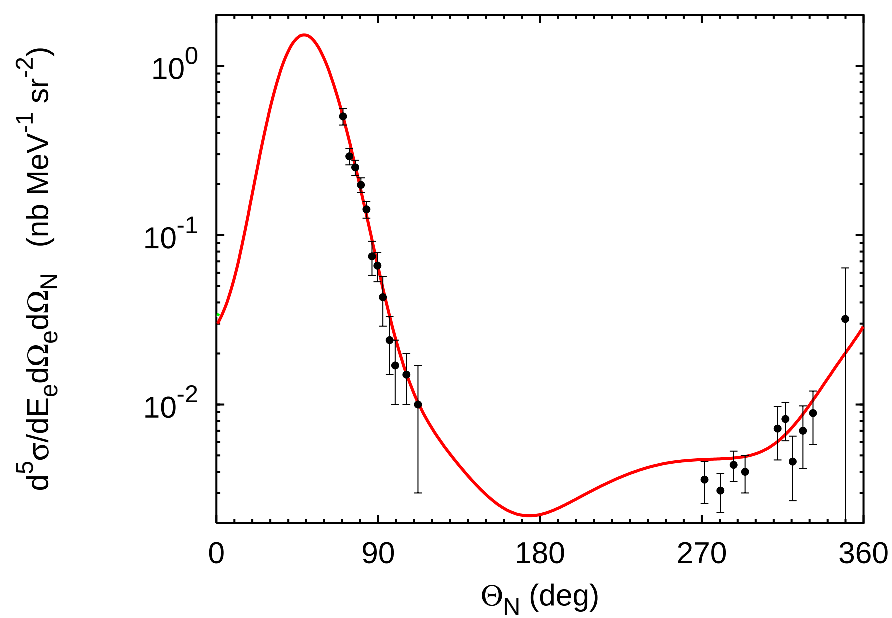
$p + d$ elastic scattering

Electromagnetic reactions



$^{1}\mathrm{H}(d,\gamma)^{3}\mathrm{He}$

$E_d = 19.8$ MeV



$^{3}\mathrm{He}(e,e'p)d$

$E_e = 390$ MeV, $\Theta_e = 39.7^\circ$

[PRC 70, 034004 (2004); 80, 064004 (2009)]

Summary

- Faddev/AGS integral equations for transition operators in momentum space
- Coulomb interaction: screening and renormalization
- 3-body nuclear reactions

Summary

- Faddev/AGS integral equations for transition operators in momentum space
- Coulomb interaction: screening and renormalization
- 3-body nuclear reactions
- low energy 4N scattering
- excitations
- electromagnetic reactions
- extensions beyond few-nucleon systems needed!