# Charge and neutron radii in an extended liquid drop approach

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# Motivation

Various precision experiments with (single) (radioactive) atoms are being planned

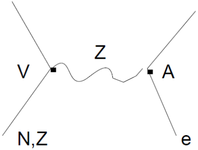
aim: test of standard model using atomic physics:
parity violation: weak charge, anapole moments;
$T-$ violation: edm...

In most cases one needs nuclear radii ($< R_c^2 >, < R_n^2 >$, but also $< R^4 >$) and atomic input (mostly $|\psi_{ns}(0)|^2$)

mostly for very heavy nuclei: Ra(Z=88), Fr(Z=89),..

# Nuclear structure in Atomic parity violation

$$\langle H\rangle = \frac{G}{2\sqrt{2}}\int d^3r[-N\rho_n(r) + Z(1-4\sin^2\theta_W)\rho_p(r)]\psi^\dagger\gamma_5\psi$$




$$<\psi_s|\gamma_5|\psi_p> = iC_Z^{sp}\mathcal{N}[u_1^s(r)u_2^p(r) - u_2^s(r)u_1^p(r)]/r^2$$

$$\mathcal{N} = \psi_s^\dagger(0)\gamma_5\psi_p(0) \sim R_p^{2\sqrt{1-(Z\alpha)^2}-2}$$

$C_Z^{sp}$: electr norm. for point nucleus

factorize m.e.: $H \sim \frac{G_F}{2\sqrt{2}} C_Z \mathcal{N} Q_W$

Nucleus: $Q_w = -Nq_n + Zq_p(1-4\sin^2\theta_W) + \Delta Q_{new}$

$q_i = \int dr\rho_{n,p}(r)f(r)$: convolution of electr. wf with finite nucleus;

expand in $(Z\alpha)^n$

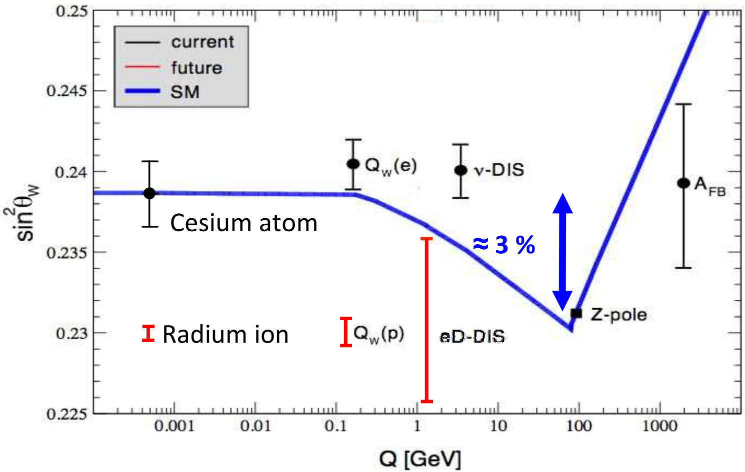
$q_p = 1 - 0.26(Z\alpha)^2 + ...$

$q_n = 1 + (Z\alpha)^2(-\frac{1}{2}\frac{R_n^2}{R_p^2} + \frac{1}{10}\frac{R_n^4}{R_p^4} + ... +$ corr. to sharp radius)

why large $Z$? since $H_{pv} \sim Z^3$ enhancement (+relat. corr.)

# Weinberg-angle

Weinberg angle $\sin^2\theta_W$ vs $Q^2$ in standard model



# present situation

aim: for Ra (Z=88), for 0.1% measurement
(to improve Cs experiment and to go after $\Delta Q_{newphysics}$ )
need: $R_p \approx 1\%$, $(R_n - R_p)/R_p \approx 25\%$ (at present no direct exp info)

Let us see whether we can meet these requirements

1. Experimental situation
   - Beyond Pb no accurate info on charge radii, only isotope shifts
   - No info on neutron skin
2. Theoretical
   - numerous microscopic calculations of $^{208}$Pb: mean field, effect of correlations
   - Properties of nuclei beyond A=208 depend on modeling: Skyrme, RMF..
   - prediction of skin depends strongly on isovector NN parameters

If one takes ratios of 2 isotopes $N, N'$ some uncertainties drop out

# outline

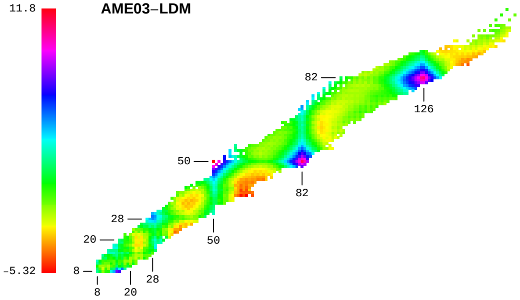
- Generalized Liquid Drop Model including Shell Corrections
  enables one to extrapolate to nuclei with unknown properties
- charge radii, isotope shifts:
  fit and prediction
- neutron skin: prediction
- Symmetry Energy in Nuclear Matter
  extrapolate from fit to nuclei to $A \to \infty$

# LDM

Conventional Bethe-von Weizsäcker (liquid drop) formula

$$E_A = -a_B A + a_{surf} A^{2/3} + S_{vol}(N - Z)^2/A + a_C \frac{Z^2}{A^{1/3}} + E_{pair}$$

- Incomplete form of symmetry energy
  Need to introduce volume and surface symmetry energy
- Coulomb need to be refined ($R_c(N, Z) \neq r_0 A^{1/3}$)
- shell corrections need to be added



# Extended Liquid Drop Model

- Symmetry Energy $E_{sym} = E(vol, surf)$

  Decompose asymmetry $N - Z = N_s - Z_s + N_{vol} - Z_{vol}$ surface+ volume

  $$E^A_{vol} = a_B A + S_{vol} \frac{(N_v - Z_v)^2}{A}$$
  $$E^A_{surf} = E^0_{surf} + S_{surf} (N_s - Z_s)^2 / A^{2/3}$$

  minimize under fixed $N - Z$ :

  $$\frac{N_s - Z_s}{N - Z} = \frac{1}{1 + y^{-1} A^{1/3}} \qquad y \equiv S_{vol}/S_{surf} \ (\sim 3)$$

  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + y A^{-1/3}} \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^{1/3}} + \ldots [a_{curv} A^{1/3}]$$

  Closely related to droplet model (Myers-Swiatecki)

  Can be viewed as leptodermous expansion (powers of $A^{1/3}$) of selfconsistent EDF for finite nuclei (P.G Reinhard et al.)

# Extended Liquid Drop Model

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  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + yA^{-1/3}} \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^{1/3}} + \ldots$$

- LDM yields also relation between skin and $S_s, S_v$ $(N_s - Z_s \ll A)$

  $$\frac{N}{N_v} = \left(\frac{R_n}{R_0}\right)^3 \rightarrow \frac{R_n - R_0}{R_0} \approx \frac{N_s}{3N}, \qquad \frac{R_p - R_0}{R_0} \approx \frac{Z_s}{3Z},$$

  $$\frac{R_n - R_p}{R} = \frac{A(N_s - Z_s)}{6NZ} \approx \frac{A}{6NZ} \frac{N-Z}{1 + A^{1/3}/y} - \frac{a_C}{12 S_v} \frac{ZA^{2/3}}{1 + A^{1/3}/y}$$

  Coulomb term: for $N = Z$ $R_p > R_n$

Danielewicz, NPA 727(2003)233; Steiner et al, Phys. Rep. 411,325

differ in the choice of condition $N_s + Z_s = 0$, or $Z_s = 0$

# shell corrections

Several methods for treating shell effects have been proposed
e.g. Strutinsky, Koura, Duflo & Zuker,....

Here we adapt the method in microscopic mass model of Duflo-Zuker
rms dev $\approx$ 500 keV, (14-28 parameters) in a simplified form

idea: count number of valence particles $(n_v, \ z_v)$
with respect to closed shells: $\Delta E_{sh} = \Delta E(n_v, z_v)$

# shell corrections(2)

$$E_{\rm shell}(N,Z) = a_1 S_2 + a_2 (S_2)^2 + a_3 S_3 + a_{\rm np} S_{\rm np}$$

$$S_2 = \frac{n_{\rm v}(D_n - n_{\rm v})}{D_n} + \frac{z_{\rm v}(D_z - z_{\rm v})}{D_z}, \qquad \xrightarrow{n_v << D} n_v + z_v$$

$$S_3 = \frac{n_{\rm v}\bar{n}_{\rm v}(n_{\rm v} - \bar{n}_{\rm v})}{D_n} + \frac{z_{\rm v}\bar{z}_{\rm v}(z_{\rm v} - \bar{z}_{\rm v})}{D_z}, \qquad \bar{n}_{\rm v} \equiv D_n - n_{\rm v} \quad \text{(number of holes)}$$

$$S_{\rm np} = \frac{n_{\rm v}\bar{n}_{\rm v} z_{\rm v}\bar{z}_{\rm v}}{D_n D_z}, \qquad \text{simulates deformation}$$

Cf: monopole force in single j-shell, degeneracy $D_j = 2j+1$, seniority $v = 0$

$$E_{pair}(n_v) = \frac{g}{D} n_v (D - n_v + 2) \equiv \frac{g}{D} n_v . \bar{n}_v + \frac{2g}{D} n_v$$

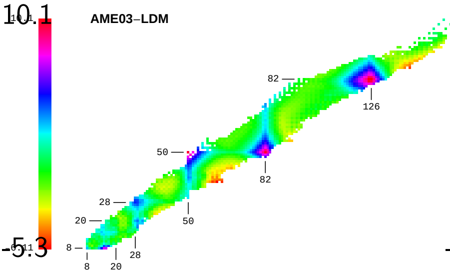
↘ absorb in core

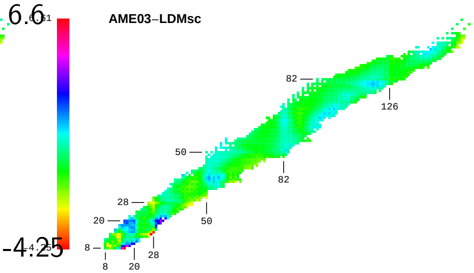
Magic numbers: 6, 14, 28, 50, 82, 126, 184

similar to terms in microscopic mass formula of Duflo and Zuker
they refer to $S_3$ as "monopole drift" (changes sign midshell)

Scaling with $A$ : $\Delta E_{shell} \sim A^{1/3}$ (However, $S_3$ does not)

# examples




without shell corr

rms deviation 2.4 MeV

with shell corr

0.790 MeV (mostly due to light A)

# More insight from differences

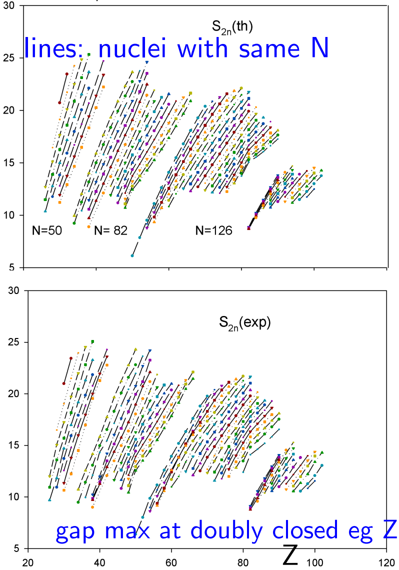
Consider differences between energies

e.g. two-neutron separation energies $S_{2n} = E(Z, N-2) - E(Z, N)$
compare LDM, microscopic mean field and experiment

# two-neutron sep. energies: $S_{2n} = E(Z, N-2) - E(Z, N)$

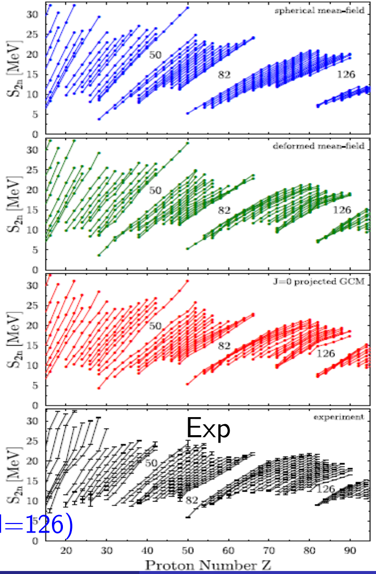
LDM+ shell corr.

lines: nuclei with same N




gap max at doubly closed eg Z=82,N=126)

Microscopic Skyrme SLy4



# radii

Crudest estimate: $R_c = r_0 A^{1/3}$

Improvement: express in terms of iso-scalar/vector parts

$$R_i(N,Z) = R_0(N,Z) \pm \frac{N-Z}{2A} R_1(N,Z) \quad \text{(i=n,p)}$$

mass radius: $R_0 = r_0 A^{1/3} + aA^{-2/3} + c\frac{(N-Z)^2}{A^2}$, in practice $c \approx 0$

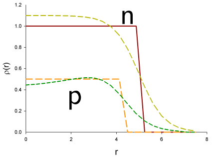
isovector radius $R_1 = b\left(\frac{1}{1+A^{1/3}/y}\right)$ $\quad y = S_v/S_s$

$R_{0,1}$ depend only weakly on $N-Z$

By charge symmetry: $R_p(N,Z)$ determines $R_n(N,Z)$

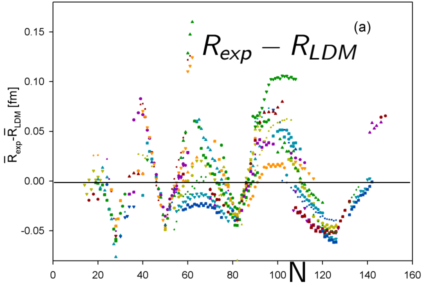
however there are Coulomb effects

1. Coulomb repulsion →for $N=Z$ $R_p > R_n$
   sharp radius: $\frac{\delta R_c}{R_0} = -\frac{a_c}{144 S_v} \frac{A^{8/3}}{NZ(1+y^{-1}A^{1/3})}$
2. polarization correction: $\rho_p(r) \neq \rho_n(r)$ in interior; (needed if $R$ is converted to rms radius)



# charge radii, shell corrections

Fit to data base (Angeli), plot $R_{exp} - R_{fit}$
At shell closures: strong binding $\leftrightarrow$ small $R$
near closed shells: decrease of binding, increase of $R$
midshell: deformation: further increase of $R$




rms dev= 0.036 fm

$$R_c(N,Z) = R_0(A) + \frac{N-Z}{A} R_1 + \delta R_c + \delta R_{shell}$$

# charge radii, shell corrections

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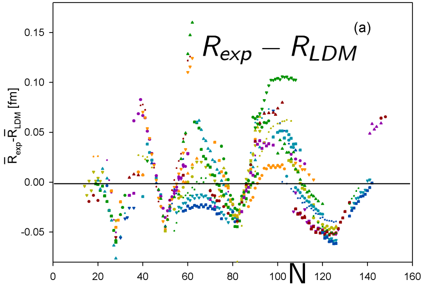
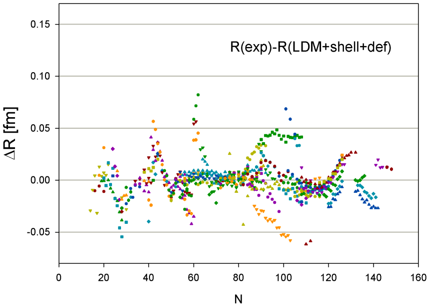



rms dev= 0.036 fm



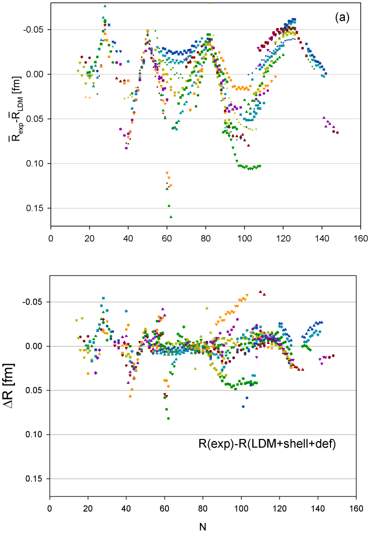

0.018 fm (only 5 par's!)

$$R_c(N,Z) = R_0(A) + \frac{N-Z}{A} R_1 + \delta R_c + \delta R_{shell}$$

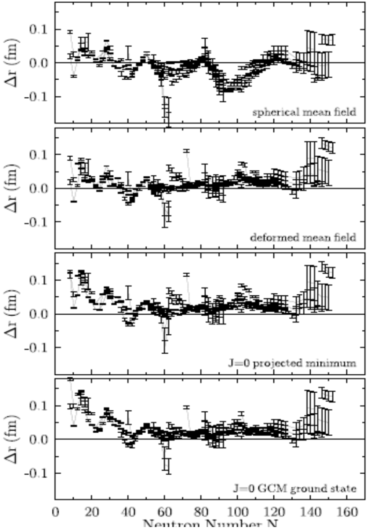
$$\delta R_{shell}/R = a_2(n_v \bar{n}_v + z_v \bar{z}_v) + a_{pn}(n_v \bar{n}_v . z_v \bar{z}_v)$$

# compare with microscopic theory

$R_{th} - R_{exp}$ LDM



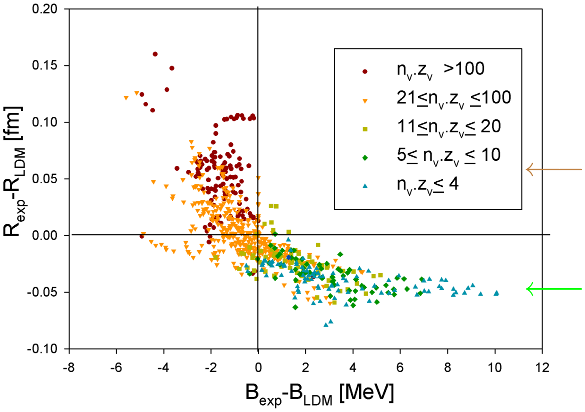

Skyrme EDF [Bender et al.]




some data (Angeli) have large error bars (mix of stat, syst, theor errors)

# correlation binding vs radius

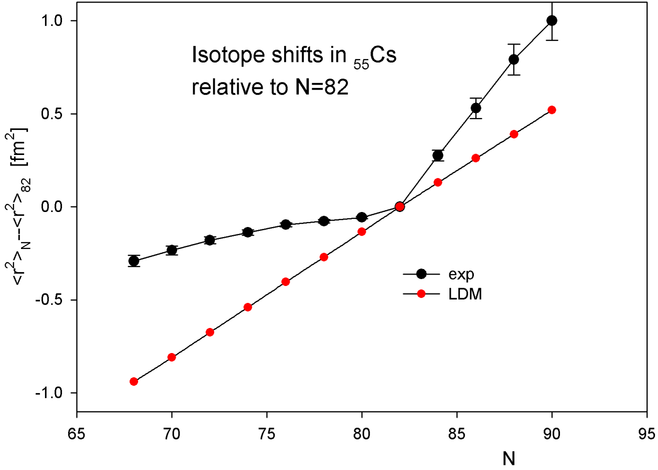
Is there a correlation between radii and binding energies?
Plot $R_{exp} - R_{LDM}$ vs $B_{exp} - B_{LDM}$ for various numbers of valence nucleons




away from closed shells: underbound, larger radii (enhanced through deformation)

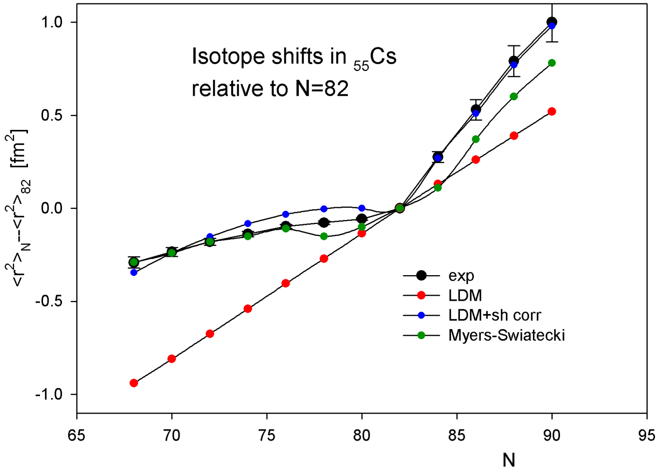
near closed shell: small valence number, strongly bound, small radii

# isotope shifts for Cs




isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

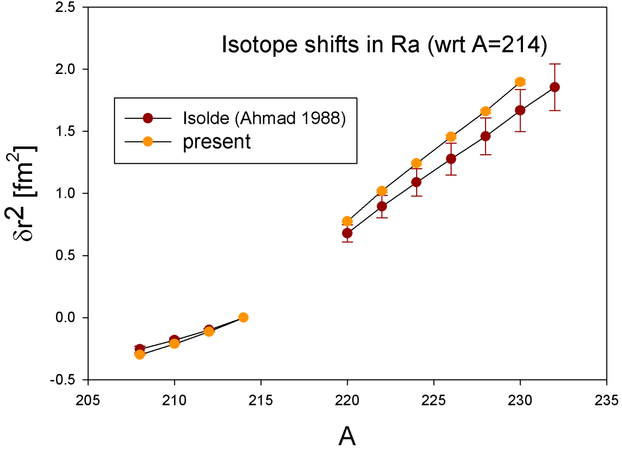
# isotope shifts for Cs




isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

# isotope shifts for Ra

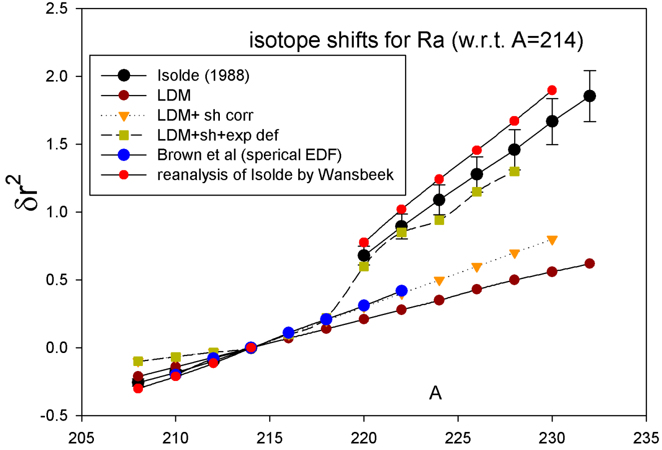
isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=126}$ from exp field shifts $\nu_A - \nu_{A'}$




Isolde (1988) obtained IS using simple sp wave functions
We performed a new analysis of the data adding some new electronic transitions and using electronic CBF

# isotope shifts for Ra

isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=126}$ from exp field shifts $\nu_A - \nu_{A'}$




Deformation effect is substantial, absent in spherical EDF

# Neutron skin and isovector chem. pot.

Skin directly related to symmetry energy
the latter can be determined best by taking derivative w.r.t. $N - Z$

$$E_{\mathrm{LDM}} \approx -a_B A + a_{surf} A^{2/3} + \frac{(N-Z)^2}{A} \frac{S_v}{1+yA^{-1/3}} + a_C \frac{Z(Z-1)}{A^{1/3}} + E_{pair} + E_{shell}$$

$$\frac{1}{2}\left(\frac{dE}{dN} - \frac{dE}{dZ}\right) = \frac{N-Z}{A} S_A + \frac{5a_c}{6} \frac{Z}{A^{1/3}} + \delta E_{shell}$$

$$\downarrow$$

$$\mu_a = \frac{1}{2}(\mu_n - \mu_p) = \frac{1}{2}[B(N-1, Z) - B(N, Z-1)] \text{ (isovector chem. pot.)}$$

$$= \frac{2(N-Z)}{A} \frac{S_v}{1+yA^{-1/3}} - \frac{5a_c}{6} \frac{Z}{A^{1/3}}$$

Note: Symmetry energy $S_A = \frac{A}{N-Z}(\mu_a - \delta E_C - \delta E_{shell})$

independent of $a_B$, $a_{surf}$

# Symmetry energy for nuclear matter

fit $\frac{S_v}{1+yA^{-1/3}}$ to $S_A(N,Z)$, $\quad y = S_v/S_s$

Plot $S_A^{-1} = \frac{1+yA^{-1/3}}{S_v}$ against $A^{-1/3}$

$S_v^{-1}$ given by fit value at $A^{-1/3} = 0$

fit values

$S_v = 32.5 \pm 1.8, \; y = 2.9 \pm 0.4$ (sh.c)

$S_v = 28.5$ MeV (no sh corr)

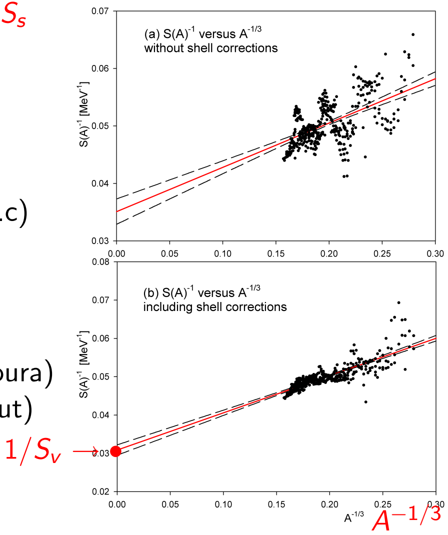
compare with Danielewicz

using IAS and sh corr from Koura

$S_v = 32.8$ y=2.8 with sh corr (Koura)

$S_v = 29 \pm 2, \quad y = 2.4 \pm 0.4$ (without)

Note correlation between $S_v$ and y



# Symmetry energy $S(\rho)$

Can be converted to nuclear matter $S(\rho)$ :
$S(\rho_0) \equiv S_v$ and $S_s$ related to some $S(\rho < \rho_0)$

Using Thomas-Fermi: $E_a = \frac{\mu_a^2}{4} \int dr \frac{\rho(r)}{S}$
$S_v/S_s = \frac{3}{r_0} \int dr \frac{\rho(r)}{\rho_0} (\frac{S_v}{S(\rho)} - 1)$

simplest case: take $S(\rho) = S_v.(\rho/\rho_0)^\gamma$

note $\gamma = 1 \rightarrow S(\rho) = \text{constant} \rightarrow S_s = \infty$

we find $\gamma \approx 0.7 \pm 0.1$
c.f. Danielewicz $\gamma = 0.65 \pm 0.1$ soft EOS

# neutron skin

options:

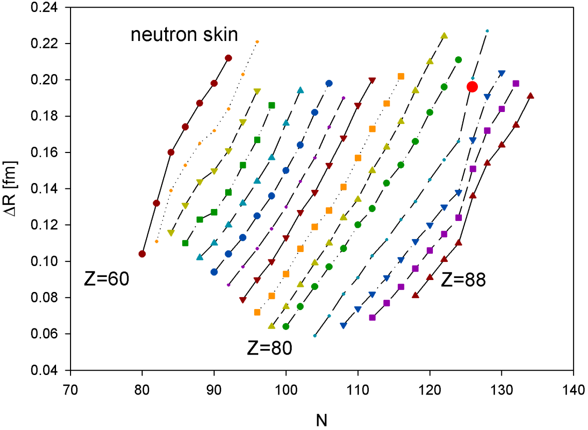
1. from isovector term in $R$: $\Delta R = \frac{N-Z}{A}\left(\frac{1}{1+y^{-1}A^{1/3}}\right)b + \delta R_C$
   fit to charge radii $\downarrow$
   e.g. $^{208}$Pb: $\Delta R = 0.18 \pm 0.03$ fm reduction of skin by 30%
2. from relation between $\Delta R$ and $\mu_a$
   (i) $\frac{R_n - R_p}{R} = \frac{A(N_S - Z_S)}{6NZ} \approx \frac{A}{6NZ}\frac{N-Z-a_c ZA^{2/3}/S_v}{1+A^{1/3}/y}$

   (ii) $\mu_a(N,Z) = \frac{2(N-Z)}{A}\frac{S_s A^{1/3}}{1+A^{1/3}/y} - \frac{5a_c}{6}\frac{Z}{A^{1/3}}$

   combining (i)+(ii) $\frac{\Delta\bar{R}}{\bar{R}_0} = \frac{\mu_a}{12S_s}\frac{A^{5/3}}{NZ} + \frac{5a_c}{72S_s}\frac{A^{4/3}}{N}$

   take $\mu_a(N,Z)$ from exp (hence shell effects implicitly included)
   overall uncertainty in $\Delta R$ from error in $S_s$ (15%)
   One accurate measurement (PREX) on $^{208}$Pb will fix that

# Results for $\Delta R$

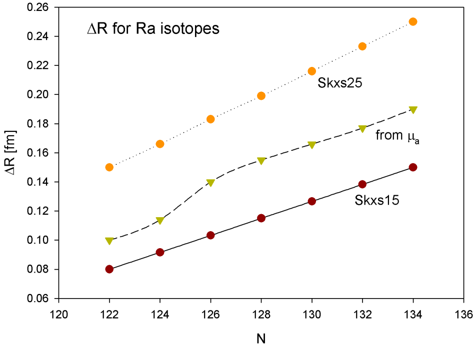



using $\mu_a$ from exp
wiggles mostly occur at magic numbers
related to loss of binding
not an effect of deformation!

$^{208}$Pb exp: $\Delta R = 0.20 \pm 0.04 \pm 0.05$ fm (anti-protonic atoms)
present: method(1): 0.18 fm; method(2): 0.20 fm
Brown: 0.15- 0.25 fm, Piekarewicz 0.22 fm, vanGiai 0.21 fm, Bender 0.16fm

# Results for $\Delta R$ in Radium

comparison with Brown et al. $\Delta R = .15\ (.25)$ fm in $^{208}$Pb for Skxs15 (25)




overall uncertainty in $\Delta R$ solely due to error in $S_s$, $\approx 15\%$?

# present status of skin

from Brown et al PRC76,2009

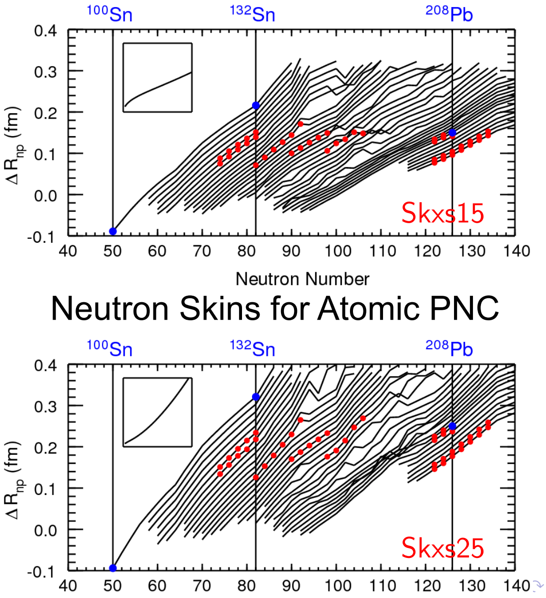
based upon spherical Skyrme EDF

$\Delta R = .15\ (.25)$ fm for Skxs(15, 25)

idea: PREX@Jlab will fix that

Questions

- Irregularities due to high spin intruder orbitals ($i13/2$)
- Effect of deformation?
- Are slopes really constant?
- Does not fit exp $R_c$ very well (except $^{208}$Pb )



# Summary

LDM can be used as a quantitative tool
by including shell effects

describes both binding energies (rms dev $\sim$ 780keV),
charge radii ($\sim$ 1%) sensitive to deformation
skin ($\sim$ 15%) insensitive to deformation effects
in particular useful for extrapolation to large $A$
symmetry energy in nm is determined

To be done: improve treatment of deformation effects
e.g by using info on spectrum, $E(2_1^+)$ or ratio $E(4_1^+)/E(2_1^+)$
Off-set of shell effects: "what are normal nuclei: open shell or closed shell"