Exploring Continuum Structures with a Pseudo-State Basis

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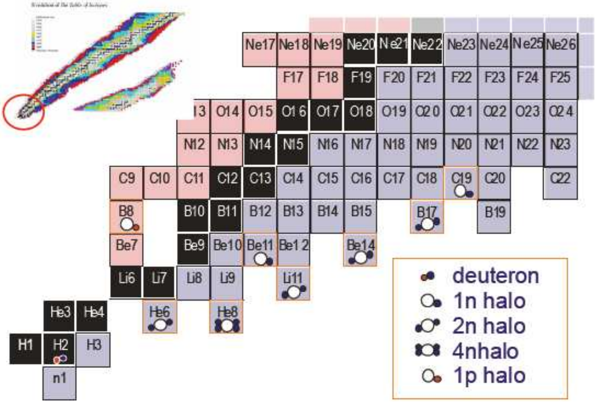
Motivation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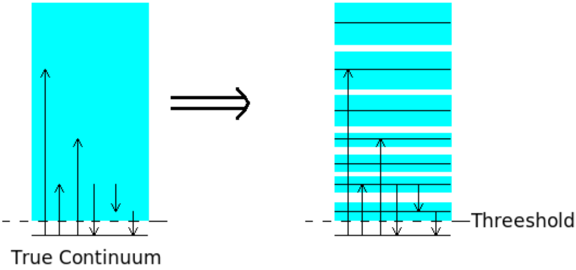
Be7
Be9
Be10
Be11
Be12
Be14
Li6
Li7
Li8
Li9
Li11
He3
He4
He6
He8
H1
H2
H3
n1
deuteron
1n halo
2n halo
4nhalo
1p halo

The role of the continuum

Coupling to BU channels play an important role in the scattering of loosely bound nuclei



True Continuum: $\begin{cases} \text{Infinite number of estates.} \\ \text{No square-integrable.} \end{cases}$

The Continuum problem in CDCC

Discretization methods:

- Binning:

$$u_l^i(k) = \sqrt{\frac{2}{\pi N_i}} \int_{k_{i-1}}^{k_i} f_i(k)\phi_l(k,r)dk.$$

- Pseudo-states.

  ⇒ Sturmian
  ⇒ Harmonic Oscillator (HO)
  ⇒ Transformed HO
  ⇒ Laguerre

Pseudo-states (PS) discretization method

- Discrete set of $\mathcal{L}^2$ functions: $|\phi_{n,\ell}(r)\rangle$

Completeness condition:

$$\sum^{N} |\phi_{i,\ell}(r)\rangle\langle\phi_{i,\ell}(r)| \approx 1$$

- To diagonalize the internal hamiltonian of a projectile $\mathcal{H}_p$

Matrix elements:

$$\mathcal{H}_p \longmapsto \sum_{n,n'} |\phi_{n,\ell}(r)\rangle\langle\phi_{n,\ell}(r)|\mathcal{H}_p|\phi_{n',\ell'}(r)\rangle\langle\phi_{n',\ell'}(r)|$$

Pseudo-states (PS) discretization method

Eigenstates of the matrix NxN:

$$|\varphi_{n,\ell}^{(N)}\rangle = \sum^{N} C_i^n |\phi_{i,\ell}(r)\rangle$$

- $\left\{ \begin{array}{l} \mathrm{n}_b \text{ states with } \varepsilon_n < 0 \text{ representing the bound states.} \\ \mathrm{N}\text{-}\mathrm{n}_b, \varepsilon_n > 0 \Rightarrow \text{discrete representation of the Cont.} \end{array} \right.$
- Orthogonal and normalizable.

Which basis may I use? Sturmian, Harmonic Oscillator?

Harmonic Oscillator basis

HO vs THO:

$$\phi(r) \longmapsto e^{-r^2} \quad \Longrightarrow \quad \phi[s(r)] \longmapsto e^{-\frac{\gamma^2}{2b^2} r}$$

- Correct asymptotic behaviour for bound states.
- Range controlled by the parameters of the LST.

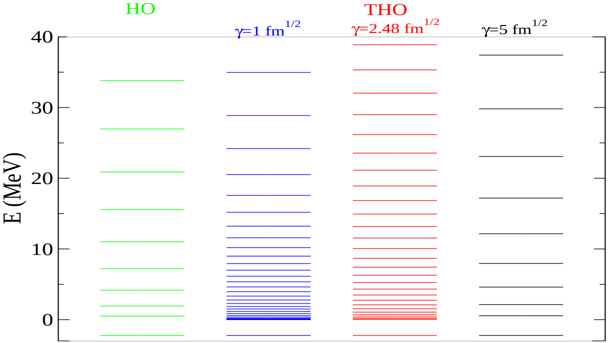
Analytic LST from Karataglidis *et al.*,PRC71,064601(2005)

$$s(r) = \frac{1}{\sqrt{2}b} \left[\frac{1}{\left(\frac{1}{r}\right)^m + \left(\frac{1}{\gamma\sqrt{r}}\right)^m} \right]^{\frac{1}{m}}$$

THO parameters

- b is treated as a variational parameter to minimize g.s. energy
- Then $\frac{\gamma}{b}$ is related to the k_{max}:

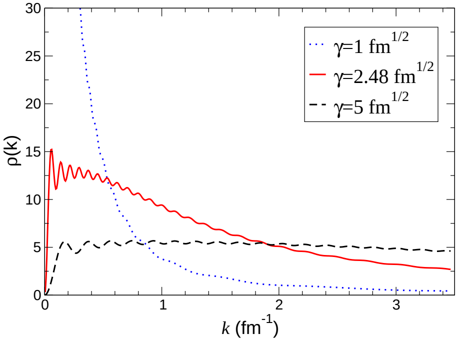
$$\frac{\gamma}{b} = \sqrt{2k_{max}}$$



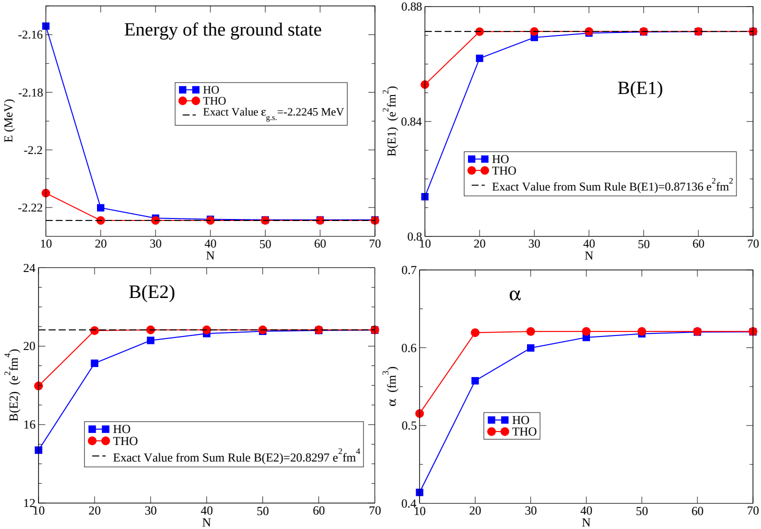
Energy distribution of pseudo-states

Density of states

$$\rho^{(N)}(k) = \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle$$



Convergence in HO vs. THO with deuteron



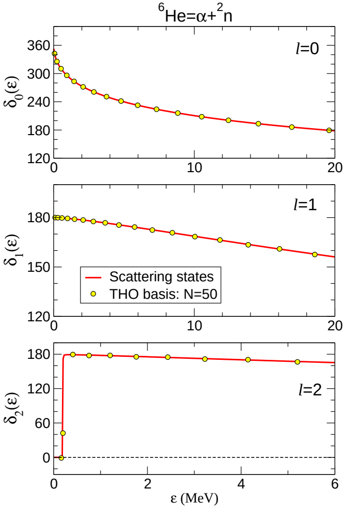
^{6}He+^{208}Pb @ 240 MeV/u

Potentials involved

1. 2n-α potential from di-neutron model
   Moro *et al.*, PRC75, 064607 (2007)
2. 2n-^{208}Pb from d-^{208}Pb
   Perey & Perey, PR132, 755 (1965)
3. α-^{208}Pb
   Khoa *et al.*, PRC59, 1252 (2002)

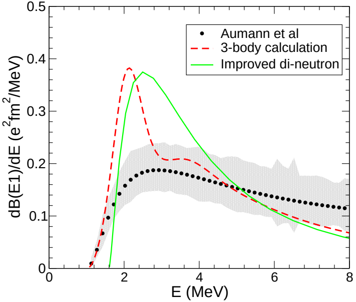
Phase-shifts

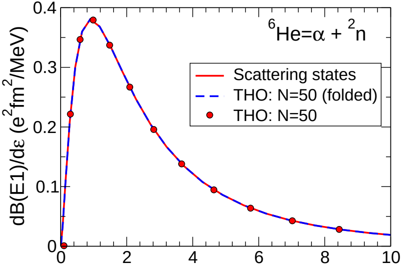
- For $\varphi_\ell(k) \Rightarrow$ Asymptotic behavior
- For $\varphi^{THO}_{n,\ell} \Rightarrow$ Integral formula



Electric Transition Probabilities

$$\frac{dB(E\lambda)}{d\varepsilon} \propto \left| \sum_{n=1}^{N} \langle \varphi_\ell(k) | \varphi_{n,\ell}^{(N)} \rangle \langle \varphi_{n,\ell}^{(N)} || \mathcal{M}(E\lambda) || \varphi_{g.s.} \rangle \right|^2$$





M. Rodríguez-Gallardo *et al.*, PRC77(2008)064609

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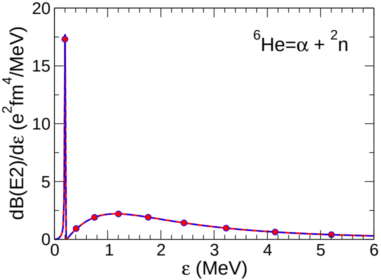
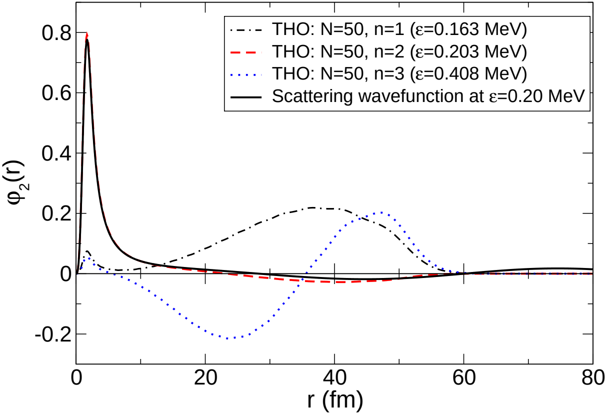
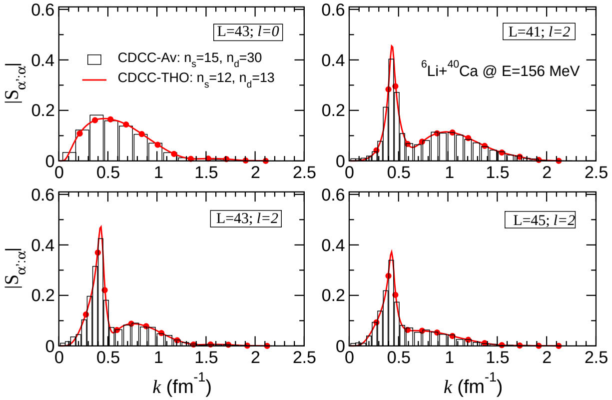


Treatment of Resonances

for N not too large, resonances tend to concentrate 1-2 eigenstates

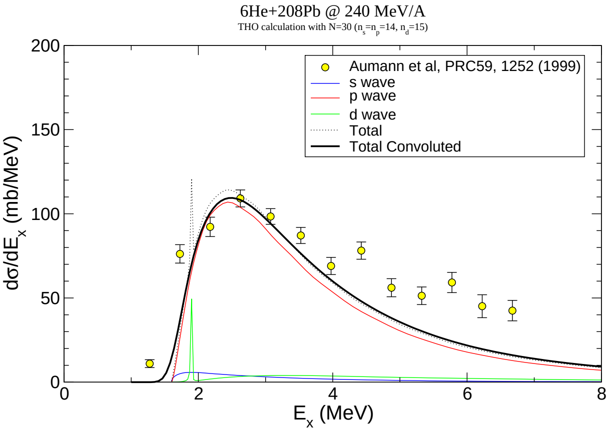


Treatment of Resonances



Moro *et al*, PRC80 (2009) 054605

From T. Aumann *et al.*



Energy distribution dominated by (Coulomb) coupling to dipole states

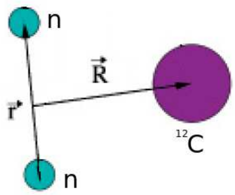
^{6}He+^{12}C @ 240 MeV/u

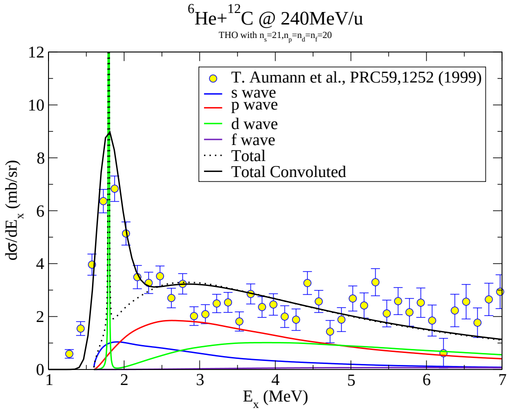
Potentials involved

1. 2n-^{12}C from d-^{12}C
2. 2n-^{12}C folding potential from Three-Body wavefunction:

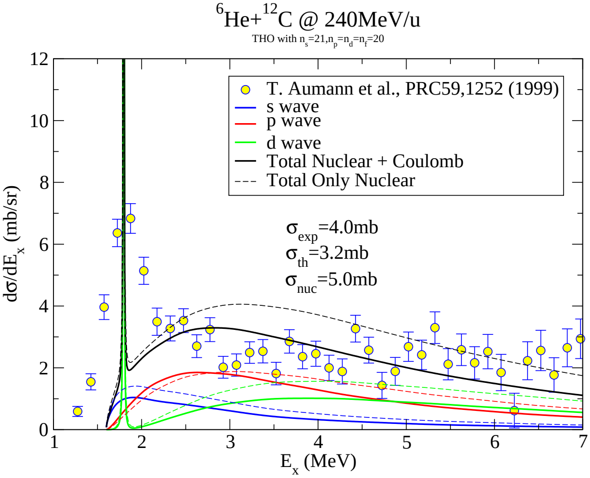
$$U(\vec{R}) = \int d\vec{r}\rho_{nn}(r)\left(V_{n-^{12}C}(\vec{R}+\frac{1}{2}\vec{r}) + V_{n-^{12}C}(\vec{R}-\frac{1}{2}\vec{r})\right)$$

ρ_{nn} obtained from Three-Body model of ^{6}He





- Low energy dominated by coupling to 2^+ resonance
- High energy *s*,*p*,*d* background



- Destructive interference with the coulomb part

Conclusions

THO PS method

Provides a suitable discrete description of the Continuum.

Dealing with Resonances

Natural and accurate treatment of narrow resonances.

Phase-shifts integral formula

Hazi and Taylor Formula, PRA1,1109 (1970)

$$\tan\delta_\ell(k) = -\frac{\int_0^\infty u_\ell(k,r)[E-H]f(r)F_\ell(kr)dr}{\int_0^\infty u_\ell(k,r)[E-H]f(r)G_\ell(kr)dr} \qquad (1)$$

Calculating energy distribution of B($E\lambda$) with PS

For B($E\lambda$):

- $$\left.\frac{dB(E\lambda)}{d\varepsilon}\right|_{\varepsilon=\varepsilon_n} \simeq \frac{1}{\Delta_n}\left|\langle\varphi_{n,\ell}^{(N)}||\mathcal{M}(E\lambda)||\varphi_{g.s.}\rangle\right|^2$$
- $$\frac{dB(E\lambda)}{d\varepsilon} \simeq \frac{\mu_{bc}k}{(2\pi)^3\hbar^2}\left|\sum_{n=1}^{N}\langle\varphi_\ell(k)|\varphi_{n,\ell}^{(N)}\rangle\langle\varphi_{n,\ell}^{(N)}||\mathcal{M}(E\lambda)||\varphi_{g.s.}\rangle\right|^2$$

$$\Delta_n = \frac{\varepsilon_{n+1}-\varepsilon_{n-1}}{2}$$

Sum Rules

For B($E\lambda$):

$$B(E\lambda) = \int d\varepsilon \frac{dB}{d\varepsilon} = \frac{2J_f+1}{2J_i+1}(D^{(\lambda)}_{J_i,J_f})^2 \langle \varphi_{\mathrm{g.s.}}|r^{2\lambda}|\varphi_{\mathrm{g.s.}}\rangle \qquad (2)$$