

Relativistic Mott criticality in graphene

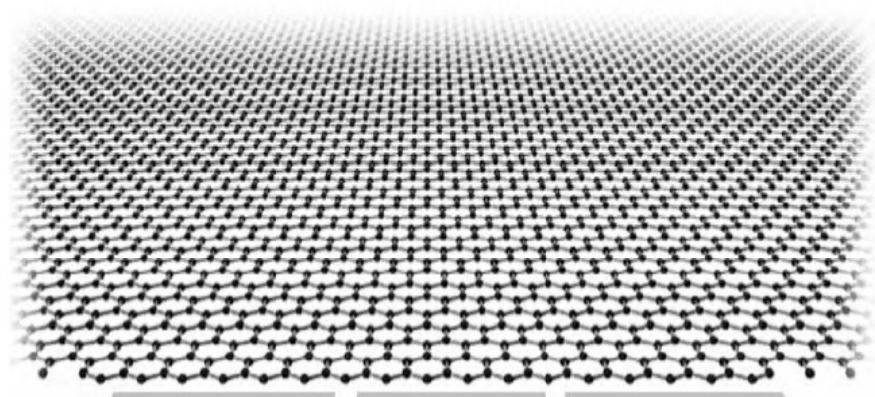
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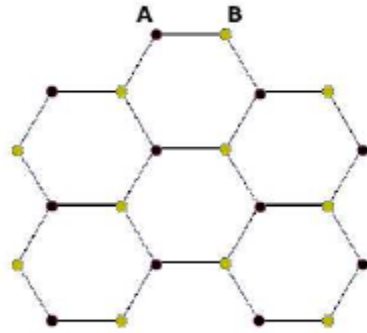
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Graphene: 2D carbon (**1s², 2s², 2p²**)



Two triangular sublattices: A and B; one electron per site (half filling)

Tight-binding model ($t = 2.5 \text{ eV}$):

$$H_0 = -t \sum_{\vec{A}, i, \sigma = \pm 1} u_{\sigma}^{\dagger}(\vec{A}) v_{\sigma}(\vec{A} + \vec{b}_i) + H.c.$$

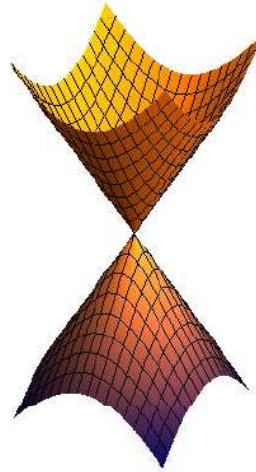
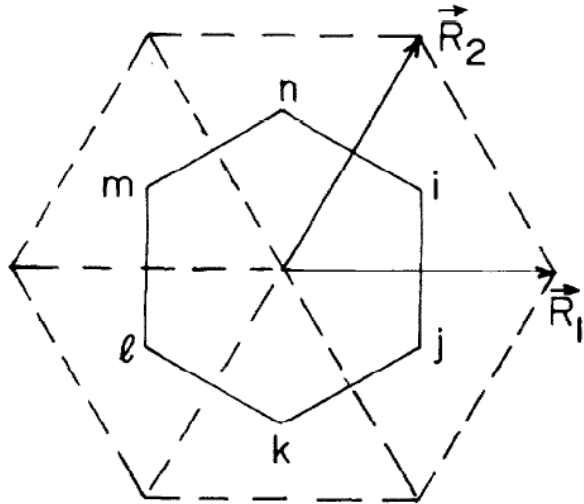
(Wallace, PR 1947, Semenoff, PRL 1984)

$$E(\vec{k}) = \pm t \left| \sum_i \exp[\vec{k} \cdot \vec{b}_i] \right|$$

The sum is complex \Rightarrow two equations for two variables for zero energy

\Rightarrow **Dirac points** (no Fermi surface)

Brillouin zone:



Two inequivalent (Dirac) points at :

+K and **-K**

Dirac fermion:

$$\Psi_{\sigma}^{\dagger}(\vec{x}, \tau) = T \sum_{\omega_n} \int^{\Lambda} \frac{d\vec{q}}{(2\pi a)^2} e^{i\omega_n \tau + i\vec{q} \cdot \vec{x}} (u_{\sigma}^{\dagger}(\vec{K} + \vec{q}, \omega_n), v_{\sigma}^{\dagger}(\vec{K} + \vec{q}, \omega_n), u_{\sigma}^{\dagger}(-\vec{K} + \vec{q}, \omega_n), v_{\sigma}^{\dagger}(-\vec{K} + \vec{q}, \omega_n)).$$

“Low - energy” Hamiltonian: $H_0 = i\gamma_0 \gamma_i (-i\partial_i - A_i) \quad i=1,2$

$$\{\gamma_{\mu}, \gamma_{\nu}\} = 2\delta_{\mu\nu} \quad \nu, \mu = 0, 1, 2$$

($v = c/300 = 1$, in our units)

Free Dirac fermions have plenty of **emergent** symmetry:

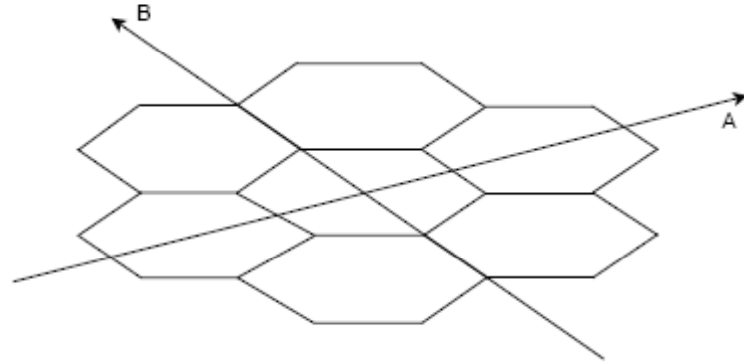
- 1) **cone's isotropy** (rotational symmetry); **Lorentz invariance**
- 2) **Chiral** (valley, pseudospin):

$$SU(2) = \{\gamma_{35}, \gamma_3, \gamma_5\}, \quad \gamma_{35} = i\gamma_3\gamma_5$$

Generators commute with the Dirac Hamiltonian (in 2D). Only two are **emergent!** (The third generates translations.)

Few exact symmetries:

(IH, Juricic, Roy, PRB 2009)



1) Time reversal

2) Sublattice exchange: $S = I \otimes \sigma_x = \gamma_2 \quad q_y \rightarrow -q_y$

3) Exchange of Dirac points: $T = i\gamma_1\gamma_5 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$
 $q_x \rightarrow -q_x$

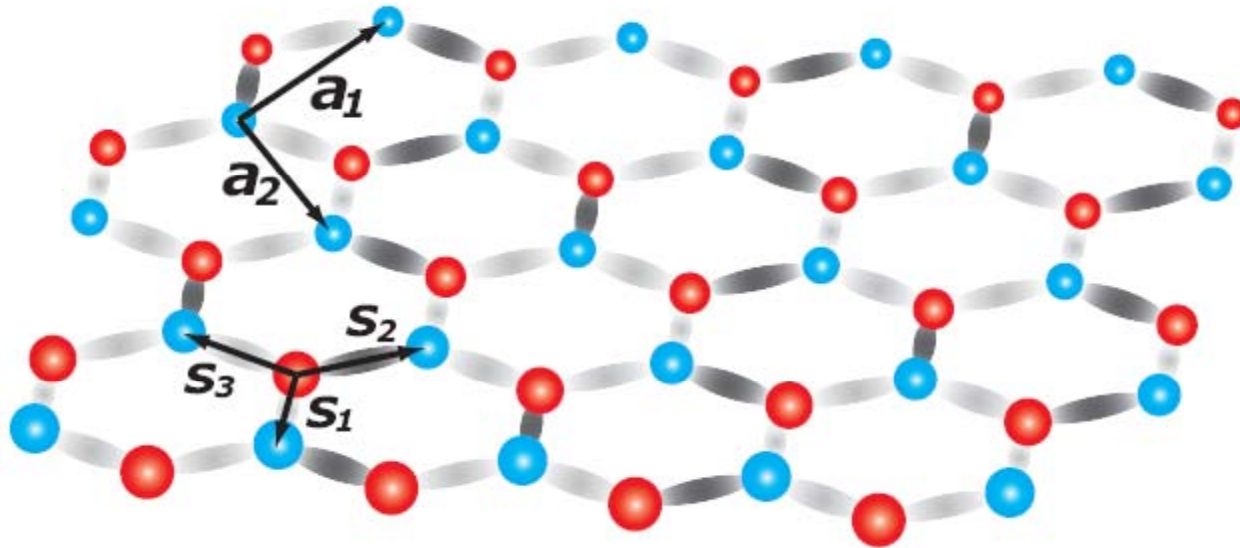
Altogether: **dihedral group** (symmetry of rectangle)

$$D_2 = \{1, S, T, ST\} = Z_2 \times Z_2$$

Masses (order parameters):

1) charge-density-wave $\langle \bar{\Psi} \Psi \rangle$ (Semenoff, PRL 1984, IH, PRL 2006 (SDW))

2) Kekule bond-density-wave $\langle \bar{\Psi} (\gamma_3 \cos \alpha + \gamma_5 \sin \alpha) \Psi \rangle$
(Hou, Chamon, Mudry, PRL 2007)

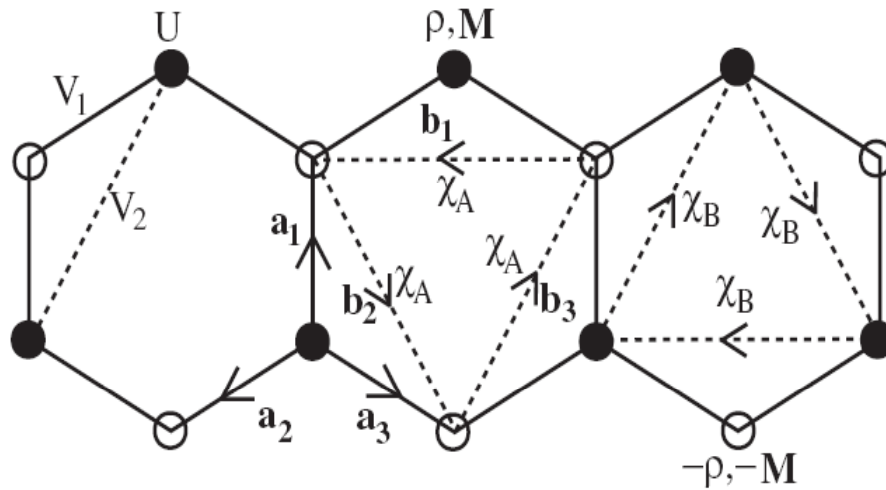


Lorentz singlets, chiral triplet!

2) Topological insulator (Haldane, PRL 1988)

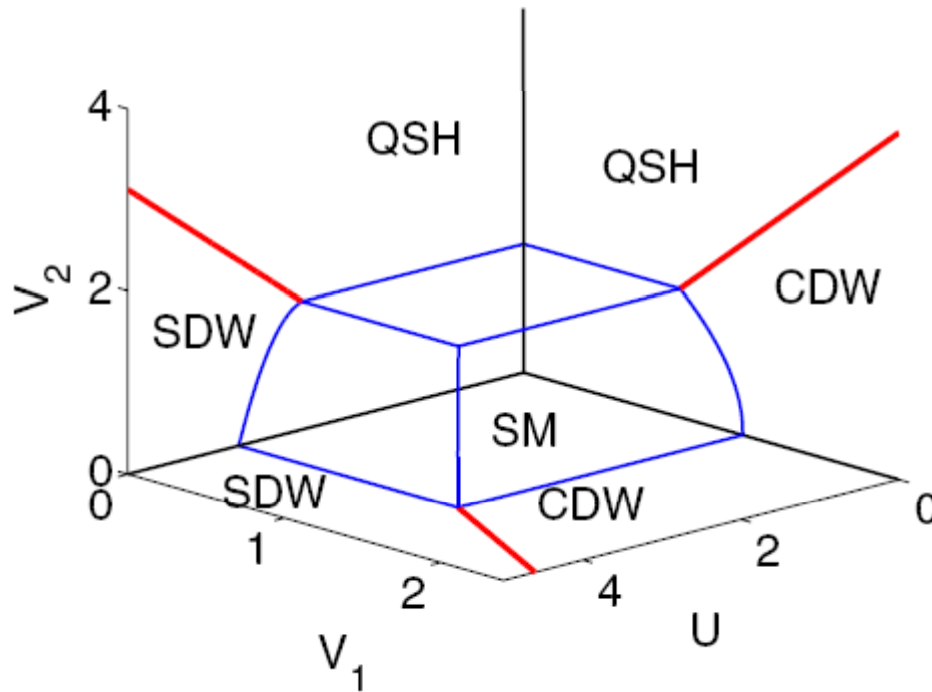
$$\langle \bar{\Psi} \gamma_{35} \Psi \rangle$$

which is both **Lorentz and chiral singlet**, but breaks **time-reversal**.



(Circulating currents in opposite directions on the two sublattices.)

Phase diagram: (Raghu et al, PRL 2008, IH, PRL 2006)



Semimetal-insulator transitions are continuous, and transitions between insulators are first-order.

Mott transition in graphene: what are the issues ?

- 1) the role and fate of Dirac fermions
- 2) symmetries of the quantum critical points (restoration?)
- 3) universality classes and exponents; is $z=1$?
- 4) (unscreened) Coulomb $1/r$ – interaction: (ir)relevant?

Short-range interactions, spinless fermions: (IH, Juricic, Roy, PRB 2009)

Most general local quartic term that obeys dihedral symmetry is:

$$L_{int} = a_{ij}(\Psi^\dagger A_i \Psi)(\Psi^\dagger A_j \Psi) + b_{ij}(\Psi^\dagger B_i \Psi)(\Psi^\dagger B_j \Psi) + c_{ij}(\Psi^\dagger C_i \Psi)(\Psi^\dagger C_j \Psi) + d_{ij}(\Psi^\dagger D_i \Psi)(\Psi^\dagger D_j \Psi)$$

where 16 matrices are grouped according to their symmetries as:

$$A \equiv \{I, \gamma_2, i\gamma_0\gamma_3, i\gamma_1\gamma_5\} \quad \text{even-even}$$

$$B \equiv \{i\gamma_0\gamma_1, \gamma_{35}, i\gamma_0\gamma_5, i\gamma_1\gamma_3\} \quad \text{even-odd}$$

$$C \equiv \{\gamma_0, i\gamma_0\gamma_2, \gamma_3, i\gamma_2\gamma_3\} \quad \text{odd-even}$$

$$D \equiv \{\gamma_1, i\gamma_1\gamma_2, \gamma_5, i\gamma_2\gamma_5\} \quad \text{odd-odd}$$

Dihedral symmetry alone allows **40** independent couplings!

Translational symmetry: (part of chiral)

$$\Psi(\vec{x}, \tau) \rightarrow e^{i(\vec{K} \cdot \vec{R})\gamma_{35}} \Psi(\vec{x} + \vec{R}, \tau) \quad \gamma_{35} = \sigma_z \otimes I_2$$

reduces the
interaction to:

$$L_{int} = \sum_{Fi} g_{Fi} X_{Fi}^2 + \sum_F g_F X_{F1} X_{F2} \\ + g_{\alpha\beta} \vec{\alpha} \times \vec{\beta} + g_{\gamma\delta} \vec{\gamma} \cdot \vec{\delta} + \sum_{\rho=\alpha,\beta,\gamma,\delta} g_\rho \vec{\rho} \cdot \vec{\rho}.$$

where $F=A,B,C,D$, and $i=1,2$,
and the vectors are :

$$\vec{\alpha} = (\Psi^\dagger A_3 \Psi, \Psi^\dagger B_3 \Psi)$$

$$\vec{\beta} = (\Psi^\dagger B_4 \Psi, \Psi^\dagger A_4 \Psi)$$

$$\vec{\gamma} = (\Psi^\dagger C_3 \Psi, \Psi^\dagger D_3 \Psi)$$

$$\vec{\delta} = (\Psi^\dagger C_4 \Psi, \Psi^\dagger D_4 \Psi)$$

This leaves **18** couplings! Time-reversal finally reduces this to **15**.

If we by hand impose more symmetry, there is further reduction in the number of couplings: **a) rotational symmetry -> 9 couplings,**
b) Lorentz -> 6, c) Lorentz + chiral -> 4.

The maximally symmetric local term is then:

$$L_{int,max} = g_{A1} S_{\mu}^2 + g_{D2} S^2 + g_{C1} \vec{V}^2 + g_{B2} \vec{V}_{\mu}^2$$

where

$$S_{\mu} = \bar{\Psi} \gamma_{\mu} \Psi,$$

$$S = \bar{\Psi} \gamma_{35} \Psi,$$

$$\vec{V} = (\bar{\Psi} \Psi, i \bar{\Psi} \gamma_3 \Psi, i \bar{\Psi} \gamma_5 \Psi),$$

$$\vec{V}_{\mu} = (\bar{\Psi} \gamma_{\mu} \gamma_{35} \Psi, \bar{\Psi} \gamma_{\mu} \gamma_3 \Psi, \bar{\Psi} \gamma_{\mu} \gamma_5 \Psi)$$

are the scalar (vector), scalar (scalar), vector (scalar) , and vector (vector) under chiral (Lorentz) group, respectively.

Fierz identities:

$$FX = 0$$

where $X^\top = (S^2, S_\mu^2, \vec{V}^2, \vec{V}_\mu^2)$ and **F** is a real asymmetric matrix.

For example, in the maximally symmetric (Lorentz + chiral) case:

$$F = \begin{pmatrix} 3 & 3 & 3 & -1 \\ 5 & 1 & 1 & 1 \\ 3 & 3 & 3 & -1 \\ 9 & -3 & -3 & 5 \end{pmatrix}$$

The dimension of the kernel of **F** gives the number of independent coupling constants! The maximally symmetric local Lagrangian can be written as:

$$L_{int,max} = \lambda_1(\vec{V}^2 - S_\mu^2) + \lambda_2(-S^2 + S_\mu^2 + \vec{V}^2 + 3\vec{V}_\mu^2)$$

with just **2** couplings!!

Then:

a) with Lorentz symmetry (no chiral) $\rightarrow 6-3 = 3$ couplings

b) with chiral + only rotational symmetry $\rightarrow 6-3 = 3$ couplings

c) rotational only $\rightarrow 9-5 = 4$ couplings

d) Dihedral group + translations + time-reversal $\rightarrow 15-9 = 6$
couplings

Gross-Neveu-Yukawa theory: epsilon-expansion (with spin, Lorentz, no chiral symmetry) (IH, PRL 2006, IH, Juricic, Vafeek, PRB 2009)

Order parameters:

$$\phi = (\phi_s, \vec{\phi}_t) = (\langle \bar{\Psi} \gamma_{35} \Psi \rangle, \langle \bar{\Psi} \vec{\sigma} \gamma_{35} \Psi \rangle), \quad \text{Haldane; singlet, triplet}$$

$$\chi = (\chi_s, \vec{\chi}_t) = (\langle \bar{\Psi} \Psi \rangle, \langle \bar{\Psi} \vec{\sigma} \Psi \rangle) \quad \text{singlet (CDW), triplet (SDW)}$$

Field theory: $L = L_f + L_y + L_b + L_c$

$$L_y = \sum_{\psi=\chi, \phi} \{ g_{\psi,s} \psi_s \bar{\Psi} M_\psi \Psi + g_{\psi,t} \vec{\psi}_t \cdot \bar{\Psi} M_\psi \vec{\sigma} \Psi \}$$

$$L_b = \sum_{\psi=\phi, \chi} \left\{ \frac{1}{2} [\psi_s (-\partial_\tau^2 - v_{\psi,s}^2 \nabla^2 + t_{\psi,s}) \psi_s + \vec{\psi}_t \cdot (-\partial_\tau^2 - v_{\psi,t}^2 \nabla^2 + t_{\psi,t}) \vec{\psi}_t] \right.$$

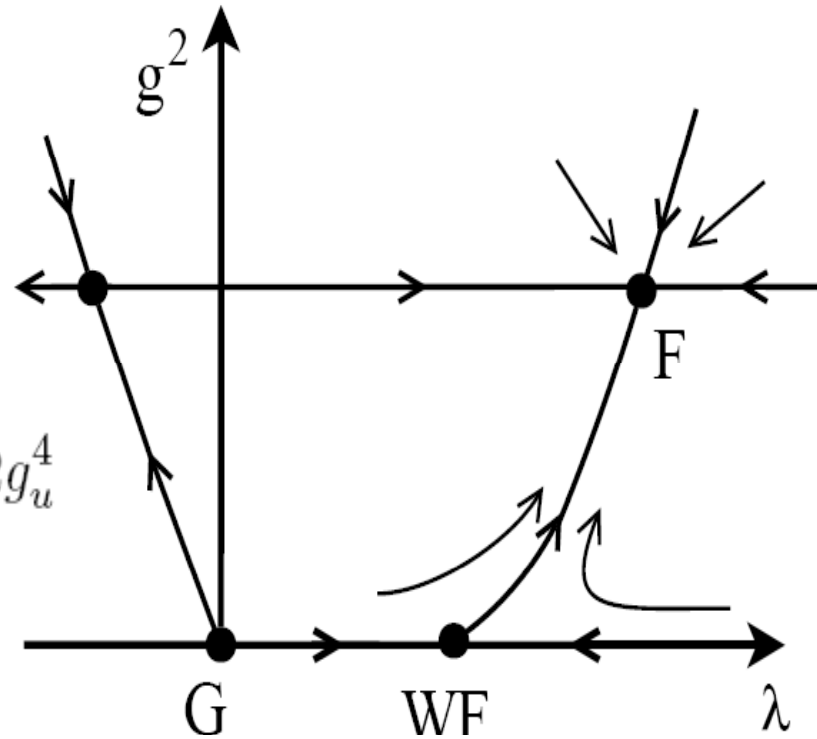
$$\left. + \lambda_{\psi,s} \psi_s^4 + \lambda_{\psi,t} (\vec{\psi}_t \cdot \vec{\psi}_t)^2 + \lambda_{\psi,st} \psi_s^2 \vec{\psi}_t^2 \right\}, \quad L_c = ia \bar{\Psi} \gamma_0 \Psi + \frac{1}{2e^2} a |\nabla|^{d-1} a$$

RG flow:

$$\frac{dg_u^2}{d \ln b} = g_u^2 (\epsilon - (7 - S)g_u^2),$$

$$\frac{d\lambda_u}{d \ln b} = \lambda_u (\epsilon - 8g_u^2) - 4(9 + S)\lambda_u^2 + 2g_u^4$$

$S = 0$ ($S = 2$) **singlet (triplet)**



Exponents:

$$\nu = \frac{1}{2} + \frac{3(5 + S)}{(7 - S)(9 + S)}\epsilon \quad \eta_f = \frac{3}{2(7 - S)}\epsilon \quad \eta_b = \frac{4}{7 - S}\epsilon$$

Long-range “charge”:

$$\frac{de^2}{d \ln b} = -\frac{4}{3}(2\delta_{d,3} + 1)e^4$$

Irrelevant!

Emergent relativity: define a small deviation

$$\delta_u = 1 - v_{\chi,u} \ll 1$$

and it is an irrelevant perturbation :

$$\frac{d\delta_u}{d \ln b} = \frac{4\epsilon}{S-7} \delta_u$$

Consequence: universal ratio of fermionic and bosonic specific heats

$$\lim_{T \rightarrow 0} \frac{C(t_{\chi,t} \rightarrow 0+)}{C(t_{\chi,t} \rightarrow 0-)} = 4(1 - 2^{-d}) \quad \left(\frac{m_b}{m_f}\right)^2 = \frac{8\lambda_u}{g_u^2} = \frac{16}{9+S} + O(\epsilon)$$

and of bosonic and fermionic masses.

Transition: from gapless (**fermions**) to gapless (**bosons**)!

Coulomb tail (1/r) : closer look

(Juricic, IH, Semenoff, PRB 2009)

Gross-Neveu with Coulomb:
$$L = \bar{\Psi}_\alpha \gamma_0 (\partial_0 + ia) \Psi_\alpha + v \bar{\Psi}_\alpha \gamma_i \partial_i \Psi_\alpha - g (\bar{\Psi}_\alpha \Psi_\alpha)^2 + \frac{1}{2e_d^2} a |\nabla|^d a.$$

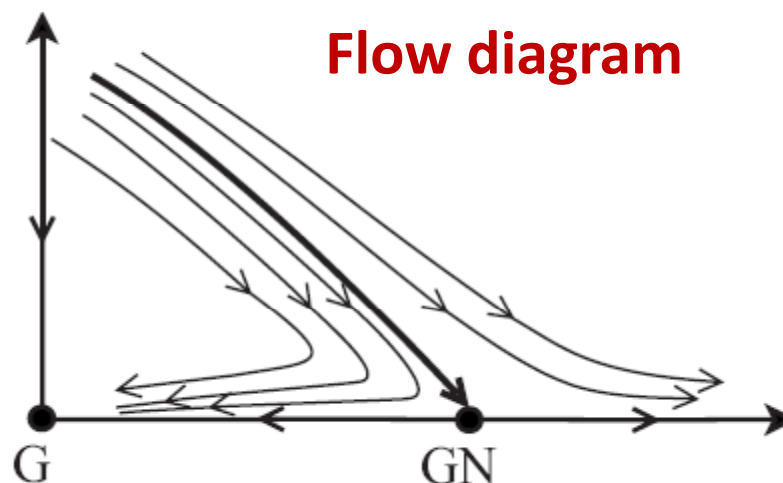
Beta-functions:

$$\frac{d\hat{e}^2}{d \ln \Lambda} = \hat{e}^4 \left(1 + \frac{\epsilon}{2(2N-1)} + O(\epsilon^2/N) \right) + O(\hat{e}^6)$$

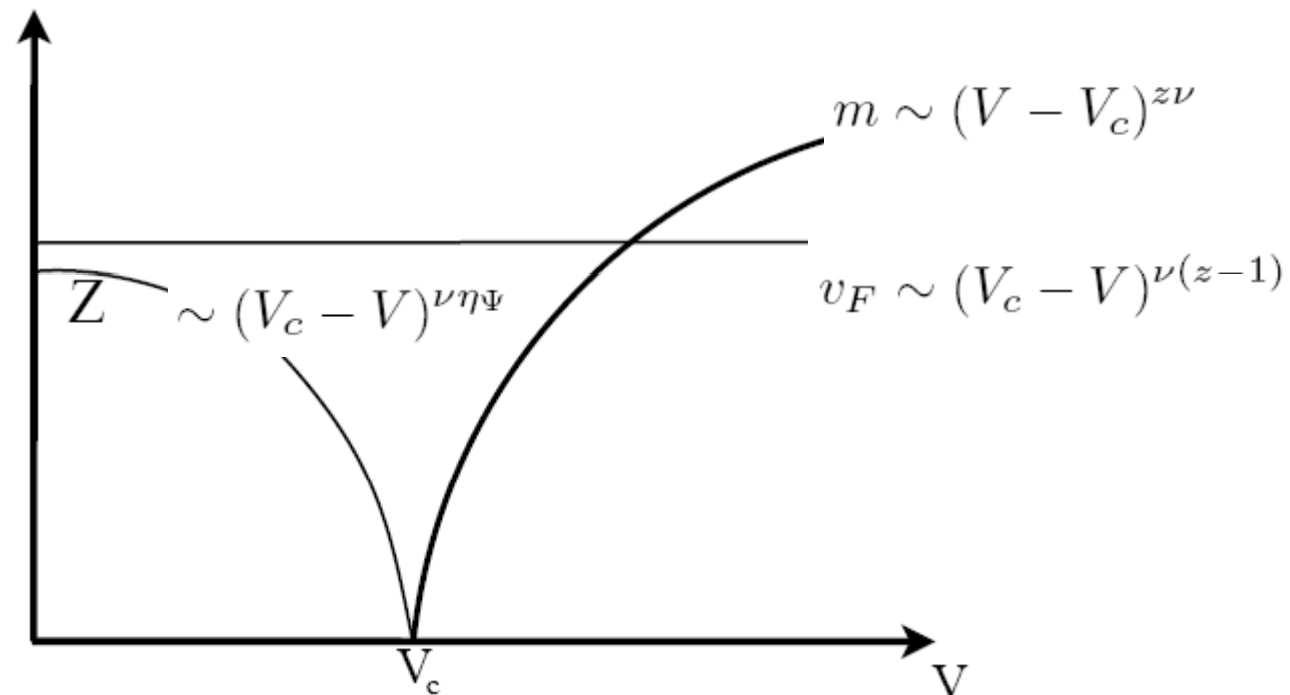
$$\frac{d\hat{g}}{d \ln \Lambda} = (\epsilon - z + 1)\hat{g} - 2(2N-1)\hat{g}^2 + c_d \hat{e}^2 \hat{g}$$

$$c_d = -2/\epsilon$$

Charge is even more irrelevant at GN critical point!



Physical picture near the criticality:



Lorentz invariance \Rightarrow **$z=1$** \Rightarrow Fermi velocity non-critical

Fermionic anomalous dimension \Rightarrow fermions **incoherent** at QCP

Magnetic catalysis:

(Gusynin, Miransky, Shovkovy, PRL 1994, Khveshchenko, PRL 2001)

“Finite-size” scaling:

$$\frac{m}{v_F \Lambda} = \left(\frac{a}{l}\right)^z G(lt^\nu)$$

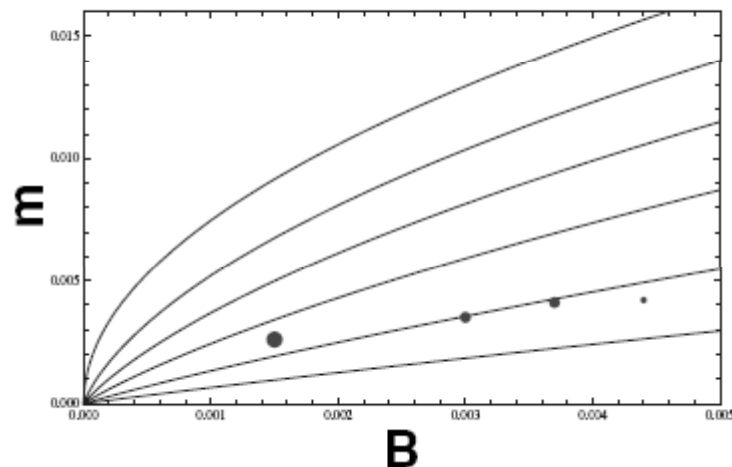
If **z=1**, at the critical point:

$$m = v_F \sqrt{B} G(0) = \frac{v_F \sqrt{2B}}{C}$$

with the **universal ratio**

$$C = 5.985 + O(1/N)$$

(IH, Roy, PRB 2008)



(Zhang et al, PRL 2006)

Instead of conclusion: (some) open questions

1) Numerics: second-order transition, unconventional exponents

$$\gamma = 1, \text{ and } \delta \approx 2.3 \quad \text{or} \quad \nu = 0.85 \text{ and } \eta_b = 0.82$$

(Drut and Lahde, PRB 2009)

$$z = 0.90(5) \quad (\text{Armour, Hands, Strouthos, PRB 2010})$$

2) **Triviality**: does the beta-function for the charge have **non-trivial zeroes**? (Or is the transition governed by a short-range critical point, like in QED4 (Kogut, Strouthos, PRD 2005).)

3) Can the critical point be **without Lorentz symmetry**? Or it is emergent!

4) **Tuning** of the transition?

5) **Superconducting QCP**? (Strack, Takei, Metzner, PRB 2010)